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On the spectra of nozzle-exit velocity disturbances in initially nominally turbulent, transitional jets

Christophe Bogey,[a] Olivier Marsden,[b] and Christophe Bailly[c]
Laboratoire de Mécanique des Fluides et d'Acoustique, UMR CNRS 5509, Ecole Centrale de Lyon, Université de Lyon, 69134 Ecully Cedex, France

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In a recent paper by C. Bogey, O. Marsden, and C. Bailly ["Large-eddy simulation of the flow and acoustic fields of a Reynolds number 10^5 subsonic jet with tripped exit boundary layers," Phys. Fluids **23**(3), 035104 (2011)], simulation results were presented for round jets with tripped boundary layers, displaying nozzle-exit conditions typical of initially nominally turbulent, transitional jets, namely laminar mean velocity profiles and high fluctuation intensities. The velocity spectra evaluated just downstream of the nozzle exit are re-examined here with respect to literature data. They agree qualitatively very well with spectra obtained in a fully turbulent pipe flow using direct numerical simulation. The wave numbers dominating in the azimuthal direction are also consistent with measurements of spanwise energy distribution in fully turbulent boundary layers. The initial turbulent structures in the jets, therefore, appear to be organized similarly to those in fully developed wall-bounded flows. © *2011 American Institute of Physics*. [doi:10.1063/1.3642642]

The effects of nozzle-exit conditions on jet flows have been investigated by many researchers since the mid-seventies. They have been found to be especially strong for laboratory jets whose diameter-based Reynolds numbers are around 10^5. In these jets at neither low nor high Reynolds numbers, a large variety of intermediate transitional initial states may indeed be encountered between the fully laminar state and the fully turbulent state, depending on the facility and on whether the jet boundary layers are tripped.[1–8] The jets can in particular be initially nominally turbulent, in Zaman's own terms,[5,6] when they exhibit, at the nozzle exit, mean velocity profiles in agreement with the Blasius laminar profile together with peak fluctuation levels around 10 % of the jet core velocity u_j. Such conditions have for instance been observed experimentally by Batt,[1] Hussain and Zedan,[3,4] and Zaman[5,6] in tripped jets characterized by initial momentum-thickness Reynolds numbers $\mathrm{Re}_\theta = u_j\delta_\theta/\nu$ around 400, between 180 and 350, and between 900 and 2250, respectively, where δ_θ is the nozzle-exit boundary-layer momentum thickness and ν is the kinematic molecular viscosity.

As it is now becoming possible to compute laboratory-scale jets thanks to the increase in computer power and to the implementation of highly accurate methods, it appears interesting, as a stepping stone between past simulations of initially laminar jets[9,10] and future simulations of initially fully turbulent jets,[11] to numerically study initially nominally turbulent jets. This was recently done by the authors[12] using large-Eddy simulation (LES) for Mach number 0.9, round jets whose boundary layers are tripped inside a pipe nozzle in order to obtain, at the exit, laminar mean velocity profiles of Reynolds number $\mathrm{Re}_\theta = 900$ and peak axial turbulent intensities $u'_e \simeq 0.09u_j$. Based on the use of different meshes and on analyses of the LES quality, the solutions determined at higher resolutions were shown to be practically grid-converged, numerically accurate as well as physically relevant. Special attention was paid to the axial velocity spectra in the shear layers close to the nozzle exit. Broadband spectra as a function of both frequency and azimuthal wave number were found. Furthermore, dominant components centered around $k_\theta\delta_\theta/r_0 \simeq 0.8$ were noticed in the azimuthal direction regardless of the boundary-layer thickness, where k_θ is the dimensionless azimuthal wave number ($k_\theta = 0$ for the axisymmetric mode, $k_\theta = 1$ for mode 1, etc.) and r_0 is the pipe radius. The origin of such features in transitional round jets was unfortunately not discussed.

In an attempt to clarify this point, the initial velocity spectra obtained in the above-mentioned LES study for two tripped jets with identical exit boundary-layer conditions are re-examined in the present letter. They are in particular compared to reference data available in the literature for fully turbulent boundary layers,[13,14] channel,[15] and pipe[16] flows, which are known to exhibit similar velocity spectra.[16–20] The two jets are those considered in the simulations referred to as Jetring1024drdz and Jetring1024drdz2δ_θ in the original paper,[12] for which satisfactory grid-convergence is achieved. For both jets, the inflow parameters are chosen to specify, at $z = 0$ at the exit of a pipe nozzle of length $2r_0$, laminar mean velocity profiles of Reynolds number $\mathrm{Re}_\theta = 900$, and peak turbulence levels $u'_e \simeq 0.09u_j$. This is performed, in practice, by imposing Blasius velocity profiles of thickness δ at the pipe inlet at $z = -2r_0$ while adding low-amplitude random vortical fluctuations inside the pipe at $z \simeq -r_0$. The boundary-layer thickness is moreover equal to $\delta = 0.15r_0$ in Jetring1024drdz but to $\delta = 0.30r_0$ in Jetring1024drdz2δ_θ, yielding $\delta_\theta = 0.018r_0$ and $\delta_\theta = 0.036r_0$, respectively.

As illustrations, the profiles of mean and rms axial velocities at the pipe exit are presented in Figure 1. The mean profiles correspond to the laminar profiles fixed at the pipe inlet, and the peak fluctuation levels are around 9 % of the jet velocity as intended. The present jets are consequently

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[a] Electronic mail: christophe.bogey@ec-lyon.fr.
[b] Electronic mail: olivier.marsden@ec-lyon.fr.
[c] Electronic mail: christophe.bailly@ec-lyon.fr.

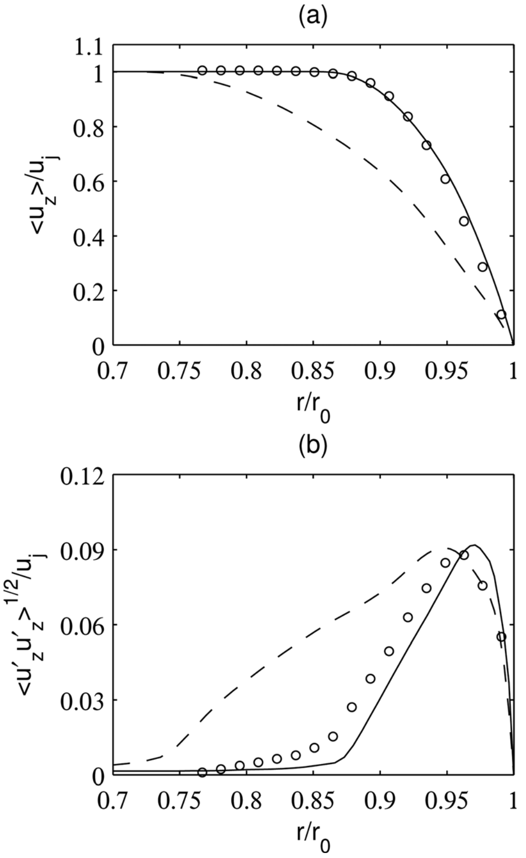

FIG. 1. Nozzle-exit profiles, at $z=0$, (a) of mean axial velocity $\langle u_z \rangle$ and (b) of the rms values of fluctuating axial velocity u_z', for (——) Jetring1024drdz and (– – –) Jetring1024drdz2δ_θ; (○) measurements of Zaman[5,6] for a tripped jet at $\mathrm{Re}_D=10^5$.

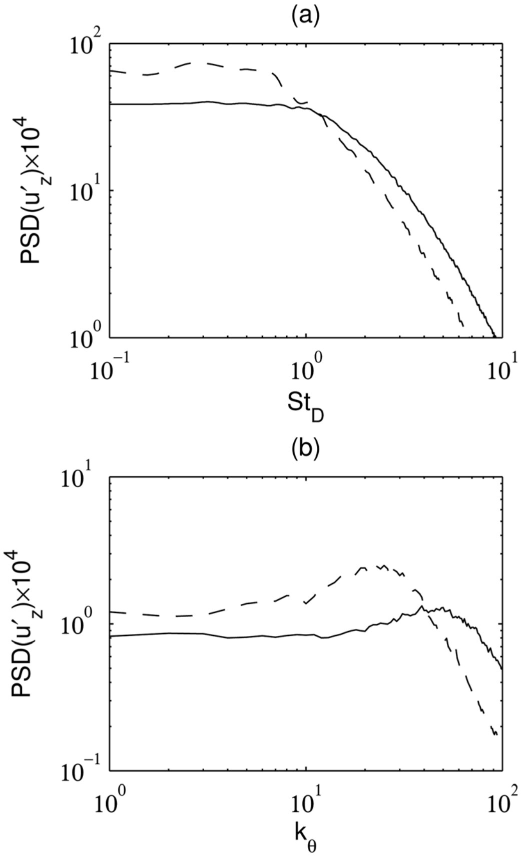

FIG. 2. Power spectral densities (PSD) normalized by u_j of axial fluctuating velocity u_z', as functions (a) of Strouhal number $\mathrm{St}_D=fD/u_j$ and (b) of azimuthal wavenumber k_θ, for (——) Jetring1024drdz at $r=r_0$ and $z=0.4r_0$ and (– – –) Jetring1024drdz2δ_θ at $r=r_0$ and $z=0.8r_0$.

initially disturbed, but not fully turbulent. A relatively good agreement can also be noted between the exit velocity profiles in Jetring1024drdz and measurements[5,6] in a tripped jet at the same Reynolds number.

The jet simulations are detailed in the associated full paper.[12] Their main parameters are nevertheless given below. The LES were carried out using a solver of the 3-D filtered compressible Navier-Stokes equations in cylindrical coordinates (r, θ, z) using low-dissipation and low-dispersion finite differences and Runge-Kutta algorithm.[21,22] The axis singularity is taken into account by the method of Mohseni and Colonius[23] in combination with a specific treatment of azimuthal differentiation to alleviate the time-step limitation near the origin.[24] Spectral-like filters[25] are applied explicitly to the flow variables every time step in order to remove grid-to-grid oscillations, as well as to relax subgrid-scale energy from scales at wave numbers close to the grid cut-off wave number while leaving larger scales mostly unaffected.[12,26,27] The grid in Jetring1024drdz contains $n_r \times n_\theta \times n_z = 256 \times 1024 \times 962 = 252 \times 10^6$ points, and is characterized by radial, azimuthal, and axial mesh spacings $\Delta r = 0.0036 r_0$ at $r=r_0$, $r_0 \Delta\theta = 0.0061 r_0$, and $\Delta z = 0.0072 r_0$ between $z=-r_0$ and $z=0$. The grid in Jetring1024drdz2δ_θ is that of Jetring1024drdz limited axially to $z=8r_0$. Because of the doubling of the boundary-layer thickness, the second simulation is thus performed at twice the effective resolution.

The properties of the jet initial disturbances are investigated by calculating spectra of the fluctuating axial velocity at $r=r_0$ near the nozzle lip, at $z=0.4r_0$ in Jetring1024drdz and $z=0.8r_0$ in Jetring1024drdz2δ_θ, corresponding to $z=22\delta_\theta=2.7\delta$ in both cases. This location is chosen very close to the nozzle exit in order to avoid the turbulence properties being appreciably affected by the mixing-layer development. The frequency spectra are represented versus the Strouhal number $\mathrm{St}_D=fD/u_j$ in Fig. 2(a), where f is the frequency and $D=2r_0$. They are rather flat up to $\mathrm{St}_D \simeq 1$ in Jetring1024drdz, and to $\mathrm{St}_D \simeq 0.5$ in Jetring1024drdz2δ_θ, and rapidly decrease for higher Strouhal numbers. The spectra as a function of the azimuthal wave number k_θ are shown in Fig. 2(b). For both jets, the energy of initial velocity fluctuations is distributed over a large range of azimuthal modes. Wave numbers centered around $k_\theta \simeq 46$ in Jetring1024drdz and $k_\theta \simeq 24$ in Jetring1024drdz2δ_θ are, however, clearly dominating.

The spectra were previously[12] represented versus $\mathrm{St}_\theta = f\delta_\theta/u_j$ and $k_\theta \delta_\theta / r_0$ instead of St_D and k_θ, respectively, to take the difference in boundary-layer thickness into account. The results for Jetring1024drdz and Jetring1024drdz2δ_θ appeared to collapse well, indicating that they are practically grid-independent. Peak components in the azimuthal spectra were also located around normalized wave numbers $k_\theta \delta_\theta / r_0 \simeq 0.8$.

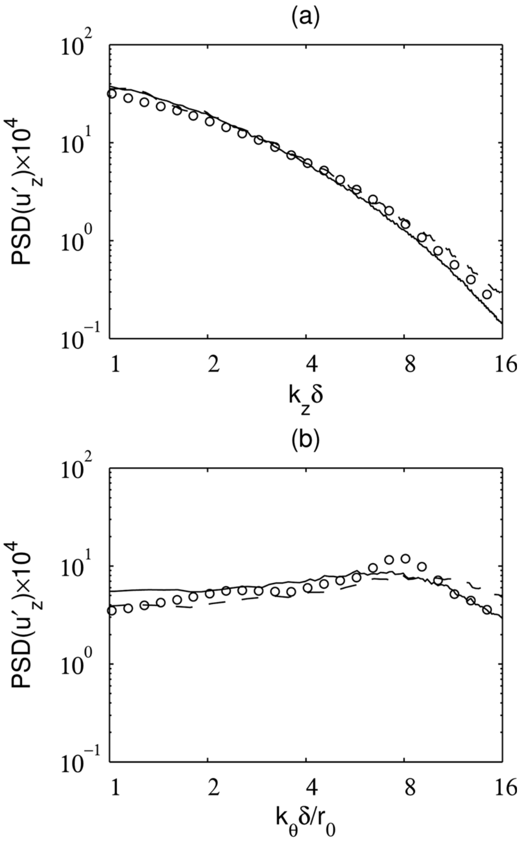

FIG. 3. PSD normalized by u_j of axial fluctuating velocity u'_z, as functions of non-dimensional axial and azimuthal wavenumbers (a) $k_z\delta$ and (b) $k_\theta\delta/r_0$, for (——) Jetring1024drdz at $r=r_0$ and $z=0.4r_0$, and (– – –) Jetring1024drdz2δ_θ at $r=r_0$ and $z=0.8r_0$; (◯) DNS data of Eggels *et al.*[16] for a fully developed turbulent pipe flow, shifted arbitrarily in magnitude for the comparison.

In order to shed light on the physics of initial turbulence in the present transitional jets, the spectra are now plotted versus axial and azimuthal wave numbers $k_z\delta$ and $k_\theta\delta/r_0$ in Figs. 3(a) and 3(b), using a scaling with the boundary-layer thickness δ frequently encountered in the literature for wall-bounded flows. The axial wave number k_z is here estimated from the Strouhal number St$_D$ using Taylor's hypothesis of a frozen turbulence being convected at the velocity $u_j/2$. As expected, the spectra from Jetring1024drdz and Jetring1024drdz2δ_θ do not differ significantly. More interestingly, their shapes are consistent with direct numerical simulation (DNS) data obtained in fully turbulent flat-plate boundary layers, channel, and pipe flows, respectively, by Spalart[13] at $\mathrm{Re}_\theta=1410$, Kim *et al.*[15] at $\mathrm{Re}_\theta=287$, and Eggels *et al.*[16] at $\mathrm{Re}_\theta=236$. To support this assertion, the spectra of fluctuating axial velocity provided by the latter authors in a fully turbulent pipe flow at a distance from the wall of $y^+\simeq 30$, or equivalently of $y/\delta\simeq 0.17$, are shown in Figs. 3(a) and 3(b), shifted arbitrarily in magnitude for the comparison. A very good qualitative agreement is noticed between the jet LES and pipe-flow DNS results, for both axial and azimuthal spectra, over the whole range of wave numbers $1\le k_z\delta, k_\theta\delta/r_0\le 16$. The wave numbers dominating in the azimuthal direction are found for $k_\theta\delta/r_0\simeq 7-8$ in all cases. The turbulent structures near the nozzle exit of the present circular jets therefore appear to be organized in a similar fashion to those in a fully turbulent pipe flow.

Regarding the strongest contributions to the azimuthal spectra, it can be worth mentioning the experimental work carried out by Tomkins and Adrian.[14] These authors succeeded in identifying the most energetic modes in turbulent boundary layers at $\mathrm{Re}_\theta=1015$ and 7705 using particle-image-velocimetry (PIV) measurements. They observed at both Reynolds numbers that the spanwise distribution of streamwise energy throughout the logarithmic region up to $y/\delta=0.2$ is dominated by large-scale elongated[28] structures with spacing $\lambda/\delta=0.75-0.9$, yielding $k\delta=7-8.4$ in terms of dimensionless spanwise wavenumber. Even if structural differences exist between velocity spectra for pipes/channels and boundary layers,[17] the peak wave numbers $k_\theta\delta/r_0\simeq 7$ emerging in the azimuthal spectra of the axial fluctuating velocity for Jetring1024drdz and Jetring1024drdz2δ_θ in Fig. 3(b) fall within that range. This suggests close similarities in the spatial arrangements of large turbulent scales at the nozzle exit of the present transitional jets and in fully turbulent boundary layers.

As a concluding remark, the nature of nozzle-exit conditions in transitional round jets, and their effects on the jet flow and sound field, continue to be the subject of much debate.[1–8,10] The resemblances reported here between the velocity spectra close to the jet exit and those in fully developed wall-bounded flows provide some new results in this field, and support that initially nominally turbulent jets can be considered as a physically justified model of initially fully turbulent jets.

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