Multi-cell digital feedback control for noise reduction through hybrid absorbers

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Abstract

This paper deals with the control aspect of a new type of hybrid acoustic absorbent developed in the LMFA Center for Acoustics. This hybrid cell aims to achieve large noise reduction over a wide frequency range by combining passive and active control. Active control is performed by using an adaptive digital feedback controller with the FXLMS algorithm applied to the IMC architecture. For extended liner surfaces, that is to say for MIMO structures, the main conclusion of the theoretical study is that acoustic coupling occurring between the hybrid cells can produce a global unstable behavior even if the optimal filter is obtained in each cell. An adaptive bandpass filter is therefore used to prevent the development of instabilities. This multi-cell algorithm has been experimentally validated for a 4-cell system located on a duct wall in the presence of flow.

1. Introduction

Noise reduction often implies the realization of an optimal impedance over a wide frequency range. To this end, the association of passive and active methods appears to be particularly effective since passive structures are completely suitable to reduce high frequency noise whereas active noise control (ANC) technologies seem to be the only way to minimize low frequency components. The LMFA Center for Acoustics of the Ecole Centrale de Lyon has developed a new concept of hybrid acoustic liner based on this principle. The objective is to minimize the pressure at the rear face of a well-suited porous layer by means of active noise control for the purpose of reaching a targeted impedance. This impedance is theoretically predetermined to produce the best noise reduction when applied to a flow duct. Using a highly resistive material as the porous layer and a piezo-electric transducer as the secondary source, a thin hybrid liner called the hybrid cell is achieved (see figure 1). The resistive layer optimization will not be considered here but is developed in another related paper. Instead, this paper deals with the control aspect of this hybrid cell.

Figure 1: Sketch of an hybrid cell.

Experiments were previously carried out using a four-cell system outfitted with a digital feedforward controller. Very important noise reductions were obtained in the frequency range of interest (700-2100 Hz). Henceforth, since the absorbent surface has to be broadened for industrial applications, it is necessary to consider hybrid cells in very large numbers. In fact, with a MIMO (multiple-input multiple-output) ANC feedforward system, the larger the secondary sources and the error sensors, the higher the memory and calculation costs. Moreover, in the targeted final applications, e.g. turbojet inlets covering, the upstream reference noise input is not highly coherent with the sound to be canceled estimated as an evolving narrowband noise. Thus, we require a system which provides an adaptive control, where the hybrid cells are self-sufficient and are able to achieve real-time noise cancelation over large ac-
This paper presents a novel algorithm which takes into account the aforementioned necessities. The paper is organized as follows. In Section 2, the extension of the single-channel problem to that of the multi-channel is developed. Section 3 gives the analysis of the proposed algorithm whose experimental validation is submitted in Section 4. Section 5 concludes the paper.

2. From the single-channel to the multi-channel ANC algorithm

2.1 Description of the single-channel problem

Considering the given restrictions, a narrowband noise adaptive feedback ANC system is to be examined. Consequently, the algorithm used for the single-channel system is the application of the normalized Filtered-X Least Mean Squares (FXLMS) algorithm to the Internal Model Control (IMC). Assuming that the off-line estimation of the secondary path $S(z)$ with a FIR filter $\hat{S}(z)$ is accurate, this technique can be viewed as an adaptive feedforward system that, in effect, uses the error sensor’s signal $e(n)$ and the adaptive control filter’s output $y(n)$ to synthesize its own reference estimation of the primary noise $d(n)$ (see figure 2 for the IMC-FXLMS architecture).

Indeed, for this feedback ANC system, the error signal can be expressed as

$$E(z) = D(z) + S(z)Y(z) = \frac{D(z)[1 + \hat{S}(z)W(z)]}{1 + [\hat{S}(z) - S(z)]W(z)}.$$  

(1)

Thus, for $S(z) \simeq \hat{S}(z)$, we have $x(n) \simeq d(n)$. The overall transfer function $H(z)$ from $d(n)$ to $e(n)$ becomes characteristic of a feedforward ANC system since rearranged as

$$H(z) = \frac{E(z)}{D(z)} \simeq 1 + S(z)W(z).$$  

(2)

2.2 Analysis

The secondary signal $y(n)$ is generated as follows:

$$y(n) = \sum_{l=0}^{L-1} w_l(n)x(n-l),$$  

(3)

where $(w_l(n))_{l=0...L-1}$ are the coefficients of the adaptive FIR control filter $W(z)$ at time $n$, and $L$ is the filter order.

2.3 Analysis of the proposed algorithm

The performance of the IMC - FXLMS algorithm depends on the predictability of the primary noise. Indeed, the optimal control filter $W_{opt}(z)$ obtained for a zero error has a phase that increases with the frequency

$$W_{opt}(z) = \frac{1}{S(z)}.$$  

(9)

For instance, figure 4 represents the optimal filter obtained for our piezo-electric transducer technology, i.e. for a resonant actuator. The sampling frequency is set to 10000 Hz.

Thus, as shown on figure 3, the control of tonal noise is efficient. We simulate here a single sine wave...
at 1000 Hz controlled with an hybrid cell located on the wall of a duct and without flow. The spectra at the error sensor of the primary noise and the error signal are plotted. The 1000-Hz tone totally disappears after 500 iterations.

Figure 3: ANC simulation for a single sine wave as primary noise in a duct.

These results are achieved with a second order FIR control filter represented on figure 4. As we can see, the optimal control filter is well realized at 1000 Hz.

Figure 4: Ideal and calculated control filter.

For MIMO structures, acoustics interactions between hybrid cells appear, due to reflections towards the wall of a duct for example. We will now see that the crossed secondary paths, as well as the direct secondary paths, have to be taken into account for ANC applications.

2.3 Extension to the multi-channel case

Most of the related studies present a multi-channel architecture that accurately estimates the primary noise by removing all the contributions, direct as well as crossed, from the measured error signal. This primary noise synthesis is used as a reference signal for the control filter. The result is that strong attenuations are obtained but real-time applications are ineffective as the calculation complexity increases with the number of references.

Nevertheless, if the crossed secondary paths are ignored in the algorithm, that is to say if the hybrid cells are made independent, acoustic coupling occurring between those cells can produce a global unstable behavior that leads to strong instabilities for certain frequencies of the primary noise (see figure 7 for instabilities triggered by a 1600-Hz tone). Moreover, those negative effects occur even if the optimal filter is realized in each cell. Indeed, as suggested in equation 9, the optimal filters depend only on their own direct secondary path.

Let us take a simplified case with the two-cell system presented in figure 5 in order to clarify these instability phenomena.

Simple estimations of the primary noise at each error sensor give

\[
\begin{align*}
X_1(z) &= E_1(z) - W_1(z)S_{11}(z)X_1(z) \\
X_2(z) &= E_2(z) - W_2(z)S_{22}(z)X_2(z)
\end{align*}
\] (10)

Here the measured error signal is

\[
\begin{align*}
E_1(z) &= D_1(z) + \sum_{i=1}^{2} W_i(z)S_{i1}(z)X_i(z) \\
E_2(z) &= D_2(z) + \sum_{i=1}^{2} W_i(z)S_{i2}(z)X_i(z)
\end{align*}
\] (11)

so, equation 10 can be recast as

\[
\begin{align*}
X_1(z) &= D_1(z) + W_2(z)S_{21}(z)X_2(z) \\
X_2(z) &= D_2(z) + W_1(z)S_{12}(z)X_1(z)
\end{align*}
\] (12)
and finally, in matrix notation and after \( n \) iterations, the reference signal can be expressed as follows

\[
X = D + M^{n+1} X
\]

where

\[
M = \begin{pmatrix}
0 & W_2 S_{21} \\
W_1 S_{12} & 0
\end{pmatrix}
\]

(14)
is the leaky matrix.

Thus, the control stability depends on the eigenvalues of \( M \) and so, intrinsically on the crossed secondary paths. The stability condition is that the eigenvalues \( \lambda_i \) \( i = 1 \ldots N \) are strictly less than unity. That leads to

\[
\lambda^2 = (W_1 S_{12}) (W_2 S_{21}) < 1
\]

(15)

In order to calculate these eigenvalues, we need the crossed secondary paths \( S_{12} \) and \( S_{21} \) and the control filters \( W_1(z) \) and \( W_2(z) \). To this end, we use the crossed secondary paths models used in the simulations and resulting from real FRF (Frequency Response Function) measurements with a two-cell system placed on the wall of a duct without flow. We have seen that the optimal filters only depend on the direct secondary paths. Thus, if we guarantee that these filters are realized in each cell, then ideal noise reduction is necessarily achieved. The stability condition is fulfilled for the SISO case (see figure 4). Thus, the optimal filters are recovered by simulating a MIMO system with zero crossed secondary paths.

Let us illustrate this point with a 1600-Hz single sine wave as primary noise. The eigenvalues are plotted on figure 6.

![Figure 6: The eigenvalues of \( M \).](image)

Figure 6: The eigenvalues of \( M \).

The stability criterion is not fulfilled. Consequently, instability frequencies can be created. Here the main instability frequency is at 1100 Hz. Since the optimal filters are known for each cell, there is no need to work with the algorithm to simulate the crossed case, that is to say we can simulate the control with \( \mu(n) = 0 \). Indeed, if the control is not realized now, it will never be better. Figure 7 illustrates these instabilities for the given ANC simulation.

![Figure 7: ANC simulation for a single sine wave as primary noise: the instabilities.](image)

Figure 7: ANC simulation for a single sine wave as primary noise: the instabilities.

This criterion is a clue to the stability issue in the simulations, but in the real-time applications developed in section 4, the crossed secondary paths are not supposed to be known. And we can see from the theoretical study that their influence is harmful for some frequencies. A robust solution must be found in order to make the control operational over the given wide frequency range.

3. The “IMC-MDFXLMS” algorithm

Practically, if we succeed in forcing each cell’s calculated control filters to be lower in amplitude than the corresponding inversed crossed secondary paths, i.e.

\[
\begin{cases}
W_1(z) < 1/S_{12}(z) \\
W_2(z) < 1/S_{21}(z)
\end{cases}
\]

(16)

then the stability condition (see equation 15) will be fulfilled.

Figure 8 represents the case where these equations are not verified (see subsection 2.3).

The first idea is to add zeros to the FIR control filters by increasing their order. Thus, for two filters with 20 taps, the stability condition is reached as shown on figure 9.

![Figure 8: The case where these equations are not verified](image)

Nevertheless, the real-time application problem re-emerges as the number of cells increases since
The basic principle of this bandpass filter rests on the adaptive notch filter (ANF) algorithm\(^6\). ANF are used to eliminate narrowband or sine wave components with unknown frequencies from observed time series. Let us consider the case of a single sine wave expressed as
\[
u(t) = \alpha \sin(w_0 t + \phi) + v(t), \quad \forall t
\]
where \(w_0 \in (0, \pi); \alpha, \phi \in \mathbb{R}\) and \(v(t)\) is a zero-mean unit-variance white Gaussian noise.

If we define
\[
A_0(z) = 1 + a_0 z + z^2, \quad a_0 = -2 \cos w_0
\]
where \(z\) denotes the unit operator, some simple trigonometry shows that equation 17 obeys the following autoregressive moving average (ARMA) process
\[
A_0(z) u(t) = A_0(z) v(t)
\]
where \(z^{-2} A_0(z)\) has its zeros located on the unit circle at \(e^{\pm i w_0}\). Thus the sine wave can be filtered out from \(u(t)\) if we define, for \(\rho \in (0, 1),
\[
\epsilon(t, a_0) = \frac{A_0(z)}{A_0(\rho z)} u(t).
\]

The aforementioned filtering is stable since \(z^{-2} A_0(\rho z)\) has its zeros located at \(\rho e^{\pm i w_0}\). For \(\rho\) close to one, \(\epsilon(t, a_0) \approx v(t)\) and the broadband component of \(u(t)\) is slightly distorted. Thus, this filter is called a notch filter since its FRF has a gain approximately equal to one at all frequencies except at the sine wave frequency where its gain is zero. The closer \(\rho\) is to one, the narrower the width of the notch.
In our application, the purpose is to extract a sinusoidal component at a known frequency from the reference signal; this algorithm is called the adaptive line enhancement (ALE) algorithm\(^6\). It can easily be done with the above ANF. Indeed, the bandpass filter is given by

\[ H(z) = 1 - \frac{A_0(z)}{A_0(\rho z)}. \] (21)

Figure 11 represents the ALE filter \( H(z) \) at 1000 Hz and where \( \rho = 0.99 \).

![Figure 11: The adaptive line enhancement filter.](image)

It is to be noted that both ANF and ALE are implemented using IIR filters which are computationally more efficient than their FIR equivalents. Moreover, this ANF technology can be applied to time-varying frequency tracking when combined with a recursive prediction error (RPE) algorithm: the ANF determines the time-varying frequency which is used in the ALE to filter the reference signal.

### 3.2 Simulations results

With the aforementioned ALE filters, the control stability is reached with second-order control filters, as we can see on figure 12.

Another advantage of this filtering is that we can now consider broadband noise. In fact, the proposed algorithm cannot intrinsically support this kind of noise due to its non-predictable aspect. Indeed, if we inject additional broadband noise in the SISO configuration, we will observe the divergence of the control unless the control filters have a very large number of taps (and in this case, the control filter is close to a bandpass filter). So, these ALE filters allow the minimization of the flow effect (which is similar to broadband noise) and consequently the noise in which we are interested (fan noise) can be controlled. Figure 13 shows the results of the simulation of a sine wave in noise (\( SNR = 15 \text{ dB} \)).

![Figure 12: Results of the simulation led thanks to the IMC MDFXLMS algorithm.](image)

![Figure 13: Sinusoid in noise.](image)

In order to verify experimentally this theoretical study, a four-cell system located on the wall of a duct was tested. Section 4 gives further informations.

### 4. Experimental validation

#### 4.1 Presentation of the laboratory experimental setup

To verify the reliability of the whole concept, flow duct experiments have to be achieved. However, the complexity of the nacelle geometry as well as the engine inlet air flow lead to consider in a first step a laboratory setup to define the optimization process and test our IMC Mimo Diagonalized FXLMS algorithm. Here, the Matisse test bench consists of a square cross-section duct with an anechoic termination to achieve insertion loss (IL) and transmission loss (TL) measurements (see figure 14).

The 66 mm × 66 mm square cross-section implies a large plane wave analysis domain until about
2500 Hz. The primary acoustic source is placed on the upper wall of the duct downstream from the silent flow generation system. So, the perturbation spectrum is composed of a pure sine signal (in the frequency range [500-2500] Hz) generated by the primary source and a broadband noise produced by the flow. Five microphones are placed at the duct wall (see figure 15) in order to deduce all the acoustics parameters necessary to evaluate the efficiency of the hybrid treatment.

The test section situated on the upper wall of the duct downstream the source consists in a 220 mm long four-cell hybrid liner prototype (see figure 16). This hybrid absorber is the result of complementary optimizations of both the cell’s active (actuator and control system) and passive (porous layer characteristics) parts. Conventional passive measurements (280 mm long) are also carried out.

4.2 Experimental results

We present here the results obtained with the IMC Mimo Diagonalized FXLMS algorithm. The experiments were carried out with three configurations: two different wiremeshes and a combination of the two. For each configuration, different flow velocities up to 50 m/s were tested in order to examine the adaptability of the algorithm with respect to tonal noise under broadband perturbed conditions. The best results were obtained with the wire mesh (WM2) which has the lowest resistance (approximately 125 rayls). This configuration is theoretically well adapted for low frequencies. Thus, important noise reductions have been achieved above 700 Hz and until 1700, in particular for low frequencies (TL up to 26 dB at 700 Hz, see figure 17).
Good attenuations in low frequencies are also observed with insertion loss results which follow the same frequency evolution as the transmission loss, see figures 18 and 19.

The new active control algorithm seems to work well with the 4 cells, even with flow up to 50 m/s in the duct: the transmission loss reaches 15 dB at low frequencies. At higher frequencies, the control efficiency is reduced most probably due to bad control loop parameters and low transducer efficiency.

To see control efficiency, measurements were carried out in the cells. Up to 55 dB noise reduction is obtained at the rear face of the porous material in the third cell, see for instance figure 20. Some harmonics can be amplified but this is limited by the reconstruction filters. In accordance with the theoretical study, experiments led with broadband noise generated by the flow show that the algorithm only reduces the pure sine primary signal without adding any harmonic perturbations, see figure 21: 36 dB noise reduction in the first cell. Control performance naturally decreases with flow velocity but works fairly well thanks to the adaptability of our algorithm (see figure 22, 25 dB noise reduction in the fourth cell).

The control algorithm behaves as expected and a rapid and high noise reduction is achieved with and without flow. The mixed layer seems to be the most sound-absorbing solution to treat separately low and high frequencies. The porous layer choice is also an important step in the realization of an appropriate adaptive liner for a given configuration. Other measurements especially with passive absorbers are available in the related paper. Comparisons with classical passive treatments such as honeycomb and active configurations are presented to see the most frequency range efficiency.
5. Conclusions

The control of some narrowband noise fields over extended regions of space requires the development of an adequate multi-channel active noise control system able to create a spatial zone of noise cancelation. In order to create this quiet zone, a new concept of hybrid absorbent combining passive absorbent properties of a porous layer and active noise control was developed in the LMFA Center for Acoustics at the Ecole Centrale de Lyon. Concerning the control aspect of these hybrid cells, the primary noise signal detection being not sufficiently coherent, an adaptive multiple-input/multiple-output feedback controller appears to be more appropriate. With real-time applications in mind, a novel algorithm has been imagined. It uses an adaptive line enhancement filter combined with the IMC FXLMS structure in order to prevent the acoustics coupling occurring between cells. Thus, each cell is independent from the others and tonal noise can therefore be controlled with a minimum number of taps for the control filter. Moreover, thanks to these filters, adding broadband noise such as flow will not disturb the active noise control. Experiments in Matisse Facility showed the adaptability and efficiency of the proposed algorithm for different flow velocities and different wire meshes. Further experiments will be carried out with more hybrid cells (27 then 54) in order to validate the concept over a large absorbing surface and so to demonstrate industrial capability.

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References


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