Un algorithme numérique pour calculer la stabilité spatiale de modes éventuellement rayonnants : Application aux écoulements supersoniques

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Local linear stability is often invoked in computational aeroacoustics to predict Mach wave radiation, or to prescribe inflow conditions in order to drive turbulent transition in large eddy simulations. In this work, the governing equations are reformulated for the non-oscillatory part of eigenfunctions. Boundary conditions can thus be explicitly enforced and moreover, the numerical cost is drastically reduced regardless of the method chosen to solve this problem. An efficient method based on a matrix formulation is proposed in this study. One single small collocation domain is indeed used even for computing the stability of supersonic flows.

1 Introduction

The linear stability theory is widely used in computational aeroacoustics. To name a few examples, Mach wave radiation can be accurately described from small flow perturbations growing in space [1, 2], as shown in the reviews by Tam [3] and Morris [4] or investigated by Oertel *et al.* [5]. Linear parabolised stability equations require an initialisation often provided by a local solution [6]. Generation of unsteady inflow conditions built on instability waves can be used to drive the transition towards turbulence in large eddy simulation, as performed by Keiderling *et al.* [7].

Apart from a small number of particular cases, the linear stability eigenvalue problem for a given base flow must be solved numerically. There are basically two main classes of numerical methods [4] : shooting techniques and global matrix methods. In the case of supersonic jet flows, and in the framework of a spatial analysis, a two-domain shooting method is commonly preferred [8, 9, 10, 11, 12, 13]. The numerical algorithm appears to be of easier implementation and generally provides the most accurate results. One of the main difficulty is represented by the highly oscillatory behaviour of the eigenfunctions and by the widening of their support as the Mach number increases.

In the present study, a new approach is proposed. The equation governing the local stability problem is reformulated to remove the oscillatory part of the eigenfunctions. This leads to a less demanding numerical problem and moreover, acoustic radiation conditions for Mach waves can be explicitly taken into account.

This paper is organised as follows. First, a brief introduction to the stability problem is presented in section 2. The numerical procedure is then explained in section 3. Some numerical results are shown in section 4 and concluding remarks are finally given. A more detailed analysis of the stability of high speed jets can be found in [14].

2 Linear stability analysis

The inviscid linear stability problem for a two-dimensional jet is considered. The jet flow is represented by the superposition of a known parallel base flow $\bar{\rho} = \bar{\rho}(y)$, $\bar{\mathbf{u}} = \bar{u}(y)\mathbf{e}_x$, $\bar{p} = 1/(\gamma M_j^2)$ and a small perturbation $(\rho', \mathbf{u}', p')$, where $(\rho, \mathbf{u}, p)$ are the density, the velocity and the pressure. All the variables are made non-dimensional using the nominal jet parameters, namely the half-width $b^\star$, the velocity $u_j^\star$, the density $\rho_j^\star$ and the pressure $p_j^\star$. Thanks to the homogeneity in the base flow direction x and in time of the partial differential system for the unknowns $(\rho', \mathbf{u}', p')$, all the physical quantities are sought in the form of normal modes

$$q'(x, y, t) = \mathcal{R}e\left\{\hat{q}'(y)e^{i(kx-\omega t)}\right\} \tag{1}$$

where the wave number k and the angular frequency ω are generally taken complex. With some mathematical manipulations, the Euler partial differential system can be reduced to a single equation for the pressure amplitude $\hat{p}'$, known as the generalized or compressible Rayleigh equation [15],

$$\begin{aligned} \mathcal{F}\left(\hat{p}'\right) \quad &\equiv \quad \frac{d^2\hat{p}'}{dy^2} - \left[\frac{1}{\bar{\rho}}\frac{d\bar{\rho}}{dy} + \frac{2k}{k\bar{u}-\omega}\frac{d\bar{u}}{dy}\right]\frac{d\hat{p}'}{dy} \; - \\ &- \quad \left[k^2 - M_j^2\bar{\rho}(k\bar{u}-\omega)^2\right]\hat{p}' = 0 \end{aligned} \tag{2}$$

where M_j denotes the jet Mach number. Spatially growing perturbations are considered here. Disturbances are periodic in time with ω taken real and positive, and one seeks the complex eigenvalues $k = k_r + ik_i$ and the corresponding eigenfunctions $\hat{p}'$. According to expression (1), a mode is thus unstable if $k_i < 0$. The appropriate boundary conditions are obtained by solving the limiting form of Rayleigh's equation (2) as $y \to \pm\infty$,

$$\frac{d^2\hat{p}'}{dy^2} - \beta^2\hat{p}' = 0 \tag{3}$$

where

$$\beta = \sqrt{k^2 - M_j^2\rho_\infty(ku_\infty - \omega)^2} \tag{4}$$

and ρ_∞ and u_∞ represent the density and velocity of the uniform free stream. The branches of the complex eigenvalue β are selected to satisfy the causality principle and to ensure that the disturbance field decreases as $y \to \pm\infty$. Consequently, the asymptotic behaviour of the pressure $\hat{p}'$ is given by $\hat{p}' \propto e^{\mp\beta y}$ as $y \to \pm\infty$, with the choice $\arg(\beta) \in]-\pi/2, \pi/2[$. This ensures that the disturbance field decays to infinity [4].

Only the discrete part of the eigenvalue spectrum is considered in this work [16]. It consists of two families of waves [17, 18]. The continuation of Kelvin-Helmholtz instability waves or vortical modes into the compressible regime and the so-called acoustic modes [19, 20, 17], which are obviously removed in the classical incompressible form of the Rayleigh equation. Following the terminology introduced by Luo & Sandham [13], acoustic perturbations are observed when $M_s \equiv M_j|1 - v_\varphi| > 1$, where $v_\varphi = \omega/k_r$ is the phase velocity in the x direction. According to Tam & Hu [17], these modes are also

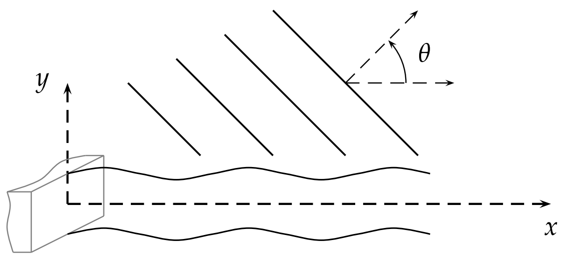

FIGURE 1 – Sketch of Mach wave fronts radiated by a sinuous instability wave, see Eq. (5).

the most unstable waves at high enough Mach numbers [18]. Both vortical and acoustic modes may radiate in the far field when their phase velocity is supersonic relative to the free stream. For a free jet with $u_\infty = 0$ for simplicity, this condition is satisfied when $M_r \equiv |v_\varphi / c_\infty| = |\rho_\infty^{1/2} M_j v_\varphi| > 1$. The radiation directivity can be determined by examinating the expression of the eigenfunction of the pressure for $y \to +\infty$

$$p'(x,y,t) \propto e^{-k_i x - \beta_r y} e^{i(k_r x - \beta_i y - \omega t)} \tag{5}$$

It can thus be observed that a Mach wave radiation is generated in the angular direction $\theta = \mathrm{atan}(-\beta_i / k_r)$ with respect to the downstream direction [3], as shown in figure 1.

As an illustration, the vortex-sheet model of a plane jet is briefly recalled. The base flow is given by,

$$\begin{cases} \bar{u}(y) = 1, \quad \bar{\rho}(y) = 1 & \text{if } y \in]-1,1[\\ \bar{u}(y) = 0, \quad \bar{\rho}(y) = \rho_\infty & \text{otherwise} \end{cases} \tag{6}$$

Symmetric and antisymmetric modes about the jet axis are solutions of the inviscid stability problem, and the two dispersion relations $\mathcal{D}_s(M_j, \omega, k)$ and $\mathcal{D}_a(M_j, \omega, k)$ can be analytically derived. The spectrum of the symmetric dispersion relation is shown in figure 2 for the case of an isothermal supersonic jet with $\omega = 0.5$, $\rho_\infty = 1$ and $M_j = 3$. This spectrum is symmetric about the k_r axis since the generalized Rayleigh equation is self-adjoint. The Kelvin-Helmholtz mode as well as the first acoustic supersonic mode are unstable, whereas the other higher acoustic modes are neutral perturbations. The dashed line indicates that the phase velocity is sonic outside of the jet, and modes on the left on this line are thus radiating modes. The dashed - dotted line represents the relation $k_r = \omega M_j / (M_j - 1)$, and supersonic acoustic modes can only exist on the right of this line. This criterion allows to separate the continuation of the Kelvin-Helmholtz mode in compressible regime from acoustic modes in this case. The eigenfunction $\hat{p}'$ can be also analytically determined. It is given by

$$\hat{p}'(y) = \begin{cases} \cosh(\beta_1 y) & y \in]0,1[\\ \cosh(\beta_1) e^{\beta(1-y)} & y \in]1,+\infty[\end{cases} \tag{7}$$

where

$$\beta_1 = \sqrt{k^2 - M_j^2 (k-\omega)^2} \qquad \beta = \sqrt{k^2 - M_j^2 \rho_\infty \omega^2}$$

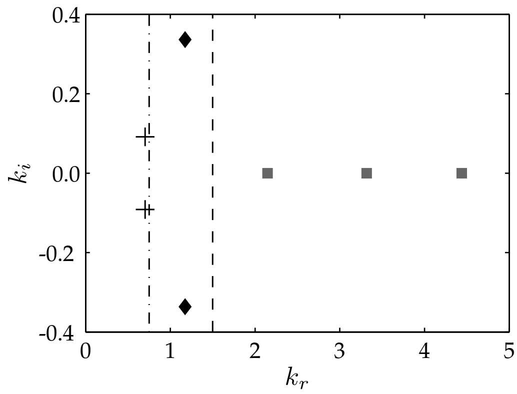

FIGURE 2 – Symmetric modes of the vortex sheet model for the case $\omega = 0.5$, $\rho_\infty = 1$ and $M_j = 3$, $+$ Kelvin - Helmholtz symmetric mode, ♦ first acoustic mode, ■ higher acoustic modes, $-\cdot-\cdot-$ $k_r = M_j \omega / (M_j - 1)$, $---$ $k_r = \rho_\infty^{1/2} M_j \omega$.

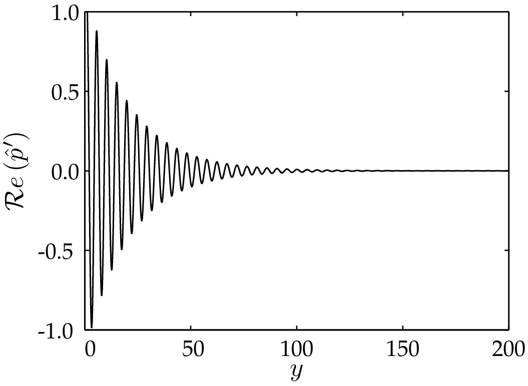

FIGURE 3 – Analytical eigenfunction (7) of the symmetric Kelvin - Helmholtz mode for the case $\omega = 0.5$, $\rho_\infty = 1$ and $M_j = 3$.

This oscillating function $\hat{p}$ is plotted in Figure 3. It has a very large support, with a transverse distance of about 150 times the jet width. This is due to the fact that for radiating modes, the term $e^{-\beta y}$ does not decay exponentially fast in the free stream unlike non-radiating unstable modes.

3 The solution technique

Apart from a small number of particular cases, the generalized Rayleigh equation (2) must be solved numerically. Early calculations on the stability of parallel flows have been made through shooting methods whereas, in recent years, also matrix methods [23] have been successfully applied to hydrodynamic stability problems. Shooting methods are best suited to obtain a single eigenvalue of the spectrum, and requires an initial guess. On the contrary, matrix methods can provide an approximation to all the eigenvalues without an initial guess. In this case, a numerical scheme allows to transform the Rayleigh equation into an algebraic polynomial eigenvalue problem [25, 26, 27, 16, 28]. As discretization procedure, the pseudospectral collocation method is often employed. The unknown function $\hat{p}'$ is approximated by a linear combination of $(N+2)$ known, but arbitrarily chosen, basis functions $\phi_i(y)$,

$$\hat{p}'(y) \approx \hat{p}'_{N+2}(y) \equiv \sum_{i=1}^{N+2} a_i \phi_i(y)$$

The $(N+2)$ unknown coefficients a_i are calculated by requiring that $\mathcal{F}(\hat{p}'_{N+2})$ vanishes at N collocation points y_n, $\mathcal{F}\left[\hat{p}'_{N+2}(y_n)\right] = 0$, and by imposing the two boundary conditions associated with (2). For a given ω and M_j, this leads to an algebraic polynomial eigenvalue problem

$$\left[\mathcal{A}_0 + k\mathcal{A}_1 + k^2\mathcal{A}_2 + k^3\mathcal{A}_3\right] \mathbf{a} = 0 \qquad \mathbf{a} = \{a_i\} \tag{8}$$

where $\mathcal{A}_0$, $\mathcal{A}_1$, $\mathcal{A}_2$ and $\mathcal{A}_3$ are $(N+2)\times(N+2)$ matrices [14]. A complete review of different approaches to compute the eigenvalues of 8 can be found in Bridges & Morris [24]. Using the companion matrix method [24, 25], equation 8 is easily transformed in a $3(N+2)\times 3(N+2)$ linear eigenvalue problem of the form $\mathcal{P}\mathbf{a} = k\mathcal{Q}\mathbf{a}$, which can be solved through a standard $\mathcal{QZ}$ algorithm.

In this work, the spatial modes of a supersonic jet are examined. As the Mach number increases, the eigenfunctions $\hat{p}'$ become more and more oscillating with a larger support than in the incompressible case, as illustrated with the vortex sheet model in figure 3. Therefore, the number of collocation points needed to accurately resolve these modes rapidly grows as the Mach number exceed the value of about $M_j = 2$. In other words, computing the spatial modes $(\omega, k, \hat{p}')$ through the above procedure turns out to be very expensive for $M_j \geq 2$. Besides, when using the collocation method, it is generally difficult to enforce the boundary conditions $\hat{p}'(y) \propto e^{\mp\beta y}$ as $y \to \pm\infty$ so that, in practice, one often imposes the condition that the derivative $d\hat{p}'/dy$ vanishes at infinity.

To overcome these problems, a new formulation of the generalized Rayleigh equation is proposed. As a starting point, it is observed that the eigenfunctions $\hat{p}'$ deviate significantly from their asymptotic behaviour $e^{\mp\beta y}$ only in the inner region of the jet. Furthermore, due to the symmetry of a jet about the axis $y = 0$, the generalized Rayleigh equation admits symmetric and anti-symmetric modes, so that it can be solved in the reduced interval $0 \leq y \leq \infty$ by enforcing appropriate boundary conditions at $y = 0$. Let us define a new function $\tilde{p}$ such that,

$$\hat{p}'(y) = \tilde{p}(y)e^{-\beta y} \tag{9}$$

This function $\tilde{p}$ varies only in the inner part of the jet and rapidly tends to a constant value outside this region. By substituting (9) in equation (2), it is straightforward to find the differential equation for $\tilde{p}(y)$,

$$\frac{d^2\tilde{p}(y)}{dy^2} + g_1\frac{d\tilde{p}(y)}{dy} + g_2\tilde{p}(y) = 0 \tag{10}$$

where

$$\begin{cases} g_1(y;\bar{u},\bar{\rho},M_j,k,\omega) &= -\dfrac{1}{\bar{\rho}}\dfrac{d\bar{\rho}}{dy} - \dfrac{2k}{k\bar{u}-\omega}\dfrac{d\bar{u}}{dy} - 2\beta \\ g_2(y;\bar{u},\bar{\rho},M_j,k,\omega) &= \beta\left(\dfrac{1}{\bar{\rho}}\dfrac{d\bar{\rho}}{dy} + \dfrac{2k}{k\bar{u}-\omega}\dfrac{d\bar{u}}{dy}\right) + \\ &+ \beta^2 - k^2 + M_j^2\bar{\rho}(k\bar{u}-\omega)^2 \end{cases}$$

Equation (10) is to be solved with the following boundary condition for $y \to \infty$,

$$\lim_{y\to\infty} \frac{d\tilde{p}(y)}{dy} = 0 \tag{11}$$

completed by

$$\left.\frac{d\hat{p}'(y)}{dy}\right|_{y=0} = \left.\frac{d\tilde{p}(y)}{dy}\right|_{y=0} - \beta\tilde{p}(y=0) = 0 \tag{12}$$

for symmetric modes, and by

$$\hat{p}'(y=0) = \tilde{p}(y=0) = 0 \tag{13}$$

for antisymmetric modes. The discretization of this problem remarkably reduces the number of collocation points needed to accurately resolve the eigenfunctions $\tilde{p}(y)$ and also simplifies the application of boundary conditions.

In the present study, the linear stability problem is solved in a finite domain $0 \leq y \leq y_\infty$, where y_∞ is taken large enough to consider $\tilde{p}$ constant. In addition, the function $\tilde{p}$ is expanded as a sum of Lagrange polynomials based on the Gauss - Lobatto points. A transformation from the computational domain $-1 \leq \xi \leq 1$ to the physical domain $0 \leq y \leq y_\infty$ is thus introduced through the following mapping,

$$y = \frac{L_1(1+\xi)}{L_2 - \xi} \tag{14}$$

where the two stretching coefficients L_1 and L_2 are given by [28]

$$L_1 = \frac{y_l y_\infty}{y_\infty - 2y_l} \qquad L_2 = 1 + \frac{2L_1}{y_\infty}$$

This coordinate transformation clusters grid points near the boundary $y = 0$ and distributes half of them in the interval $0 \leq y \leq y_l$.

The generalized Rayleigh equation (2) exhibits a singularity at the locations $y = y_c$ such that $k\bar{u}(y_c) - \omega = 0$. The collocation method is particularly sensitive to their proximity to the computational domain. The closer the singularities are, the slower the convergence will be, and for critical points on the real axis, the numerical approximation does not hold anymore. In the context of the spatial stability analysis of this work, the singularity y_c inhibits the computation of neutral and damped modes. As proposed by Boyd [29] and Gill & Sneddon [30], a mapping in the complex plane can be introduced to bypass the singularities. The new contour must pass below the real axis when $d\bar{u}(y)/dy$ is positive and above the real axis when $d\bar{u}(y)/dy$ is negative

ω	k ref.[31]	k present	Error
0.1	$0.270462 - 0.206506i$	$0.270462 - 0.206510i$	$\approx 10^{-6}$
0.8	$1.449709 - 0.134110i$	$1.449709 - 0.134109i$	$< 10^{-6}$
1.2	$1.871369 - 0.029338i$	$1.871369 - 0.029340i$	$\approx 10^{-6}$
$1.\bar{3}$	$2.000000 - 0.000000i$	$2.000000 - 0.000000i$	$< 10^{-6}$

Table 1 – Incompressible stability of the Bickley jet : wavenumber k as a function of the frequency ω for the anti-symmetric mode.

ω	k ref[31]	k present	Error
0.2	$0.241420 - 0.043023i$	$0.241420 - 0.043023i$	$< 10^{-6}$
0.6	$0.901124 - 0.026220i$	$0.901124 - 0.026220i$	$< 10^{-6}$
$0.\bar{6}$	$1.000000 - 0.000000i$	$1.000000 - 0.000000i$	$< 10^{-6}$

Table 2 – Incompressible stability of the Bicklet jet : wavenumber k as a function of the frequency ω for the symmetric mode.

[29, 30, 4]. In this work, the following mapping is chosen

$$z = y + i\delta(1 - \xi^2) \tag{15}$$

where δ is a real factor controlling the distance from the real axis. In practice, one could chose δ so that the distance of the new contour from the critical point is sufficient to guarantee the convergence of the algorithm, but it is also necessary to pay attention to other singularities eventually induced by the base flow.

Equation (10) can be recast in the form an a nonlinear eigenvalue problem

$$(m_0 + km_1 + k^2 m_2 + k^3 m_3 + \beta k m_4 + \beta m_5)\tilde{p} = 0$$

where the m_i's represent differential operators. This equation is transformed into an algebraic problem by replacing the derivatives of $\tilde{p}$ with the differentiation matrices with respect to the variable z, and the functions $\bar{u}$ and $\bar{\rho}$ with diagonal matrices whose terms represents their values at the collocation points. One finally finds a non linear eigenvalue problem $\mathcal{M}(k, \omega, M_j)\tilde{\mathbf{p}} = \mathbf{0}$ of the form

$$\begin{aligned}\mathcal{M}(k,\omega,M_j) &= \sum_{i=0}^{3} k^i \mathcal{M}_i(\omega, M_j) + \beta k \mathcal{M}_4(\omega, M_j) \; + \\ &+ \; \beta \mathcal{M}_5(\omega, M_j)\end{aligned} \tag{16}$$

Boundary conditions are enforced by replacing the first and last lines of the system with equations (11), and (12) or (13). The present eigenvalue problem is nonlinear and non polynomial in k, which does not allow the use of a linearized form [25, 16, 27, 26, 4, 28]. An iterative procedure must be used. Here, the new formulation of the generalized Rayleigh equation (16) is solved through the method of successive linearization proposed by Ruhe [?]. Starting with an approximation $k^{(0)}$ of k, a correction $h^{(0)}$ to $k^{(0)}$ is sought to satisfy $\mathcal{M}(k^{(0)} + h^{(0)})\tilde{\mathbf{p}} = \mathbf{0}$. By using Taylor's formula, one can write

$$\mathcal{M}(k^{(0)} + h^{(0)})\tilde{\mathbf{p}} \simeq \left[\mathcal{M}(k^{(0)}) + h^{(0)}\mathcal{M}'(k^{(0)})\right]\tilde{\mathbf{p}}$$

where the symbol $'$ indicates the derivative with respect to k. Finally, the expression

$$\left[\mathcal{M}(k^{(0)}) + h^{(0)}\mathcal{M}'(k^{(0)})\right]\tilde{\mathbf{p}} = \mathbf{0}$$

represents a generalized linear eigenvalue problem in the unknown $h^{(0)}$ and can be solved through a standard $\mathcal{QZ}$ algorithm. Due to the truncation of the series, the correction $h^{(0)}$ is however not exact, and a sequence is then built where $h^{(n)}$ is chosen as the absolutely smallest eigenvalue. The convergence is quadratic and at the end, for a relative error less than $\epsilon = 10^{-6}$, the initial non linear system $\mathcal{M}(k)\tilde{\mathbf{p}} = \mathbf{0}$ becomes equivalent to the following linear eigenvalue problem

$$\mathcal{M}(k^{(n)})\tilde{\mathbf{p}} = -h^{(n)}\mathcal{M}'(k^{(n)})\tilde{\mathbf{p}}$$

Thus, the sought eigenfunction corresponds to the one associated with the absolutely smallest eigenvalue $h^{(n)}$. In summary, for ω and M_j, application of this algorithm allows to find only one eigenvalue k, the closest to a certain initial guess.

4 Results

The numerical approach presented in the previous section is first applied to an academic incompressible case, and our numerical results are compared to the calculations by Betchov & Criminale [31]. The velocity profile of the Bickley jet, $\bar{u}(y) = 1/\cosh^2(y)$, has two critical points $y_c = \pm\cosh^{-1}(\sqrt{3/2})$ corresponding to the two inflexion points. Consequently, two neutral modes can be found, the symmetric mode $(\omega, k) = (2/3, 1)$, and the antisymmetric mode $(\omega, k) = (4/3, 2)$. The present calculations are obtained with $N = 200$ collocation points, and with the parameters $L_1 = 6$, $L_2 = 1.001$ and $\delta = 1$ for the contour (15). Our results as well as those of Betchov & Criminale [31] are listed in Tables 1 and 2. The relative error is always less than or equal to 10^{-6}. The eigenvalues have also been calculated without using the complex mapping ($\delta = 0$). For frequencies

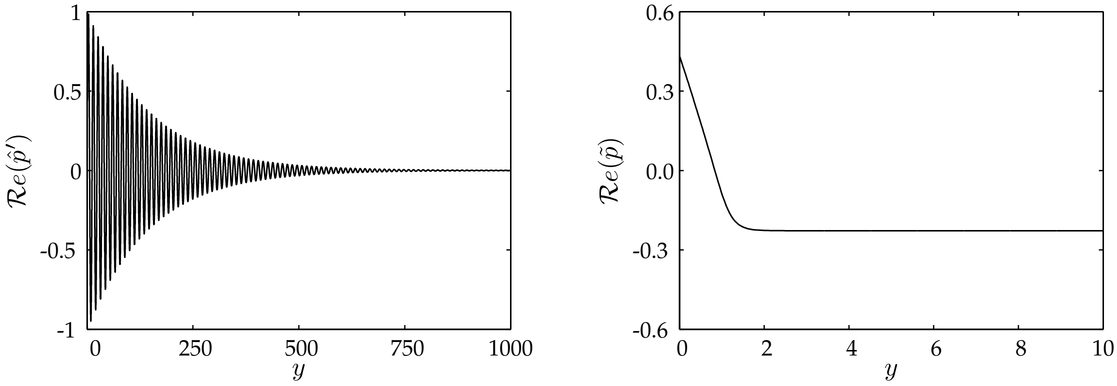

FIGURE 4 – Kelvin - Helmholtz symmetric mode : eigenfunctions $\tilde{p}$ and $\hat{p}'$ for $\delta_\theta = 1/8$, $T_\infty = 1$, $M_j = 3$ and $\omega = 0.2$.

ω much lower than the neutral one, the relative error has been found to be lower than 10^{-6}. In this case, the singularities are sufficiently far from the real axis so that one can solve the problem without using a complex transformation. Nevertheless, a complex mapping is required for angular frequencies near the neutral one, for computing the neutral and damped modes.

A detailed analysis of the jet profile

$$\bar{u}(y) = \frac{1}{2}\left\{1 + \tanh\left[\frac{1}{4\delta_\theta}\left(\frac{1}{y} - y\right)\right]\right\} \quad y > 0$$

for different Mach numbers can be found in [14]. Here, a small insight into the numerical performance of the proposed formulation is provided. All the variables are still dimensionless. In particular, the momentum thickness δ_θ

$$\delta_\theta = \int_0^\infty \bar{u}(y)\,[1 - \bar{u}(y)]\,dy$$

is normalized by the half width of the jet, defined as the distance from the axis at which the velocity $\bar{u}$ is equal to the half of the speed on the axis. The density profile is calculated with the the Crocco - Busemann relation,

$$\frac{1}{\bar{\rho}(y)} = T_\infty - (T_\infty - 1)\,\bar{u}(y) + M_j^2\frac{\gamma - 1}{2}\,[1 - \bar{u}(y)]\,\bar{u}(y)$$

where γ and T_∞ represent respectively the ratio of specific heats and the ratio between the ambient temperature and the nominal jet temperature. Figure 4 displays the eigenfunctions $\hat{p}'$ and $\tilde{p}$ relative to the radiating Kelvin-Helmholtz symmetric mode for $\delta_\theta = 1/8$, $T_\infty = 1$, $M_j = 3$ and $\omega = 0.2$. The support of $\hat{p}'$ is very large, $0 \leq y \leq 1000$, whereas $\tilde{p}$ is nearly constant for $y > 2$ since $\hat{p}'$ rapidly tends to its asymptotic form just outside the jet flow region. The importance of the change of variable is clearly demonstrated in this case. The calculation for $\tilde{p}$ has been carried out with $N = 200$, $L_1 = 3$, $L_2 = 1.001$ and $\delta = 0.3$. Instead, a direct computation of the eigenfunction $\hat{p}'$ would have required more than 2000 grid points.

5 Conclusion

A reformulation of the generalized Rayleigh equation is proposed, to efficiently compute the spatial stability of high-speed flows characterized by radiating Kelvin-Helmholtz and supersonic acoustic modes. The oscillating part of eigenfunctions is removed through a change of variable, which allows a reduced computational domain to be considered. Radiation boundary conditions are also explicitly enforced. To demonstrate the numerical robustness of this approach, the stability of a two-dimensional supersonic jet has been revisited for high Mach numbers.

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