HOMEWORK #3

Time average of a turbulent signal

Consider a turbulent signal u composed of a slowly varying component $U(t) = U_0 e^{-t/\tau}$, where U_0 and τ are two constants, and of a rapidly varying component $u'(t) = aU_0 \cos(2\pi t/\epsilon^2\tau)$ where a is a constant and ϵ a small parameter such that $0 < \epsilon \ll 1$. We want to show that the time average defined by

$$\overline{U}(\boldsymbol{x}) = \frac{1}{T}\int_t^{t+T} u(\boldsymbol{x}, t)\, dt$$

makes sense, and thus allows to extract the slowly varying part of the signal u when $\epsilon^2\tau \ll T \ll \tau$.

1. Calculate the time average $\overline{U}$ of the signal $u = U + u'$.
2. By taking $T = \epsilon\tau$ in the computation of the time average, show that

   $$\overline{U} = U(t) + \mathcal{O}(\epsilon)$$

   where $\mathcal{O}(\epsilon)$ denotes a quantity that goes to zero as $\epsilon \to 0$.
3. Use Matlab (for instance) to illustrate your results, by plotting the different signals, the time average signal and the three characteristic times τ, T and $\epsilon^2\tau$.
4. Repeat the two first questions for du/dt and comment your results.

Oscillating boundary layer

Consider an incompressible boundary layer developing along a flat plate, and an imposed freestream (external) velocity given by

$$U_{e1} = U_0\{1 - ax[1 - \cos(2\pi f t)]\}$$

where a is a constant, dimension of inverse length, U_0 is a constant reference velocity and f is the oscillation frequency.

1. Compute the pressure gradient dP_e/dx in the external flow, and its mean value $d\bar{P}_e/dx$ by integrating over one period $T = 1/f$.
2. Compare the expressions obtained for $f = 0$ and for $f \neq 0$. What conclusion can you draw about the separation of the boundary layer?

(Adapted from Wilcox, D.C., 2006, Turbulence modeling for CFD)