AIAA 93-4412

Jet Noise Predictions Using a K-Epsilon Turbulence Model

C. Bailly, W. Béchara, P. Lafon
Département AMV, EDF-DER
1, avenue du Général de Gaulle
92141 Clamart Cedex, France

S. Candel
Ecole Centrale Paris
Grande Voie des Vignes
92295 Châtenay-Malabry Cedex, France

15th AIAA Aeroacoustics Conference
October 25-27, 1993 / Long Beach, CA

AIAA-93-4412

JET NOISE PREDICTIONS USING A K-EPSILON TURBULENCE MODEL

C. Bailly*, W. Béchara*, P. Lafon**
Département AMV, EDF-DER
1, avenue du Général de Gaulle
92141 Clamart Cedex, France

S. Candel†
Ecole Centrale Paris
Grande Voie des Vignes
92295 Châtenay-Malabry Cedex, France

Abstract

This paper deals with the prediction of the noise radiated by subsonic and supersonic jets using methods derived from the Lighthill acoustic analogy. The jet noise models of Ribner and Goldstein (mixing noise) and Ffowcs-Williams (Mach wave noise) are adapted and used in combination with aerodynamic data obtained with k-ε turbulence codes. The results are compared to experimental data. A good agreement is found for subsonic as well as supersonic Mach numbers.

Nomenclature

c_0	ambient speed of sound
k	turbulent kinetic energy
M	nominal Mach number
M_c	convective Mach number
p_0	ambient pressure
u_i	instantaneous velocity components
u_{ti}	turbulent velocity components
U_i	mean velocity components
U_j	exit jet mean velocity
U_{1axis}	mean centerline velocity
x	observer coordinates
y	source coordinates
ε	dissipation rate of k
ρ_0	ambient density
ξ,η	spatial correlation separator vectors
ω_f	characteristic pulsation of turbulence
θ	angle between the jet axis and the observer

I. Introduction

The Lighthill acoustic analogy[1,2] which is a reformulation of the conservation laws of mass and momentum reduces the aerodynamic sound problem to an acoustic wave equation associated with a source term. With some suitable assumptions, this source term can be determined from the flow, assumed to be known a priori[3].

Many different approaches can be used in order to apply Lighthill's analogy. The simplest one consists in introducing empirical assumptions to estimate analytically the source term. On the other hand, the exact solution of the Lighthill equation requires the knowledge of the time dependent fluctuations of the flow. This information can be obtained with direct or large eddy simulations of the flow but these techniques are expensive. An alternative way is to develop a statistical approach by defining the space time correlation tensor[3,4]. By an adequate introduction of assumptions obtained from the turbulence theory[5,6] and a decomposition in terms of second order correlations, Ribner[4] showed that a model for jet noise could be formulated. Hence, it is possible to introduce aerodynamic data provided by the application of a numerical code with a k-ε turbulence model[7,8].

When the convective Mach number is subsonic, the noise radiation is due to the intrinsic fluctuations of the turbulent eddies which behave like compact quadrupoles. The influence of convection is to increase the sound intensity radiated when the source moves towards the observer. When the eddies are convected supersonically (i.e. when the source approaches the observer at the speed of sound), the source loses its compactness and behaves like monopoles convected supersonically. This changes the nature of radiation because Mach waves appear and the efficiency of the noise emission is increased in their direction. Ffowcs Williams[9] and Ffowcs Williams and Maidanik[10] developed a reformulation of the Lighthill source term that can be applied to supersonic flows. This model can be used for the prediction of supersonic jet associated with a k-ε turbulence model[11].

This paper provides a synthesis of several studies concerning jet noise modeling[7,8,11] and all details cannot be included here. In the next section, we will only present the main features for the derivation of the Ribner model for the jet mixing noise and how it is used in combination with k-ε turbulence data. Comparisons with experimental results are exhibited for simple and coaxial jets. In the third section, we briefly present the Ffowcs Williams model for Mach wave radiation and comparisons with experimental data for jets. It is then shown that by associating these two types of models one may obtain a hybrid description of noise emission for all Mach numbers. The noise radiated by shocks is however not included at this time.

II. Subsonic jets

Derivation of the model

The Lighthill theory shows that the density fluctuations observed in the far field and generated by a volume V of turbulence are given by:

* Ph. D. Student
** Research Engineer, Member AIAA
† Professor of Fluid Mechanics, Member AIAA

$$(\rho - \rho_0)(x,t) = \frac{1}{4\pi c_0^4} \frac{x_i x_j}{x^3} \int_v \frac{\partial^2}{\partial t^2} T_{ij}\left(y, t - \frac{|x-y|}{c_0}\right) dy$$

T_{ij} is the instantaneous Lighthill stress tensor which reduces to $\rho_0 u_i u_j$ with the classical assumptions (high Reynolds number, incompressible turbulence and no entropy fluctuations). Then the autocorrelation function $C_{pp}(x,\tau)$ of the pressure fluctuations which is defined by:

$$C_{pp}(x,\tau) = \overline{[p(x,t) - p_0][p(x,t+\tau) - p_0]}$$

takes the following form:

$$C_{pp}(x,\tau) = \frac{\rho_0 x_i x_j x_k x_l}{16\pi^2 c_0^5 x^6} \frac{\partial^4}{\partial \tau^4} \iint_v \mathbb{R}_{ijkl}\left(y, \eta, \tau + \frac{x.\eta}{c_0 x}\right) dy d\eta$$

$\mathbb{R}_{ijkl}$ is the fourth order space-time correlation tensor. In order to neglect the influence of the retarded time, a moving reference frame is introduced and the expression of the autocorrelation function becomes:

$$C_{pp}(x,\tau) = \frac{\rho_0 x_i x_j x_k x_l}{16\pi^2 c_0^5 x^6} \frac{1}{C^5} \iint_v \frac{\partial^4}{\partial t^4}\left[R_{ijkl}\left(y, \xi, t\right)\right]_{t=\tau/C} dy d\xi$$

where $C = 1 - M_c \cos\theta$ is the convection factor. It is standard to take for the convective velocity $0.67\ U_{1axis}$.

Ribner[4] developed some formulations of the Lighthill theory adapted to jet configurations. Introducing the decomposition of the instantaneous velocity field in a mean stationary field and a turbulent field:

$$u_i = U_1 \delta_{1i} + u_{ti}$$

the space time correlation tensor is written as:

$$R_{ijkl} = \overline{u'_{ti} u'_{tj} u''_{tk} u''_{tl}} + U'_1 U''_1 \left(\delta_{1i}\delta_{1k} \overline{u'_{tj} u''_{tl}} + \delta_{1j}\delta_{1k} \overline{u'_{ti} u''_{tl}} + \delta_{1i}\delta_{1l} \overline{u'_{tj} u''_{tk}} + \delta_{1j}\delta_{1l} \overline{u'_{ti} u''_{tk}} \right)$$

The first term is known as the self noise contribution and the others as the shear noise contribution. Using an homogeneous and isotropic description of the turbulent field[4], it is possible to find some simplified expressions for the velocity correlations and the evaluation of $C_{pp}(x,\tau)$ at $\tau = 0$ gives the directional intensities radiated by a unit source volume located at **y** to a point in the far field located at **x**[7,8]:

$$I_{self}(x|y) = \frac{3\rho_0 L^3 \omega_f^4 \overline{u_t^2}^2}{2\sqrt{2}\pi^2 c_0^5 C^5 x^2}$$

$$I_{shear}(x|y) = \frac{3\rho_0 L^5 \omega_f^4 \overline{u_t^2}}{8\pi^3 c_0^5 C^5 x^2} \left(\frac{\partial U_1}{\partial y_2}\right)^2 \frac{1}{2}\left(\cos^4\theta + \cos^2\theta\right)$$

Finally, the intensity radiated in the far field is written as:

$$I(x) = \int_v \left[I_{self}(x|y) + I_{shear}(x|y)\right] dy$$

In these expressions, L (longitudinal integral scale of turbulence), ω_f (turbulence characteristic pulsation) and $\overline{u_t^2}$ are defined using the results of the aerodynamic calculation[7,8]:

$$L = \frac{k^{3/2}}{\varepsilon}\ ,\ \omega_f = 2\pi\frac{\varepsilon}{k}\ ,\ \overline{u_t^2} = \frac{2}{3}k$$

The convection factor C has a singularity when $M_C \cos\theta = 1$. A more sophisticated analysis developed by Ffowcs-Williams[9] leads to the following modified convection factor:

$$C_m = \left[\left(1 - M_c \cos\theta\right)^2 + \alpha^2 M_c^2\right]^{1/2}$$

Ribner[4] proposed an expression for the factor α:

$$\alpha^2 M_c^2 = \frac{\omega_f^2 L^2}{\pi c_0^2}$$

Goldstein and Rosenbaum[12] developed a modification to the Ribner model by introducing an anisotropic description of the turbulent field.

For these two models, a global and unique adjustable factor is determined by a single comparison with experimental data[7,8].

Applications

The details of the aerodynamic calculations and the validations of the results are fully documented in ref 7 and 8.

round jet

The configuration examined is that investigated experimentally by Lush[13]. Figures 2 and 3 show the intensity predicted by the models of Ribner and Goldstein for $M = 0.56$ and $M = 0.86$. Ribner model overestimates the noise level for small angles. The improvement of the turbulence description in the Goldstein model leads to results closer to the experimental values. The drop-off of experimental intensity at very small angles is due to refraction effects which are not included in the models. Figure 4 gives a similar comparison for $M = 1.33$ using experimental data of Tanna *et al.*[14]. It is shown that the directivity is not well predicted. In fact for such high Mach numbers, the basic assumptions used for the two models (retarded time neglected, weak interaction with flow) are no longer valid, the influence of supersonically convected eddies appears and the models used here do not match all the physics of the jet.

Figure 5 gives the total acoustic power predicted by Ribner and Goldstein models as a function of the nominal Mach number. It appears that the models are in good agreement with results of Lush and also with those of Tanna which concern higher Mach numbers. In fact, the mixing noise models succeed in predicting total acoustic power for high Mach numbers because of the convection factor which in the limit of high M yields the U^3 law.

coaxial jet

Figure 6 presents the intensity radiated at an angle of 90° as a function of the ratio of the secondary exit velocity to the primary exit velocity (U_{1s}/U_{1p}). This result is compared to the experimental data of Juvé *et al.*[15]. The exit velocity of the primary jet is 130 m/s. One finds that the numerical results obtained with the Goldstein model give the correct value of U_{1s}/U_{1p} for which the acoustic intensity is minimum. The level of this minimum is also close to the experimental point.

III. Supersonic jets

Derivation of the model

The supersonic convection of eddies produces Mach waves which are one of the main noise sources in supersonic jets. Ffowcs-Williams and Maidanik[10] transformed the source term of the Lighthill theory in order to account for Mach wave radiation. In the far field, the density fluctuations are given by:

$$(\rho - \rho_0)(x,t) = -\frac{1}{4\pi c_0^2}\int_v \frac{(x_i-y_i)(x_j-y_j)}{|x-y|^2}\left[\frac{1}{c^2}\frac{\partial P}{\partial t}\frac{\partial U_i}{\partial y_j}\right]\left(y, t-\frac{|x-y|}{c_0}\right)\frac{dy}{|x-y|}$$

When applied to jets, this formulation leads to an expression for the intensity radiated in the far field[10,11]:

$$I(x) = \frac{c_0^2\rho_0^2}{32\pi^2\rho_0 x^2}\times \int_v \sin^2\theta\,\cos^2\theta\, D\left(M_c,\theta\right)\left[\frac{1}{c^2}\frac{\partial U_1}{\partial y_2}\right]^2 \tau^*(y)\, M_c^3(y)\,dy$$

where $D\left(M_c,\theta\right) = \left[\frac{\alpha^2 M_c^2}{\left(1-M_c\cos\theta\right)^2+\alpha^2 M_c^2}\right]^{5/2}$ and $\tau^* \sim \frac{k}{\varepsilon}$.

τ^* is the local characteristic time of the turbulence. The source term contains the local speed of sound; thus, the influence of the temperature on the noise radiation can be estimated.

For this model, as for the two previous ones, an adjustable factor is necessary[11]. This unique and global factor is determined by comparison with experimental data of Tanna *et al.*[14].

Application

Figure 7 presents the case of a cold round supersonic free jet with a nominal Mach number of 2. This configuration was experimentally studied by Tanna *et al.*[14]. The acoustic intensity predicted by the Ffowcs Williams model is plotted and a good agreement is found with experimental data.

Figure 8 displays the acoustic power predicted by a hybrid model for different exit Mach numbers. This hybrid model is a combination of the Goldstein model and the Ffowcs Williams model which is applied when the local convective Mach number is supersonic. The experimental results of Lush[13], Tanna *et al.*[14], Seiner *et al.*[16,17] and Norum and Seiner[18] are plotted. One observes that the subsonic law follows closely the variations of the acoustic power for an exit Mach number less than 1.5, whereas the supersonic law is valid for an exit Mach number greater than 2.

IV. Conclusion

In this paper, several numerical approaches for the prediction of jet noise are developed. These approaches are based on extensions of Lighthill's theory: Ribner and Goldstein models (mixing noise), Ffowcs Williams model (Mach Wave noise). To apply these models, it is necessary to know the local mean velocity and some statistical informations about the turbulence. Aerodynamic computations, carried out using a k-ε turbulence code, supply the mean velocity, the turbulent kinetic energy and the rate of dissipation.

Three types of validations are presented: subsonic simple and coaxial round jets for the mixing noise models and supersonic round jet for the Mach wave model.
The Goldstein model gives better results than Ribner's model especially for the directivity pattern. The predictive ability of this model is shown by the application on coaxial jets.
The Ffowcs Williams model retrieves the change in the nature of the acoustic radiation induced by Mach waves.

The combination of these two models into a hybrid allows calculations for all Mach numbers (however excluding shock radiation). This hybrid model may be used in other configurations: coaxial subsonic-supersonic jets, supersonic hot jets.

References

1 Lighthill M. J., "On Sound Generated Aerodynamically. I-General Theory", *Proceedings of The Royal Society of London*, Vol. 211, Ser. A, 1107, 1952, pp.564-587
2 Lighthill M. J., "On Sound Generated Aerodynamically. II-Turbulence as a Source of Sound", *Proceedings of The Royal Society of London*, Vol. 222, Ser. A, 1148, 1954, pp.1-32
3 Goldstein M. E., "Aeroacoustics", Mc-Graw-Hill International Book Company, New York, 1976
4 Ribner H. S., "Quadrupole Correlations Governing the Pattern of Jet Noise", *Journal of Fluid Mechanics*, Vol.38(1), 1969, pp.1-24
5 Batchelor G. K., "The Theory of Homogeneous Turbulence", Cambridge University Press, 1953
6 Hinze J. O., "Turbulence", Second Edition, Mc-Graw-Hill International Book Company, New York, 1975
7 Béchara W., Lafon P., Candel S., "Application of a k-ε Model to the Prediction of Noise for Simple and Coaxial Free Jets" *JASA* (to be published)
8 Bailly C., Lafon P., Candel S., "Computation of Subsonic and Supersonic Jet Mixing Noise Using a Modified k-ε Model for Compressible Free Shear Flows", *Acta Acustica* (submitted)
9 Ffowcs Williams J. E., "The Noise from Turbulence Convected at High Speed", *Philosophical Transactions of The Royal Society of London*, Vol.225, Ser. A, 1963, pp.469-503
10 Ffowcs Williams J. E., Maidanik G., "The Mach Wave Field Radiated by Supersonic Turbulent Shear Flows", *Journal of Fluid Mechanics*, Vol.21(4), 1965, pp.641-657
11 Bailly C., Lafon P., Candel S., "Eddy Mach Wave Noise Modeling Using a k-ε Turbulence Model" (in preparation)
12 Goldstein M. E., Rosenbaum B., "Effect of Anisotropic Turbulence on Aerodynamic Noise", *Journal of the Acoustical Society of America*, Vol.54(3), 1973, pp.630-645

13 Lush P. A., "Measurements of Subsonic Jet Noise and Comparison with Theory", *Journal of Fluid Mechanics*, Vol.46(3), 1971, pp.477-500
14 Tanna H. K., Dean P. D., Burrin R. H., "The Generation and Radiation of Supersonic Jet Noise. Vol.III: Turbulent Mixing Noise Data", AFAPL-TR-76-65, 1976
15 Juvé D., Bataille J., Comte-Bellot G., "Bruits de Jets Coaxiaux Froids Subsoniques", *J. Méc. Appl.*, Vol.2(3), 1978, pp.385-398
16 Seiner J. M., McLaughlin D. K., Liu C. H., "Supersonic Jet Noise Generated by Large-Scale Instabilities", NASA, Technical Paper, 2072, 1982
17 Seiner J. M., Ponton M. K., Jansen B. J., Lagen N. T., "The Effects of Temperature on Supersonic Jet Noise Emission", *DGLR/AIAA 14th Aeroacoustic Conference*, AIAA-92-02-046, 11-14 May 1992
18 Norum T. D., Seiner J. M., "Measurements of mean static pressure and far field acoustics of shock containing supersonic jets", NASA TM *84521, 1982*

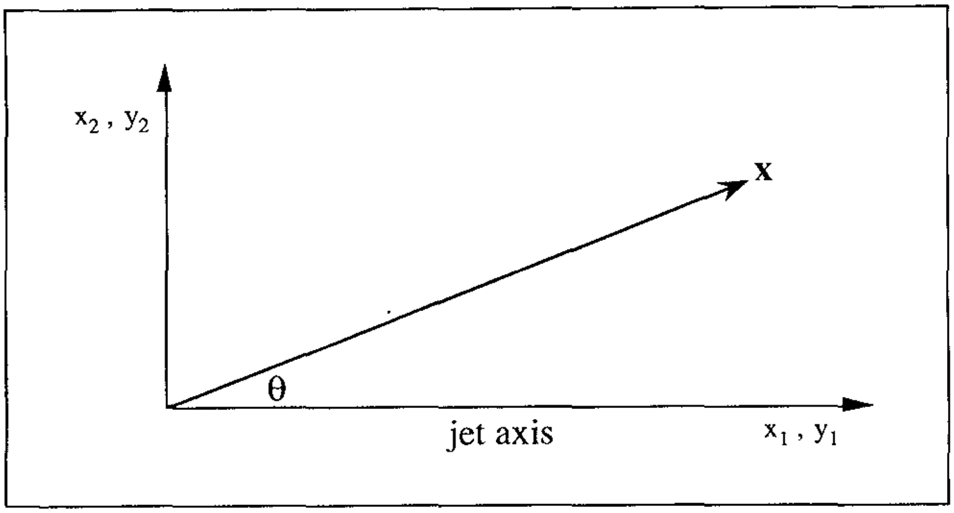

Fig. 1 - sketch of the jet configuration

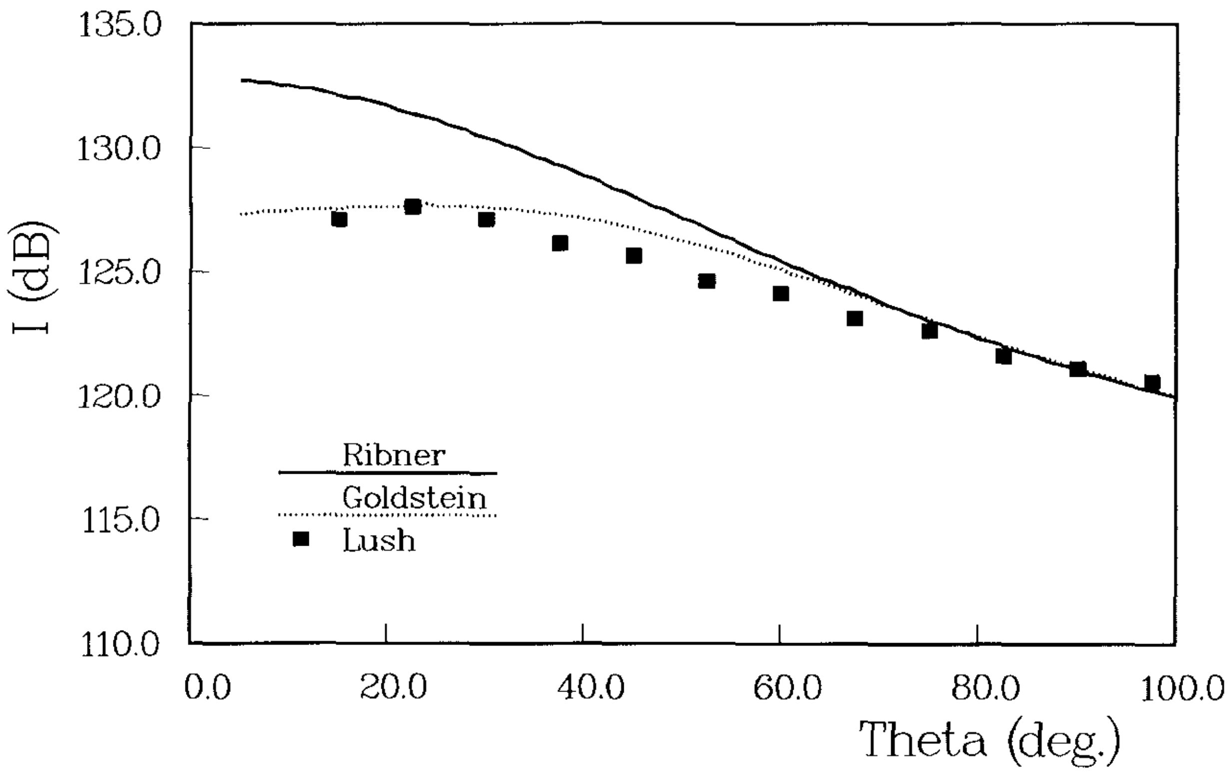

Fig. 2 - acoustic intensity for M = 0.56

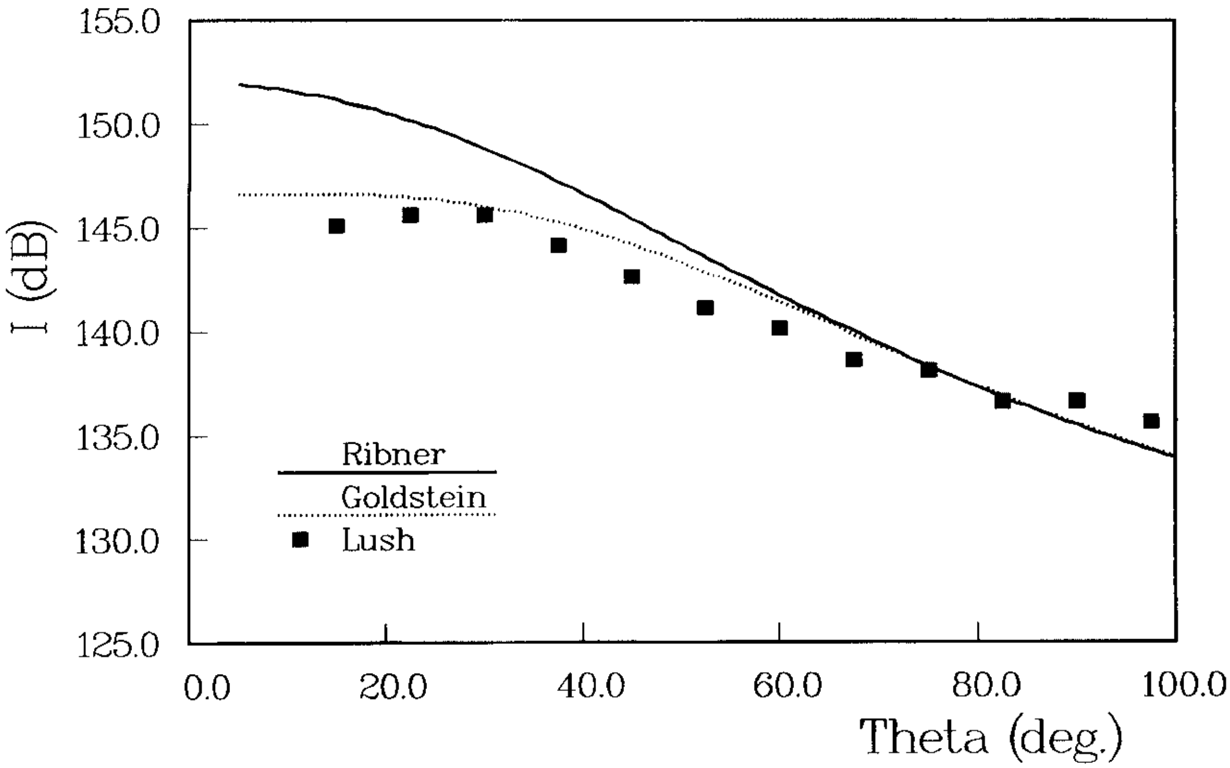

Fig. 3 - acoustic intensity for M = 0.86

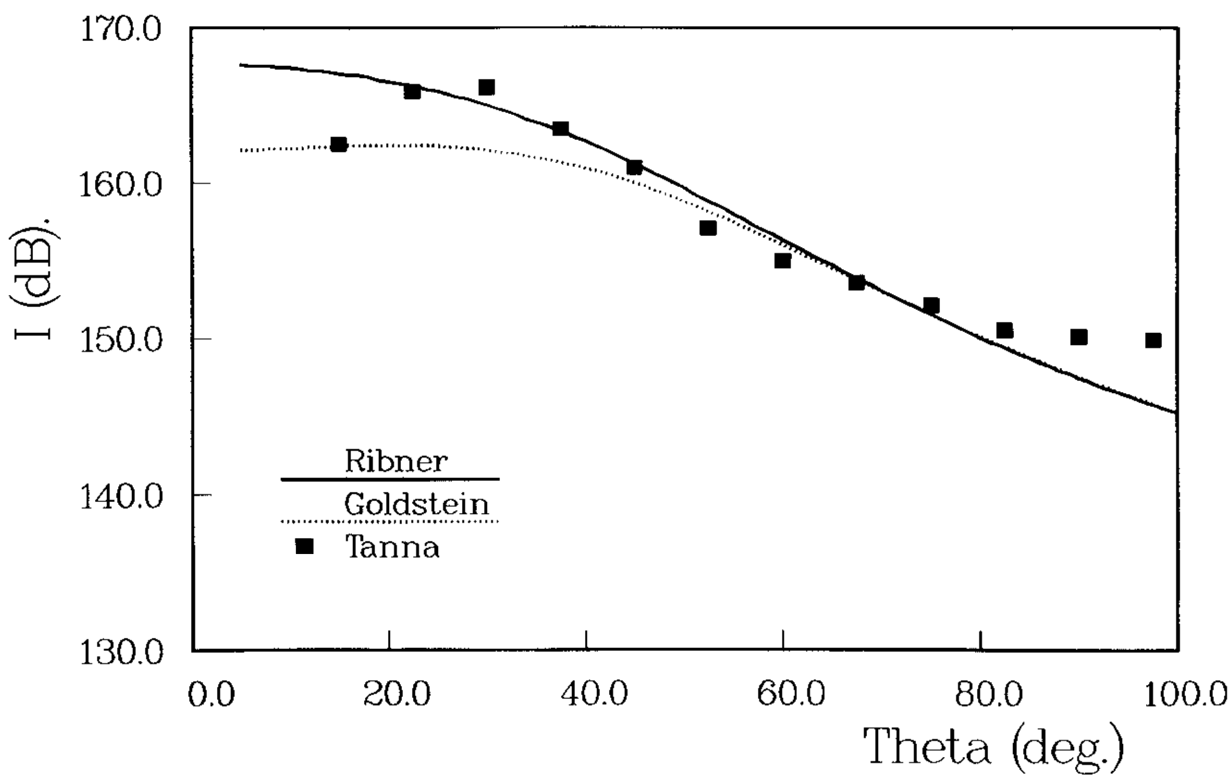

Fig. 4 - acoustic intensity for M = 1.33

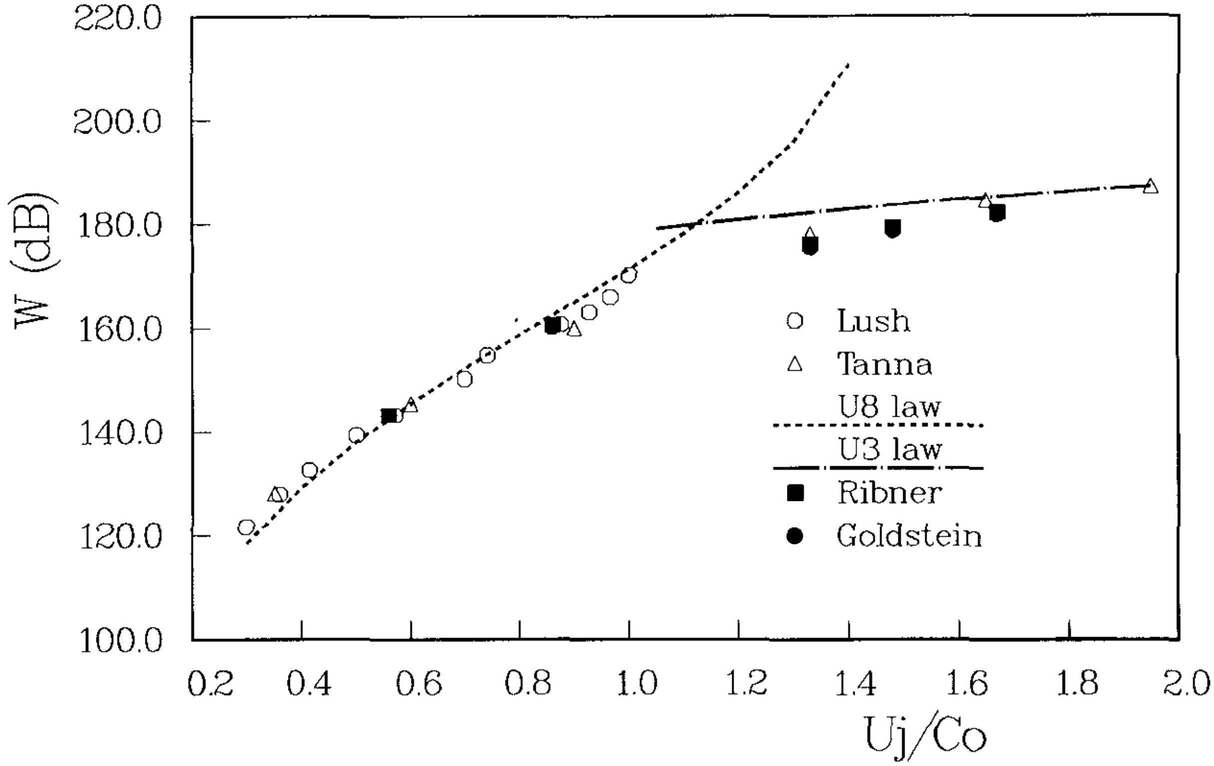

Fig. 5 - total acoustic power as a function of jet Mach number: comparison of mixing noise models to experimental data

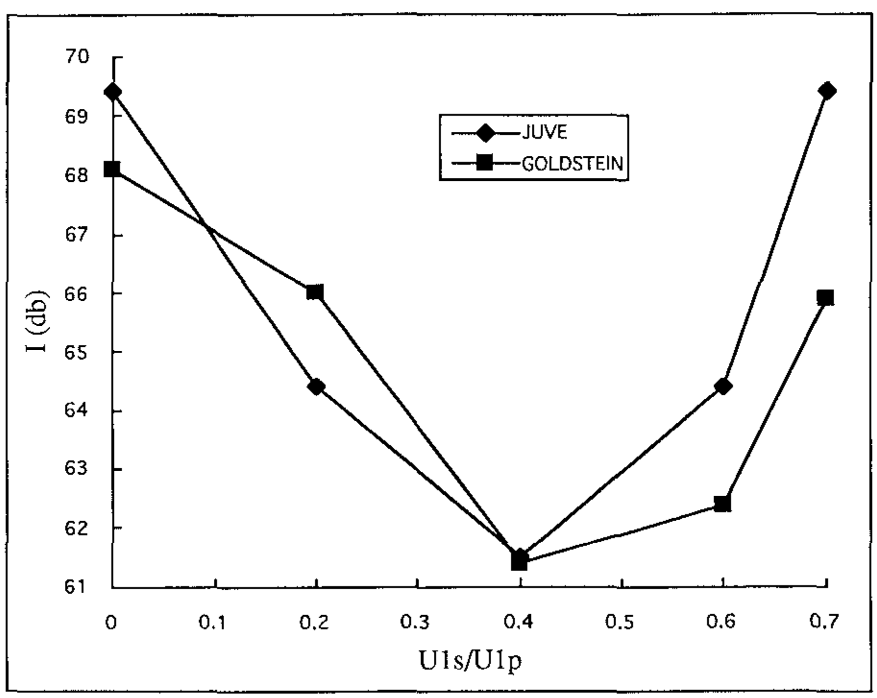

Fig. 6 - acoustic intensity of coaxial jets (theta = 90°)

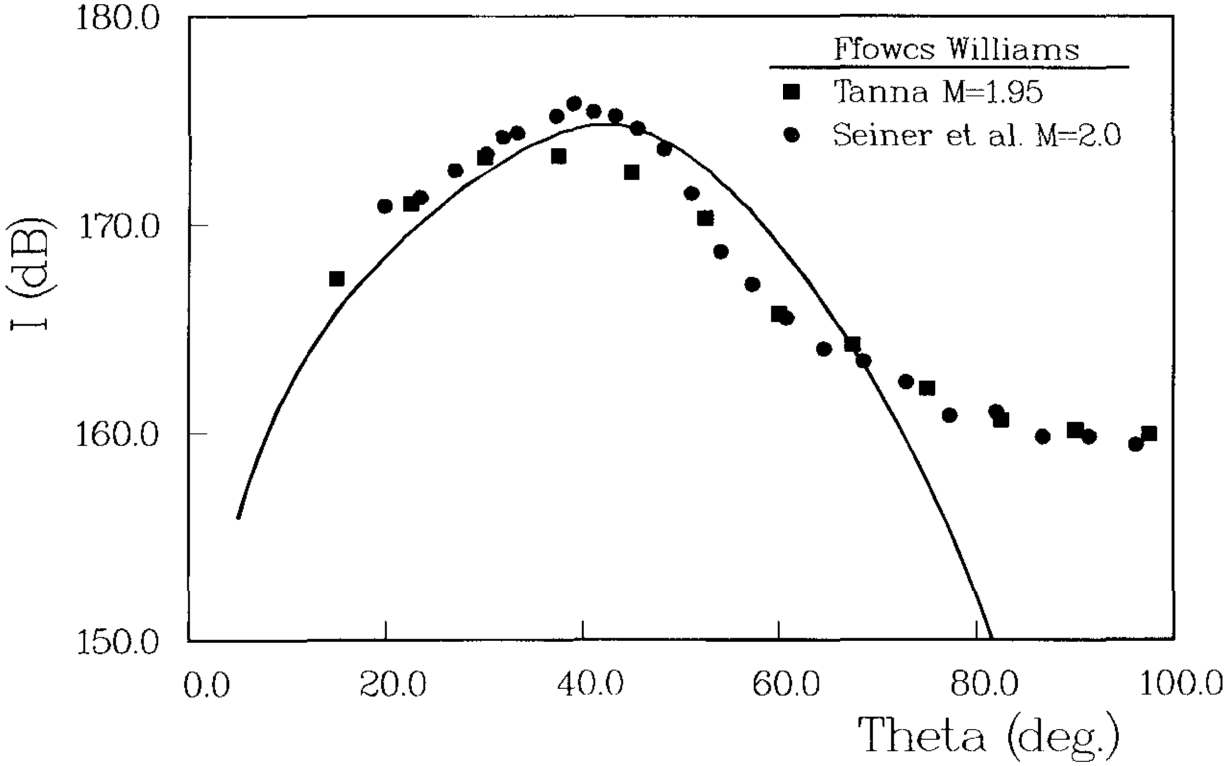

Fig. 7 - acoustic intensity for M = 2

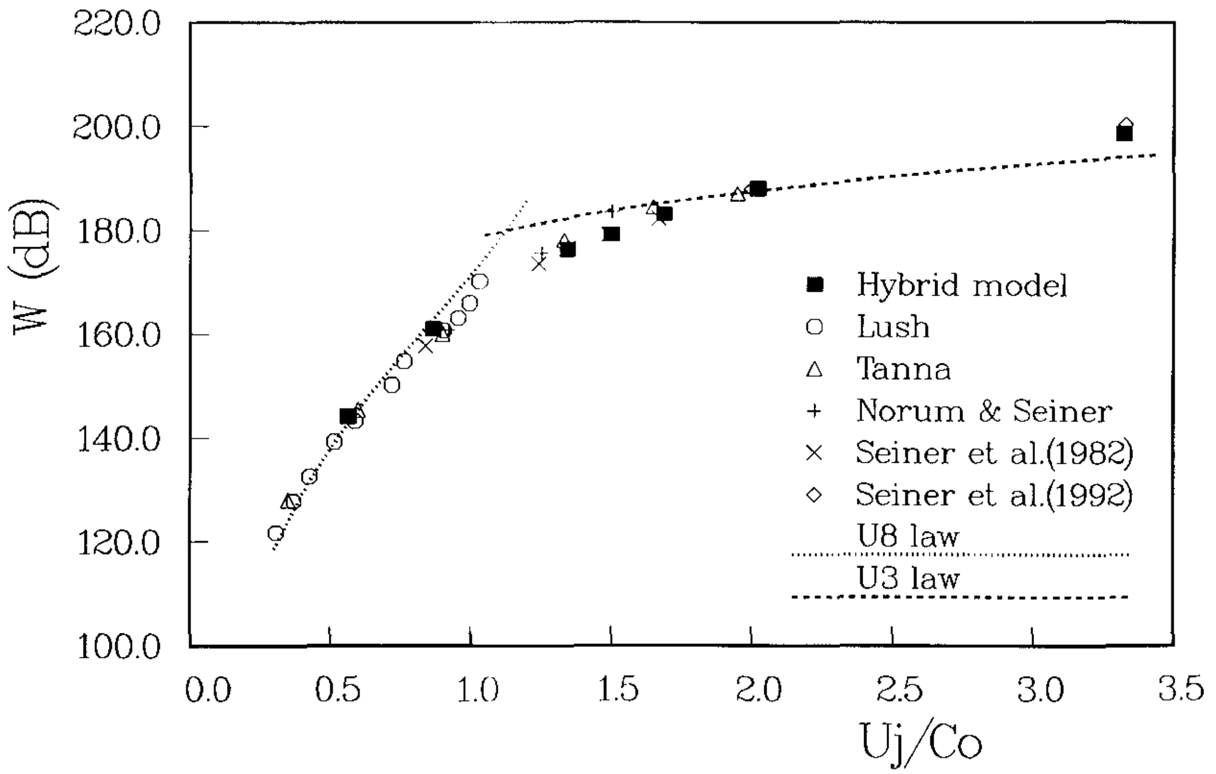

Fig. 8 - total acoustic power as a function of jet Mach number: comparison of hybrid noise model to experimental data