A brief overview in array processing for infrasonic sources

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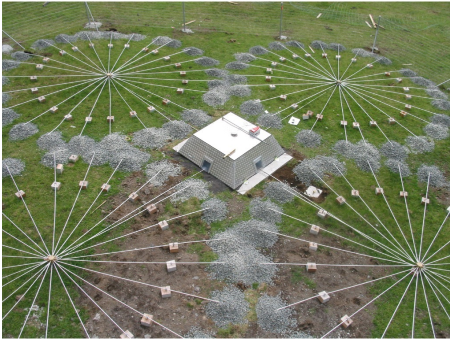
1. Array processing
2. Infrasound context
3. Numerical results/Comments

Sensor array

Array processing is based on a set of sensors. The sensors could be radio receivers, microphones, accelerometers, telescopes, etc.

International Monitoring System (IMS)



Very Large Telescope (VLT)

- The wave is propagating both in time and space
- In most cases, the signals of interest (SOI), unlike the noise, are spatially coherent.

Applications

- This research domain is very active for the last 50 years.
- Multiple applications :
  - Source geolocalisation,
  - Denoising acoustical signals,
  - Image improvment,
  - Geophysical layer analysis,
  - Focalisation in a direction of interest,
  - Source separation.

Physical aspects

- Acoustical vs electromagnetical propagation,
- Phase array vs Received Signal Strength (RSS),
- Spherical vs planar front wave,
- Narrow band vs wide band,
- Monosource vs multiple sources,
- Fixed sources vs mobile sources,
- Direct path vs multiple paths, even more absence of direct path

Only red points are considered in this short presentation in the context of infrasound, i.e. frequency under 10 Hz.

Geometrical aspects in infrasound

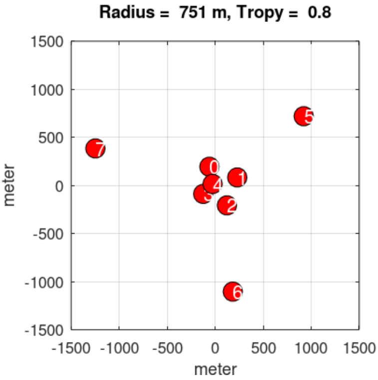


FIGURE – *IS 31*

- About less than a dozen of sensors,
- Aperture/radius : about 1000 m,
- Sensor relative locations : often almost isotropic

Main issues

In the context of infrasounds, the main issues are :

- Estimation of the source number (not presented here [20]),
- Detection : test of the noise-only hypothesis (denoted H_0),
- Estimation of the propagation parameters,
- Estimation of the SNRs,
- Determination of the confidence region of the DOA estimates,
- ROC curves and AUC of the test of H_0,
- Determination of the p-value expression of test of H_0,
- Loss of coherence,
- Data integrity : outliers, missing data, sensor failures detection,
- Array design.

Key parameters

- Signal to Noise Ratio (SNR) : larger the SNR better the performances,
- Observation time : longer the observation time better the performances, provided that the signals stay stationary. In practical cases, the observation time is large enough to be in asymptotic condititons.
- Frequency bandwidth : wider the frequency bandwidth better the performances,
- Array aperture impact :
  - Larger the aperture, better the performances of the direction of arrival (DOA) estimates. The counterpart is the possible loss of spatial coherence,
  - In high frequencies, the distance c/f_0 may be small compared to the sensor interdistances, leading to possible mis-estimation.

Wideband signal model

The signal on the m-th sensor writes :

$$x_m(t) \quad = \quad \sum_{k=1}^{K} s_k(t - \tau_{m,k}) + b_m(t) \tag{1}$$

where

- The signal of interest (SOI) $s_k(t)$ is a wideband stationary process in the band $[f_1, f_2]$. By wideband we mean that the central frequency is of the same order of magnitude than the bandwidth.
- The SOIs are assumed to be independent, zero-mean, with diagonal spectral density matrix $\Gamma^s(f)$ of size K,
- The noise $b(t)$ is a zero-mean process, with diagonal spectral density matrix $\Gamma^b(f)$ of size M.

Spectral density matrix

The spectral density matrix of the M-ary process $x(t)$ writes :

$$C(f;\mu) \quad = \quad G(f)\Gamma^s(f)G^H(f) + \Gamma^b(f)$$

where $G(f)$ is a $M \times K$ matrix whose entries write $e^{-2j\pi f \tau_{m,k}}$. Assuming that the array is in the horizontal plane, we write :

$$\tau_{m,k} \quad = \quad v^{-1}\left(r_{m,1}\cos(\pi a_k/180) + r_{m,2}\sin(\pi a_k/180)\right)$$

where $(a, v) \in (0^\circ, 360^\circ) \times (300 \text{ m/s}, 700 \text{ m/s})$.

The column $G_k(f)$ describes a manifold in $\mathbb{C}^M$, that is the response of the array at the frequency f.

Spectral estimation

Non parametric estimation of $C(f)$ can be achieved by smoothing periodogram or by averaging periodogram. Based on N observations, the periodogram writes

$$P(f) = \frac{1}{N}\sum_{n=0}^{N-1}\sum_{m=0}^{N-1} x_n x_m^H e^{-2i\pi f(n-m)}$$

In the following the spectral estimation is performed by smoothed periodogram, using a Bartlett windowing with width $N^{0.3}$.

Spatially and temporally white case

Assuming that $\Gamma^b(f) = \sigma^2 I_M$ and $\Gamma^s = \mathrm{diag}\left[s_1^2, \cdots, s_K^2\right]$, the spectral density matrix of the M-ary process $x(t)$ writes :

$$C(f;\mu) = \underbrace{G(f)\Gamma^s G^H(f)}_{\mathrm{rank} \leq K} + \sigma^2 I_M$$

where

- $G(f)$ is a $M \times K$ matrix whose entries write $e^{-2j\pi f \tau_{m,k}}$,
- $\mu = (a_1, \cdots, a_K, v_1, \cdots, v_K, s_1^2, \cdots, s_K^2, \sigma^2)$.

It follows that $C(f;\mu)$ has at least $(M-K)$ eigenvalues equal to σ^2.

Detection : function of test of H_0 (noise only)

Non exhaustive list of detectors for infrasound :

- Consistance [3],
- MCCM (mean of cross-correlation maxima) [21],
- Fisher statistic (GLRT on deterministic model) [17],
- Generalized Fisher statistic (gaussian prior) [16],
- Short Time Average over Long Time Average (STA/LTA) [2],
- Sum of the squares of variance ratio [9],
- GLRT on stochastic model [6],
- Cumulative sum (CUSUM) for onset detection [18],
- Hough transform [11] [1],

DOA estimation

In the following we assume that the observation time is very large (asymptotic regime) and about a dozen of sensors. Therefore the performances are mainly determined by observation time, the array aperture and the SNR.

- Mono-source : TDOA [3],
- Mono-source : Estimator, as Fisher, derived from MLE deterministic or stochastic likelihood, [18], [17], [6], [14]
- Wideband MUSIC [23], [24], [12], [19], [15] : CSS, WAVES, TOPS
- Multiple sources : from beamforming approach, as Bartlett, Capon (MVDR), [18], [4]
- Multiple sources : CLEAN, deconvolution approach [8], [22]

Monosource constraint

A commonly used way to achieve monosource constraint is to divide the time-frequency domain in small cells. The sizes of the cells are derived from empirical knowledges. For example PMCC uses a logarithmic filter bank.

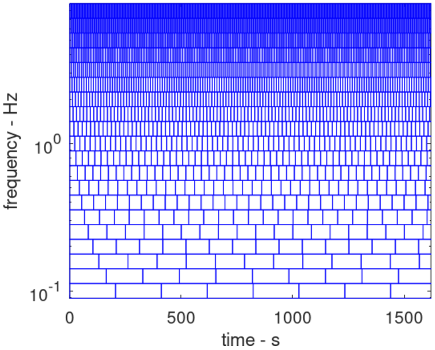


FIGURE – *PMCC filter bank : constant ratio of 4.36 between the central frequency and bandwidth (wideband). The BT values are between 5 et 10. We observe between 20 and 40 oscillations per window.*

TDOA

The Time Difference Of Arrivals (TDOA), using correlation, writes for each pair of sensors :

$$\hat{\tau}_{ij} \quad = \quad \arg\max_{\tau} \int x_i(t)x_j(t-\tau)dt$$

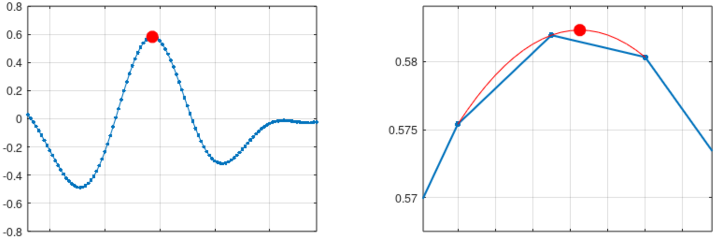


FIGURE – *Delay estimation and parabolic interpolation*

The major advantage is a vey low computational load,
But delay is performed without taking into account the location of sensors.

DOA - estimation based on TDOA

- If Z denotes the matrix built from the sensor locations and θ the slowness vector we can write :

$$\tau = Z\theta + \text{noise}$$

  The ordinary least square (OLS) estimate of θ writes :

$$\hat{\theta}_{\text{OLS}} = (Z^T Z)^{-1} Z^T \tau$$

  In [7], a weighted least square (WLS) estimate is proposed based on the covariance expression of the noise at the correlator output, which is not diagonal.

- Assuming the antenna in the horizontal plane, we can derive the azimut and the horizontal velocity estimates as :

$$\begin{cases} \hat{v} = (\hat{\theta}_1^2 + \hat{\theta}_2^2)^{-1/2} \\ \hat{a} = \arg(\hat{\theta}_1 + j\hat{\theta}_2) \end{cases}$$

Likelihood

- The likelihood function is based on Gaussian assumption for the observation $x_n = x(nT_s)$. From Eq. (1) the likelihood writes

$$\mathcal{L}(s^2, \sigma^2, a, v) = -\log \det R - \text{Trace}\left\{R^{-1}\widehat{R}_N(a, v)\right\} \quad (2)$$

where $R = s^2 \mathbb{1}_M \mathbb{1}_M^T + \sigma^2 I_M$ and

$$\widehat{R}_N(a, v) = \frac{1}{N}\sum_{n=1}^{N} \tilde{x}_n(a, v)\tilde{x}_n^T(a, v) \quad (3)$$

where $\tilde{x}_n(a, v)$ is performed from the signals x_n by applying the delays derived from the couple (a, v) and the sensor locations.

The parameter set is $\{s, \sigma, a, v\} \in \Omega = \mathbb{R}^+ \times \mathbb{R}^+ \times [0, 360] \times [300, 700]$.

MLE

- The maximum likelihood estimator writes

$$\hat{\mu}_{\mathrm{MLE}} \quad = \quad \arg\max \mathcal{L}(s^2, \sigma^2, a, v)$$

Closed form expression can be expressed for the maximum w.r.t. s^2 and σ^2. Unfortunately the maximization w.r.t. (a, v) has no closed form expression. The maximum can be achieved with a fine grid and then refine it by local research.

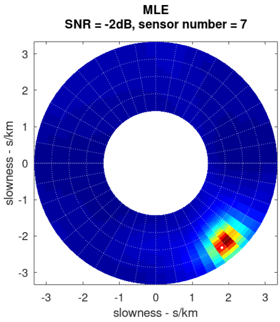


FIGURE – *1 source with SNR* $= -2dB$

MLE performances

The asymptotic distribution of the (a, v) MLE writes

$$\sqrt{\mathbf{T}} \left(\begin{bmatrix} \hat{a} \\ \hat{v} \end{bmatrix} - \begin{bmatrix} a^* \\ v^* \end{bmatrix} \right) \Rightarrow^{d}_{T \to \inf} \mathcal{N}\left(0, J\Gamma_{\mathrm{MLE}} J^T\right)$$

where

$$J = \begin{bmatrix} -v\cos(a) & v\sin(a) \\ -v^2\sin(a) & v^2\cos(a) \end{bmatrix}$$

Expression of Γ_{MLE} is given in [13] for general spectral shapes. In the particular case where that the SOI and the noise are white in the filter bandwidth $[f_{\min}, f_{\max}]$:

$$\Gamma_{\mathrm{MLE}} = \frac{3\ (M\,\mathrm{SNR}+1)}{8\pi^2\, M\,\mathrm{SNR}^2\ \left(f_{\max}^3 - f_{\min}^3\right)} \left(Z^T \Pi_M^{\perp} Z\right)^{-1} \tag{4}$$

This expression can be used to evaluate the confidence region of the estimation, replacing the a, v and SNR by consistent estimates.

MLE/TDOA

The expression (4) of the covariance MLE estimates is to compare to the asymtotic covariance of TDOA estimation given by :

$$\Gamma_{\mathrm{TDOA}} = \frac{3(2\,\mathrm{SNR}+1)}{4\pi^2\,\mathrm{SNR}^2(f_{\max}^3 - f_{\min}^3)}(Z^T Z)^{-1}\, Z^T\, C\, Z\,(Z^T Z^{-1})$$

where Z is a matrix with dimension $M(M-1)/2 \times 2$ built with the all pairs of sensor location and C is given in [13] :

$$\mathbb{E}\left[\eta_{k,m}\eta_{k',m'}\right] = \begin{cases} 3 & \text{if } k = k' \text{ and } m = m' \\ 1 & \quad \text{if } k = k' \text{ and } m \neq m' \\ & \text{or if } m = m' \text{ and } k \neq k' \\ -1 & \quad \text{if } k = m' \text{ and } m \neq k' \\ & \text{or if } m = k' \text{ and } k \neq m' \\ 0 & \text{elsewhere} \end{cases}$$

where η_{ij} is the error at the TDOA estimate for the pair (i,j).

MLE performances as function of sensor number, etc

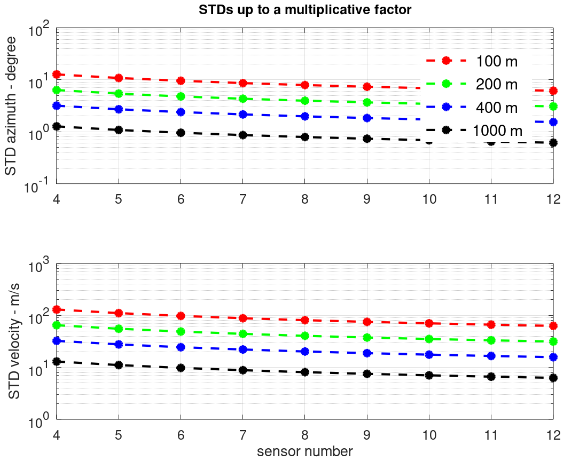


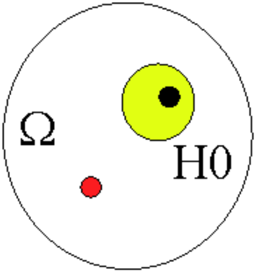
FIGURE – *Asymptotic STDs of MLE estimates as a function of sensor number for different apertures. Sensor number has low effect.*

GLRT

By definition an hypothesis is any non empty set of the parameter set.

The noise only hypothesis we want to test is :

$$H_0 \quad = \quad \{s = 0\} \subset \Omega$$



The Generalized Likelihood Ratio Test (GLRT) is defined by

$$\mathrm{GRLT} \quad = \quad \frac{\sup_{\Omega} \mathcal{L}}{\sup_{H_0} \mathcal{L}}$$

GLRT

In our case, eq. (2), the GLRT writes (after a few calculations)

$$T = \sup_{a,v} \left(\frac{M S_N(a,v)}{T_N(a,v)} - 1 \right)_+ \tag{5}$$

where S_N and T_N are respectively the sum and the trace of $\hat{R}_N(a,v)$, eq. (3).

It is worth to notice that, because the sup function on (a,v), the r.v. T under H_0 is not Fisher distributed.

p-value of GLRT

The p-value is a statistic performed by inversion of the cumulative function of the function of test under H_0. Hence the p-value is between 0 and 1.

The p-value has a clear meaning : it says the probability to incorrectly accept H_0.

Typically a p-value less than 10^{-2} means that the H_0 hypothesis is very unlikely.

In [13] the asymptotic distribution of the GLRT under H_0 is determined.

p-value of GLRT - sketch

In short, it is shown that :

$$\left(\sqrt{N}\left(\widehat{R}_N(a,v) - \sigma^2 I_M\right)\right)_{(a,v)} \implies (\mathcal{R}(a,v))_{(a,v)}$$

where $(\mathcal{R}(a,v))_{(a,v)}$ is a gaussian process with covariance given in [13]. By *delta method* we derive that the asymptotic distribution of

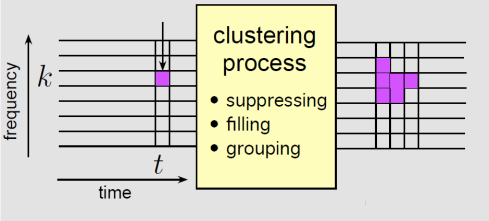
$$(T(a,v))_{(a,v)} = \left(\frac{MS_N(a,v)}{T_N(a,v)} - 1\right)_{(a,v)}$$

is Gaussian. If G denotes the number of elements of a given discrete bidimensional grid on the region (a,v), the GLRT appears as the maximum over the G components of $(T(a,v))_{(a,v)}$.

Unfortunately because the covariance matrix is not diagonal, there is no closed form expression for the distribution of the maximum. In [13] a Monte-Carlo approach is proposed for approaching the asymptotic distribution.

Using filter bank

- The main avantage is to achieve mono source constraint,
- The major drawback is that the analysis is carried out independently cell by cell, postponing the clustering of cells associated with the same event to a later step.



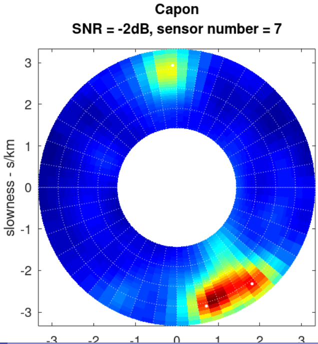
That suggests to consider longer window time ad larger frequency bandwidth and use multiple source approaches.

Multiple sources : beam forming approach

Based on an averaging of the Capon beamformer (Minimum Variance Distortionless Response - MVDR) and an estimate of the spectral matrix $\widehat{C}(f)$, we perform :

$$\widehat{P}(\theta) = \frac{1}{\sum_{f=f_{\min}}^{f_{\max}} G^H(f;\theta)\widehat{C}^{-1}(f)G_k(f;\theta)}$$

where $G(f;\theta)$ is the antenna response for the slowness vector θ. The figure 6 reports an illustration for 3 sources.



Multiple sources : MLE built from "monosource"

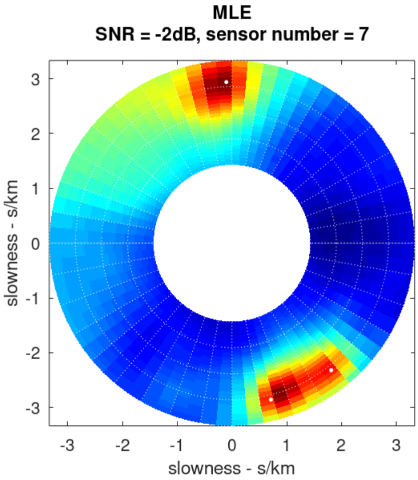


FIGURE – *Simulation : 3 sources, mean SNR* $= 4$ *dB. SOI bandwith* $[0.1\text{Hz} - 0.3\text{Hz}]$, *observation time* $T = 1000$s. *Station :* $IS31$ $(8$ *sensors). True azimuth : 90, 280, 300, True velocities : 340 m/s.*

Multiple sources : Subspace method with non coherent approach

The Incoherent MUSIC writes :

$$(\hat{a}, \hat{v}) = \arg\min_{a,v} \sum_{f=f_{\min}}^{f_{\max}} G(f; a, v) W(f; a, v) W^H(f; a, v) G^H(f, a, v)$$

where W is the noise subspace basis at the frequency f and $G(f; a, v)$ is the antenna response at the frequency f for a DOA (a, v).

Therefore the non coherent approach has an heavy computational cost because we have to do an eigendecomposition for each frequency bin.

Moreover the method provides poor results for low SNRs and/or near DOAs.

Multiple sources : Subspace method with focusing

Coherent MUSIC : The key idea is to design a focusing matrix $T(f)$ which transforms the array manifold at frequency f to that at a common given frequency f_0.

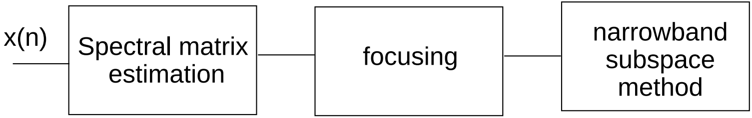


FIGURE – *Focusing idea*

For keeping white property for noise, we assume that $T(f)T^H(f) = I_M$ [15], [24].

Multiple sources : Subspace method with focusing

Typical algorithms are :

- Coherent Signal Subspace (CSS) : needs an initialisation step. CSS is an iterative method, evaluatinng only one direction at each step.
- Weighted Average of Signal Subspace (WAVES) : needs an initialisation step. It performs better than CSS but suffering from complexity.
- Steered Covariance (STC) avoids initialisation step by partioning the DOA domain. The accuracy increases with the number of partitions used.
- Test of Orthogonality of Projected Subspaces (TOPS) [24] : no initialisation step

Multiple sources : deconvolution in the spectral domain

The CLEAN algorithm [10] was originally developed to remove sidelobe artefacts in radio astronomy images. It and its variations are still extensively used in radio astronomy.

In [22], authors applied CLEAN for acoustic source localization.

In [8], authors applied CLEAN for estimation of DOA of multiples infrasonic sources.The "cleaned' spectral density matrix is updated as it follows :

$$C^{p+1}(f) \quad = \quad C^{p}(f) - \phi P_{\max}(f) w_{\max}(f) w_{\max}^{H}(f)$$

where the maximum is taken w.r.t. (a, v). For example $P(f; a, v)$ and $w(f; a, v)$ can be derived from Capon approach.

A stopping condition is performed using the p-value of the Fisher distribution, allowing a way to determine the number of sources.

Numerical results

One source

RMSE	MUSIC-TOPS	MLE	Capon
azimut (°)	0.87	**0.35**	0.52
hor. velocity (m/s)	7.81	**3.05**	3.76

Table – *RMSE on azimut and horizontal velociy. Simulation : monosource, SNR* $= 0$ *dB. SOI bandwith* $[0.2\text{Hz} - 0.4\text{Hz}]$*, simulation run number* $= 100$*, observation time* $= 500$ *s. Station :* $IS31$ *(*8 *sensors).*

Three sources

RMSE	MUSIC-TOPS	MLE	Capon
azimut (°)	**0.45**	5.5	52
hor. velocity (m/s)	**2.04**	2.64	12.9

Table – *RMSE on azimut and horizontal velociy. Simulation : 3 sources, mean SNR* $= 4$ *dB. SOI bandwith* $[0.1\text{Hz} - 0.3\text{Hz}]$*, simulation run number* $= 30$*, observation time* $T = 1000$ *s. Station :* $IS31$ *(*8 *sensors). True azimuth : 90, 280, 300, True velocities : 340 m/s.*

A few comments

- For monosource
  - → MLE is better than TDOA for low SNRs, providing that the model correctly explains the observations,
  - → Mono source approach, by filtering in the time-frequency domain, needs a grouping method.
    To improve (or not) the grouping step, we suggest to modelize the observations directly in the time-frequency-azimut-velocity domain.
- For multiple sources
  - → At this step, we retain MLE (monosource), TOPS.
    CLEAN algorithm is well documented in [8]. Authors provide a large simulation and also results on real data.
  - → An other approach that could be investigated is the source separation based on Independent Component Analysis (ICA), as e.g. Joint Approximation Diagonalization of Eigen-matrices (JADE) algorithm proposed in [5] for ECG.

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