Homework #7

Modèle de turbulence $k_t - \omega_t$

The $k_t - \epsilon$ turbulence model is widely used, but to fix some shortcomings of this model now well established in the literature Saffman and Wilcox,[3,4] then Menter[1,2] have proposed alternative models. For this family of turbulence models, the transport equation for the turbulent kinetic energy k_t is still solved, but the turbulent viscosity μ_t is now computed as $\mu_t = \rho k_t/\omega_t$, where ω_t is a new variable. For high-Reynolds number turbulent flow, the $k_t - \omega_t$ model of Wilcox reads

$$\left\{ \begin{aligned} \frac{\partial(\rho k_t)}{\partial t} + \frac{\partial(\rho k_t \overline{U}_i)}{\partial x_i} &= \frac{\partial}{\partial x_j}\left[\left(\mu + \sigma^\star \mu_t\right)\frac{\partial k_t}{\partial x_j}\right] + \mathcal{P} - \beta^\star \rho k_t \omega \\ \frac{\partial(\rho \omega_t)}{\partial t} + \frac{\partial(\rho \omega_t \overline{U}_i)}{\partial x_i} &= \frac{\partial}{\partial x_j}\left[(\mu + \sigma \mu_t)\frac{\partial \omega_t}{\partial x_j}\right] + \alpha\frac{\omega_t}{k_t}\mathcal{P} - \beta\rho\omega_t^2 \end{aligned} \right. \tag{1}$$

where the values of the constants are

$$\alpha = 5/9 \qquad \beta = 3/40 \qquad \beta^\star = 9/100 \qquad \sigma = 1/2 \qquad \sigma^\star = 1/2$$

As usual, incompressible flow with uniform density is considered and $\mathcal{P}$ stands for the production term of the turbulent kinetic energy.

1. By applying the change of variable $\epsilon = C_\mu \omega_t k_t$ in the $k_t - \epsilon$ turbulence model, derive the transport equation for ω_t, and show that this equation has some differences with respect to the Wilcox model (1). Identify, nevertheless, the constants of the model to obtain the expressions of α, β, σ and $\sigma^\star$ as a function of $C_\mu, C_{\epsilon_1}, C_{\epsilon_2}$, σ_k and σ_ϵ, that is

$$\alpha = C_{\epsilon_1} - 1 \qquad \beta = C_\mu\left(C_{\epsilon_2} - 1\right) \qquad \beta^\star = C_\mu \qquad \sigma = \frac{1}{\sigma_\epsilon} \qquad \sigma^\star = \frac{1}{\sigma_k}$$

   Compare the numerical values with the values above, provided by the authors, and comment.

2. Write the transport equations of the $k_t - \omega_t$ model for the case of decaying isotropic turbulence, and provide the evolution of k_t, ω_t and ν_t (refer to homework #6).

3. Write the transport equations of the $k_t - \omega_t$ for the log-law of a turbulent boundary layer (refer again to homework #6 for the assumptions).

References

[1] Menter, F. R., 1994, « Two-equation eddy-viscosity turbulence models for engineering applications », *AIAA Journal*, **32**(8), 1598-1605.

[2] Menter, F. R., 1997, « Eddy viscosity transport equations and their relation to the $k - \epsilon$ model », *J. Fluid Eng.*, **119**, 876-884.

[3] Saffman, P. G. & Wilcox, D. C., 1974, « Turbulence-model predictions for turbulent boundary layers », *AIAA Journal*, **12**(4), 541-546.

[4] Wilcox, D. C., 1988, « Reassessment of the scale-determining equation for advanced turbulence models », *AIAA Journal*, **26**(11), 1299-1310.