MOD 3^{e}A, ECL'16 & École doctorale MEGA

Physique des écoulements turbulents

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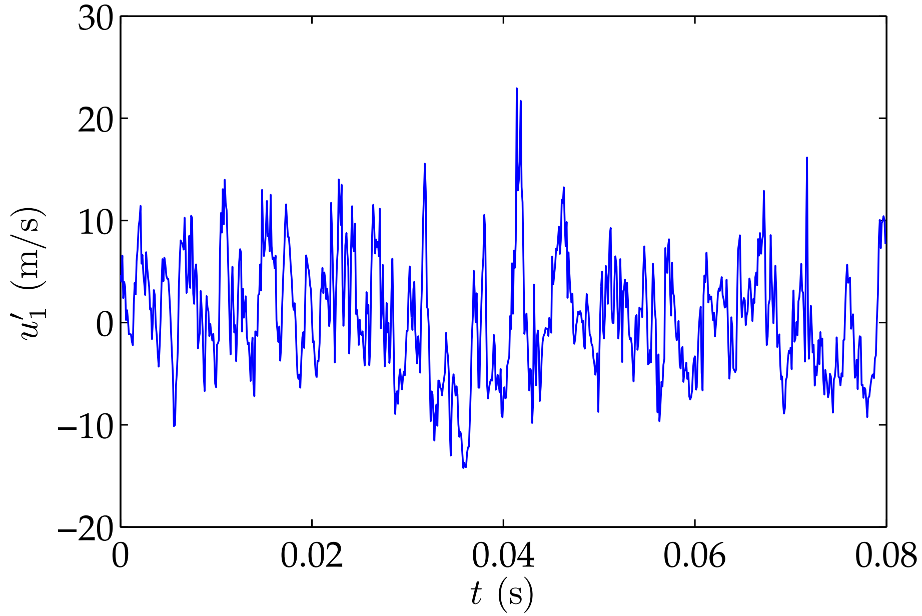
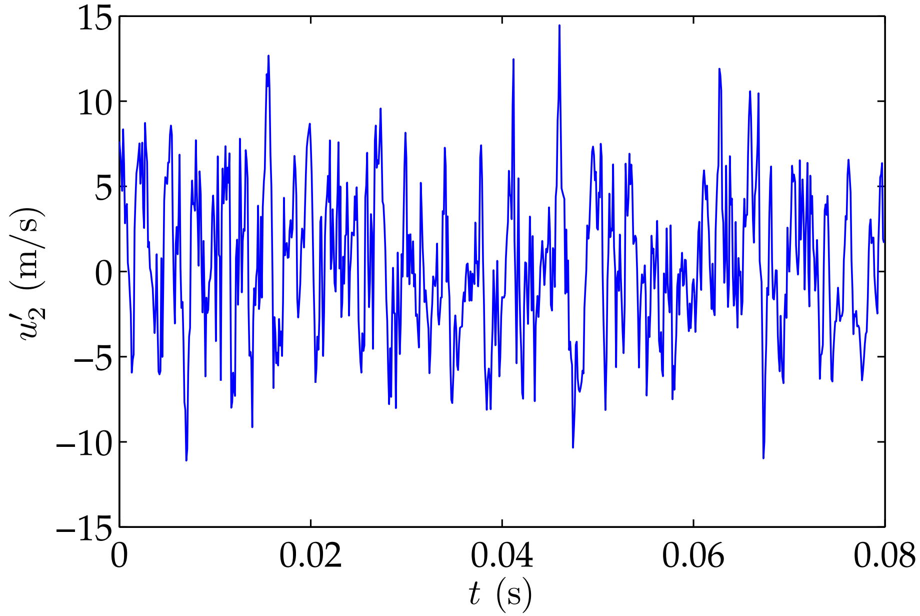
- **Subsonic round jet**, nozzle diameter $D = 50$ mm, exit velocity $U_j = 30$ m.s^{-1}
  $\rightsquigarrow$ Reynolds number $\mathrm{Re}_D = 10^5$

$u_1'(t)$ and $u_2'(t)$ at $x_2 = D/2$ and $x_3 = 0$ $\quad (x_1 = 2D)$

measured by crossed-wire probes (hot wire anemometer)





Courtesy of Emmanuel Jondeau (LMFA) for the data

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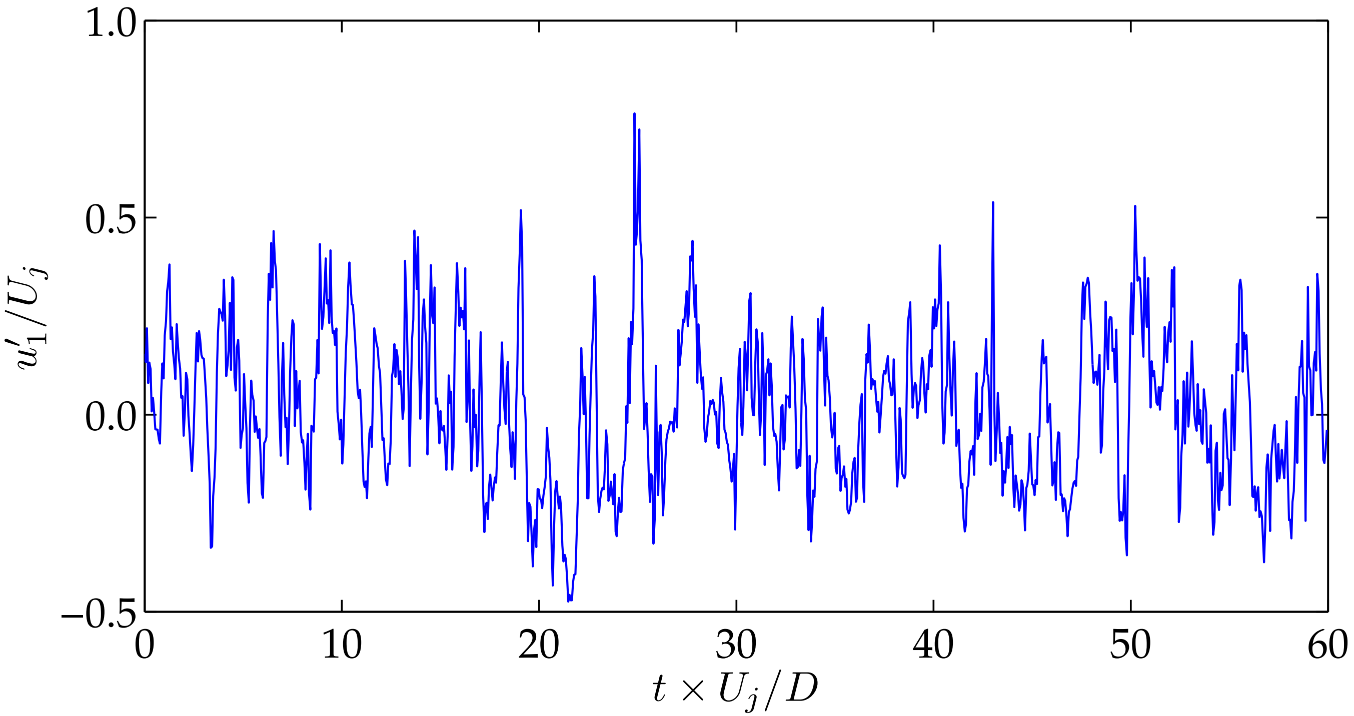
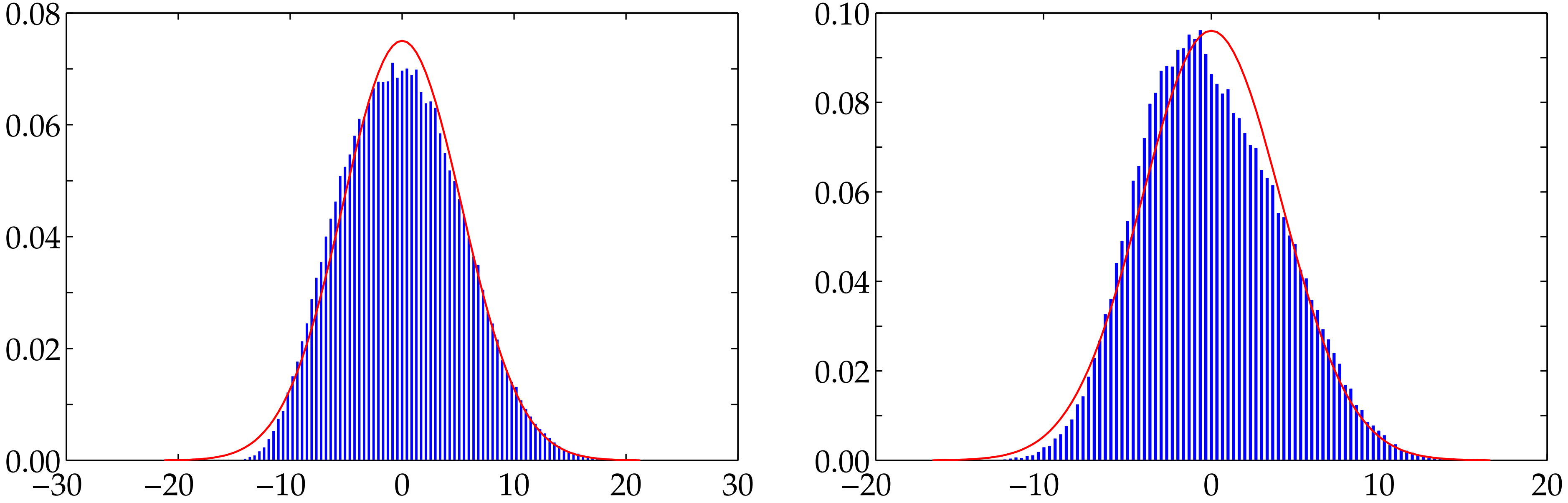
$u_1'(t)$ at $x_2 = D/2$ and $x_3 = 0$ $\qquad$ ($x_1 = 2D$)



Probability density functions

pdf of $u'_1(t)$ and $u'_2(t)$ at $x_2 = D/2$ and $x_3 = 0$



$S_{u'_1} \simeq 0.21 \qquad T_{u'_1} \simeq 2.74 \qquad\qquad S_{u'_2} \simeq 0.24 \qquad T_{u'_2} \simeq 2.76$

— Gaussian or normal distribution $\quad p(\xi) = \dfrac{1}{\sqrt{2\pi}\sigma_\xi} \exp\left(-\dfrac{\xi^2}{2\sigma_\xi^2}\right)$

$$\sigma_\xi \equiv \sqrt{\overline{(\xi - \bar{\xi})^2}} \qquad S_\xi = 0 \qquad T_\xi = 3$$

- **Skewness and flatness or kurtosis factors**
  centered variable $x'_i = x_i - \bar{X}_i$, $x'_{i,\mathrm{rms}} = \sqrt{\overline{x'^2_i}} = \sigma_{x_i}$

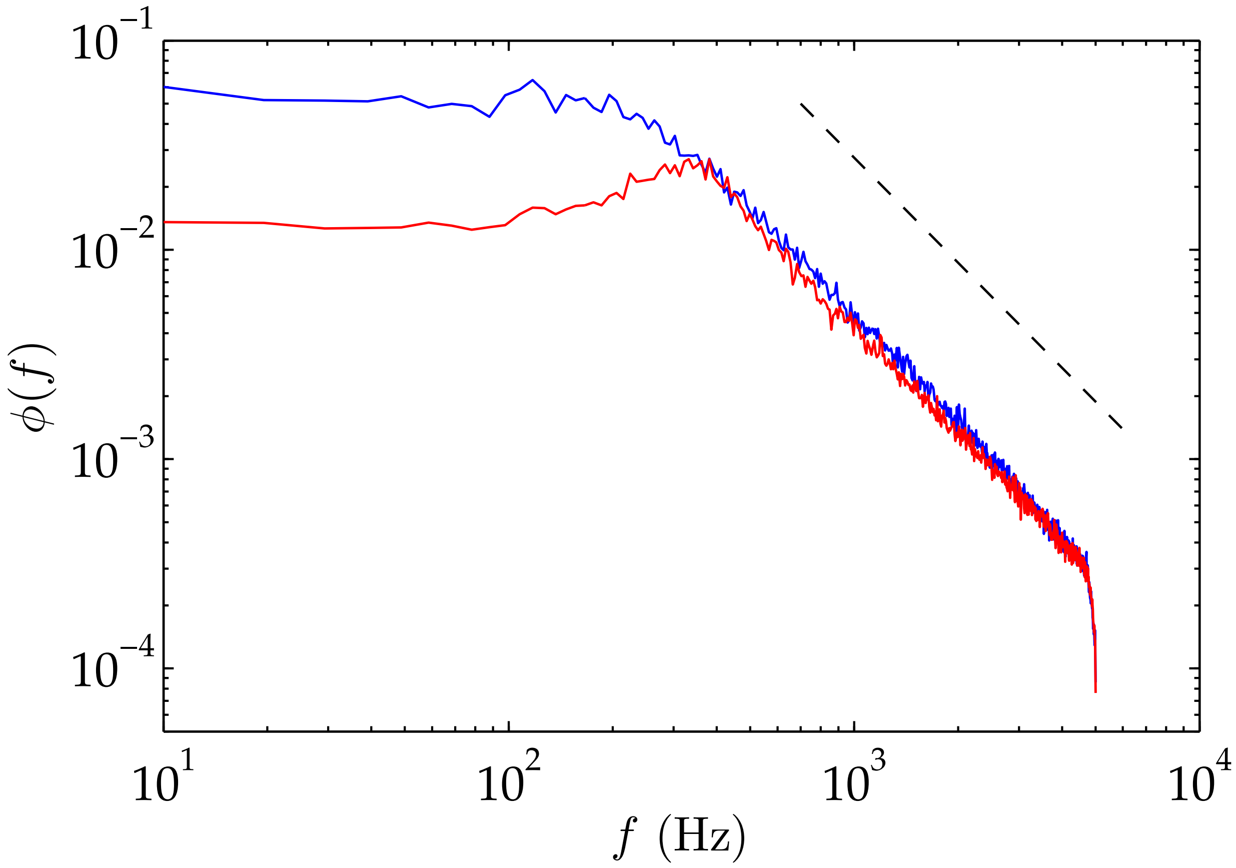
$$S_{x_i} \equiv \frac{\overline{x'^3_i}}{x'^3_{i,\mathrm{rms}}} \qquad T_x \equiv \frac{\overline{x'^4_i}}{x'^4_{i,\mathrm{rms}}}$$

- **Reynolds stress tensor in the shear layer**

$$\frac{\sqrt{|\overline{u'_i u'_j}|}}{U_j} \simeq \begin{pmatrix} 0.18 & 0.10 & 0.10 \\ 0.10 & 0.14 & - \\ 0.10 & - & 0.13 \end{pmatrix}$$

  - $\overline{u'_1 u'_2} > 0$, in agreement with Boussinesq's hypothesis
    $-\overline{u'_1 u'_2} = \nu_t \partial \overline{U}_1 / \partial x_2$ (okay, at least for the sign !)
  - The Schwarz inequality is (must be !) satisfied, $|\overline{u'_1 u'_2}|^2 \leq \overline{u'^2_1} \times \overline{u'^2_2}$
  - Turbulence intensity $u'/U_j \simeq 0.18$

- **Spectra of u'_1 and u'_2**
  $f_s = 1/\Delta t = 10^4$ Hz, $n_{\text{fft}} = 1024$, $\Delta f = f_s/n_{\text{fft}}$, $f_{\text{max}} = f_s/2 = 5$ kHz

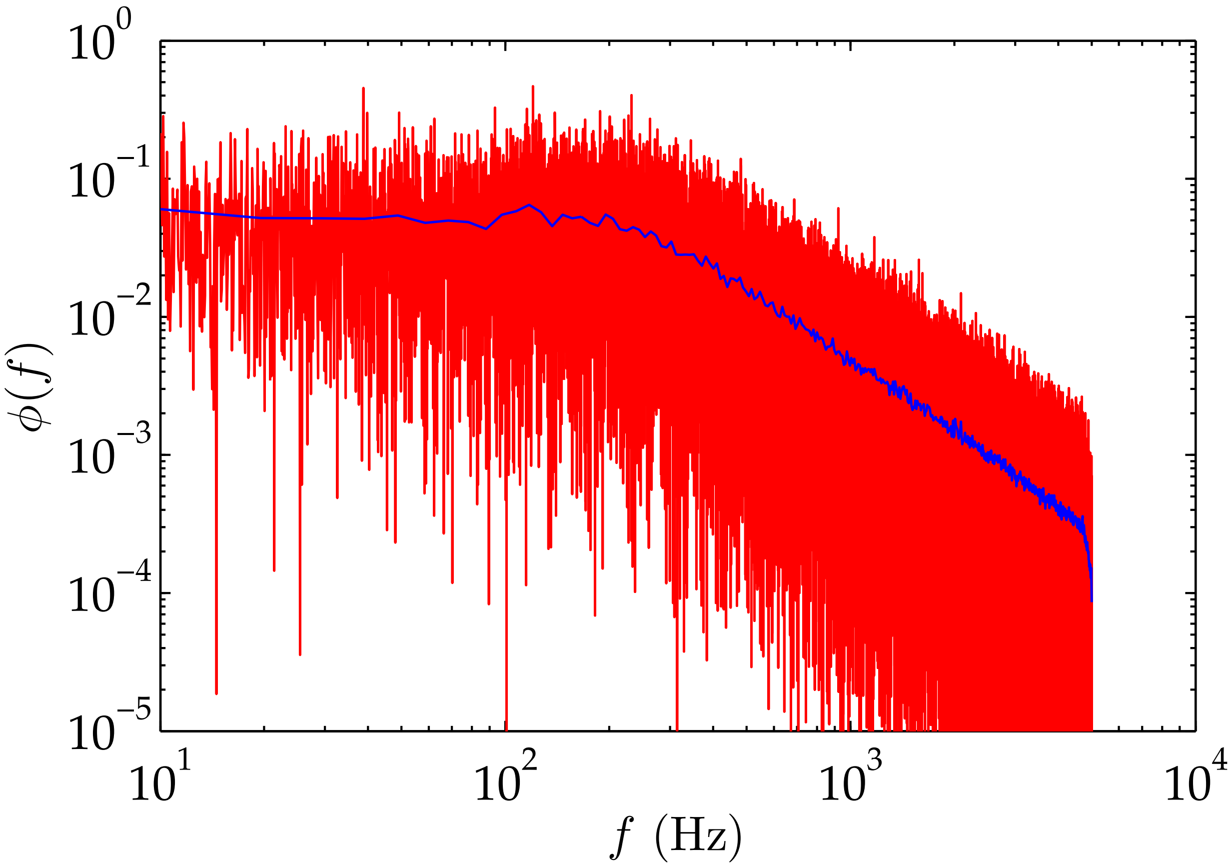


- **Spectra of u'_1 and u'_2**

```
%.. number of points for the Fourier transform
nfft = 1024;

dt = t(2)-t(1);
fs = 1./dt;

%.. Power spectral density
window = ones(nfft,1);
noverlap = nfft/2;

[PSDu f] = pwelch(u,window,noverlap,nfft,1./dt);
PSDu_int = trapz(f,PSDu);

disp(['PSD - Welch E = ',num2str(PSDu_int)]);
disp(['df = ',num2str(1/(nfft*dt)),' fmax = ',num2str(1/(2*dt))]);
```

- FFT^2 of $u'_1(t)$ over $n_{\mathrm{fft}} = 150000$ points (all the signal)

  FFT^2 $\neq$ PSD power spectral density $\phi(f)$!



- **Power spectral density for a stationary random process**

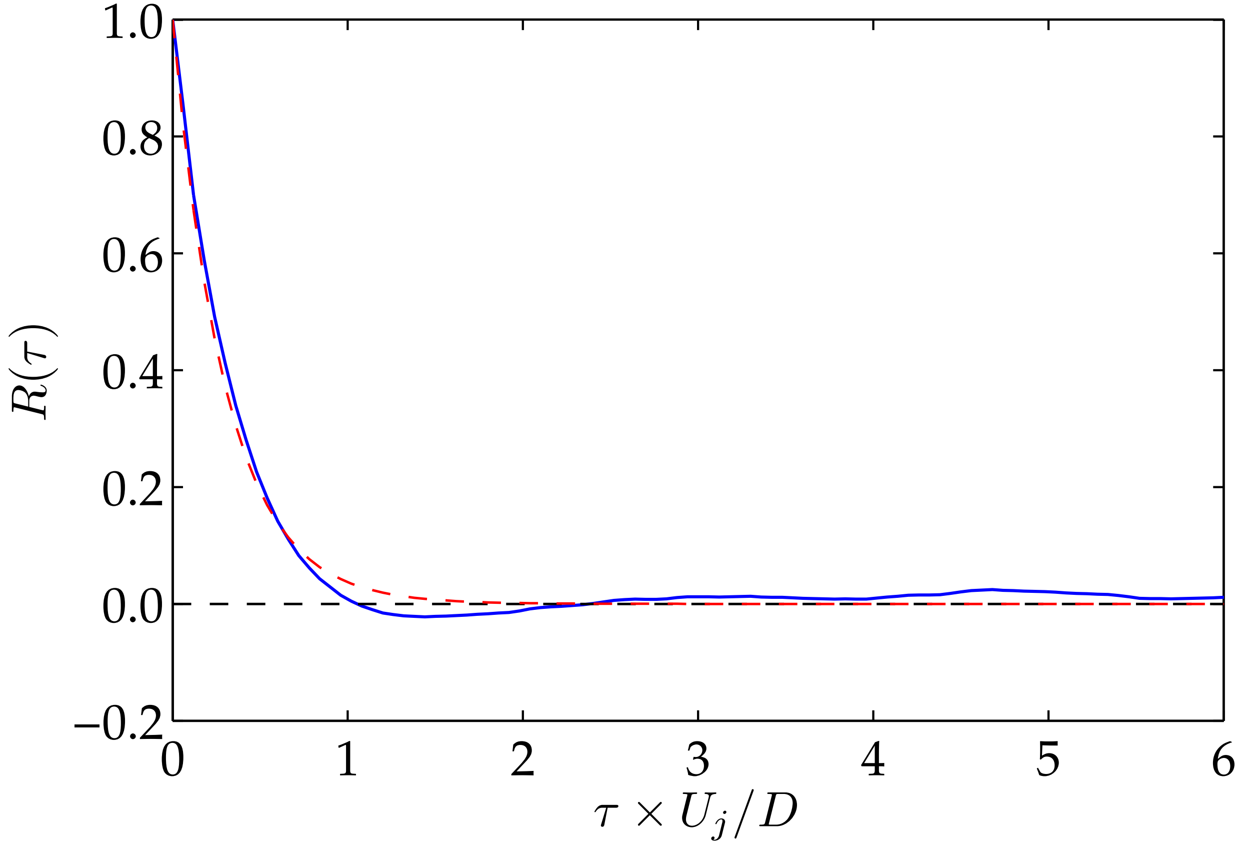
  $\hat{u}_1(f,k)$ is the DFT of $u_1(t)$ for the k-th block, n_K block averages

One-sided power spectral density ($f > 0$)

$$\phi_{u_1u_1}(f) = \lim_{T\to\infty} \frac{2}{T} E\left[\hat{u}_1^\star(f,k)\,\hat{u}_1(f,k)\right] \simeq \frac{2}{n_K T} \sum_{k=1}^{n_K} \hat{u}_1^\star(f,k)\,\hat{u}_1(f,k)$$

$$\int_0^\infty \phi_{u_1u_1}(f)df = u'^2_{u_1,\mathrm{rms}} \qquad \text{(Parseval)}$$

- Autocorrelation function of u_1'



$$R(\tau) = \frac{\overline{u_1'(t)u_1'(t+\tau)}}{u'^2_{1\text{rms}}}$$

— $R(\tau)$

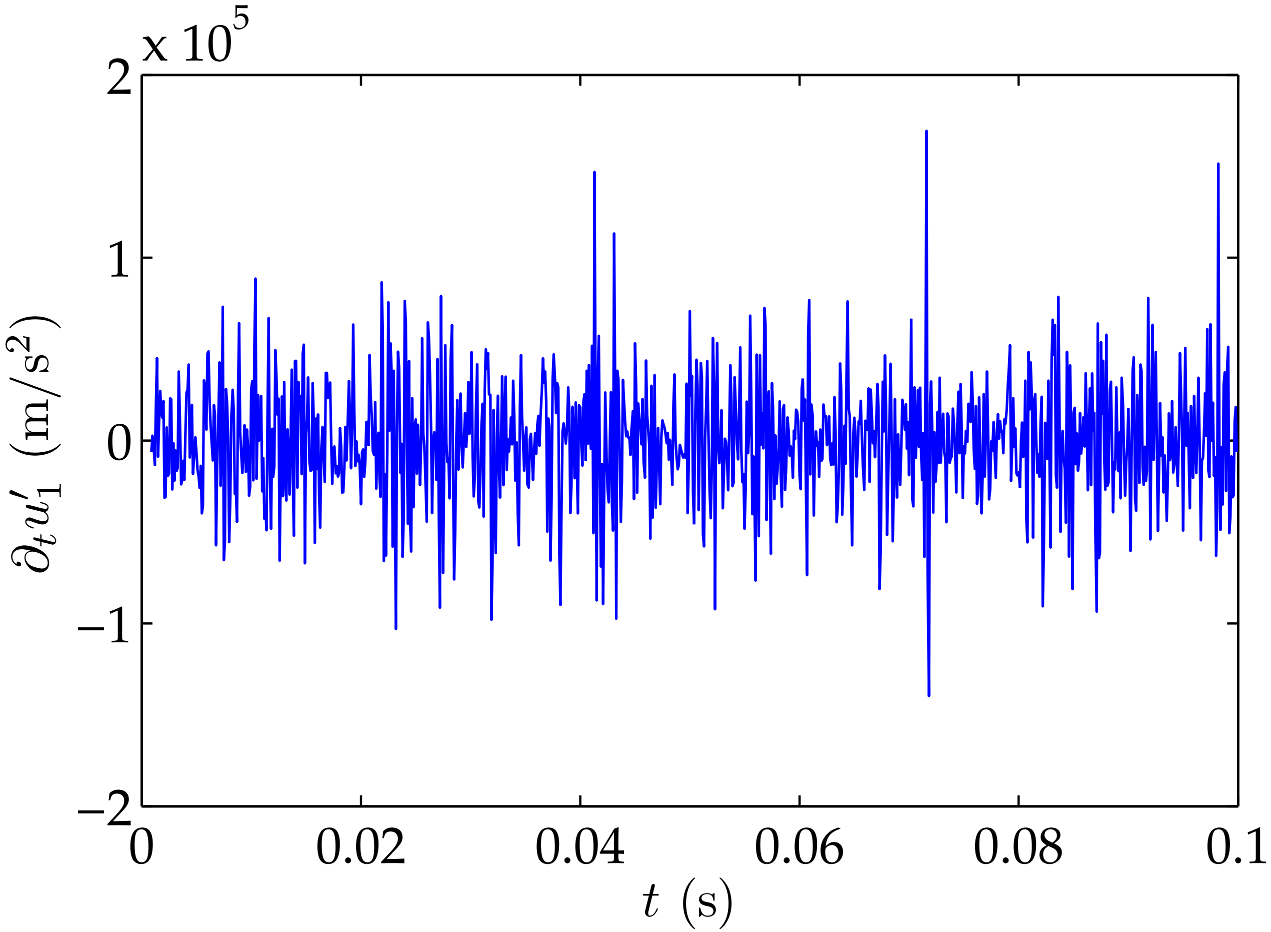
--- $\exp(-\tau/\Theta)$

$$\theta = \int_{-\infty}^{+\infty} R(\tau)\, d\tau$$

$\Theta = 5 \times 10^{-4}$ s, $L_f \simeq \overline{U}_1 \times \Theta \simeq 1.5 \times 10^{-2}$ m

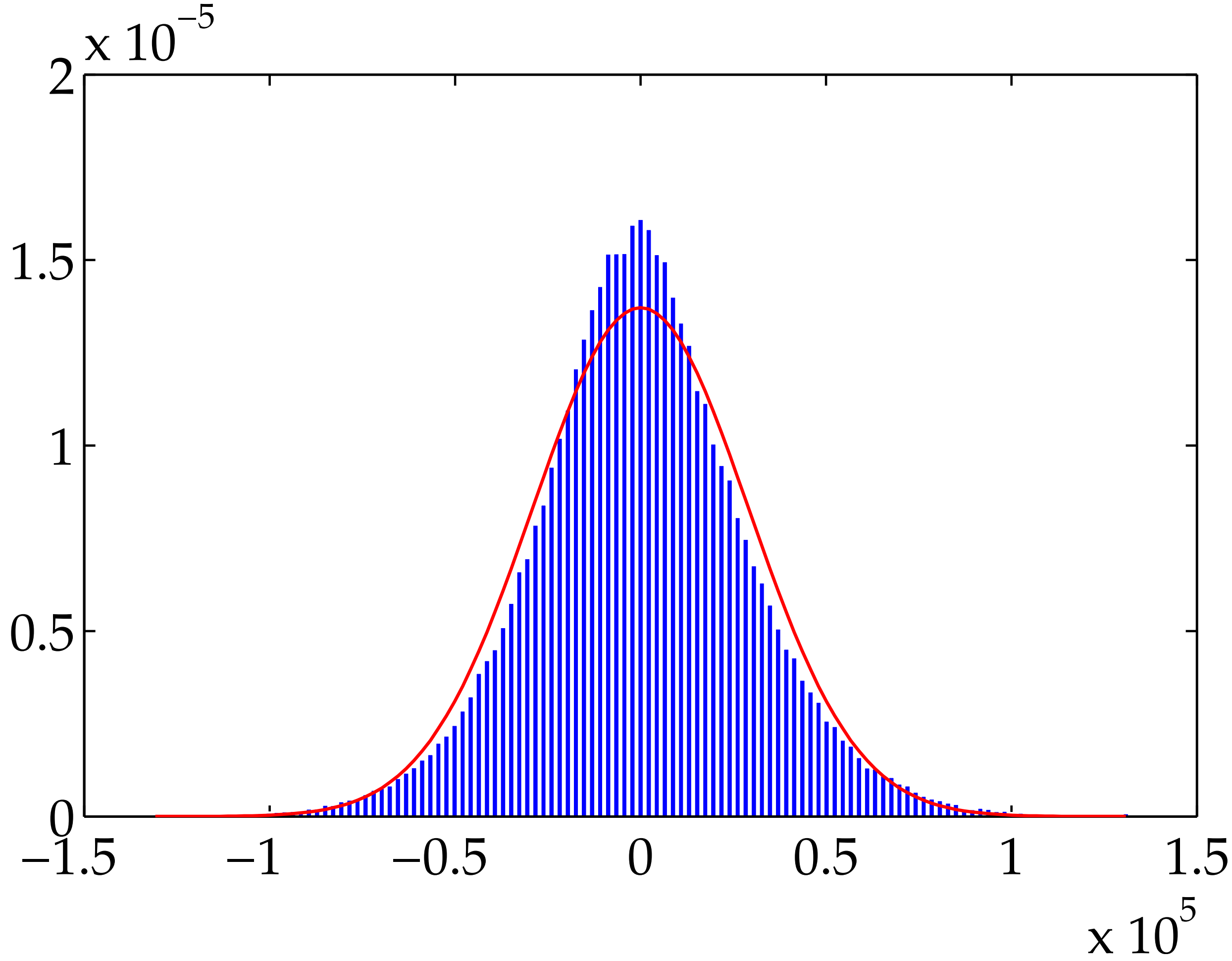
$l_\eta \simeq 2.4 \times 10^{-5}$ m, $f_\eta = 2.6 \times 10^4$ Hz

- **Time derivative of u_1'**



- Probability density function of du'_1/dt

$$S_{\partial_t u'_1} \simeq 0.07 \qquad T_{\partial_t u'_1} \simeq 4.03$$



- **Spectrum of du_1'/dt**
  directly computed from du_1'/dt using a finite difference scheme (11 pts, 4th order),
  and from the spectrum of u_1' using $\phi_{\partial_t u_1'}(f) = (2\pi f)^2 \phi_{u_1'}(f)$