

Coherence function of a spherical acoustic wave after passing through a turbulent jet

Vladimir E. OSTASHEV^{a*}, Philippe BLANC-BENON^b, Daniel JUVÉ^b

^a Department of Physics, New Mexico State University, Las Cruces, NM 88003-8001, USA

^b Centre acoustique, équipe LMFA/UMR CNRS 5509, École centrale de Lyon, 36, avenue Guy-de-Collongue, BP 163, 69131 Écully, France

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Abstract. The paper deals with a comparison between experimental data obtained by Blanc-Benon (1984) and theoretical predictions of the coherence function of a spherical sound wave after passing through a turbulent jet. The predictions are based on the theory of sound propagation in moving random media recently developed by Ostashev (1994, 1997), which correctly takes into account the effects of an isotropic vector random field of the medium velocity fluctuations on line-of-sight sound propagation. It is shown that the theoretical predictions fit the data well. © Académie des Sciences/Elsevier, Paris

Keywords: acoustics / wave / turbulence / propagation / random media / coherence

Cohérence spatiale d'une onde acoustique sphérique après traversée d'un jet d'air turbulent

Résumé. Dans ce travail, nous nous intéressons à la perte de cohérence spatiale d'une onde acoustique sphérique traversant un jet d'air turbulent. Nous présentons l'approche théorique d'Ostashev (1997) dans laquelle les fluctuations turbulentes de vitesse, qui apparaissent dans l'indice de réfraction, sont prises en compte de façon exacte à partir du spectre tridimensionnel de l'énergie cinétique. Une comparaison avec l'expérience de Blanc-Benon (1984) réalisée en laboratoire, permet de valider cette approche récente. © Académie des Sciences/Elsevier, Paris

Mots clés : acoustique / onde / turbulence / propagation / milieux aléatoires / cohérence

Version française abrégée

La propagation des ondes acoustiques dans l'atmosphère ou les océans est affectée par les fluctuations de température ou de vitesse du milieu traversé. On s'intéresse ici à l'effet des fluctuations de vitesse, pour lequel la récente approche théorique d'Ostashev (1997) fournit des résultats analytiques, et les essais de Blanc-Benon (1984), des données expérimentales dans des conditions de laboratoire bien contrôlées. Une confrontation théorie-expérience est donc conduite et porte sur la fonction de cohérence spatiale des ondes transmises.

Les résultats présentés dans ce travail sont relatifs au cas d'une onde acoustique sphérique qui se propage selon l'envergure d'un jet d'air bidimensionnel. Comme nous pouvons le voir sur la *figure 1*, la source émettrice est placée à 150 cm de la buse, et elle est située hors du jet ($SO \approx 25$ cm). La

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* On leave from Institute of Atmospheric Physics, Moscow 109017, Russia.

distance de propagation dans le jet est $OC \approx 100$ cm. Dans cette région, la demi-largeur du jet est $\delta \approx 8$ cm. Deux récepteurs, R_1 et R_2 , constitués par des microphones BK 1/4", sont disposés symétriquement par rapport au point C, près du bord du jet. $R_1 R_2$ est perpendiculaire à OC et parallèle à la petite dimension de la buse. La séparation entre les récepteurs est $r = R_1 R_2 = 2,4, 8, 10, \text{ ou } 14$ cm. Les signaux reçus permettent de déterminer la fonction de cohérence normalisée $I(r)/I(0)$. $I(r)$ est le moment d'ordre deux de la pression acoustique p en deux points $I(r = |\mathbf{r}_1 - \mathbf{r}_2|) = \langle p(x, \mathbf{r}_1)p^*(x, \mathbf{r}_2) \rangle$. Deux fréquences de la source S sont possibles, $f = 25$ et 45 kHz, et la vitesse U du jet est ajustable entre $7,9$ et $12,2$ ms^{-1} . La *figure 2* donne un exemple du résultat obtenu pour $I(r)/I(0)$ lorsque $f = 40$ kHz et $U = 7,9$ ms^{-1} .

La théorie classique de Tatarskii (1971) ou Ishimaru (1978), est rappelée dans le paragraphe 3, d'abord pour des fluctuations de température, [éq. (1)], puis pour des fluctuations de vitesse [éq. (3)]. Nous noterons que l'équation (3) est obtenue à partir de l'équation (1), par une simple substitution du facteur C_T^2/T_0^2 par $4C_v^2/c_0^2$, – où C_T^2 et C_v^2 sont les coefficients de structure respectifs des deux types de turbulence. Sur la *figure 2*, on observe que les prédictions théoriques obtenues pour $I(r)/I(0)$ se situent toujours au-dessus des données expérimentales, et qu'ainsi, la décorrélation spatiale de l'onde transmise est sous-estimée. Pour ce tracé, on utilise les équations (4) et (5), qui sont directement déduites de l'équation (3) par introduction de valeurs adimensionnelles, d'un spectre de von Karman modifié pour décrire la turbulence, et du coefficient d'atténuation de l'onde cohérente γ . Ajoutons que des inexactitudes sur la valeur du niveau de turbulence ou sur la valeur de la vitesse de référence c_0 , fonction de la température à laquelle les essais sont conduits, ne peuvent expliquer l'écart observé entre théorie et expériences.

La théorie développée par Ostashev (1994, 1997), est utilisée dans le paragraphe 4. La solution (1) est reprise, mais cette fois, les fluctuations de vitesse sont prises en compte de façon exacte en introduisant le spectre tridimensionnel $F(\kappa)$ [éq. (6)]. Le résultat obtenu a, alors, la forme dimensionnelle (7) dont on déduit aisément la forme adimensionnelle (8). La *figure 2* fait état de la nouvelle courbe théorique ainsi obtenue – tracé en trait continu – et illustre déjà le bon accord obtenu avec une série d'expériences.

Pour confronter cette nouvelle approche théorique à tous les résultats expérimentaux disponibles dans (Blanc-Benon, 1984), on transforme l'équation (8), qui exprime $I(r)/I(0)$, en l'équation (10) qui fournit $(I(r)/I(0))^{\gamma_0/2}$. Le coefficient γ_0 est alors une valeur moyenne du coefficient d'atténuation des ondes sur les différents essais. La *figure 3* illustre le bien-fondé de la nouvelle approche. La courbe continue, qui correspond à l'équation (10), est très proche des résultats expérimentaux. Le désaccord avec la théorie classique est à nouveau observable, si l'on considère le tracé en pointillés de la *figure 3*. Il correspond à l'expression (9) déduite de l'équation (4) pour fournir $(I(r)/I(0))^{\gamma_0/2}$.

Sur la *figure 3*, seul le dernier groupe de points correspondant à $r = 14$ cm, demeure légèrement au-dessus du résultat prédit par l'expression (10). Cet écart pourrait être dû au fait que le calcul considère toujours un chemin OC complètement immergé dans la turbulence, ce qui n'est pas tout à fait vérifié lorsque r est du même ordre ou supérieur à la demi-largeur du jet δ .

1. Introduction

Sound waves, propagating in a turbulent atmosphere or ocean, are affected by temperature and medium velocity fluctuations. In early theories of waves in random media (e.g. Tatarskii, 1971), it was assumed incorrectly that the contributions of temperature and medium velocity fluctuations to the statistical moments of a sound field for line-of-sight sound propagation are the same provided that the

corresponding contributions to the variance of acoustic refractive-index fluctuations are also the same. However, in a more recent theory developed by Ostashev (1994, 1997), it has been shown that the temperature and velocity contributions to the statistical moments of a sound field can differ significantly. This difference, revealed in the latter theory, is due to the fact that the temperature fluctuations are a scalar random field and have an isotropic correlation function for isotropic turbulence, while the medium velocity fluctuations in the direction of sound propagation are a component of a vector random field and have an anisotropic correlation function even for the case of isotropic turbulence. Some predictions of this theory have been verified experimentally for the cases of sound propagation in a turbulent atmosphere and ocean (Di Iorio and Farmer, 1996; Mellert, 1996). The atmosphere and the ocean are very complicated media in which characteristics of turbulence change in time and along the path of sound propagation. This fact complicates comparisons between theory and experiment. The purpose of the present paper is to compare experimental data on sound propagation in a turbulent jet, obtained in a well-controlled experiment (Blanc-Benon, 1984), with predictions based on the early theories and the theory developed by Ostashev (1994, 1997).

2. Experiment

Figure 1 represents the scheme of the experimental setup. A 2D jet of air issued from the nozzle via a slot of span 100 cm and height 8 cm. The path of sound propagation from source S to the center C of two receivers R_1 and R_2 was perpendicular to the direction of flow and was 150 cm downstream of the nozzle. At this distance, the estimated jet thickness in the direction perpendicular to the line segment SC was $\delta \sim 8$ cm. (δ is defined as the distance from the center of the jet to the point where the flow velocity decreases by a factor of 2.) The mean speed of the jet U could be varied. The measured spectrum of the velocity fluctuations was approximated fairly well by a von Karman spectrum with the outer and inner scales given by $L_0 = 8.97$ cm and $l_0 = 0.1$ cm (for more details, see Blanc-Benon, 1981). The variance of the velocity fluctuations σ_v^2 was proportional to U^2 : $\sigma_v^2 = \beta^2 U^2$. The source emitted a spherical wave of frequency $f = 25$ or 40 kHz and was located at a distance $x_1 = SO = 25$ cm from the jet. Two receivers (1/4-inch B& K microphones)

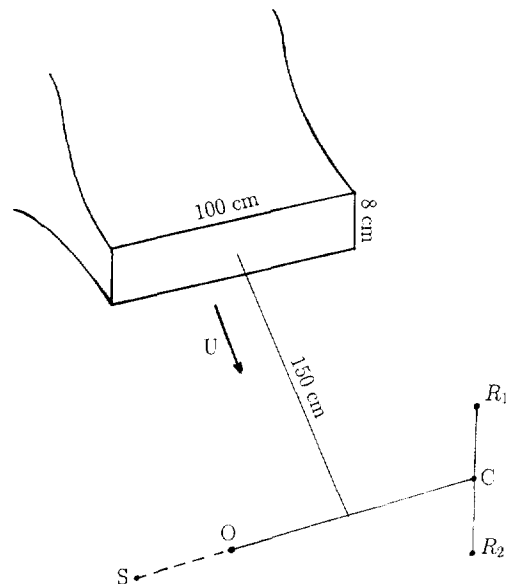


Figure 1. The scheme of the experimental setup. S is the source, R_1 and R_2 are the microphones.

Figure 1. Schéma du dispositif expérimental. S est la source, R_1 et R_2 sont les microphones.

were located on the opposite side of the jet near its boundary. The distance of sound propagation through the turbulent jet was $OC = 100$ cm, resulting in a total distance between source and the center C of $x = SC = 125$ cm. The separation between receivers was $r = R_1 R_2 = 2, 4, 6, 8, 10$ or 14 cm, the line segment connecting the receivers $R_2 R_1$ was perpendicular to the direction of jet propagation, and $R_2 C = CR_1$. Data, recorded by the receivers, were used to compute the coherence function $\Gamma(r) = \langle p(x, \mathbf{r}_1) p^*(x, \mathbf{r}_2) \rangle$. Here p is the sound pressure, the vectors \mathbf{r}_1 and \mathbf{r}_2 determine the positions of the receivers R_1 and R_2 , and $r = |\mathbf{r}_1 - \mathbf{r}_2|$. Figure 2 gives an example of the measured normalized coherence function, $\Gamma(r)/\Gamma(0)$ for $U = 7.9$ m/s and $f = 40$ kHz.

3. Early theories

For the experiment in question, the sound wavelengths $\lambda = 1.37$ and 0.86 cm were much smaller than L_0 . This allows one to employ the parabolic equation method. The coherence function of a spherical wave propagating in a continuous random medium with temperature fluctuations, calculated by using the parabolic equation method, is given (Ishimaru, 1978) by

$$\Gamma(r) = (4\pi x)^{-2} \exp \left\{ -\pi^2 k^2 \int_0^x dx' \int_0^\infty d\kappa \kappa [1 - J_0(\kappa r x'/x)] \Phi_T(\kappa)/T_0^2 \right\}. \quad (1)$$

Here J_0 is the Bessel function, $\Phi_T(\kappa)$ is the three-dimensional spectral density of temperature fluctuations, T_0 is the mean value of the temperature, and $k = 2\pi f/c_0$ is the wavenumber, where c_0 is the mean value of the adiabatic sound speed. In equation (1), the integral over x' corresponds to the integration along the path of sound propagation. For the von Karman spectrum of temperature fluctuations,

$$\Phi_T(\kappa)/T_0^2 = A(\kappa^2 + \kappa_0^2)^{-11/6} e^{-\kappa^2/\kappa_m^2} C_T^2/T_0^2, \quad (2)$$

where $A = 0.033$, C_T^2 is the structure parameter of temperature fluctuations, $\kappa_0 = 1/L_0$ and $\kappa_m = 5.91/l_0$.

According to the early theories (Tatarskii, 1971), the coherence function Γ of a spherical wave after passing through a turbulent jet is still given by equations (1) and (2) if C_T^2/T_0^2 is replaced by $4C_v^2/c_0^2$, where C_v^2 is the structure parameter of the velocity fluctuations. Thus,

$$\Gamma(r) = (4\pi x)^{-2} \exp \left\{ -\frac{4\pi^2 A k^2 C_v^2}{c_0^2} \int_{x_1}^x dx' \int_0^\infty d\kappa \kappa \frac{1 - J_0(\kappa r x'/x)}{(\kappa^2 + \kappa_0^2)^{11/6}} e^{-\kappa^2/\kappa_m^2} \right\}. \quad (3)$$

In this equation, the limits of the integration over x' correspond to the part of the sound wave path which is located within the turbulent jet. It is worthwhile to express equation (3) in the following form:

$$\Gamma(r) = (4\pi x)^{-2} \exp \left\{ -\frac{10\gamma x L_0}{3r} \int_{rx_1/(L_0 x)}^{rL_0} d\xi \int_0^\infty d\eta \eta \frac{1 - J_0(\eta \xi)}{(1 + \eta^2)^{11/6}} e^{-\eta^2/\alpha^2} \right\}. \quad (4)$$

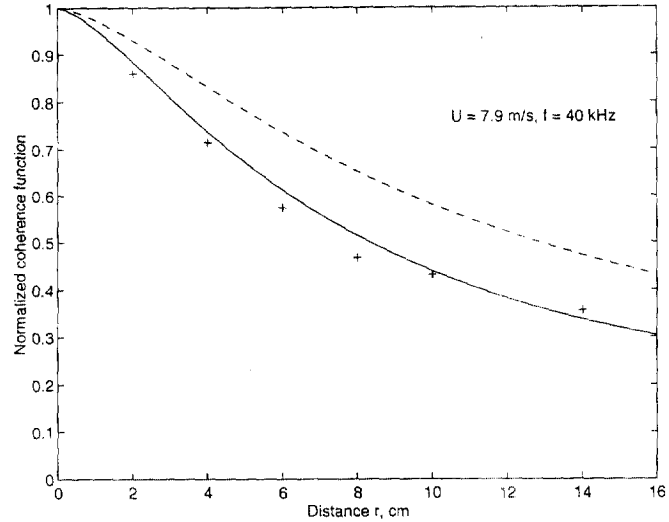
Here $\xi = x'\kappa_0 r/x$, $\eta = \kappa/\kappa_0$, $\alpha = \kappa_m/\kappa_0$, and γ is the attenuation coefficient of the mean sound field for the von Karman spectrum, given (Ostashev, 1997) by

$$\gamma = (6/5) \pi^2 A k^2 \kappa_0^{-5/3} C_v^2/c_0^2 = 29.48 f^2 \beta^2 U^2 L_0/c_0^4. \quad (5)$$

In this equation, we take into account that C_v^2 can be expressed in terms of σ_v^2 , and hence U^2 : $C_v^2 = 1.91 \kappa_0^{2/3} \sigma_v^2 = 1.91 \kappa_0^{2/3} \beta^2 U^2$.

Figure 2. Normalized coherence function $\Gamma(r)/\Gamma(0)$ versus the separation distance r . Experimental data are indicated by +. The dashed and solid lines are theoretical predictions based on equations (4) and (8).

Figure 2. Évolution de la fonction de cohérence normalisée $\Gamma(r)/\Gamma(0)$ avec la séparation entre capteurs r : expérience « + »; équation (4) - -; équation (8) —.



In Blanc-Benon (1984), the measured coherence functions were compared with the predictions based on equation (4). The experimental parameters required in equations (4) and (5) are $x = 125$ cm, $x_1/x = 0.2$, $L_0 = 8.97$ cm, $\alpha = 530.1$, f and/or U varied with each run. Also it was assumed that $c_0 = 340$ m/s and $\beta = 0.24$. The value of β was chosen in an attempt to fit the experimental data to predictions based on equation (4). But even with $\beta = 0.24$, the values of the normalized coherence function calculated from equation (4) overpredicted the experimental data. Let us reexamine the values of $c_0 = 340$ m/s and $\beta = 0.24$. $c_0 = 340$ m/s corresponds to $T_0 = 14.3$ °C and is appropriate for outdoor sound propagation. The experiment was performed in an anechoic chamber. Although the temperature in the chamber was not monitored we shall assume that T_0 was 20 °C, which results in $c_0 = 343.3$ m/s. Using the same apparatus the value of β was measured by Blanc-Benon (1981) to be $\beta = 0.217$. This value of β is hereafter used when comparing experimental data and theoretical predictions. The dashed line in *figure 2* is the theoretical prediction of the normalized coherence function $\Gamma(r)/\Gamma(0)$ based on equation (4). It is clearly seen that all experimental data are located below this line.

4. Theory developed by Ostashev (1994, 1997)

According to the theory of sound propagation in moving random media, presented by Ostashev (1994, 1997), the coherence function of a spherical wave is given by equation (1) if $\Phi_T(\kappa)/T_0^2$ is replaced by $4F(\kappa)/c_0^2$. Here $F(\kappa)$ is the three-dimensional spectral density of the velocity fluctuations. For the von Karman spectrum

$$F(\kappa) = \frac{11 A \kappa^2 C_v^2}{6(\kappa^2 + \kappa_0^2)^{17/6}} e^{-\kappa^2/\kappa_m^2}, \quad (6)$$

so that the equation for the coherence function takes the form

$$\Gamma(r) = (4\pi x)^{-2} \exp \left\{ -\frac{22\pi^2 A \kappa^2 C_v^2}{3c_0^2} \int_{x_1}^x dx' \int_0^\infty d\kappa \kappa^3 \frac{1 - J_0(\kappa r x'/x)}{(\kappa^2 + \kappa_0^2)^{17/6}} e^{-\kappa^2/\kappa_m^2} \right\}. \quad (7)$$

The limits of integration over x are the same as those in equation (3). Equation (7) can be expressed in the following form:

$$F(r) = (4\pi x)^{-2} \exp \left\{ -\frac{55 \gamma x L_0}{9 r} \int_{rx_1/(L_0 x)}^{r/L_0} d\xi \int_0^\infty d\eta \eta^3 \frac{1 - J_0(\eta \xi)}{(1 + \eta^2)^{17/6}} e^{-\eta^2 / \alpha^2} \right\}. \quad (8)$$

Equation (8) differs significantly from equation (4). The solid line in *figure 2* is the normalized coherence function $F(r)/F(0)$ calculated from equation (8), which is in good agreement with the experimental data.

In addition to the experimental data presented in *figure 2*, the coherence function was also measured for five runs corresponding to different values of U and f : $U = 7.9$ m/s, $f = 25$ kHz; $U = 10.8$ m/s, $f = 25$ kHz; $U = 9.6$ m/s, $f = 40$ kHz; $U = 11.6$ m/s, $f = 40$ kHz and $U = 12.2$ m/s, $f = 40$ kHz. (The latter of these runs was presented in Blanc-Benon, 1984; the others are available from laboratory records.) U and f appear only in equation (5), resulting in different values of γ for each run. For U and f given above, γ is in the range $0.351 \text{ m}^{-1} < \gamma < 2.13 \text{ m}^{-1}$, and its mean value is given by $\gamma_0 = 1.2 \text{ m}^{-1}$. Because of the different γ , the values of $F(r)/F(0)$, measured in all six runs, cannot be presented as an experimental ‘curve’. However, such an experimental curve can be obtained if one raises the measured values of $F(r)/F(0)$ to the power γ_0 / γ .

Indeed, it follows from equation (4) that

$$\left(\frac{F(r)}{F(0)} \right)^{\gamma_0 / \gamma} = \exp \left\{ -\frac{10 \gamma_0 x L_0}{3 r} \int_{rx_1/(L_0 x)}^{r/L_0} d\xi \int_0^\infty d\eta \eta \frac{1 - J_0(\eta \xi)}{(1 + \eta^2)^{11/6}} e^{-\eta^2 / \alpha^2} \right\} \quad (9)$$

in the early theories, and it follows from equation (8) that

$$\left(\frac{F(r)}{F(0)} \right)^{\gamma_0 / \gamma} = \exp \left\{ -\frac{55 \gamma_0 x L_0}{9 r} \int_{rx_1/(L_0 x)}^{r/L_0} d\xi \int_0^\infty d\eta \eta^3 \frac{1 - J_0(\eta \xi)}{(1 + \eta^2)^{17/6}} e^{-\eta^2 / \alpha^2} \right\} \quad (10)$$

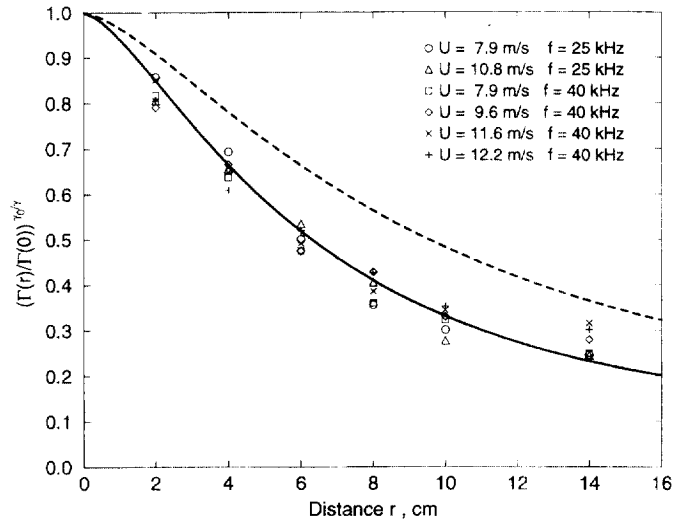
in the theory presented by Ostashev (1994, 1997). It is evident from these equations that $(F(r)/F(0))^{\gamma_0 / \gamma}$ does not depend on γ and must be the same for all six runs. The same result is expected to be valid for the experimental data.

The experimental data $(F(r)/F(0))^{\gamma_0 / \gamma}$ for all six runs are plotted in *figure 3*. The dashed and solid lines in *figure 3* are the functions $(F(r)/F(0))^{\gamma_0 / \gamma}$ plotted on the basis of equations (9) and (10), respectively. It is clearly seen that the theoretical predictions based on equation (10) are in good agreement with the experimental data, while those based on the early theories are well above the experimental data. An experimental error bar is not available from the data set we have now. One may speculate that this error might be of the order of the magnitude of spread of the experimental data for each r , shown in *figure 3*. The solid line passes through all these spreads except for $r = 14$ cm. All experimental data for $r = 14$ cm are located above the theoretical predictions based on equation (10). This can be explained by the fact that the microphones were located at a distance $r/2 = 7$ cm from the center of the jet, which was of the order of the jet thickness δ . In this case, sound propagated through less intense turbulence than that near the jet center, resulting in nearly the same values of the coherence functions as those for $r = 10$ cm.

It should be noted that the effect of the inner scale of turbulence l_0 on all numerical values of the coherence functions in *figures 2* and *3* is negligible. This supports the predictions of the theory presented in Ostashev (1994, 1997) that, when calculating the statistical moments of the sound field, l_0 can be equated to 0 as long as $l_0 \ll \lambda$. For $l_0 = 0$, the integrals over η in equations (4), (6), (9) and (10) can be calculated analytically as has been done elsewhere (Ostashev, 1997).

Figure 3. The function $(\Gamma(r)/\Gamma(0))^{20/7}$ versus the separation distance r . Experimental data are indicated by points of different shape. The dashed and solid lines are theoretical predictions based on equations (9) and (10).

Figure 3. Évolution de la fonction $(\Gamma(r)/\Gamma(0))^{20/7}$ avec la séparation entre capteurs r pour différentes vitesses du jet : expérience « + » ; équation (9) - - ; équation (10) —.



5. Conclusions

We have presented the experimental data for the coherence function of a spherical wave after passing through a turbulent jet, obtained by Blanc-Benon (1984). These data have been compared with predictions based on early theories of sound propagation in moving random media and those based on the theory developed by Ostashev (1994, 1997). The latter theory allows one to correctly account for the effects of an isotropic vector random field of the velocity fluctuations on sound propagation. It is shown that the early theories do not allow one to well explain the experimental data. On the other hand, the predictions of the theory presented in Ostashev (1994, 1997) fit the experimental data well. It would be worthwhile to measure other statistical moments of a sound wave after passing through a turbulent jet and to compare them with predictions of the latter theory. We are currently planning to do such experiments.

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