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Addendum to the back-scattering correction of Amiet's trailing-edge noise model

Michel Roger [a], Stéphane Moreau [b,*]

[a] École Centrale de Lyon, Laboratoire de Mécanique des Fluides et Acoustique, UMR CNRS 5509, 36 avenue Guy de Collongue, 69134 Ecully, France
[b] Université de Sherbrooke, Groupe d'Acoustique de l'Université de Sherbrooke, Faculté de Génie, 2500 boulevard de l'université, Sherbrooke, QC, Canada J1K2R1

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ABSTRACT

A minor correction to a previous analytical formulation aimed at predicting broadband trailing-edge noise of subsonic airfoils is provided. An improved numerical implementation free of mathematical singularities is proposed, and a revised plot of the final results found in the reference paper is given.

In [1], a back-scattering correction was proposed for the trailing-edge noise radiation of subsonic airfoils and solutions were provided for general three-dimensional supercritical and subcritical pressure gusts over the trailing edge. The numerical implementation of the radiation integrals $I(\overline{K_1},\overline{K_2})$, with $\overline{K_1}$ and $\overline{K_2}$ the reduced streamwise and spanwise wave numbers respectively, involved mathematical singularities when square roots of a vanishing quantity in a denominator combined with a numerator that also approached zero. This typically occurs when the Fresnel integral $E^*(z)$ or the complex error function $\Phi^{(0)}(\sqrt{\mathrm{i}z})$ produced in the derivations of [1] is divided by $\sqrt{z}$. Directly dealing with such ratios in three-dimensional applications may induce inaccuracy or divergence in the solution for some angles of observation and/or for some oblique, either subcritical or supercritical gusts. Stable computations can be ensured if the functions are extended to complex arguments and if the following new function is introduced:

$$\mathrm{ES}^*[z] = \frac{E^*(z)}{\sqrt{z}} = \frac{1-\mathrm{i}}{2}\frac{\Phi^{(0)}(\sqrt{\mathrm{i}z})}{\sqrt{z}} \tag{1}$$

This function is implemented using standard expansions [2]. It is found regular in the entire complex plane, even though the numerators and denominators independently exhibit branch cuts, as shown by the plots of Fig. 1.

If the new function given by Eq. (1) is introduced, the radiation integral involving the main contribution for the supercritical gusts, Eq. (13) in [1], becomes

$$\int_{-2}^{0} f_1(X)e^{-\mathrm{i}CX}\,\mathrm{d}X = -\frac{e^{2\mathrm{i}C}}{\mathrm{i}C} \tag{2}$$

$$\{(1+\mathrm{i})e^{-2\mathrm{i}C}\sqrt{2B}\,\mathrm{ES}^*[2(B-C)]-(1+\mathrm{i})E^*[2B]+1-e^{-2\mathrm{i}C}\}$$

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* Corresponding author. Tel.: +1 819 437 6751; fax: +1 819 821 7163.
E-mail address: Stephane.Moreau@USherbrooke.ca (S. Moreau).

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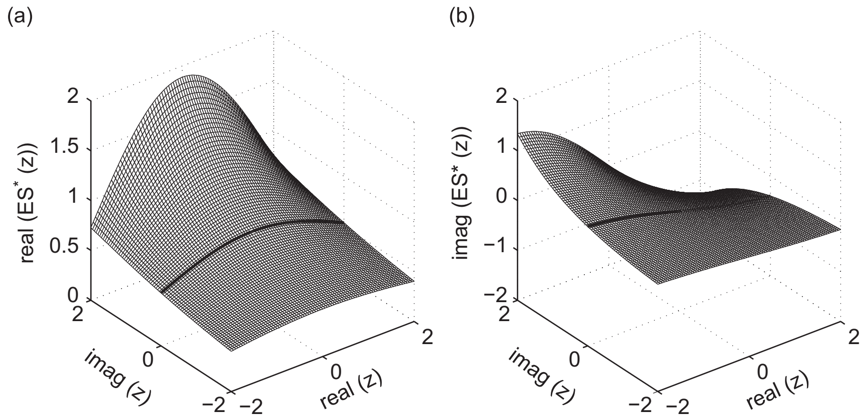

Fig. 1. Real and imaginary parts of the function ES*[z] with complex argument. The ordinary reduction for real arguments is plotted as a thick line.

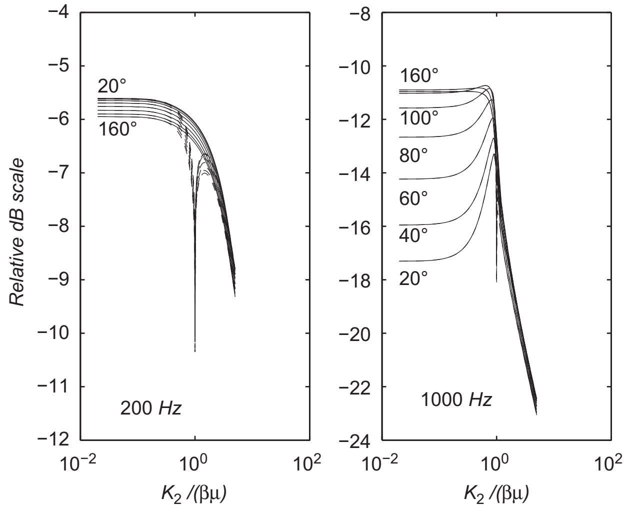

Fig. 2. Radiation integral profiles in nondimensional variables. $M=0.05$. Dashed line regularization off, solid line regularization on. Radiation angles from 20° to 160° by steps of 20°.

Similarly, for the back-scattering correction, Eq. (14) in [1] gives

$$\frac{1}{H}\int_{-2}^{0} f_2(X)e^{-\mathrm{i}CX}\,\mathrm{d}X = \{e^{4\mathrm{i}\overline{\kappa}}[1-(1+\mathrm{i})E^*(4\overline{\kappa})]\}^c - e^{2\mathrm{i}D} + \mathrm{i}[D+\overline{K}+M\overline{\mu}-\overline{\kappa}]G \tag{3}$$

with

$$\begin{aligned} G = {} & (1+\varepsilon)e^{\mathrm{i}(2\overline{\kappa}+D)}\frac{\sin(D-2\overline{\kappa})}{D-2\overline{\kappa}} + (1-\varepsilon)e^{\mathrm{i}(-2\overline{\kappa}+D)}\frac{\sin(D+2\overline{\kappa})}{D+2\overline{\kappa}} \\ & + \frac{(1+\varepsilon)(1-\mathrm{i})}{2(D-2\overline{\kappa})}e^{4\mathrm{i}\overline{\kappa}}E^*(4\overline{\kappa}) - \frac{(1-\varepsilon)(1+\mathrm{i})}{2(D+2\overline{\kappa})}e^{-4\mathrm{i}\overline{\kappa}}E(4\overline{\kappa}) \\ & + \frac{e^{2\mathrm{i}D}}{\sqrt{2}}\sqrt{2\overline{\kappa}}\,\mathrm{ES}^*[2D]\left[\frac{(1+\mathrm{i})(1-\varepsilon)}{D+2\overline{\kappa}} - \frac{(1-\mathrm{i})(1+\varepsilon)}{D-2\overline{\kappa}}\right] \end{aligned}$$

Equivalent modifications can be introduced for the subcritical gusts. The radiation integral of the main contribution, Eq. (15) in [1], yields

$$\int_{-2}^{0} f'_1(X)e^{-\mathrm{i}CX}\,\mathrm{d}X = -\frac{e^{2\mathrm{i}C}}{\mathrm{i}C}\left\{e^{-2\mathrm{i}C}\sqrt{2A'_1}(1+\mathrm{i})\mathrm{ES}^*[2(\overline{\mu}(x_1/S_0)-\mathrm{i}\overline{\kappa}')] - \Phi^0([2\mathrm{i}A'_1]^{1/2})+1\right\} \tag{4}$$

The radiation integral for the back-scattering correction, Eq. (16) in [1], now reads

$$\int_{-2}^{0} f'_2(X) e^{-\mathrm{i}CX}\,\mathrm{d}X = \frac{e^{-2\mathrm{i}D'}}{D'} H' \left\{ A'(e^{2\mathrm{i}D'}[1-\mathrm{erf}(\sqrt{4\overline{\kappa}'})]-1) + 2\sqrt{2\overline{\kappa}'}\left(\overline{K} + \left(M - \frac{x_1}{S_0}\right)\overline{\mu}\right)\mathrm{ES}^*[-2D'^*] \right\} \tag{5}$$

with

$$H' = \frac{(1+\mathrm{i})(1-\Theta'^2)}{2\sqrt{\pi}(\alpha-1)\overline{K}\sqrt{A_1}'}, \quad D' = \frac{\overline{\mu}x_1}{S_0} - \mathrm{i}\overline{\kappa}'$$

In Eq. (5), it should be emphasized that a new notation D' is introduced and defined. In [1] it had been confused with the notation B used in Eq. (2). None of the results are however modified. In Figs. 9 and 10 in [1], the radiation integral $|I(\overline{\omega}/U_c,\overline{K}_2)|/2$ is plotted rather than $|I(\overline{\omega}/U_c,\overline{K}_2)|$ but the profiles are preserved using Eqs. (2)–(5). The only modification introduced by using the new function is found in Fig. 11 in [1]. It yields smoother radiation integral profiles in dimensionless variables as seen in Fig. 2.

References

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