RADIATION AND REFRACTION OF SOUND WAVES THROUGH A TWO-DIMENSIONAL SHEAR LAYER

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INTRODUCTION

As a starting point, we consider the linearized Euler equations which govern sound propagation in a steady mean flow $(\bar{\rho}, \bar{\mathbf{u}}, \bar{p})$ where ρ, $\mathbf{u} = (u, v)$ and p denote the density, the velocity and the pressure respectively. Using Cartesian coordinates $\mathbf{x} = (x, y)$, these equations read

$$\begin{cases} \dfrac{\partial \rho'}{\partial t} + \nabla.\left(\rho' \bar{\mathbf{u}} + \bar{\rho}\mathbf{u}'\right) = 0 \\[2ex] \dfrac{\partial \left(\bar{\rho}\mathbf{u}'\right)}{\partial t} + \nabla.\left(\bar{\rho}\bar{\mathbf{u}}\mathbf{u}'\right) + \nabla p' + \left(\bar{\rho}\mathbf{u}' + \rho'\bar{\mathbf{u}}\right).\nabla\bar{\mathbf{u}} = 0 \\[2ex] \dfrac{\partial p'}{\partial t} + \nabla.\left[p'\bar{\mathbf{u}} + \gamma\bar{p}\mathbf{u}'\right] + (\gamma - 1)\left[p'\nabla.\bar{\mathbf{u}} - \mathbf{u}'.\nabla\bar{p}\right] = \Lambda \end{cases} \tag{1}$$

where γ is the specific heat ratio, r the gas constant, T the temperature, for a perfect gas $p = \rho r T$, and $f = \bar{f} + f'$ for any variable f. In the present work, these equations are solved in the following form

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{E}}{\partial x} + \frac{\partial \mathbf{F}}{\partial y} + \mathbf{H} = \mathbf{S} \tag{2}$$

where the vectors $\mathbf{U}$, $\mathbf{E}$, $\mathbf{F}$ and $\mathbf{H}$ are given by

$$\mathbf{U} = \begin{pmatrix} \rho' \\ \bar{\rho}u' \\ \bar{\rho}v' \\ p' \end{pmatrix} \quad \mathbf{E} = \begin{pmatrix} \rho'\bar{u} + \bar{\rho}u' \\ \bar{u}\bar{\rho}u' + p' \\ \bar{u}\bar{\rho}v' \\ \bar{u}p' + \gamma\bar{p}u' \end{pmatrix} \quad \mathbf{F} = \begin{pmatrix} \bar{\rho}v' \\ 0 \\ p' \\ \gamma\bar{p}v' \end{pmatrix} \quad \mathbf{H} = \begin{pmatrix} 0 \\ \bar{\rho}v'\dfrac{d\bar{u}}{dy} \\ 0 \\ 0 \end{pmatrix}$$

for a parallel base flow $\bar{u}(y)$, $\bar{\rho}(y)$, $\bar{v} = 0$ and $\bar{p} = p_\infty =$ constant. A Gaussian function is taken for the unidirectional sheared mean flow

$$\frac{\bar{u}}{u_j} = \exp\left(-\log(2)\left(\frac{y}{b}\right)^2\right) \tag{3}$$

and the density profile is deduced from the Crocco-Busemann relation

$$\frac{\rho_j}{\bar{\rho}} = \frac{T_\infty}{T_j} - \left(\frac{T_\infty}{T_j} - 1\right)\frac{\bar{u}}{u_j} + \frac{\gamma - 1}{2}\mathrm{M}_j^2\,\frac{\bar{u}}{u_j}\left(1 - \frac{\bar{u}}{u_j}\right)$$

where the variables with subscript j are defined at the jet axis. The time-harmonic acoustic source term $\mathbf{S}^T = (0, 0, 0, \Lambda)$ is given by $\Lambda = S(x, y)\cos(\omega_0 t)$ with $S = Ae^{-\alpha_x x^2 - \alpha_y y^2}$. For the numerical resolution, the equations are made dimensionless by using:

$$\tilde{\mathbf{x}} = \frac{\mathbf{x}}{b} \qquad \tilde{u} = \frac{u}{u_j} \qquad \tilde{\rho} = \frac{\rho}{\rho_j} \qquad \tilde{t} = \frac{t}{b/u_j} \qquad \tilde{p} = \frac{p}{\rho_j u_j^2}$$

The parameters of the problem are $M_j = 0.756$, $T_j = 600$ K, $T_\infty = 300$ K, $\gamma = 1.4$, $r = 287$ m^2.s^{-2}.K^{-1}, $b = 1.3$, $\alpha_x = 0.04 \log(2)$ m^{-2}, $\alpha_y = 0.32 \log(2)$ m^{-2}, $A = 10^{-3}$ kg.m^{-1}.s^{-3} and $\omega_0 = 76$ rad.s^{-1}. The mean pressure is $\bar{p} = 103330$ kg.m^{-1}.s^{-2}. A Strouhal number based on the jet exit velocity and an estimation of the jet diameter $2b$ can be defined as St= $f_0\, 2b/u_j = \tilde{\omega}_0/\pi \simeq 0.085$ where $\tilde{\omega}_0 = \omega_0 b/u_j$.

In the present simulations a dimensionless amplitude $\tilde{A}_0 = 10^{-3}$ has been used rather than $\tilde{A} = A \times b/(\rho_j u_j^2)$. As a consequence dimensional pressure of the test case is obtained by multiplying $\tilde{p}$ by a factor $b/u_j \simeq 3.5 \times 10^{-3}$ since the problem is linear.

SIMPLIFIED LINEARIZED EULER'S EQUATIONS

The linearized Euler equations (1) govern the propagation of small acoustical disturbances through a steady mean flow in taking into account all the mean flow effects such as refraction and convection of the sound by the mean flow. However, spatially growing instability waves are also solutions of these equations. The fluctuating velocity and density can be eliminated by differentiating (1) for a parallel mean flow and the resulting equation for the pressure reads

$$\bar{D}^3 p' - \bar{c}^2 \left(\bar{D} \nabla^2 p' - \frac{1}{\bar{\rho}} \frac{d\bar{\rho}}{dy} \bar{D} \frac{\partial p'}{\partial y} - 2 \frac{d\bar{u}}{dy} \frac{\partial^2 p'}{\partial x \partial y} \right) = \bar{D}^2 \Lambda \tag{4}$$

where $\bar{D} = \partial/\partial t + \bar{u}\partial/\partial x$ is the convective derivative. Equation (4) can be seen as a wave equation, but represents also the generalization of the Rayleigh equation for a compressible perturbation. Figure 1 shows the spatial growth rate and the phase velocity as a function of the frequency of the Kelvin-Helmholtz instability wave obtained by seeking solutions of the homogeneous equation in the form $p'(\mathbf{x}, t) = \hat{p}(y) \exp(ikx - \omega t)$ where $k = k_r + ik_i$ is the complex wave number. The wave number corresponding to the angular frequency $\tilde{\omega}_0$ of the acoustic forcing is found to be $\tilde{k}_H \simeq 0.53883 - i0.04906$.
The objective of the present problem is to compute only the acoustic part of the solution, and not the associated instability wave. The authors have developed an acoustic analogy based on the Linearized Euler Equation (LEE) with an *ad hoc* source term reducing to Lilley's analogy for a parallel mean flow.[3] In order to prevent the development of instability waves which can overwhelm the radiated acoustic field and thus prohibit the use of the hybrid approach, a simplified formulation of LEE has been proposed. This simplified formulation is obtained in removing arbitrarily the term $d\bar{u}/dy$ in the vector $\mathbf{H}$. As a result, the equation for the pressure reads now

$$\bar{D}^3 p' - \bar{c}^2 \left(\bar{D} \nabla^2 p' - \frac{1}{\bar{\rho}} \frac{d\bar{\rho}}{dy} \bar{D} \frac{\partial p'}{\partial y} - \frac{d\bar{u}}{dy} \frac{\partial^2 p'}{\partial x \partial y} \right) = \bar{D}^2 \Lambda \tag{5}$$

The only difference between equations (4) and (5) is the factor 2 in the velocity gradient term. The consequences for the description of the acoustic propagation are not easy to describe. Note however that this approximation is not the usual high-frequency approximation of (4), namely the ray-tracing equations, as given by Candel[6] for instance. Note also that this formulation has been successfully applied to the noise generated by an isothermal mixing layer,[3] where vortex pairings produce a relatively low-frequency sound field.

NUMERICAL METHOD

LEE (2) are solved with the Sprint-2d solver developed for aeroacoustic applications.[2,3] The numerical algorithm has been updated with numerical schemes optimized in the wave-number space ensuring accuracy

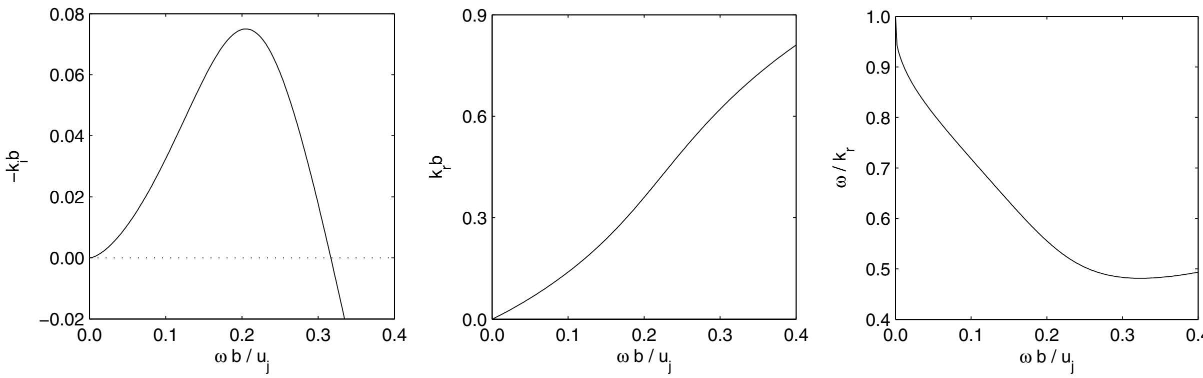

Figure 1: Spatial growth rate $-k_i b$, real wave number $k_r b$ and phase velocity $v_\varphi / u_j = \omega / k_r$ as a function of the frequency $\omega b / u_j$. The compressible Rayleigh equation is solved using a 4th-order Runge-Kutta shooting method and the integration path is deformed in the complex plane to avoid the critical point.

up to five points per wavelength. A 11-point-stencil finite-difference scheme and a 6-step low storage Runge-Kutta algorithm are used for the spatial derivation and time integration.[5] Grid-to-grid oscillations are removing by filtering the unknow vector using a highly selective filter on 11 points. The radiation boundary conditions of Tam & Webb are implemented on the three last points of the computational domain and a small buffer zone is added at the inflow boundary condition to prevent the possible generation of instability waves by the outgoing acoustic field.[4] A symmetry boundary condition is applied along the line $x = 0$.

After preliminary calculations, two grids of different size have been adopted and are presented in the paper. The first one is a regular grid of 601×301 points with $\Delta \tilde{x} = 0.4/b$ and $\Delta \tilde{y} = 0.25/b$. The whole computational domain is $-63 \leq \tilde{x} \leq 175$ and $0 \leq \tilde{y} \leq 74$. The domain of the second grid is similar but the step size in the two directions is divided by 2 yielding a grid of 1201×601 points. Thus there are 10 points in the mean shear flow with the finest grid. The time of the simulation corresponds to 18 periods of the source with a CFL number of 0.9 and the average is calculated at the last period to obtain the root mean square pressure. The computations are performed on a Nec-Sx5 with a CPU time per time iteration and by mesh point of around 3.8×10^{-7} s. Using a Dec α server 1280 GS MARVEL (EV7, 1.15 GHz) the CPU time per time iteration and by mesh point is 6.3×10^{-6} s.

RESULTS AND DISCUSSION

Numerical results obtained with the simplified formulation of LEE associated to wave equation (5) are presented in figures 2 to 5. Figure 2 shows the pressure at the start of a cycle along the two lines $\tilde{y} = 15$ and $\tilde{y} = 50$. No appreciable difference has been observed between the two grids which emphasizes the good accuracy of the optimized schemes. Figure 3 displays the pressure along the line $\tilde{x} = 100$. Time evolution and root mean square pressure profiles are reported in figures 4 and 5.

The simplified wave equation (5) prevents the development of instability waves as observed and discussed in early works.[3] But only an approximate high frequency acoustic field is obtained, not appropriate for the low frequency source of the test case corresponding to a Strouhal number St < 0.1. The pressure field obtained with the simplified formulation of LEE is shown in figure 6. A ray-tracing has been superimposed to show that the wave equation (5) associated to the simplified LEE is not a full high-frequency approximation. As expected the comparison with the analytical solution in figure 7 also displays some significant differences especially in the shadow zone for $\tilde{x} > 50$.

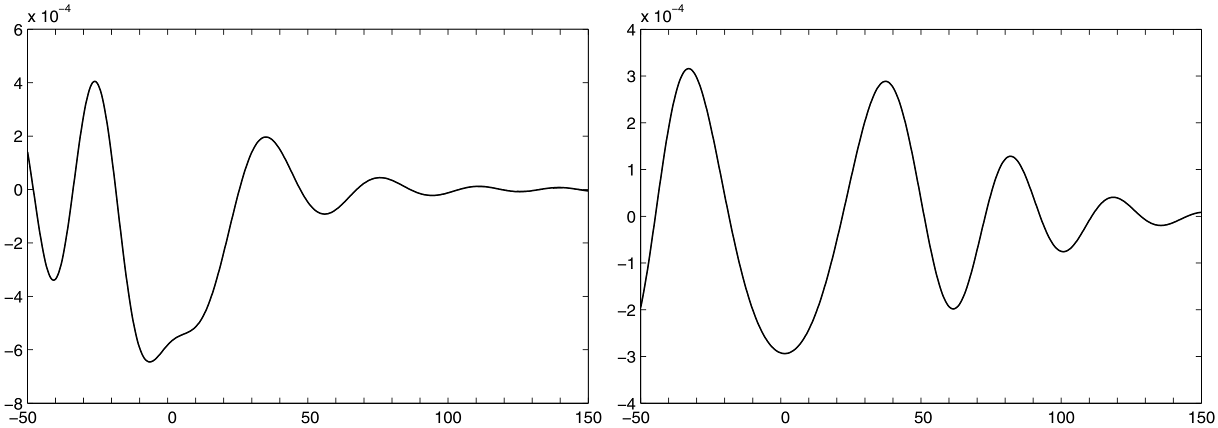

Figure 2: Pressure $\tilde{p}'$ at the start of a cycle along the line $\tilde{y} = 15$ (left) and the line $\tilde{y} = 50$ (right): ——— coarse grid — — — fine grid.

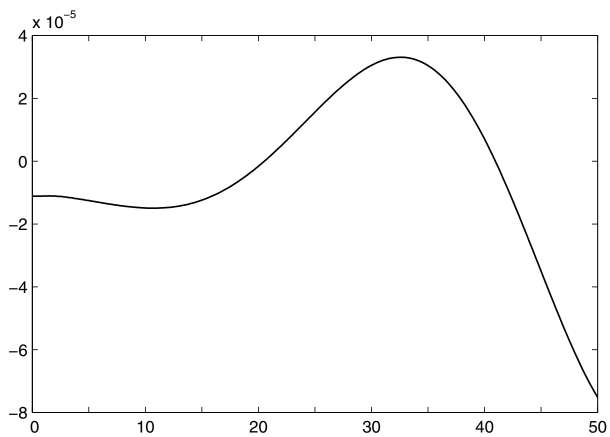

Figure 3: Pressure $\tilde{p}'$ at the start of a cycle along the line $\tilde{x} = 100$.

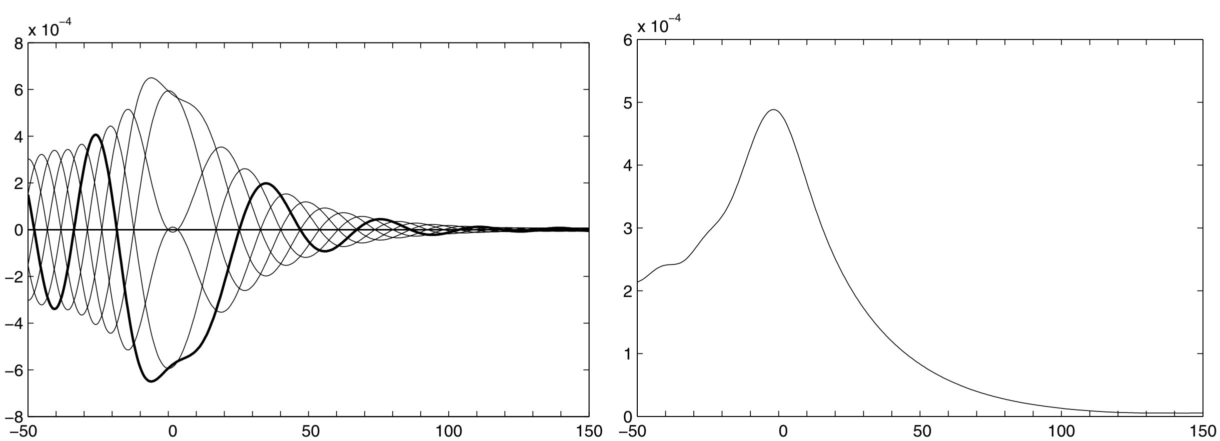

Figure 4: Time history of the pressure $\tilde{p}'$ and root mean square pressure $\tilde{p}_{rms}$ along the line $\tilde{x} = 15$.

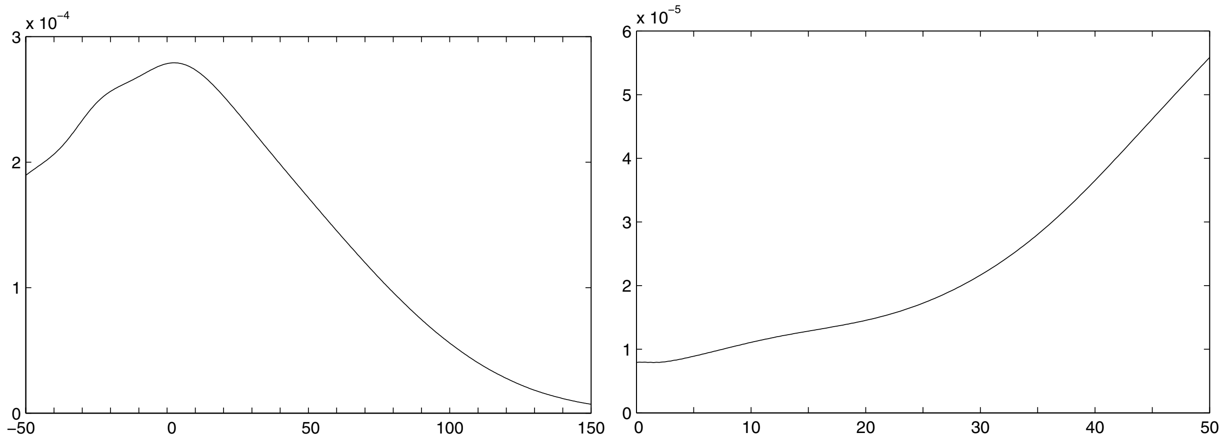

Figure 5: Root mean square pressure $\tilde{p}_{rms}$ along the line $\tilde{y} = 50$ (left) and along the line $\tilde{x} = 100$ (right).

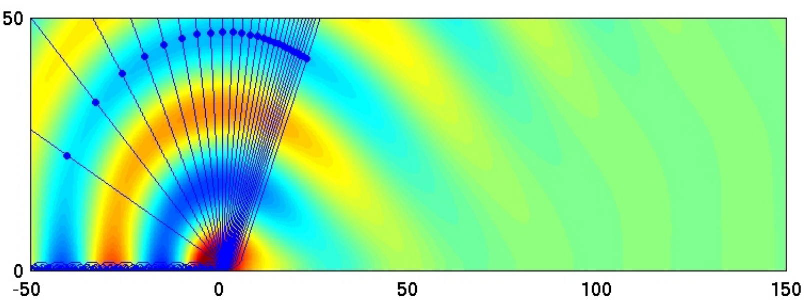

Figure 6: Pressure field $\tilde{p}'$ at the start of a cycle and ray-tracing from the source: 32 rays are plotted from $0 \leq \theta \leq \pi$.

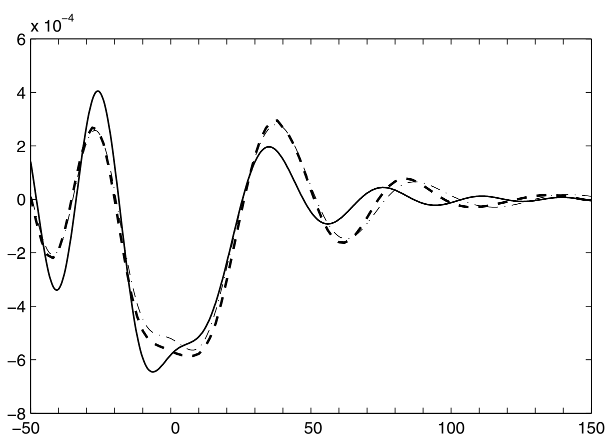

Figure 7: Pressure $\tilde{p}'$ at the start of a cycle along the line $\tilde{y} = 15$: — — — analytical solution,[1] ——— simplified formulation of LEE associated to wave equation (5), — · — · LEE (2) + source term (6).

Finally, Eschricht, Thiele and their coauthors[7] have presented during the workshop a new formulation to remove the instability wave in the time domain. LEE (2) are solved forced by following source term in the right hand side

$$\mathbf{S}^T = \left(0, S_x = \frac{\partial \bar{v}}{\partial x}\, u', S_y = \frac{\partial \bar{u}}{\partial y}\, v', \Lambda\right) \tag{6}$$

with $S_x \equiv 0$ for the workshop test case. The corresponding wave equation writes as

$$\bar{D}^3 p' - \bar{c}^2 \left(\bar{D}\nabla^2 p' - \frac{1}{\bar{\rho}}\frac{d\bar{\rho}}{dy}\bar{D}\frac{\partial p'}{\partial y} - 2\frac{d\bar{u}}{dy}\frac{\partial^2 p'}{\partial x \partial y}\right) = \bar{D}^2\Lambda - \gamma\bar{p}\bar{D}\left(\frac{\partial S_x}{\partial x} + \frac{\partial S_y}{\partial y}\right) + 2\gamma\bar{p}\frac{d\bar{u}}{dy}\frac{\partial S_y}{\partial x} \tag{7}$$

The computed pressure thus obtained is reported in figure 7. The solution compares better with the analytical one than the solution based on the simplified LEE.

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