Computation of noise generation and propagation for free and confined turbulent flows

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This paper deals with the application of the SNGR (Stochastic Noise Generation and Radiation) model to compute turbulent mixing noise generated in a duct obstructed by a diaphragm. Two problems must be solved in the framework of an acoustic analogy. First, a wave operator must be derived for sound waves traveling in any mean flow. In the SNGR model, the system of linearized Euler equations is used. An expression of the source term is then deduced from the conservation laws of motion and can be simplified with classical assumptions of aeroacoustics in the case of subsonic mixing noise. Secondly, the knowledge of the turbulence velocity field is required to compute this source term. In the SNGR model, the space-time turbulence velocity field is generated by a sum of random Fourier modes. Finally, the radiated acoustic field is calculated numerically by solving the inhomogeneous propagation system. The method is applied to the case of a 2D duct obstructed by a diaphragm. Numerical results are compared with available experimental ones. Computed noise levels closely match experimental ones and follow the expected U4 law. (Author)
Computation of noise generation and propagation for free and confined turbulent flows

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Abstract
This paper deals with the application of the SNGR (Stochastic Noise Generation and Radiation) model to compute turbulent mixing noise generated in a duct obstructed by a diaphragm. Two problems must be solved in the framework of an acoustic analogy. First, a wave operator must be derived for sound waves travelling in any mean flow. In the SNGR model, the system of linearized Euler equations is used. An expression of the source term is then deduced from the conservation laws of motion, and can be simplified with classical assumptions of aeroacoustics in the case of subsonic mixing noise. Secondly, the knowledge of the turbulence velocity field is required to compute this source term. In the SNGR model, the space-time turbulence velocity field is generated by a sum of random Fourier modes. Finally, the radiated acoustic field is calculated numerically by solving the inhomogeneous propagation system. The method is applied in this paper to the case of a two-dimensional duct obstructed by a diaphragm. Numerical results are compared with available experimental ones. Computed noise levels closely match experimental ones and follow the expected $U^4$ law.

Introduction
Aeroacoustic calculations usually require a wave equation and its source term. It is also necessary to provide some informations about the turbulence in the flow. Classical aeroacoustic approaches based on the Lighthill acoustic analogy generally use the free-space Green’s function in order to solve the wave equation, but the application of such techniques to confined configurations are not straightforward. In this paper, we use the Stochastic Noise Generation and Radiation model (SNGR), which solves the system of linearized Euler equations instead of a wave equation and uses as input a synthesized turbulent field providing the source term.

The equations of the SNGR model are presented in section 2. It is shown in section 3 how one may synthesize a space-time turbulent velocity field with suitable statistical properties. The application to the case of a duct obstructed by a diaphragm is discussed in section 4.

Propagation in nonuniform mean flow
The simplest wave equation that one can exactly derive from the fundamental conservation laws of motion is Lighthill’s equation:

$$\frac{\partial^2 \rho}{\partial t^2} - c_0^2 \frac{\partial^2 \rho}{\partial x_i \partial x_i} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}$$  \hspace{1cm} (1)$$

where $\rho$ is the density and $T_{ij}$ is Lighthill’s tensor $T_{ij} = \mu \nu_{ii} + (p/c_0^2) \delta_{ij} - \tau_{ij}$. In this last expression, $u$, $p$, and $\tau$ designate the velocity, pressure and viscous stress tensor. Here one assumes that the medium external to the flow is homogeneous and at rest, $c_0$ being the constant speed in
this ambient medium. The free-space Green's function of this wave operator is known allowing easy application of Lighthill's analogy in many studies.\textsuperscript{1-3,7,12-14,25} The method does not account for mean flow effects on acoustic wave propagation. Such effects are known to modify the aerodynamic noise spectrum and directivity. Phillips\textsuperscript{24} replaced Lighthill's equation by a convected wave equation where a part of the mean flow effects were included in the wave operator rather than in the source term. Introducing the logarithm of the pressure variable $\pi = \ln p$, Phillips' equation reads:

\[
\frac{d^2 \pi}{d^2 t} - \frac{\partial}{\partial x_i} \left( c_o^2 \frac{\partial \pi}{\partial x_i} \right) = \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} + \frac{2}{\gamma} \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} (c_o^2 \frac{\partial \pi}{\partial x_i})
\]

where $\gamma = c_p / c_v$ is the specific heats ratio. One sees that the main source term for jet noise only contains components of the velocity field unlike Lighthill's source term. At high Reynolds numbers, the viscous stress tensor can be neglected. Furthermore, one assumes viscous dissipation and heat conduction effects are negligible in sound generation and propagation. Then assuming a parallel sheared mean flow: $u_i = U (x_j) \delta_{ij} + u'_i$, Phillips' equation (3) may be written in the form:

\[
\frac{D^2 \pi'}{D t^2} - \frac{\partial}{\partial x_i} \left( c_o^2 \frac{\partial \pi'}{\partial x_i} \right) = 2 \gamma \frac{\partial u'_i}{\partial x_j} \frac{dU}{\partial x_i} \frac{dU}{\partial x_j} + \gamma \left( \frac{\partial u'_i}{\partial x_j} \frac{\partial u'_j}{\partial x_i} - \frac{\partial u'_i}{\partial x_j} \frac{\partial u'_j}{\partial x_i} \right)
\]

where $D / Dt = \partial / \partial t + U \partial / \partial x_i$ is the convective time derivation operator. It is also known that linearized Euler's equations govern acoustic wave propagation. So, the associated wave equation should be identical to the previous homogeneous equation. But this is not exactly the case.\textsuperscript{10,22} Indeed, assuming a global isentropic relation $dp = c_d d\rho$, the wave equation derived from the linearized Euler's equation is given by:

\[
\frac{D^2 \pi'}{D t^2} - \frac{\partial}{\partial x_i} \left( c_o^2 \frac{\partial \pi'}{\partial x_i} \right) - 2 \gamma \frac{dU}{d x_2} \frac{dU}{d x_1} = 0
\]

In order to eliminate the term containing velocity fluctuations of the wave operator, one must again apply the $D / Dt$ operator to the last equation. This finally yields:

\[
\frac{D}{Dt} \left\{ \frac{D^2 \pi'}{D t^2} - \frac{\partial}{\partial x_i} \left( c_o^2 \frac{\partial \pi'}{\partial x_i} \right) \right\} + 2 \frac{dU}{d x_2} \frac{\partial}{\partial x_1} \left( c_o^2 \frac{\partial \pi'}{\partial x_1} \right) = 0
\]

This equation shows that Phillips' wave operator does not contain all the terms that appear in (5). On the other hand, in the case of a sheared mean flow, the simplest wave equation for the acoustic variable $\pi'$ is a third order differential equation. Lilley\textsuperscript{22} derived such a third order wave equation from Phillips' equation with this idea. Thus, in applying the convective $D / Dt$ operator to Phillips' equation (3), it follows:

\[
\frac{d}{dt} \left\{ \frac{d^2 \pi'}{d^2 t} - \frac{\partial}{\partial x_i} \left( c_o^2 \frac{\partial \pi'}{\partial x_i} \right) \right\} + 2 \frac{\partial u'_i}{\partial x_j} \frac{\partial u'_j}{\partial x_i} (c_o^2 \frac{\partial \pi'}{\partial x_i}) = 0
\]

where the viscous contribution and the entropy fluctuations are neglected. The free-space Green function of the Lilley's equation is unknown and it is difficult to solve numerically a third order wave equation, unlike the linearized Euler's equations. Moreover, in the case of a nonuniform mean flow, acoustic and hydrodynamic fluctuations cannot be clearly separated to form a wave operator.\textsuperscript{8,23,28} Thus, computation of the acoustic field by solving linearized Euler's equations constitutes an interesting alternative. An analysis of the acoustic analogy associated with linearized Euler's equations was developed.\textsuperscript{2,6} It was found that the following system of two first-order equations has to be retained:

\[
\begin{align*}
\frac{\partial p}{\partial t} + u_j \frac{\partial p}{\partial x_j} + u'_j \frac{\partial p_0}{\partial x_j} + \gamma p_0 \frac{\partial u'_j}{\partial x_j} + \frac{\partial p}{\partial x_j} (c_o^2 \rho \frac{\partial u'_j}{\partial x_j}) &= 0 \\
\frac{\partial u'_j}{\partial t} + u_j \frac{\partial u'_j}{\partial x_j} + u'_j \frac{\partial u_0}{\partial x_j} + \frac{1}{\rho_0} \frac{\partial p}{\partial x_j} - \frac{\rho' \partial p_0}{\partial x_j} &= S_i
\end{align*}
\]

where the source term reads as follows:

\[
S_i = - \left\{ u_j \frac{\partial u'_i}{\partial x_j} - u'_j \frac{\partial u'_i}{\partial x_j} \right\}
\]

The subscript 0 designates a value of the mean flowfield and the subscript t a value of the turbulent field. $u$ and $p'$ are the acoustic velocity and pressure. The left hand side of (7) is the system of the linearized Euler's equations around a stationary mean flow $(U_0, p_0, \rho_0)$. The right hand side of this system is the acoustic source term, which is nonlinear in velocity fluctuations. However, the time average of the source term is zero. The previous system (7) is derived under the following assumptions. Acoustic pressure fluctuations are isentropic, the turbulent velocity is incompressible, and only the first order interaction between the mean flow and the acoustic field is retained. In other words, phenomena such as scattering of sound by turbulence are assumed to be negligible.
Finally, to compute the sound field, one carries out the three following steps:

(i) An aerodynamic calculation of the mean flow is performed by solving the averaged Navier-Stokes equations with a $k - \epsilon$ turbulence closure.

(ii) A space-time stochastic turbulent velocity field is generated as a sum of random Fourier modes.

(iii) The propagation system (7) is solved. In the left hand side, one uses values of the mean flow-field calculated in the first step as coefficients of the two first-order differential equations. On the right hand side, the acoustic source term $S$ is calculated from the synthetized turbulent field.

\[
\epsilon = 2\nu \int_0^\infty k^2 E(k, t)dk
\]

where $k$ is the kinetic energy per mass unit and $\epsilon$ the rate of dissipation, that is to say the rate of transfer of kinetic energy per mass unit and per time unit. These two local values of the turbulent field may be used to estimate the integral length scale $L$:

\[
L = \int_0^\infty f(\tau) d\tau \simeq \frac{k^{3/2}}{\epsilon}
\]

A characteristic angular frequency for the turbulence is given for each mode by the relation:

\[
\omega_n = \epsilon^{1/3} k_n^{2/3}
\]  

**Application to the case of a duct obstructed by a diaphragm**

The SNGR model was first applied to cases of axisymmetric subsonic jets.\textsuperscript{2,4,6} The configuration of the 2D duct obstructed by a diaphragm (see figure 1) has already been studied\textsuperscript{27} and we here use the same aerodynamic results.

![Figure 1: Sketch of the geometry.](image)

**Aerodynamic results**

The mean flowfield is calculated as a numerical solution of the average Navier-Stokes equations associated with a $k - \epsilon$ turbulence closure. These calculations are carried out with the ESTET code developed by the "Laboratoire National Hydraulique" of the "Direction des Etudes et Recherches d'Electricité de France" and are performed on a grid of $190 \times 83$ points. The mesh is finer close to the diaphragm. The smallest mesh size is $1.0 \times 10^{-3}$ m and the largest is $5.0 \times 10^{-3}$ m. The computational domain is displayed in figure 2 and a view of the grid close to the diaphragm is shown in figure 3. It is not necessary to discretize a large part of the duct upstream of the diaphragm. The two main parameters for these calculations are the aperture $e$ of the diaphragm and the mean flow velocity in the duct $U_0$. The configurations studied are summarised in table 1.
Figure 2: Global view of the computational domain used in the ESTET calculations.

Figure 3: Partial view of the grid used in the ESTET calculations.

Figure 4 shows fields of turbulent kinetic energy for three configurations. For the smallest aperture \((e = 15 \text{ mm})\), the flows are always non-symmetrical. For the largest aperture \((e = 55 \text{ mm})\), the flows are always symmetrical. For the aperture of 35 mm, the flow is symmetrical up to 14 m/s.

**Acoustic results**

The acoustic propagation computations are carried out with the EOLE code developed by the "Acoustique et Mécanique Vibratoire" Department of the "Direction des Etudes et Recherches d'Electricité de France". The EOLE code solves the Euler linearized Equations using a fractional step scheme and relies on solutions of one-dimensional propagation problems in terms of a weak formulation. Numerical tests indicate that sound wave propagation is calculated with little dispersion and dissipation. Some applications to the propagation in hot jets show that the effects of convection and refraction are retrieved in the predicted sound field.

The acoustic calculations are performed on a grid of \(601 \times 32\) points. The mesh size is constant \((3.0 \times 10^{-3} \text{ m})\). The acoustic computational domain is displayed on figure 5 and a view of the grid close to the diaphragm is presented on figure 3.

Figure 5: Global view of the computational domain used in the EOLE calculations.

Aerodynamic and acoustic computations require different grids and mesh sizes because:

- local mesh refinements on the aerodynamic grid are needed because of the viscous effects
- larger upstream and downstream discretized domains are needed by the acoustic calculation in order to capture the farfield
- the accuracy of the acoustic computation requires a regular grid.

<table>
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<th>(U_0) (m/s)</th>
<th>14</th>
<th>23</th>
<th>32</th>
<th>55</th>
<th>75</th>
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<tr>
<td>(e = 15) mm</td>
<td>☒</td>
<td>☒</td>
<td>☒</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(e = 35) mm</td>
<td>☒</td>
<td>☒</td>
<td>☒</td>
<td>☒</td>
<td>-</td>
</tr>
<tr>
<td>(e = 55) mm</td>
<td>-</td>
<td>-</td>
<td>☒</td>
<td>☒</td>
<td>☒</td>
</tr>
</tbody>
</table>

Table 1: Configurations analysed in this study.
• the accuracy of the acoustic computation requires a regular grid.

The isentropic Euler linearized equations bear acoustic fluctuations which are propagative ones but also vorticity fluctuations which are convective ones. It is known that in a mean sheared flow these two modes of fluctuations are coupled. But, even in an uniform flow, vorticity fluctuations will appear if the source term is not irrotational.

In the duct problem, the radiated acoustic power is obtained by recording the pressure fluctuations at both ends of the computation domain. Thus, in the general case, acoustic fluctuations but also vorticity fluctuations would be recorded at the downstream boundary of the domain and the calculation of the radiated acoustic power would be difficult because vorticity fluctuations are much stronger than acoustic fluctuations.

It follows that three kinds of acoustic propagation calculations may be carried out:

• without taking into account the influence of the mean flow in order to avoid the development of the vorticity mode

• taking into account the influence of the mean flow but neglecting in the system of the Euler linearized equations the coupling terms between the two modes. These terms are identified as those containing mean velocity gradients. 19

• solving the full system of the linearized Euler equations.

Results obtained with the first and second methods of calculation are presented in this paper.

Without mean flow

Effects of the mean flow on the propagation are not taken into account but in the low subsonic range we may expect that their influence will not be essential.

Figures 7, 8, 9 present the acoustic power radiated with respect to the mean velocity in the duct. The experimental results obtained in a previous study 27 and the expected $U^4$ law typical of the acoustic radiation in such confined configurations are also plotted.

Figure 10 displays the intensity spectrum for one configuration (e = 35 mm and $U_0 = 14$ m/s). Here again the result given by the SNGR model is close to the experimental one and the evolution of the computed levels with respect to the frequency is well retrieved.

With mean flow

In these calculations, the vorticity mode can develop. The simplest way to separate acoustic and vorticity fluctuations is to use their difference in speed of propagation. The computational domain is chosen long enough to be able to record acoustic fluctuations before vorticity fluctuations arrive. Table 2 compares the acoustic results for three configurations with and without mean flow. The differences are quite small.
Figure 7: Acoustic power radiated by the diaphragm $e = 15$ mm with respect to the mean flow velocity.

Figure 8: Acoustic power radiated by the diaphragm $e = 35$ mm with respect to the mean flow velocity.

Figure 9: Acoustic power radiated by the diaphragm $e = 55$ mm with respect to the mean flow velocity.

Figure 10: Comparison of acoustic intensity results for the configuration $e = 35$ mm and $U_0 = 14$ m/s

<table>
<thead>
<tr>
<th>Configuration aperture (mm)</th>
<th>acoustic power (dB) without flow</th>
<th>acoustic power (dB) with flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e = 15/U_0 = 23$</td>
<td>107.0</td>
<td>105.3</td>
</tr>
<tr>
<td>$e = 35/U_0 = 32$</td>
<td>104.6</td>
<td>102.8</td>
</tr>
<tr>
<td>$e = 55/U_0 = 55$</td>
<td>102.8</td>
<td>102.9</td>
</tr>
</tbody>
</table>

Table 2: Comparison of acoustic power results obtained with and without mean flow.
Conclusion

The SNGR model was used in a previous study to calculate acoustic radiation from free jets. It is here applied to estimate acoustic radiation in a confined configuration. It is shown that the SNGR model provides useful results with good agreement with experimental data.

The interaction of the mean flow with the acoustic radiation is not clearly highlighted. Numerical and physical problems have to be assessed in order to improve the interpretation of the results given by the SNGR model.

References


Figure 4: Comparison between turbulent kinetic energy for three configurations: (a) $e = 15 \text{ mm et } U_0 = 14 \text{ m/s}$, (b) $e = 35 \text{ mm et } U_0 = 32 \text{ m/s}$, (c) $e = 55 \text{ mm et } U_0 = 55 \text{ m/s}$. 