Gravity waves in the atmosphere

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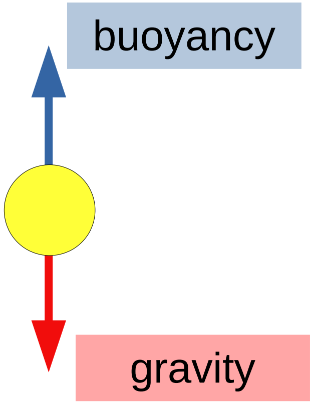
Outline

- Introduction: What are gravity waves?
- General characteristics (linear wave theory)
- Impacts on the atmosphere
- Gravity waves and general circulation models

What is a gravity wave?

- The term «gravity wave» refers to an oscillatory motion with gravity and buoyancy as the restoring forces.

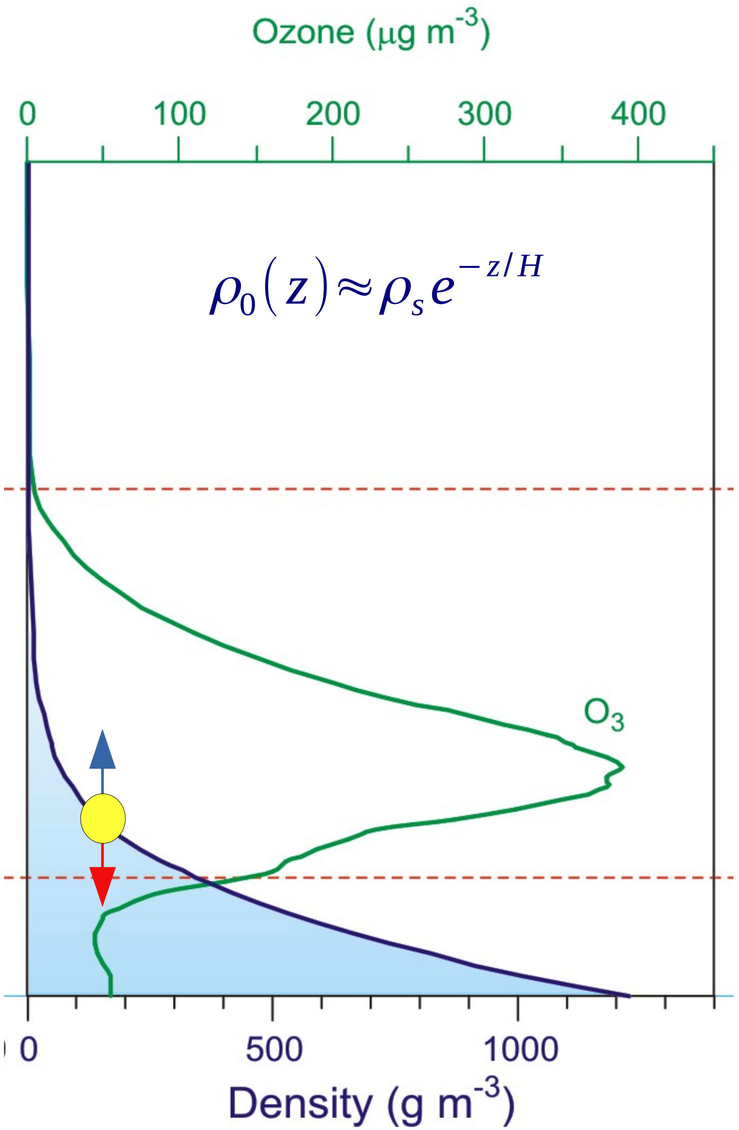


- The interface between two fluids with different densities is well suited to maintain gravity waves

What is a gravity wave?

Wind waves on the ocean surface are a type of gravity waves

Vertical stratification of the atmosphere



A stable stratification (density decreasing with height) is key to maintain gravity waves within the atmosphere

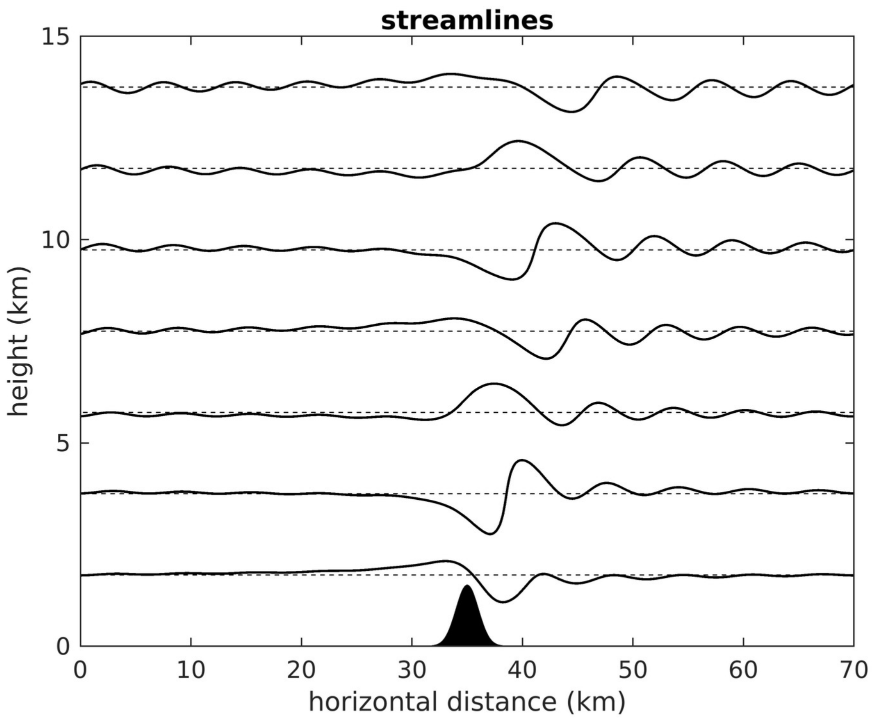
N: Frequency of vertically oscillating air parcels

$$N^2 = -\frac{g}{\rho_0}\frac{d\rho_0}{dz}$$

Buoyancy frequency

Processes that generate vertical displacements in a stable atmosphere are primary sources of gravity waves:

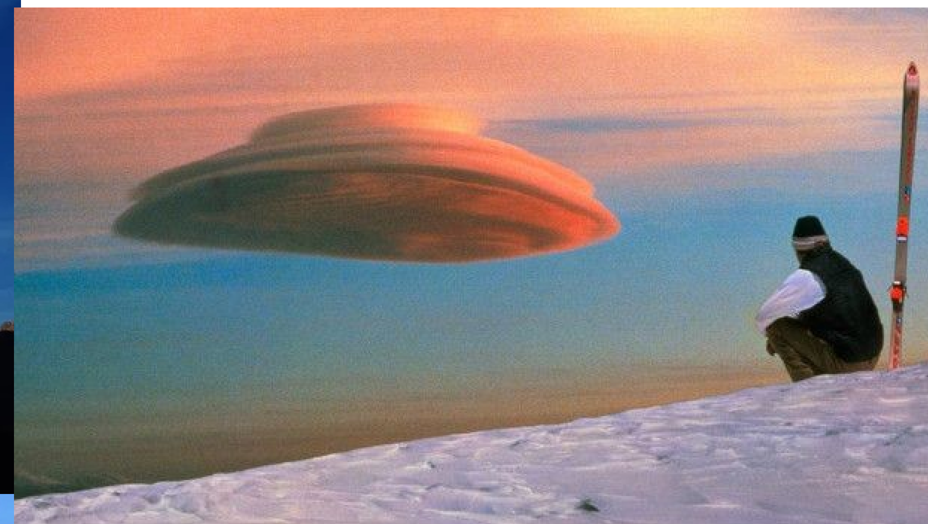
- Flow over topography
- Flow over convection (thunderstorms, tropical storms, ...)
- Unbalanced flows in mid-high latitudes (e.g. frontal systems)

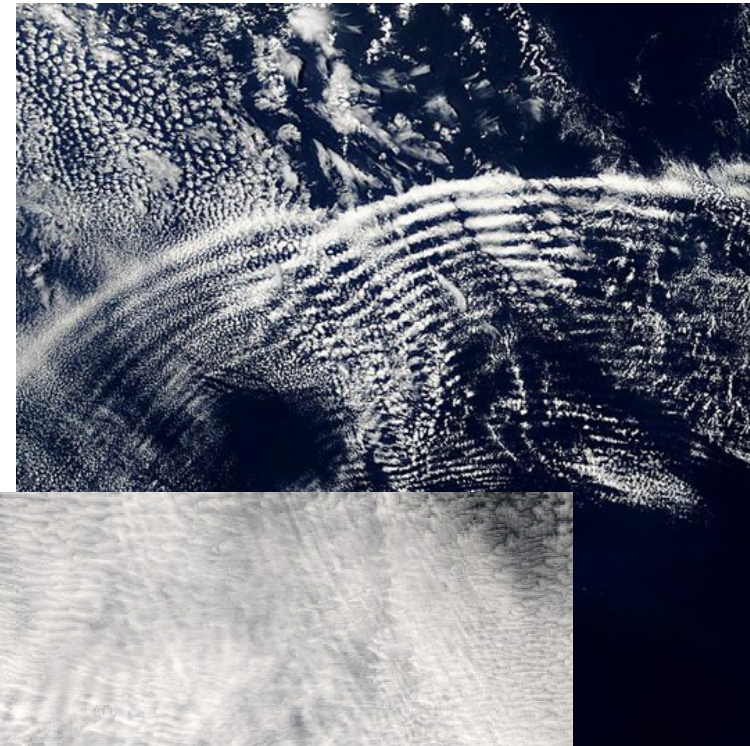


Gravity waves in the atmosphere

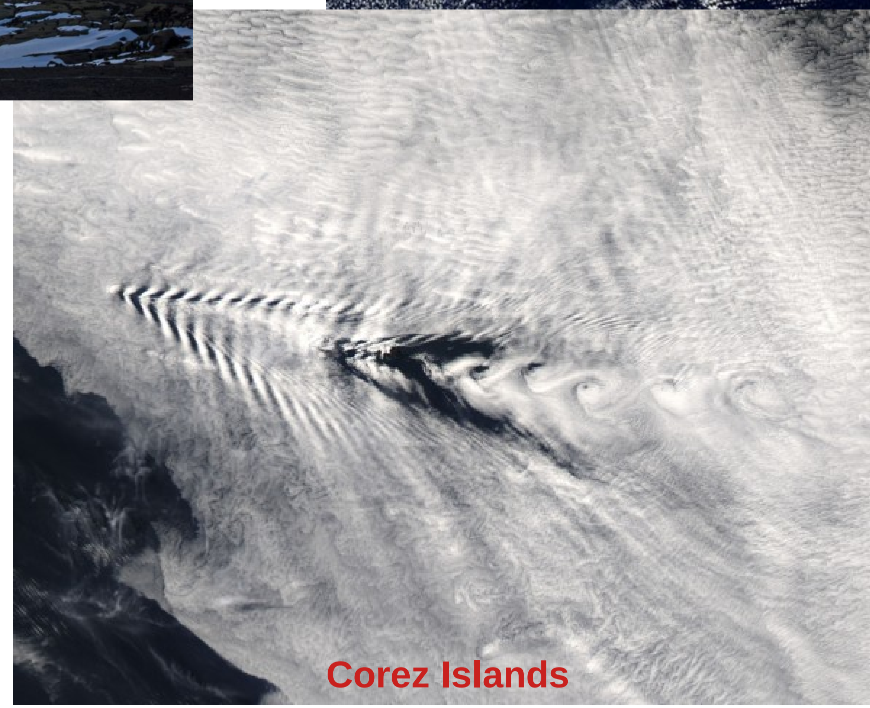

Credit: Doug Bone / BAS Photo Database

Lenticular clouds over crests of orographic (mountain) gravity waves

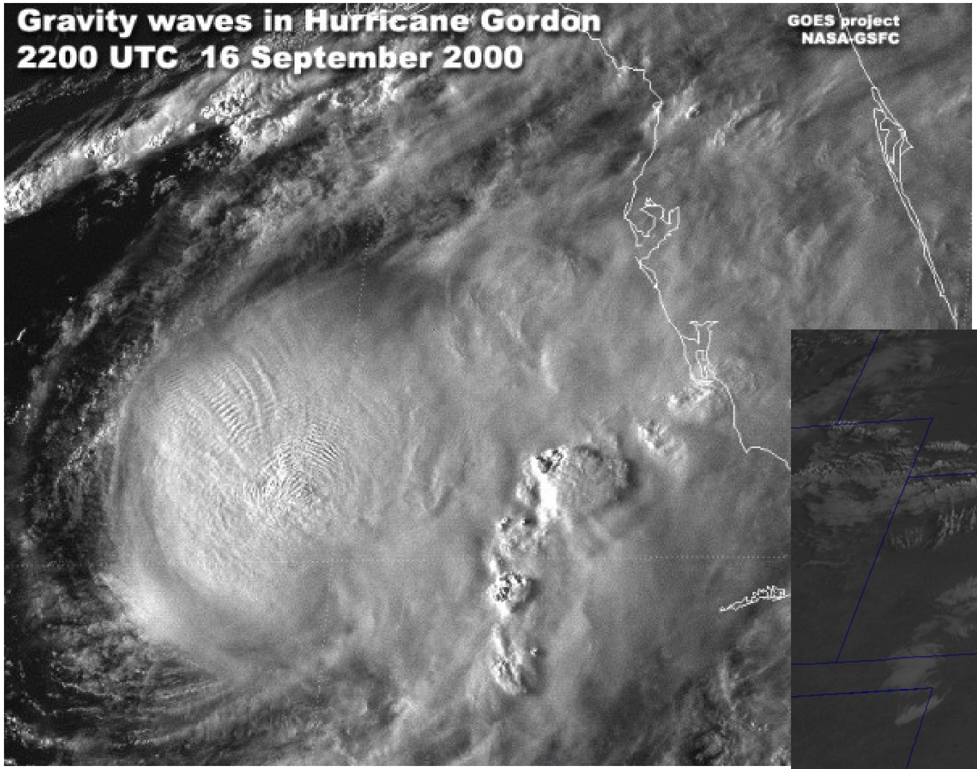


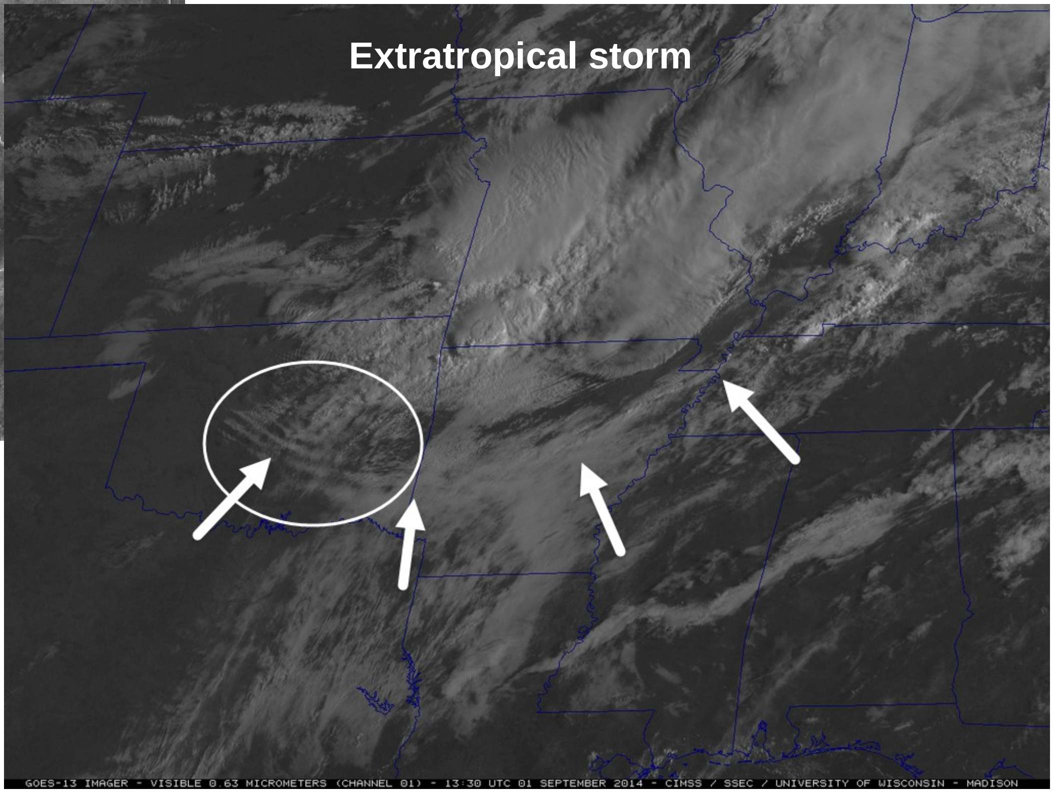


Orographic gravity waves

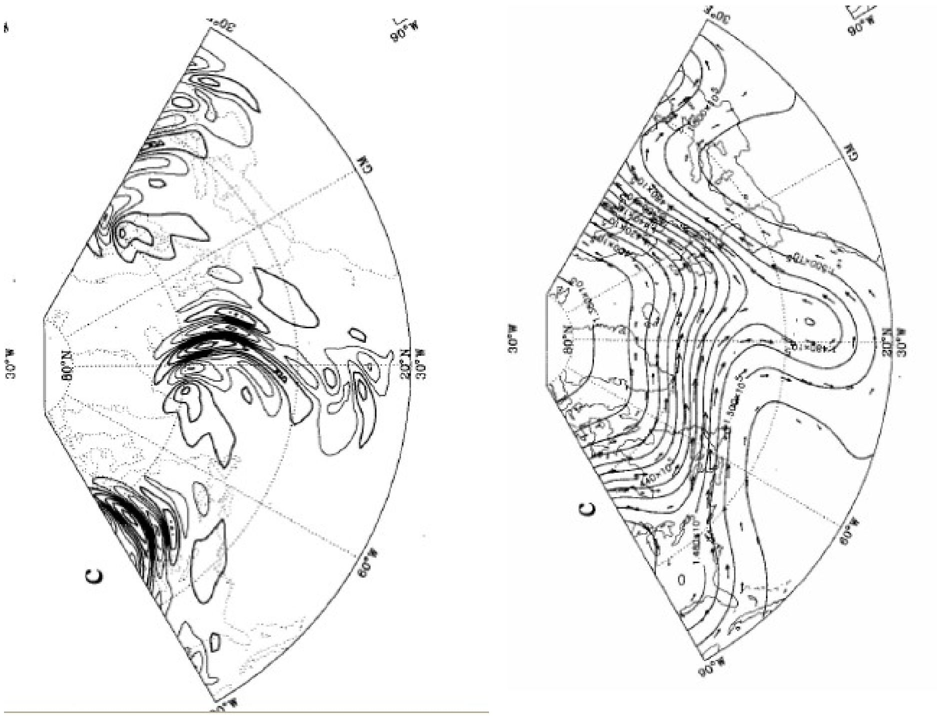


Convection / tropical and extratropical storms

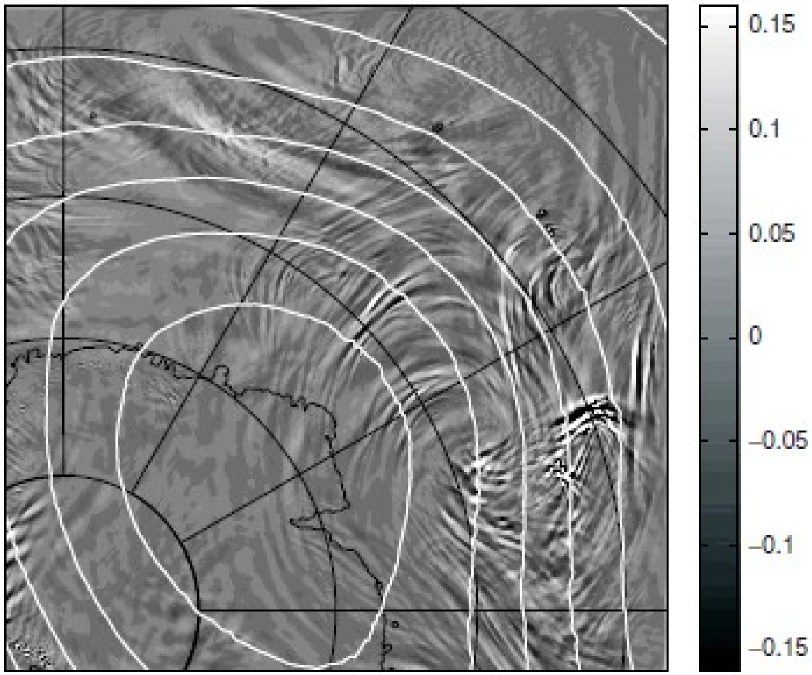




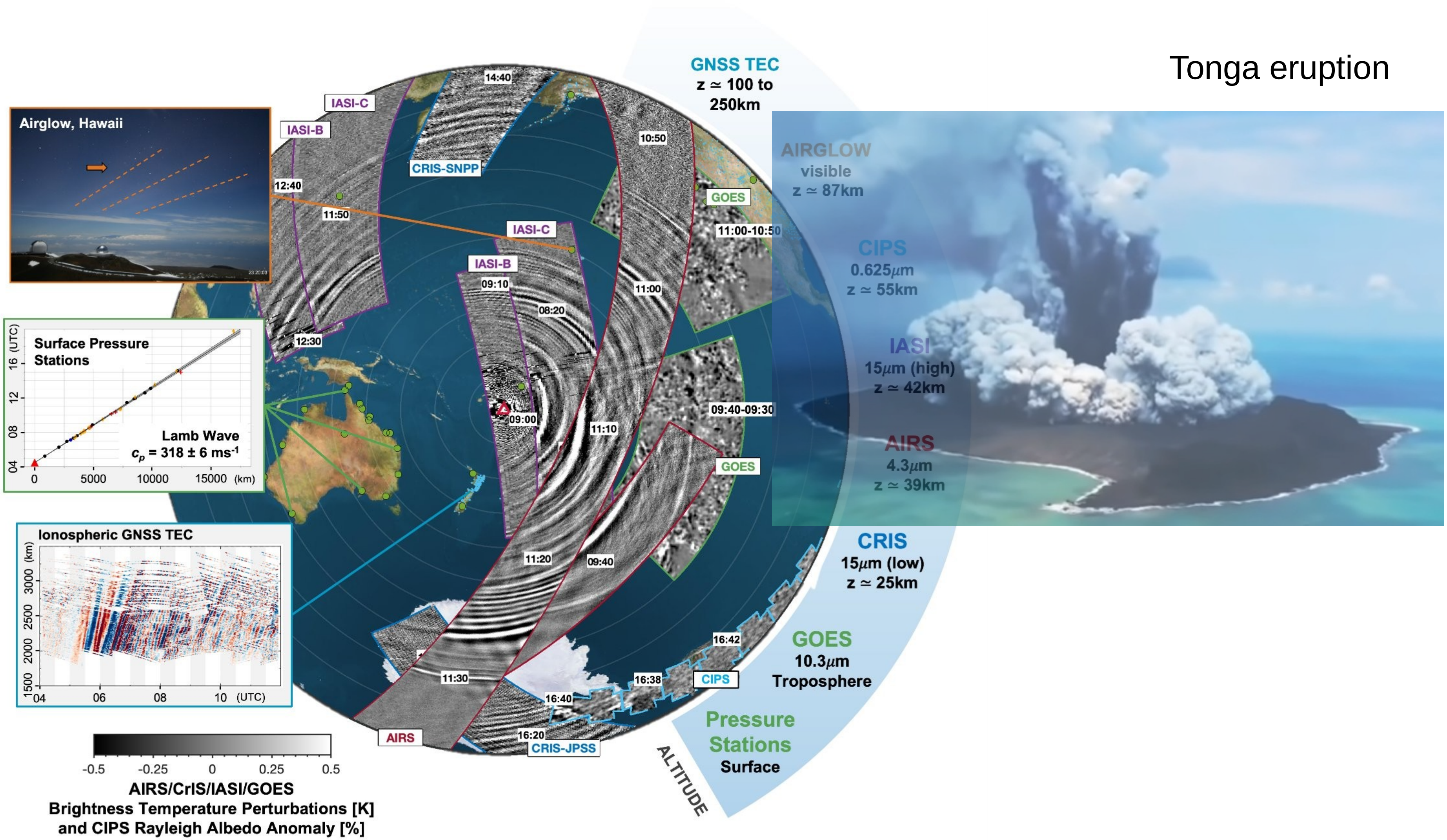
Unbalanced flows in the midlatitudes



O'Sullivan and Dunkerton, 1995 *J Geophys Res*



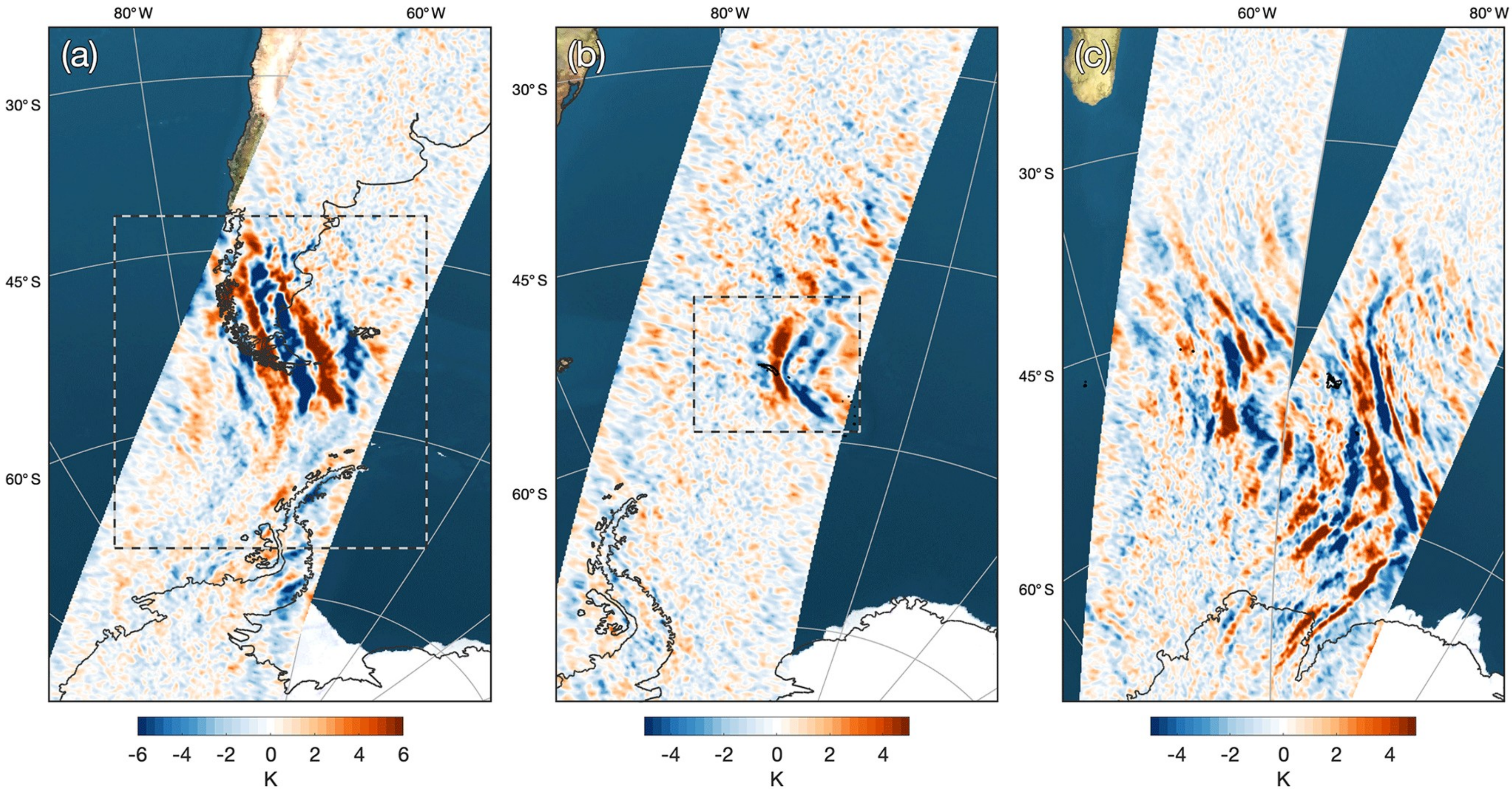
Plougonven et al., 2013 *Q J R Meteorol Soc*



Wright et al 2022 EOS

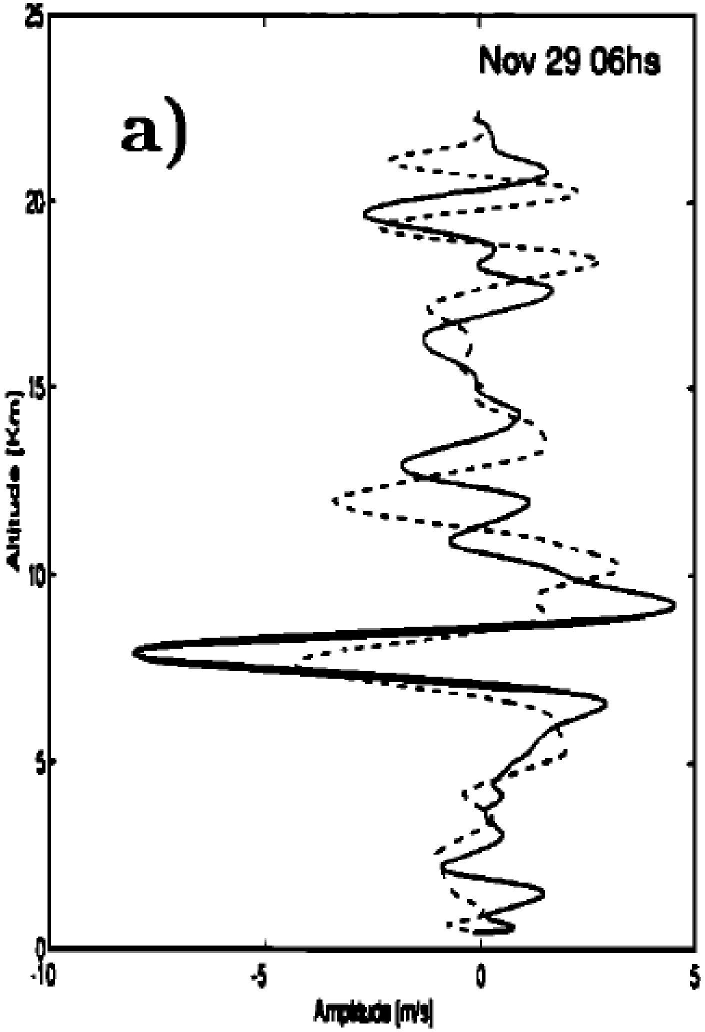
Observations of gravity waves

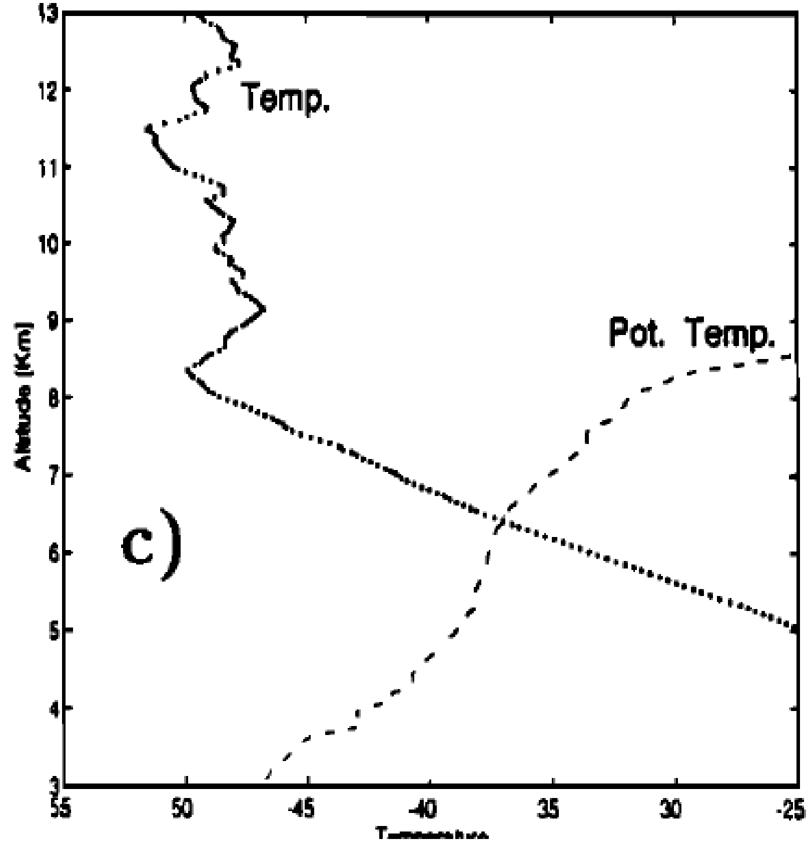
Temperature perturbations, satellite obs (z = 40 km)



Hindley et al. 2019 Atmos Chem Phys

Wind and temperature perturbations, soundings





Scavuzzo et al. 1998 J Geophys Res

General characteristics of gravity waves (linear wave theory)

Primitive equations in the atmosphere:

$$\left.\begin{aligned} \frac{D\vec{v}}{Dt} + 2\vec{\Omega} \times \vec{v} &= -\frac{\vec{\nabla} p}{\rho} - \vec{\nabla}\phi \\ \frac{D\theta}{Dt} &= Q \\ \frac{D\rho}{Dt} + \rho \vec{\nabla} \cdot \vec{v} &= 0 \end{aligned}\right\}$$

$\Rightarrow$

Governing equations for adiabatic, inviscid, non-rotating flow in the Boussinesq approximation:

$$\left.\begin{aligned} \frac{Du}{Dt} &= -\frac{1}{\rho}\frac{\partial p}{\partial x} \\ \frac{Dv}{Dt} &= -\frac{1}{\rho}\frac{\partial p}{\partial y} \\ \frac{Dw}{Dt} &= -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \\ \frac{D\theta}{Dt} &= 0 \\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} &= 0 \end{aligned}\right\}$$

* *Material derivative:*

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

Linear equations

- Decompose variables: Basic state (zero order) + (small, first order) perturbations $X = \bar{X} + X'$
- Assume small perturbations → multiplication of *prime* variables is second order and negligible.
- Linearized equations in the and X-Z plane around an basic flow at rest: $\bar{u} = \bar{w} = 0$, constant stratification (N^2 = const)

$$
\left.
\begin{aligned}
\frac{\partial u'}{\partial t} &= -\frac{1}{\rho_0}\frac{\partial p'}{\partial x} \\
\frac{\partial w'}{\partial t} &= -\frac{1}{\rho_0}\frac{\partial p'}{\partial z} + b' \\
\frac{\partial b'}{\partial t} + N^2 w' &= 0 \\
\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} &= 0,
\end{aligned}
\right\}
$$

Buoyancy perturbation:

$$b' = g\theta'/\bar{\theta} = -g\rho'/\rho_0$$

Dispersion relation

- We seek wave solutions of the form

$$u'(x,z,t) = \hat{u}(z)e^{i(kx-\omega t)},\ w'(x,z,t) = \hat{w}(z)e^{i(kx-\omega t)}, \ldots$$

- Substituing in our set of linearized eqs, we obtain:

Taylor-Goldstein equation

$$\boxed{\frac{d^2\hat{w}}{dz^2} + m^2\hat{w} = 0}$$

with

$$m^2 = k^2\frac{N^2}{\omega^2}$$

- If $m^2 < 0$ → exponentially growing or decaying solutions
- If $m^2 > 0$ → wave solutions with m the vertical wavenumber

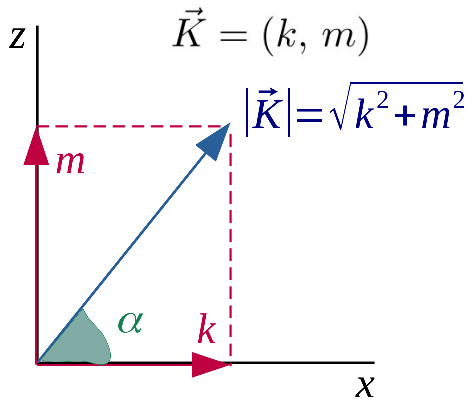
$$w'(x,z,t) = \hat{w}e^{i(kx+mz-\omega t)}$$

The dispersion relation:

$$\boxed{\omega^2 = N^2\frac{k^2}{k^2+m^2}}$$

Dispersion relation

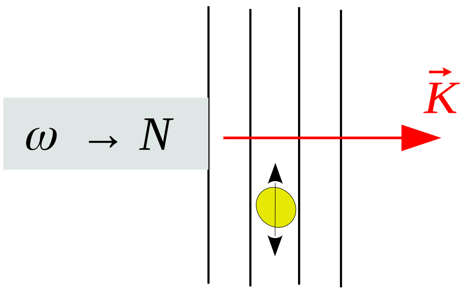
$$\boxed{\omega^2 = N^2 \frac{k^2}{k^2 + m^2}}$$

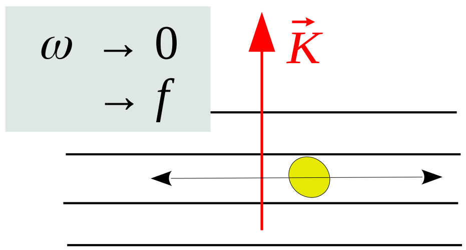


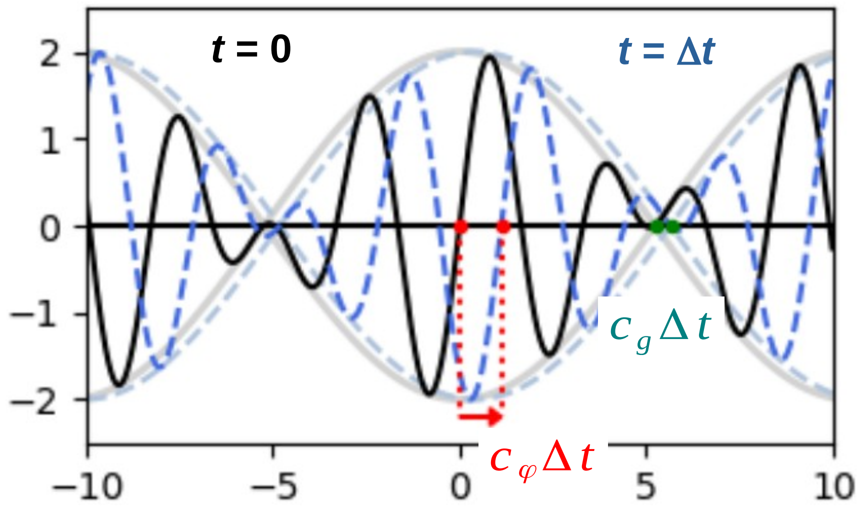
- The frequency is determined by the inclination of the wavenumber relative to the vertical, and is independent of the norm of the wavenumber:

$$\omega^2 = N^2 \cos^2 \alpha$$

$0 < |\omega| < N$







Phase speed*:

Speed of propagation of wave fronts, in the direction of the wavenumber vector $\vec{K}$

$$c_\varphi = \frac{\omega}{|\vec{K}|}\frac{\vec{K}}{|\vec{K}|} \qquad c_x = \frac{\omega}{k}, \quad c_z = \frac{\omega}{m}$$

Group velocity:

Velocity of propagation of wave *envelopes* or *packets*. It is also the velocity of propagation of energy

$$\vec{c}_g = \left(\frac{\partial\omega}{\partial k}, \frac{\partial\omega}{\partial m}\right) = \frac{N^2}{\omega}\frac{2km}{k^2+m^2}(m, -k)$$

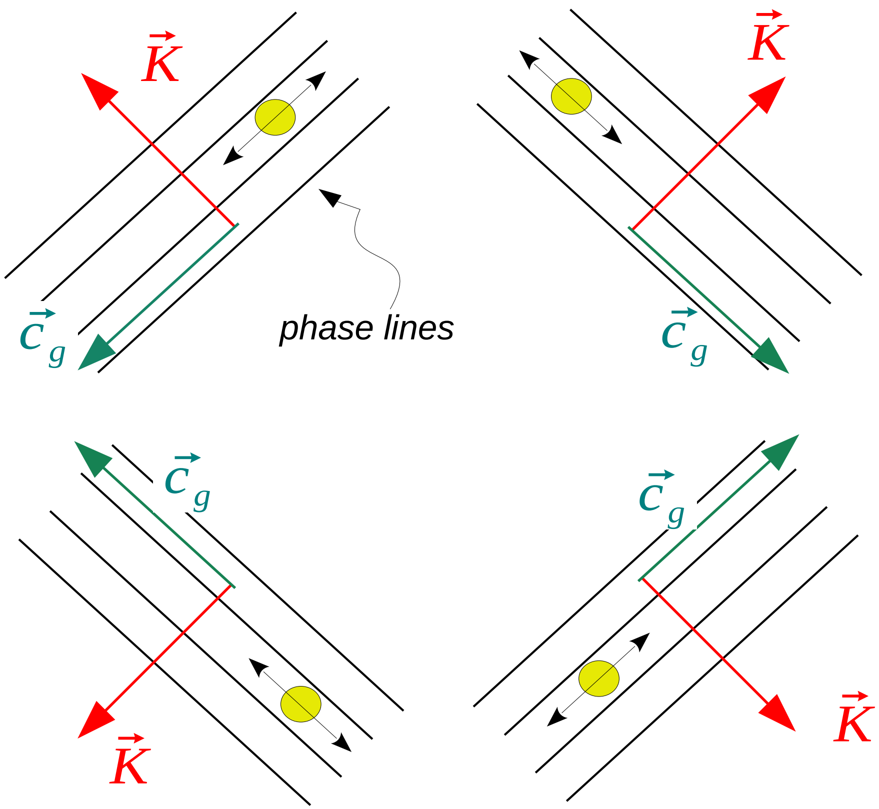
**Phase speed is not a vector, its (x,z) components are not the speeds of the wave in the respective directions (i.e. c_x and c_z).*

Phase speed and group velocity

$$\boxed{\vec{K} \cdot \vec{c}_g = 0}$$

Energy propagates along phase lines, but perpendicular to phase propagation

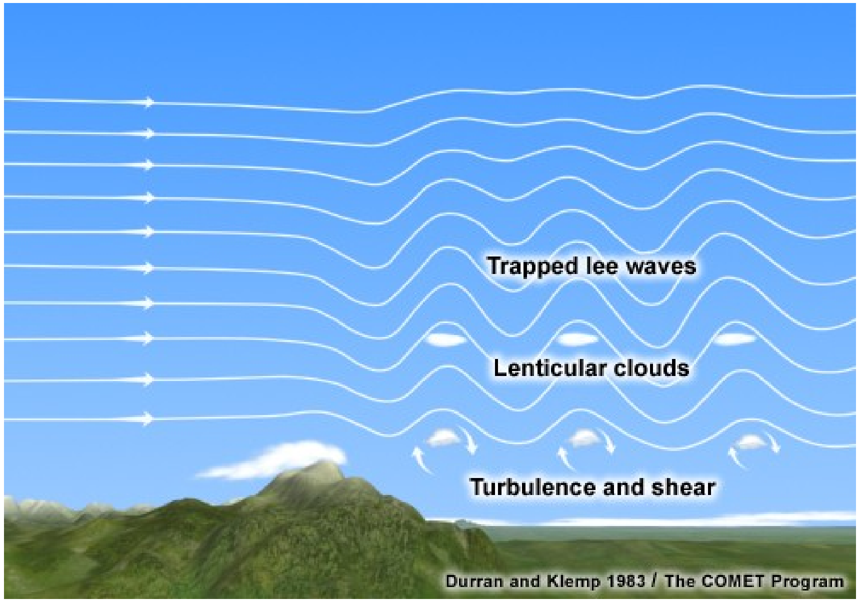
If the phase moves upward, energy (and wave packets) propagates downward!

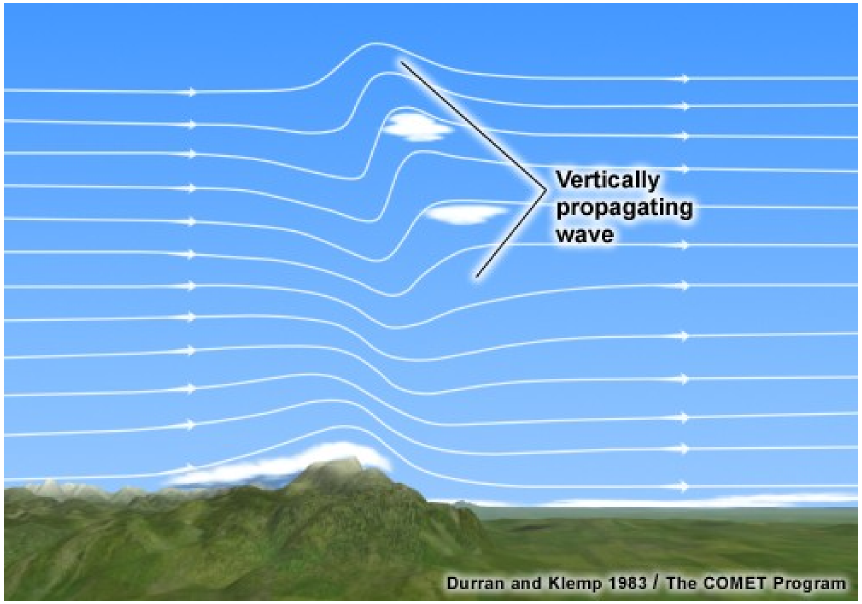


$$\vec{c}_g = \frac{N^2}{\omega} 2 \sin\alpha \cos\alpha \,(m, -k)$$

If the wavevector is either vertical or horizontal, there is no transfer or energy

Vertically propagating waves have phase tilts with height

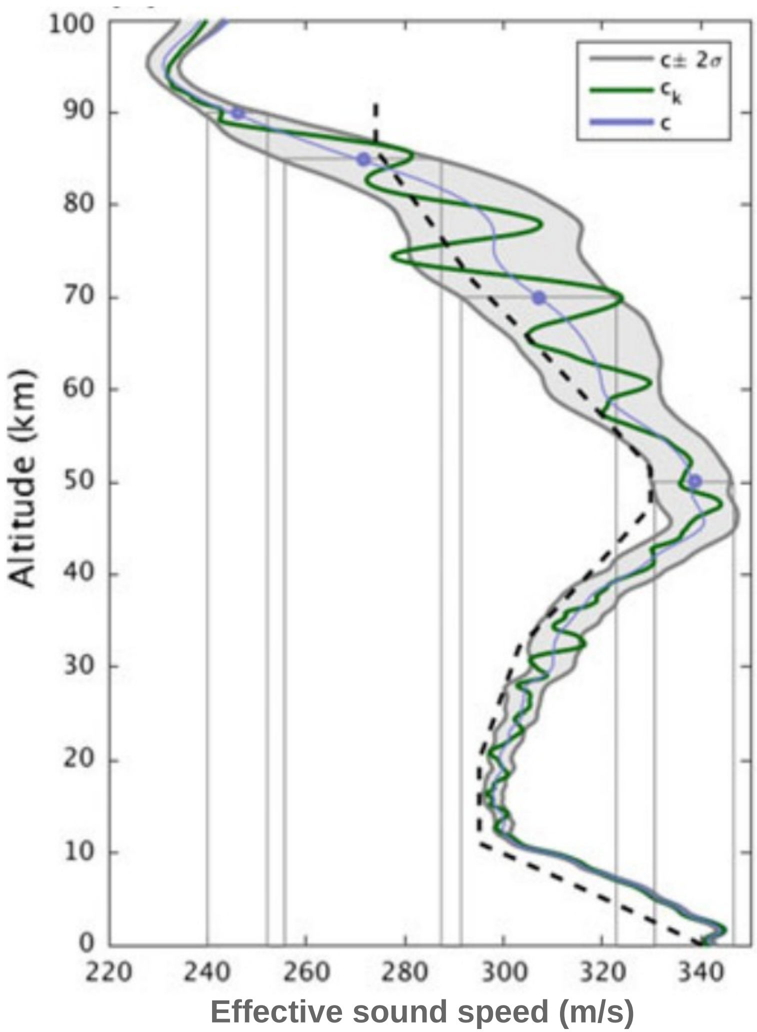




Why do we care about gravity waves?

Propagation of infrasound

- The effective infrasound propagation speed depends on the temperature structure of the atmosphere
- T perturbations due to the presence of gravity waves modify the vertical T structure



Cugnet et al 2019
https://doi.org/10.1007/978-3-319-75140-5_27

Wave mean-flow interaction / critical levels

From energetics considerations, the following expressions can be derived

$$(\bar{u} - c)\frac{\partial}{\partial z}\overbrace{\left(\rho_0 \overline{u'w'}\right)}^{\text{Momentum flux}} = 0 \qquad \boxed{\frac{\partial \bar{u}}{\partial t} = -\frac{1}{\rho_0}\frac{\partial}{\partial z}\left(\rho_0 \overline{u'w'}\right)}$$

Non-acceleration theorem

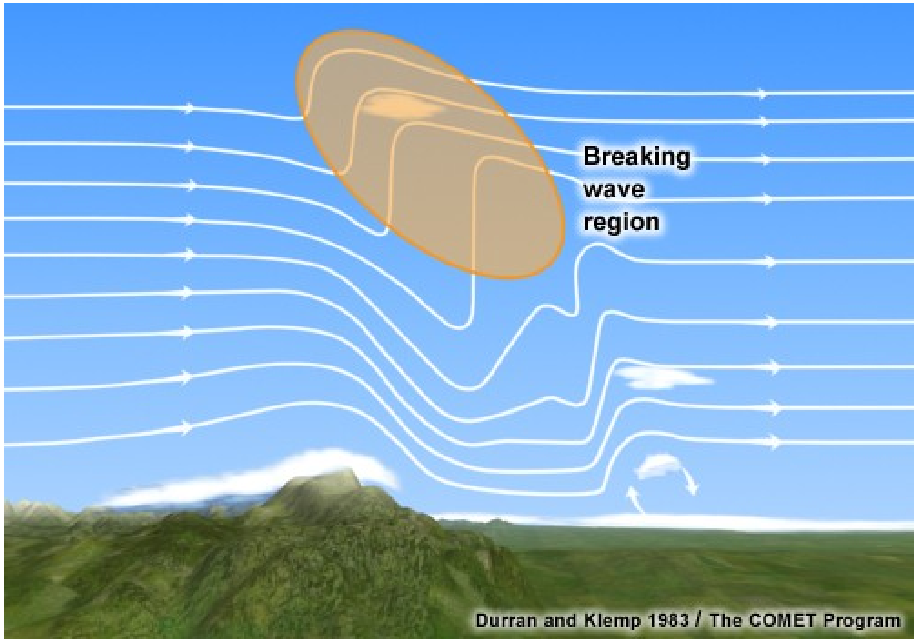
Gravity wave momentum flux is conserved in the vertical away from critical levels (where c = U)

Linear, adiabatic and frictionless waves do not interact with the mean flow even in second order, unless they hit a critical level.

Wave mean-flow interaction / wave breaking

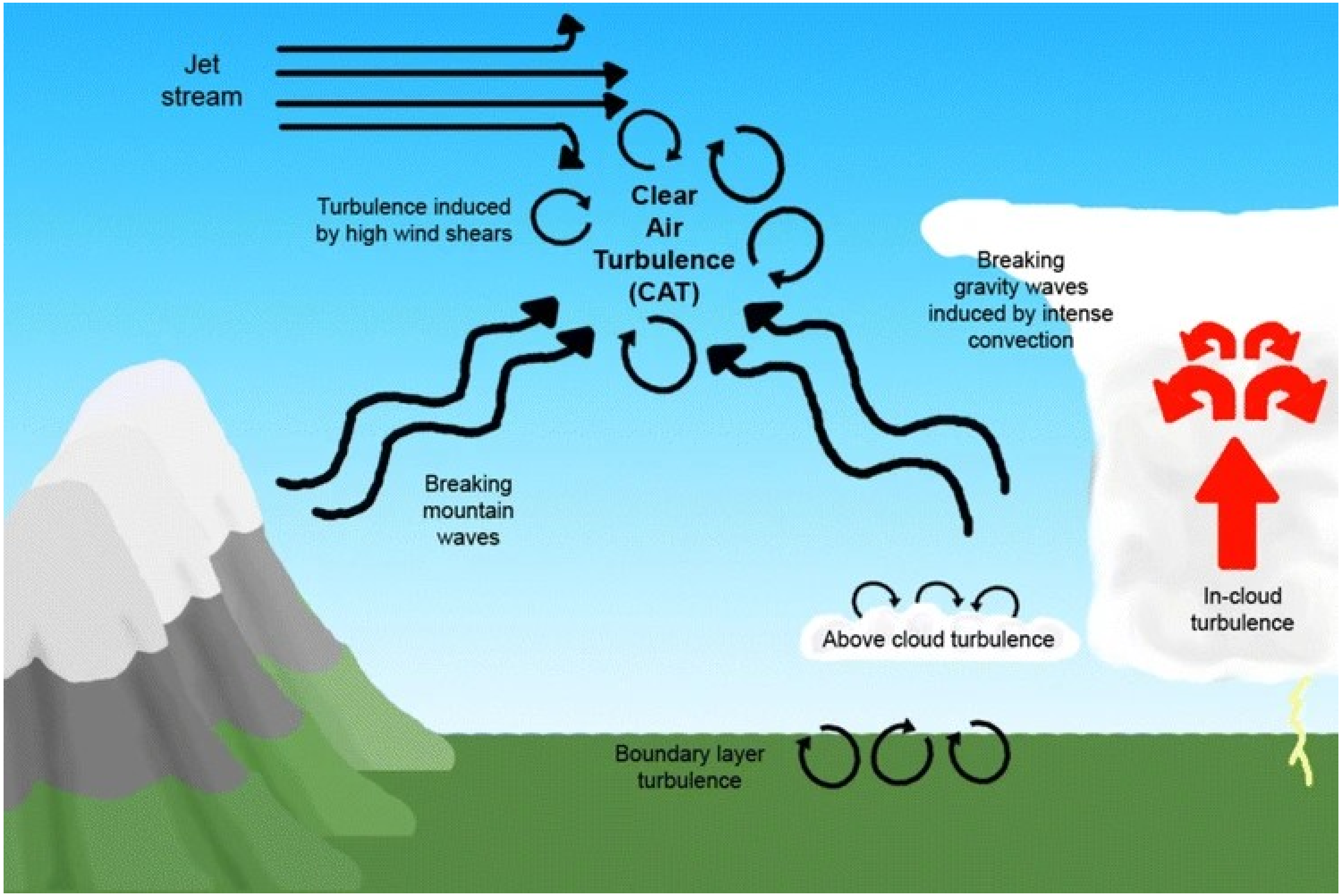
Wave amplitude grows as waves propagate upward, eventually leading to *convective instability* where

$$\frac{\partial \rho}{\partial z} = \frac{\partial(\rho_0 + \rho')}{\partial z} > 0$$



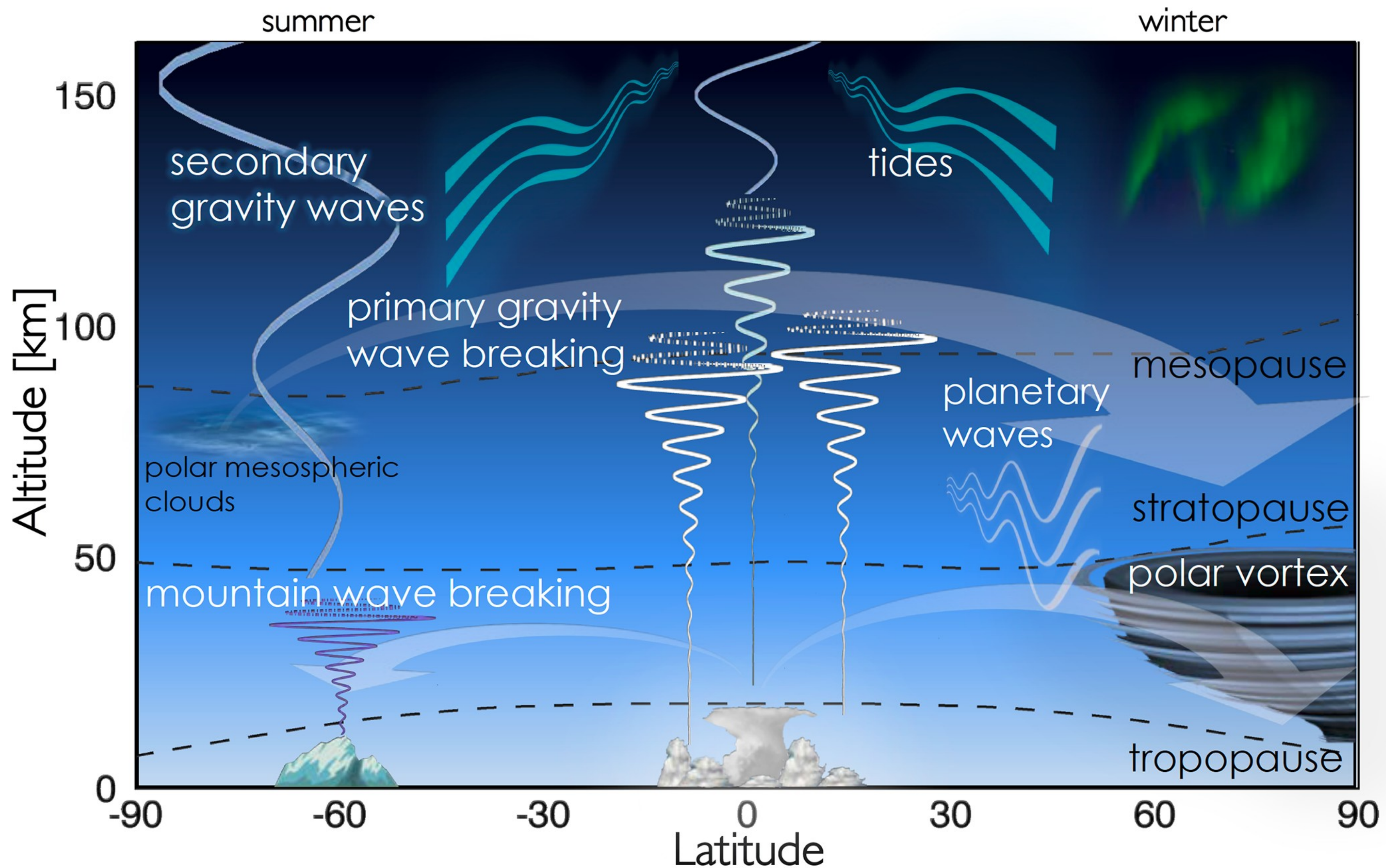
- Wave breaking limits the amplitude growth of gravity waves, and induces a change of momentum flux with height the exerts a drag on the background flow.

- Wave breaking also generates a cascade of energy leading to turbulence, which also modifies the basic flow! (and generates mixing of trace gasses)



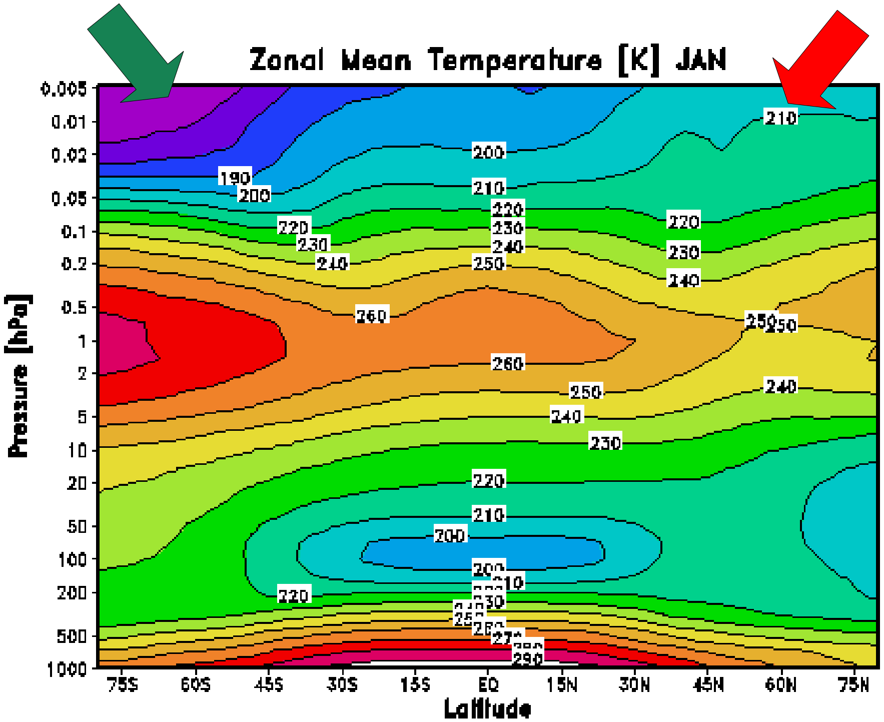
Storer et al. 2019 Pure Appl Geophys

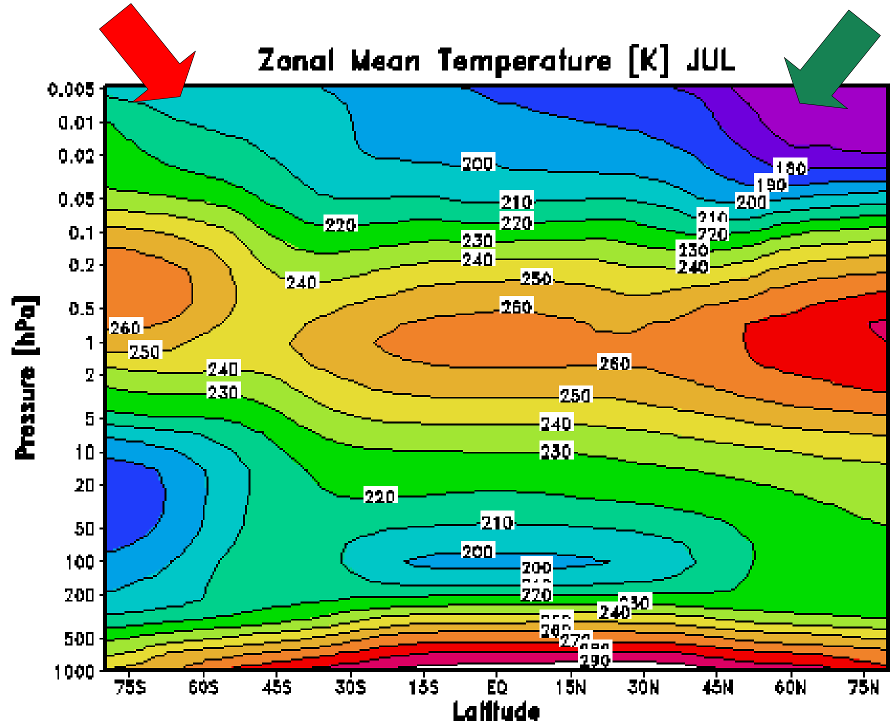
Impacts on the general circulation



McCormack et al. 2021 Atmos Chem Phys

Gravity waves maintain temperatures in the winter mesopause much warmer than in the summer mesopause



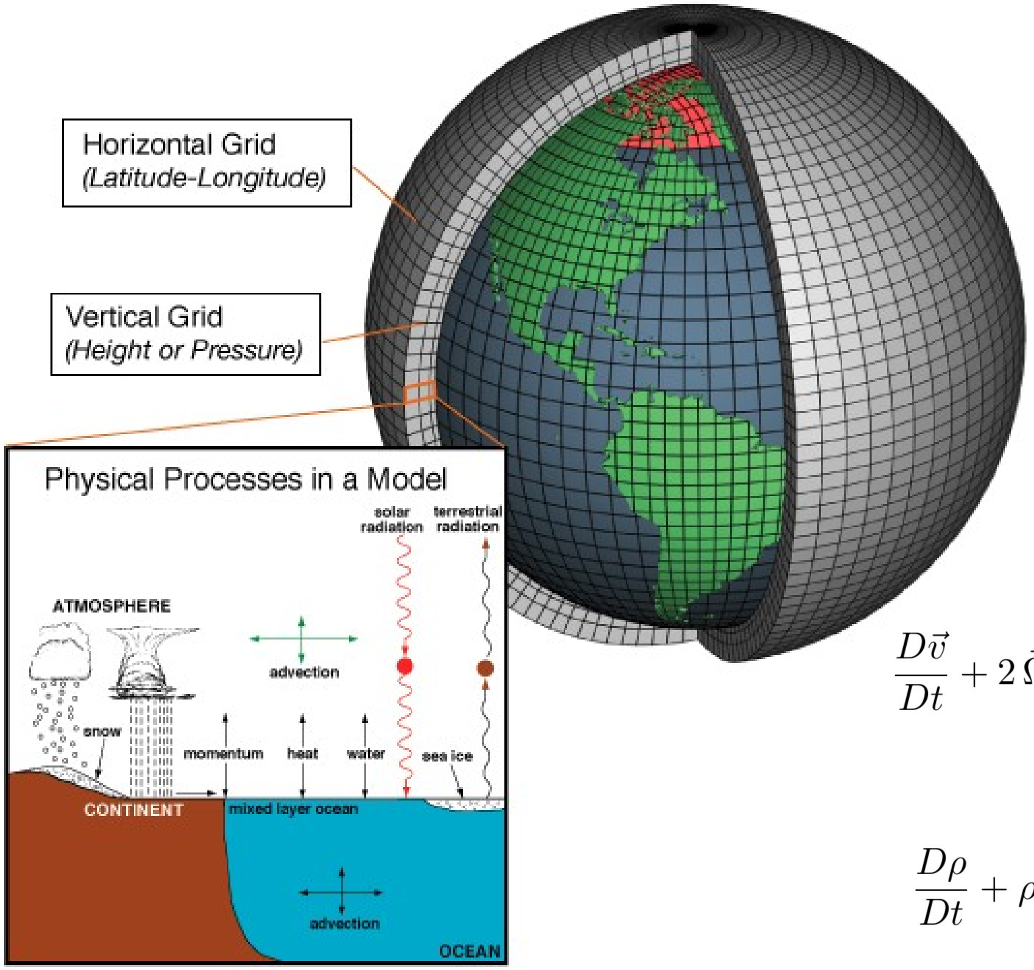


SPARC climatology – Randel et al 2004 J Clim

Gravity waves in general circulation models

General circulation models

- The primitive equations are numerically solved on a discrete grid
- Processes that occur in shorter spacial scales (including gravity waves) are parameterized → aka *model physics*

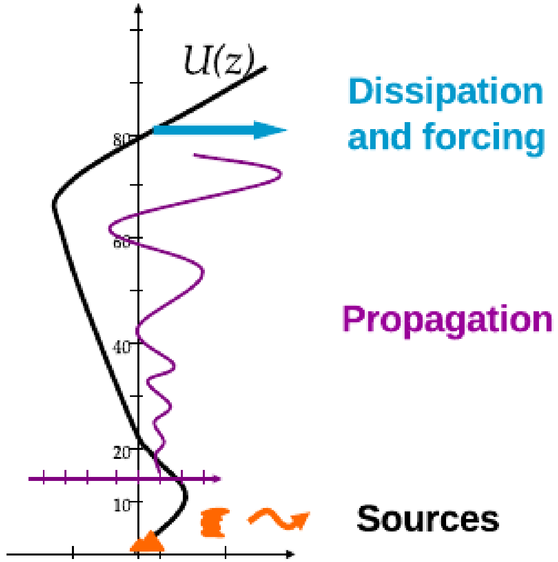


$$\frac{D\vec{v}}{Dt} + 2\vec{\Omega} \times \vec{v} = -\frac{\vec{\nabla} p}{\rho} - \vec{\nabla}\phi$$

$$\frac{D\theta}{Dt} = Q$$

$$\frac{D\rho}{Dt} + \rho\vec{\nabla} \cdot \vec{v} = 0$$

- Goal of GW parameterizations: represent the effects of *subgrid* gravity waves on the grid-scale dynamics of the model.



Courtesy of Riwal Plougonven

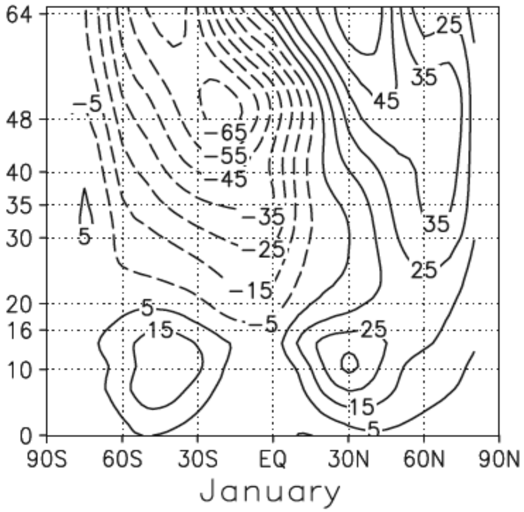
GW parameterizations include 3 components:

- Specification of GW sources
- Vertical propagation
- Description of the forcing from (and criteria for) GW dissipation

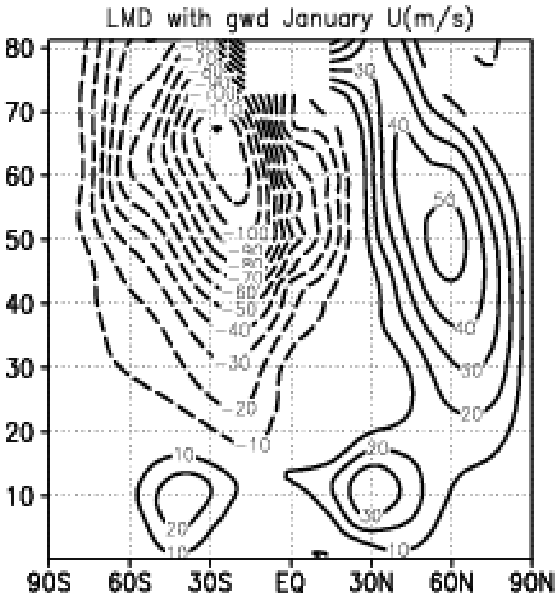
$$\frac{\partial \bar{u}}{\partial t} = -\frac{1}{\rho_0}\frac{\partial}{\partial z}\left(\rho_0 \overline{u'w'}\right)$$

Climatology of zonal mean zonal wind (U) in January:

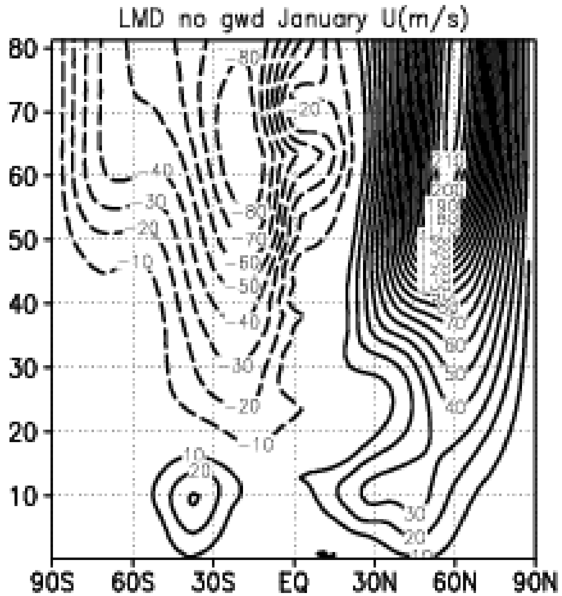
Observations



LMDz-GCM **with** GWs



LMDz-GCM **w/o** GWs



Lott and Millet 2010

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Textbooks:

- Nappo C. (2012): An introduction to atmospheric gravity waves. *Elsevier Science*, 400 pp.
- Encyclopedia “Sciences” – ISTE-Wiley *in preparation.*

Review papers:

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- Plougonven and Zhang (2014): Internal gravity waves from atmospheric jets and fronts. *Rev. Geophys.*, 52(1), 33-76.

Some classical papers:

- Eliassen and Palm (1962): On the transfer of energy in stationary mountain waves. *Geophys. Publ.*, 22, 1-23.
- Bretherton (1969): On the mean motion induced by internal gravity waves. *J. Fluid. Mech.*, 36, 785-803.