Atmosphere structure and dynamics

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Aims & Scope

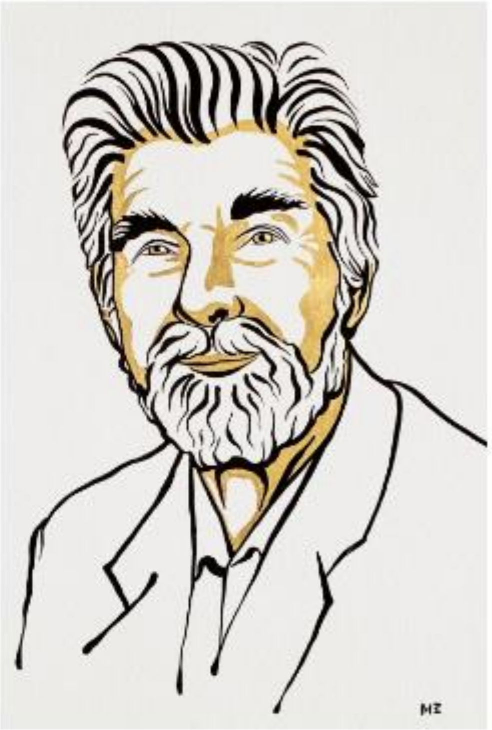
1. Introduce to the most striking atmospheric phenomena that are responsible for the climatic and meteorological conditions having a direct impact on our every day life.
2. Make use of the minimal amount of physical arguments & mathematical formalisms.

Plan

1. Radiative transfer and energy budget
2. Role of convection: sensible and latent heat transfer
3. Effects of rotation: geostrophic motion, jet stream, waves

The Nobel Prize in Physics 2021

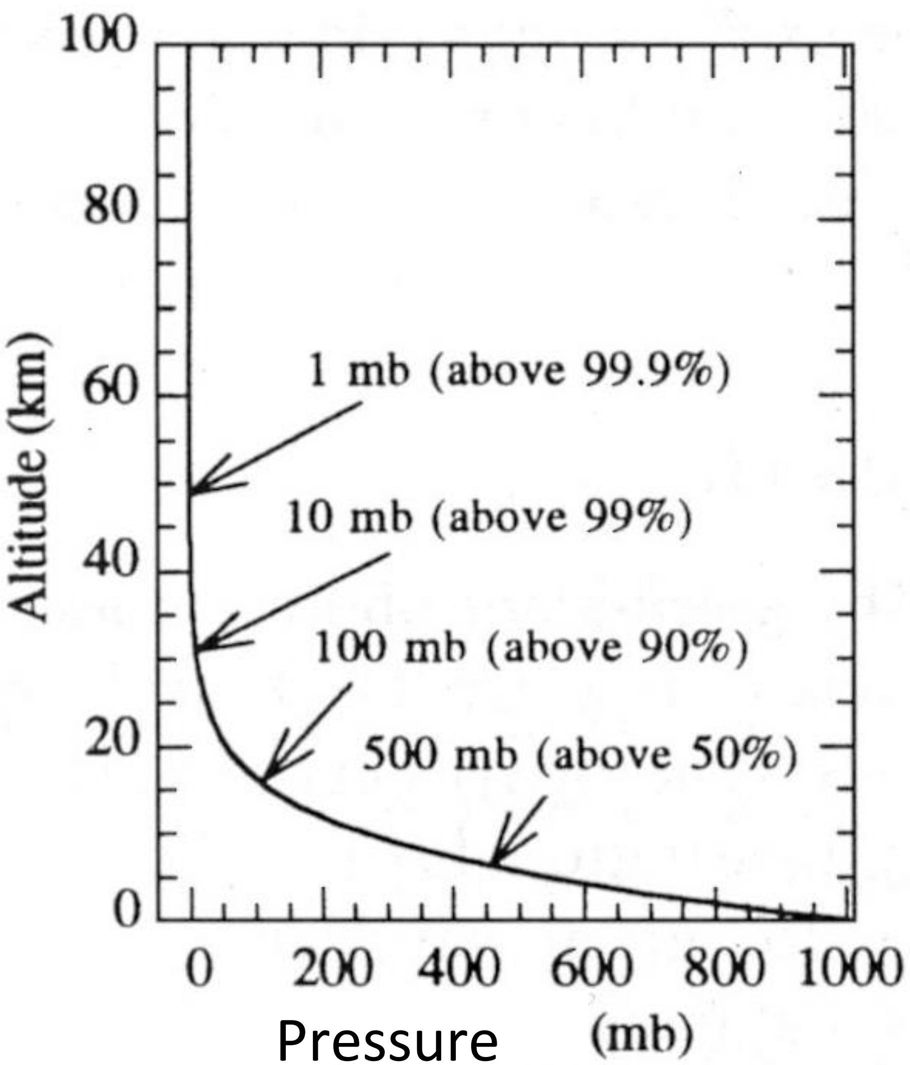
Syukuro Manabe

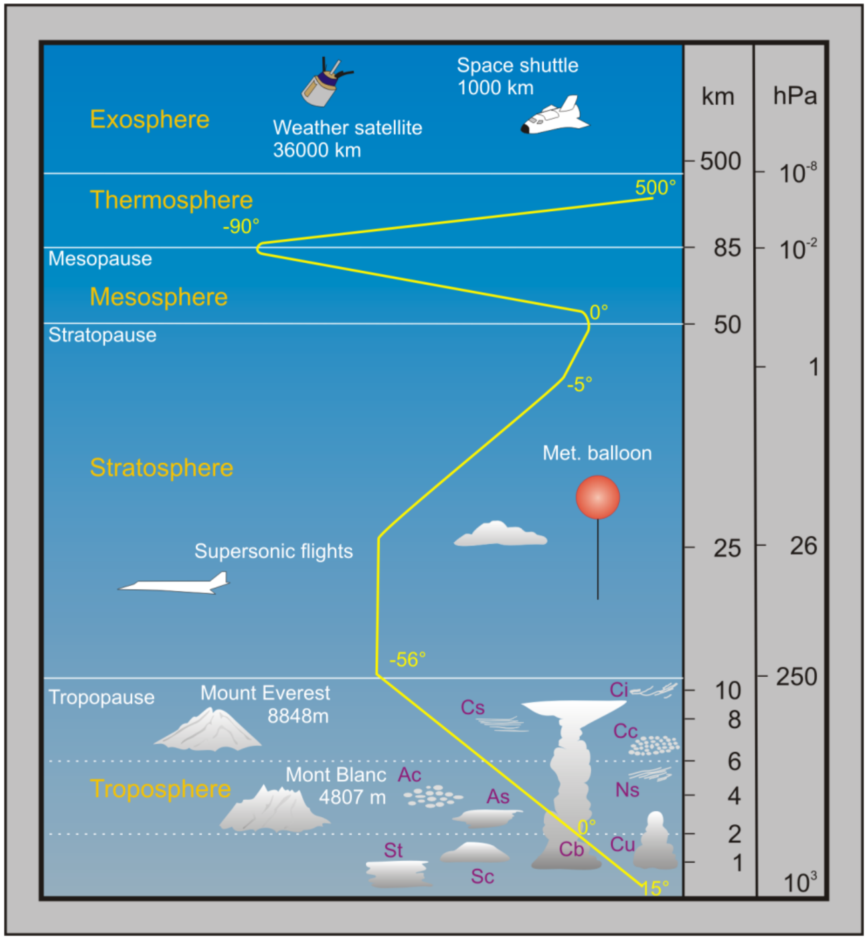


Klaus Hasselmann

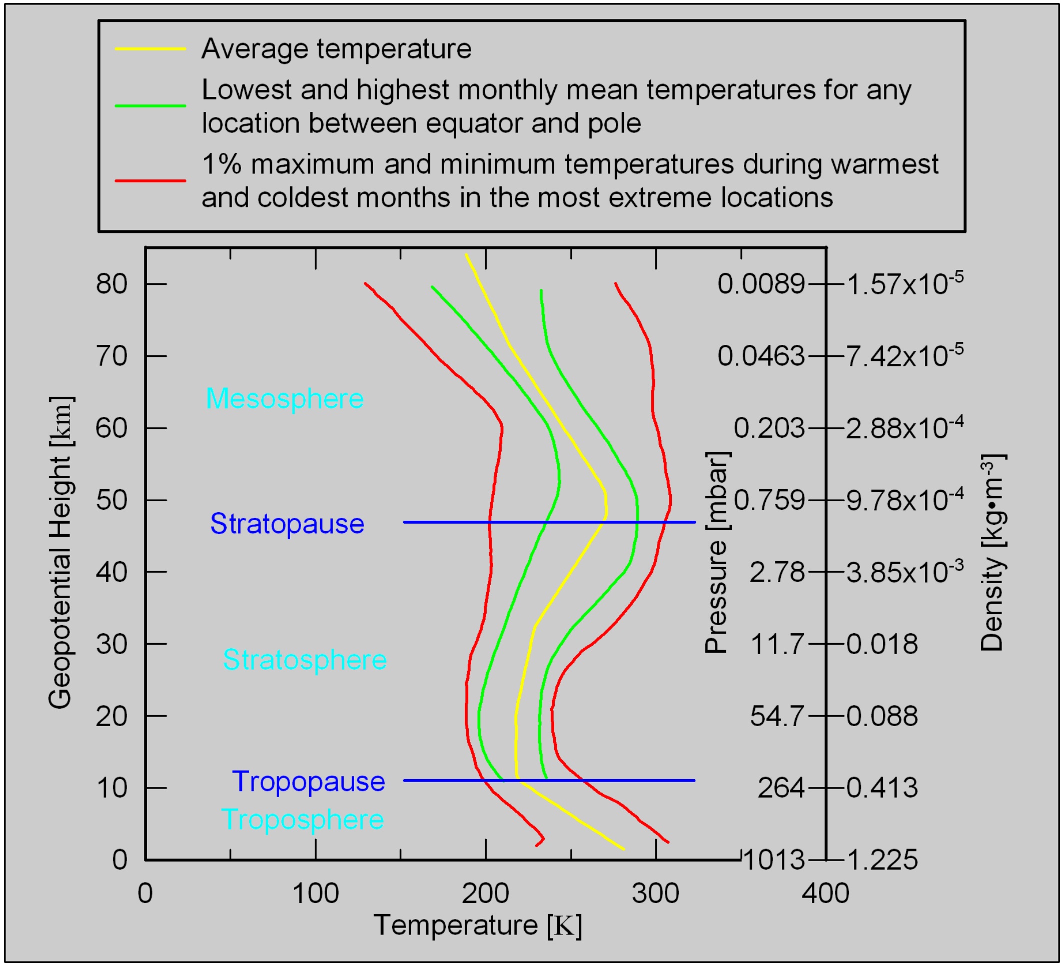
Giorgio Parisi

The atmosphere



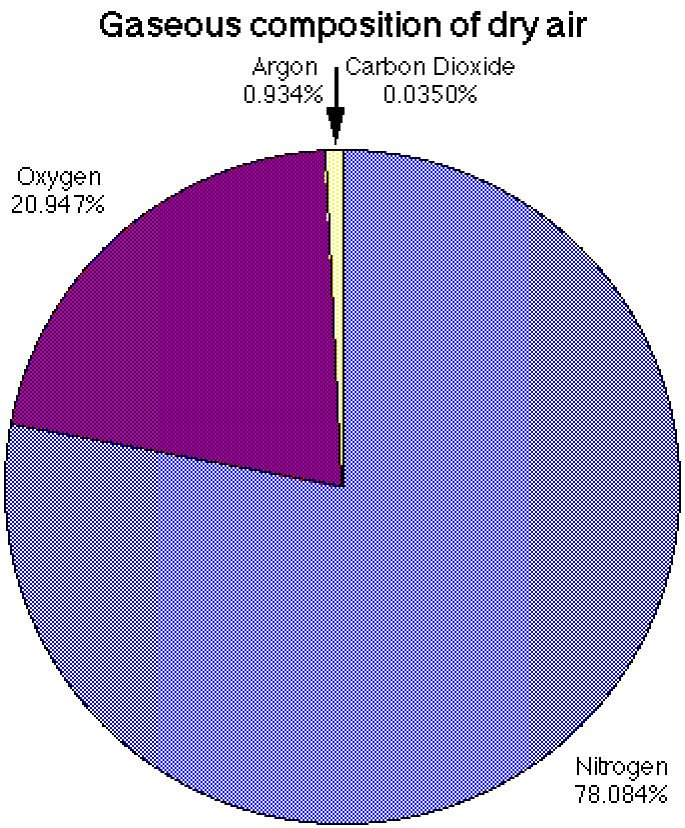


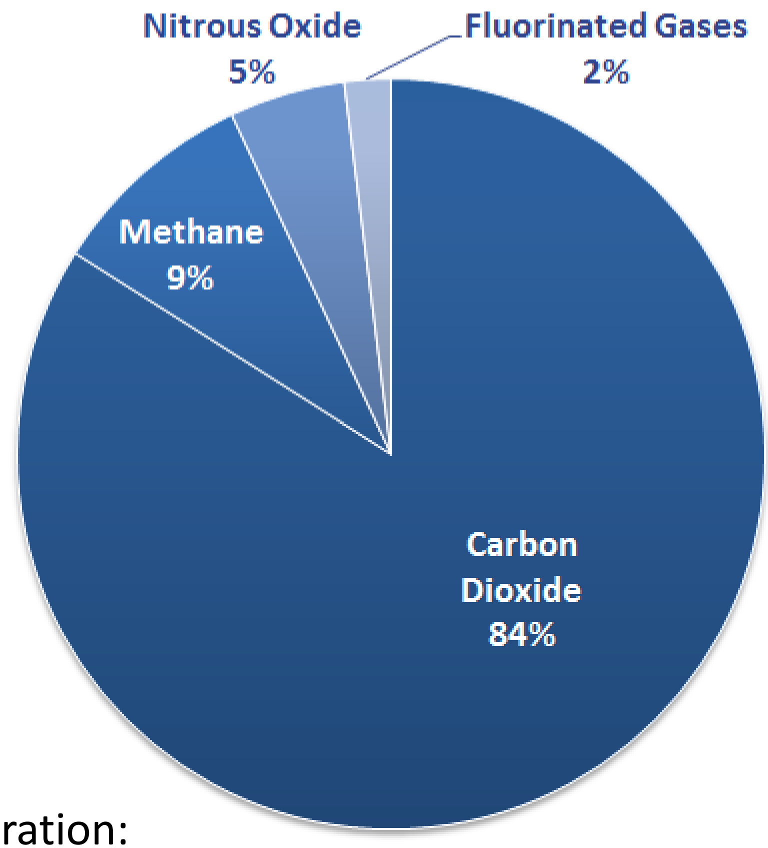
The atmosphere



Species	% by volume
N_2	78.084
O_2	20.9476
Ar	0.934
CO_2	0.0314
Ne	1.818×10^{-3}
He	5.24×10^{-4}
CH_4	2.0×10^{-4}
Kr	1.14×10^{-4}
H_2	5.0×10^{-5}
N_2O	3.0×10^{-5}
CO	1.0×10^{-5}
Xe	8.7×10^{-6}

The atmosphere





Concentration:

CO_2 400 ppm

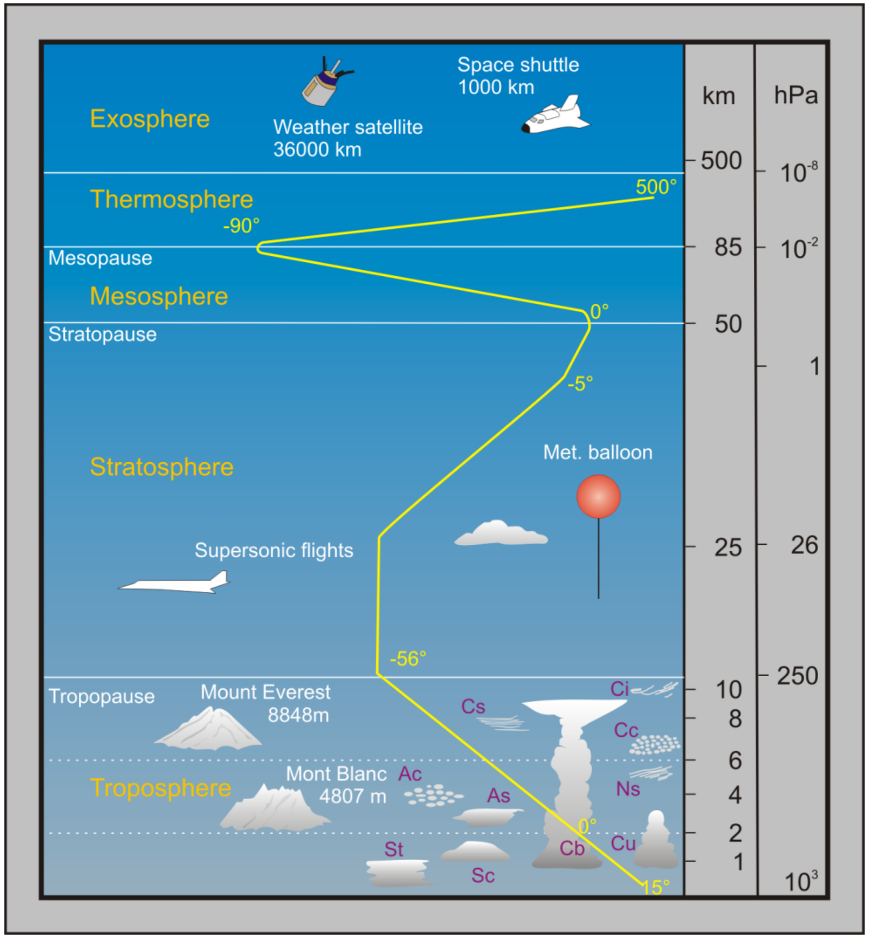
CH_4 2 ppm

The atmosphere

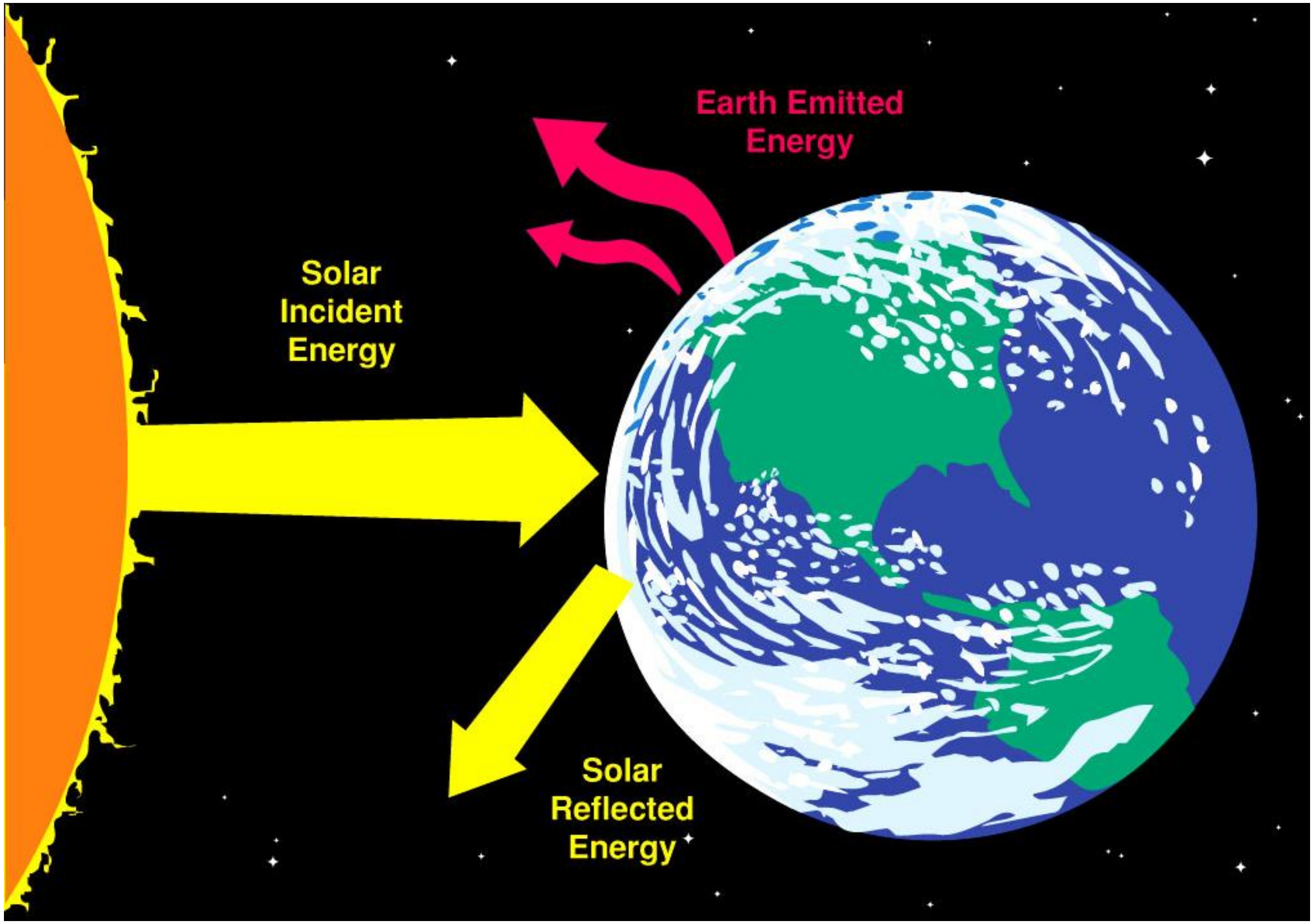
Temperature

The stratosphere has the same composition of gases as the rest of the atmosphere, with the exception of the **ozone layer**.

Temperature decreases with height in the troposphere and increases in the stratosphere (due to the presence of the **ozone layer**).



Atmosphere & Climate



Radiation

Black body radiation

Wave length

The energy of a photon basically depends on its wave length λ

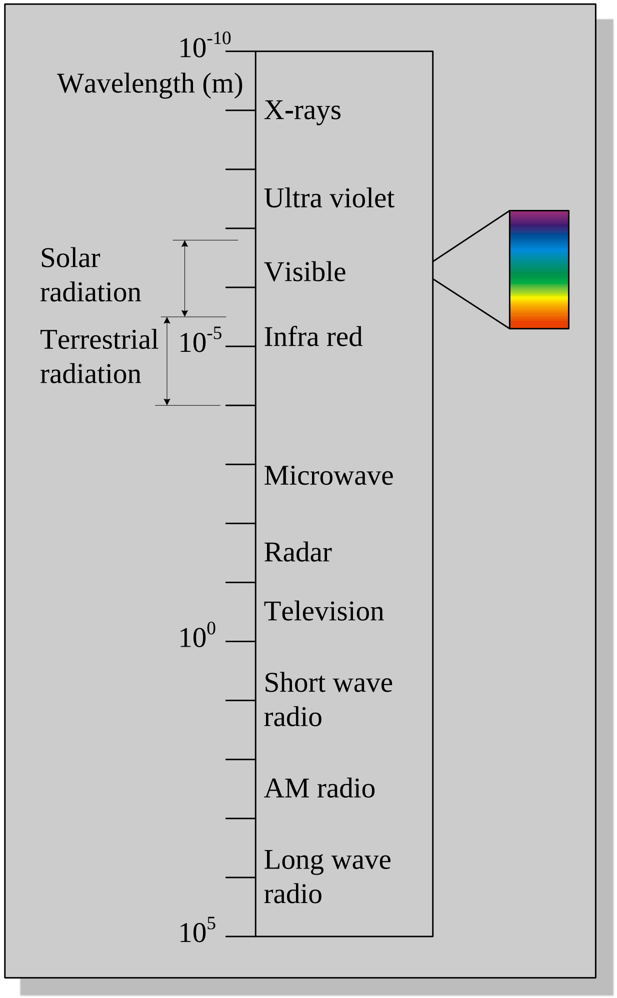
$$\left(e = \frac{hc}{\lambda}\right)$$

where

- h = Planck const. (6,63.10^{-34} J.s)
- c = speed of light (2.998.10^{8} m.s^{-1})

Black body - definition

A black body is an " ideal" body absorbing all the radiation it recieves



Radiation

Black body radiation

Planck law

Power emitted by unit surface, per unit of solide angle for a given wave length λ [W.m^{-2}.m^{-1}.sr^{-1}]

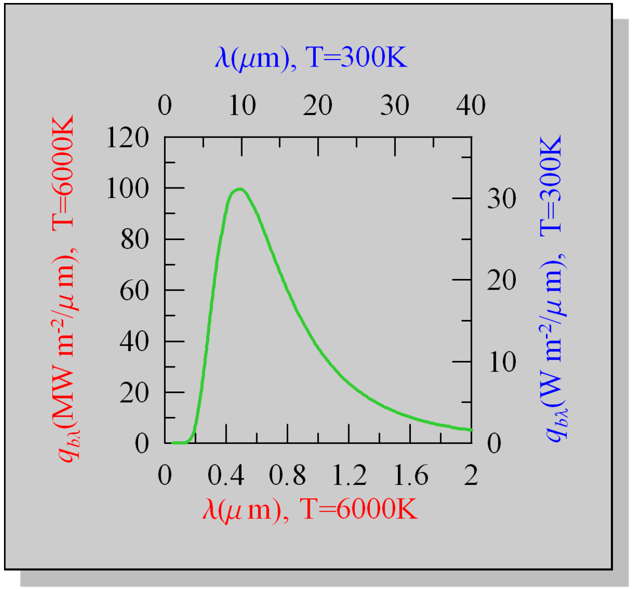
$$\boxed{q_{\lambda,T} = \frac{2hc^2}{\lambda^5\left[\exp\left(\frac{hc}{\lambda k_B T}\right) - 1\right]}}$$

where

- h = Planck const. (6,63.10^{-34} J.s)
- c = speed of light (2.998.10^{8} m.s^{-1})
- k_B = Boltzmann const. (1,38.10^{-23} J. K^{-1})
- λ = wave length [m]
- T = temperature [K]

$T_{sur} \approx 6000K$

$T_{earth} \approx 300K$



Radiation

Black body radiation

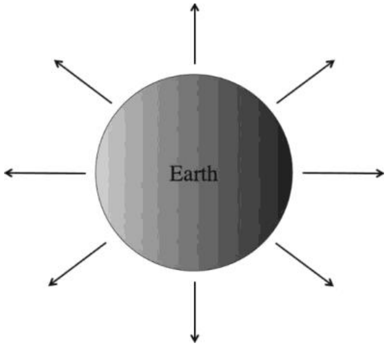
Stefan-Boltzmann law

Power emitted per unit surface, for all wave lengths [W.m^{-2}]

$$\boxed{E_e = \sigma T^4 \text{ avec } \sigma = \frac{2k_B^4\pi^4}{15h^3c^2} = 5{,}67.10^{-8}\, W.m^{-2}.K^{-4}}$$

Stefan-Boltzmann law is obtained by integrating Planck law over all wave lengths and solid angles

Radiation



Grey body radiation

A grey body absorbs only a fraction of the radiation it recieves

$E_{absorbed} = a\,E_{recieved}$ with $0 \leq a \leq 1$ absorption coefficient

Radiation that is not absorbed is reflected at the surface or transmitted within it

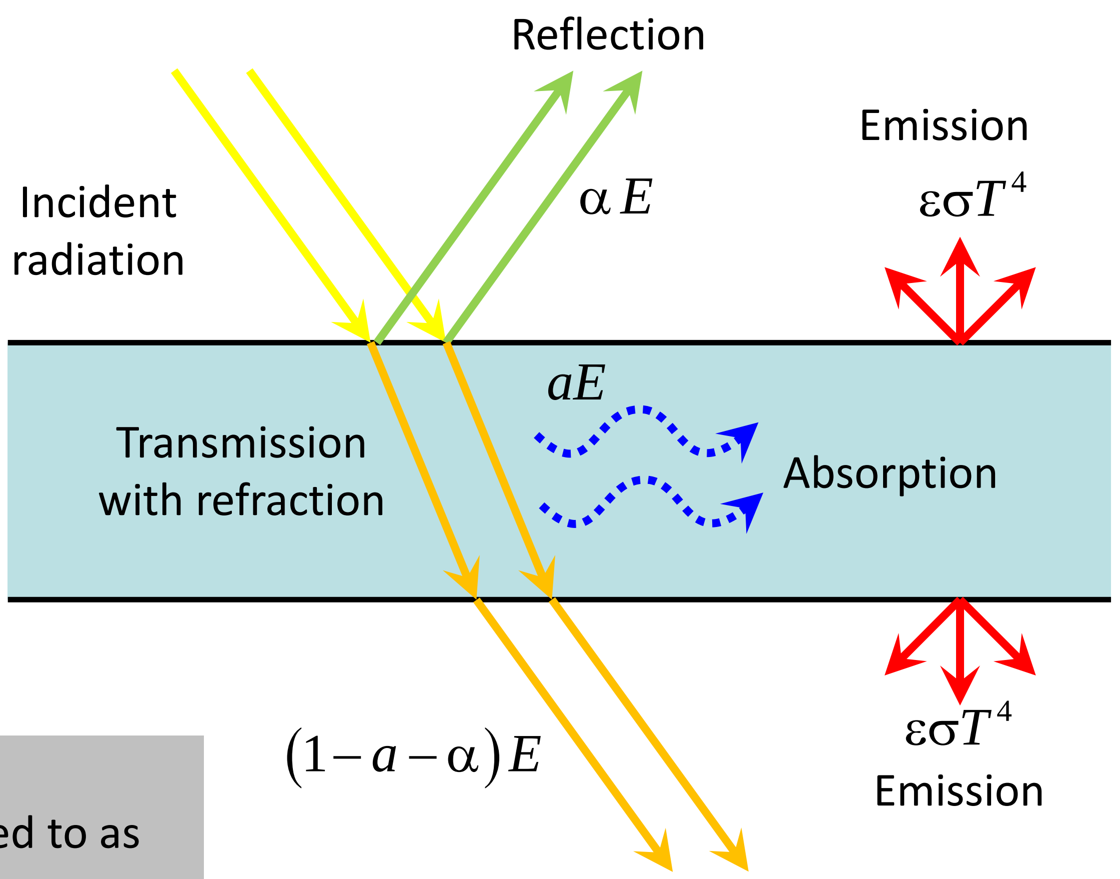
Grey body emission $\boxed{E_e = \varepsilon \sigma T^4 \text{ with } \varepsilon = \text{emissivity}}$

Kirchoff law : $\boxed{a = \varepsilon}$

Note *that* a *and* ε *depend on the wave length* λ

Radiation

Electromagnetic waves propagation

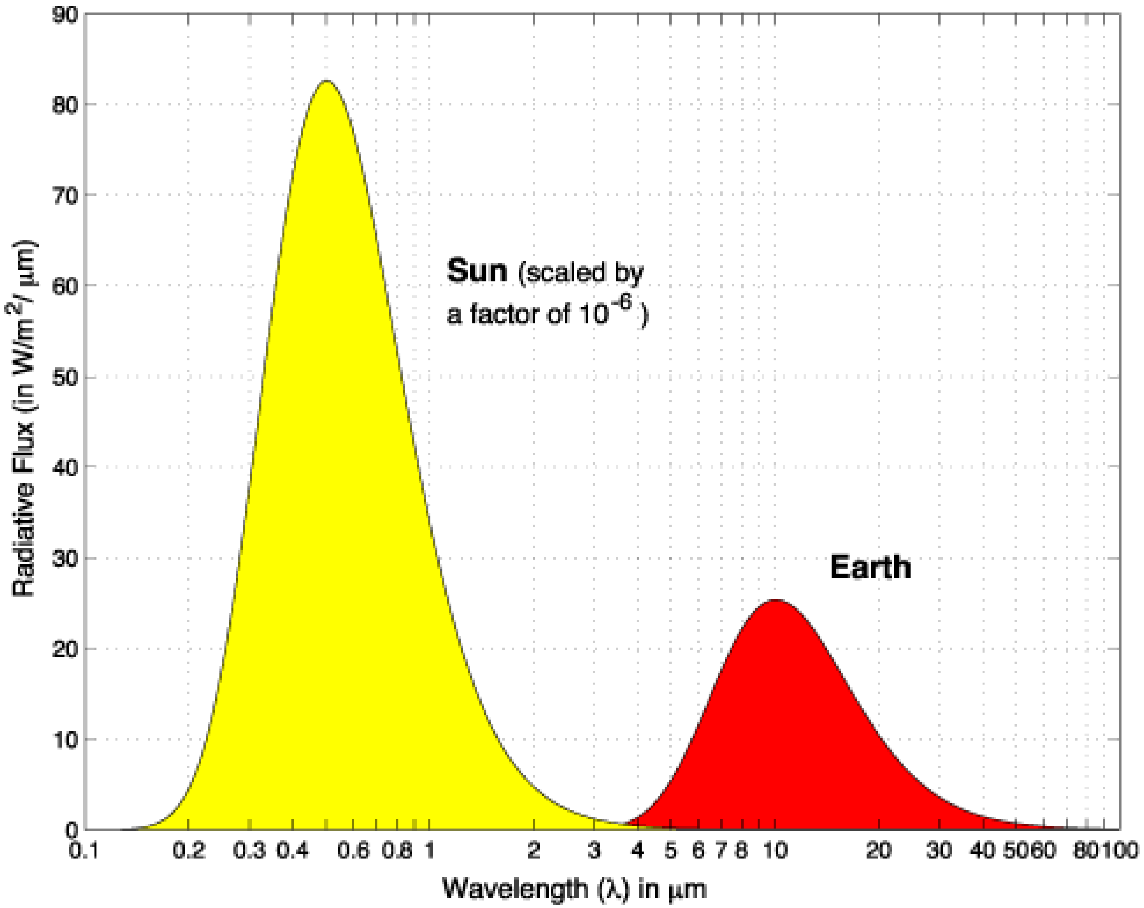


α is the reflection coefficient, referred to as ***albedo***

Sun and Earth Radiation Emission Spectrum

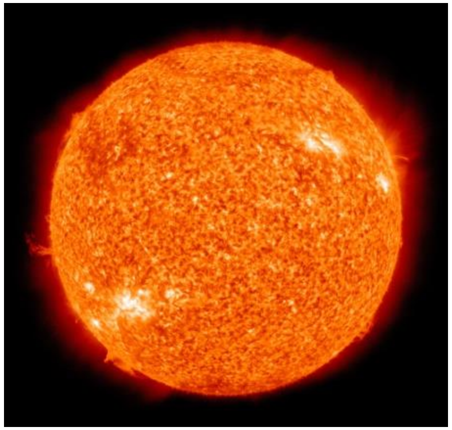
- Sun $T_s = 6000K$
- Earth $T_e = 255K$

Black body emissions curve of the Sune and the Earth

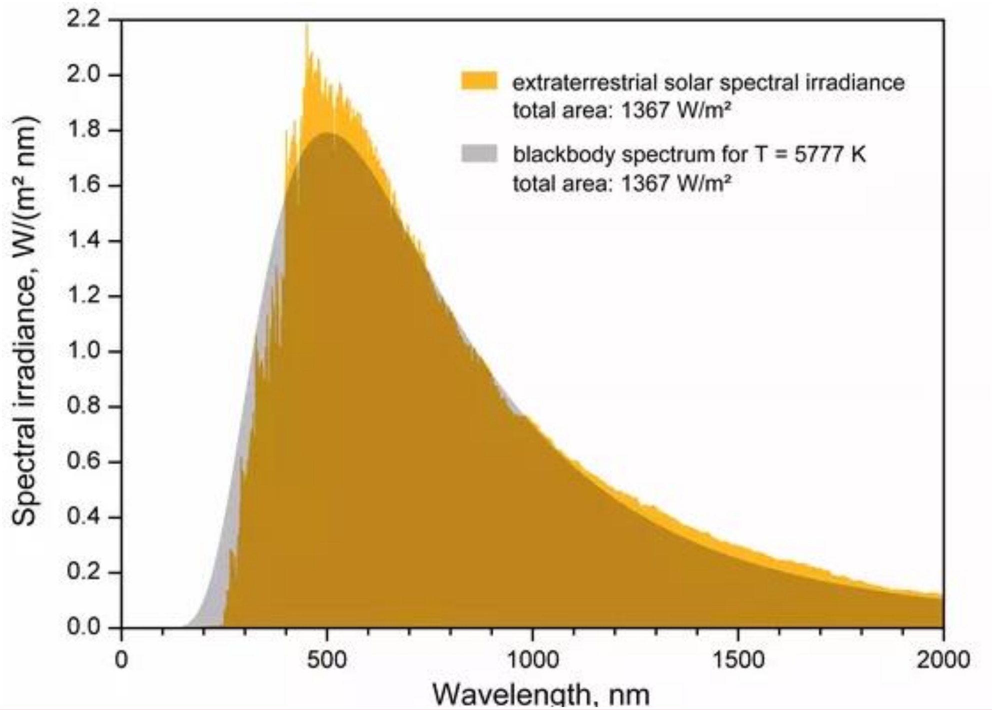


The Sun

- Emission occurs at the **photosphere** – visible sun surface
- Photosphere Layer thickness 10 [km] – 100 [km]
- Temperature ~ 6000 [K], opaque to visible light
- Above the photosphere, visible light propagates away from the sun, into space

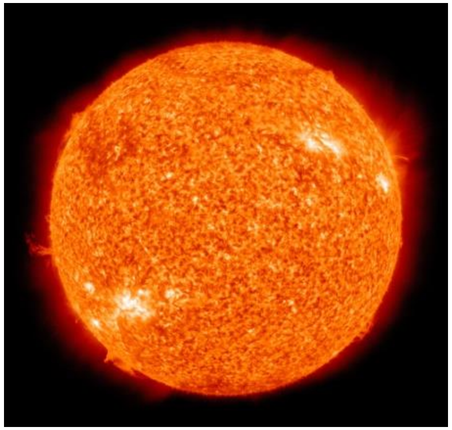


- Visible light produced by the reaction between electrons and Hydrogen atoms, to produce H^- ions
- It takes several **thousands of years** for a photon to reach the photosphere from the core



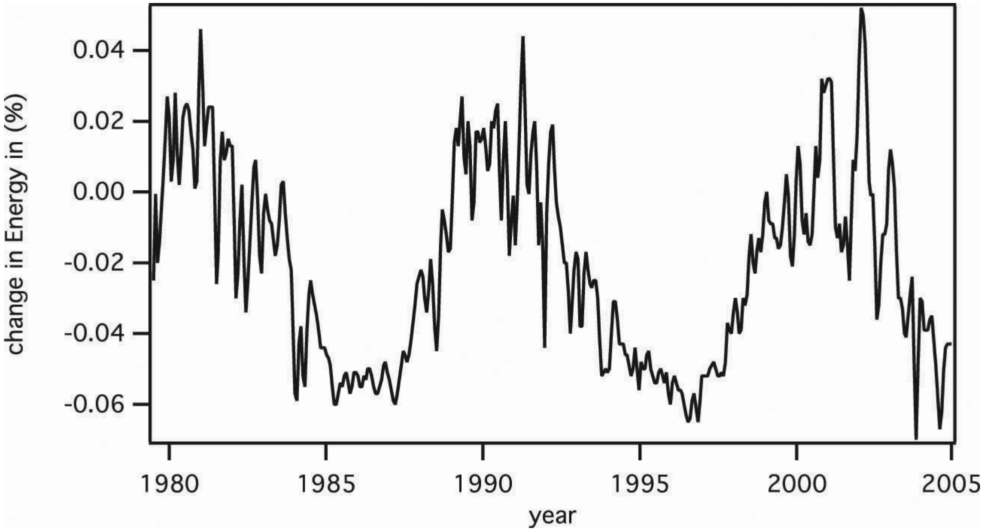
The Sun

- Sun's output varies on many scales and has brightened of about 30% during its existence.
- Since 1970 satellites measure the solar emissions showing variations of about 0.1%.
- Energy recieved from the sun (10^{17}W) is four orders of magintude larger than our energy conumption (10^{13}W)



Satellites measure of the solar constant S, i.e. the average radiation (per unit surface) at the location of the Earth.

The solar constant varies little, i.e. $\pm$ 1 W.m^{-2} over 11 years



Radiation interaction with atmosphere: Scattering/diffusion/reflection

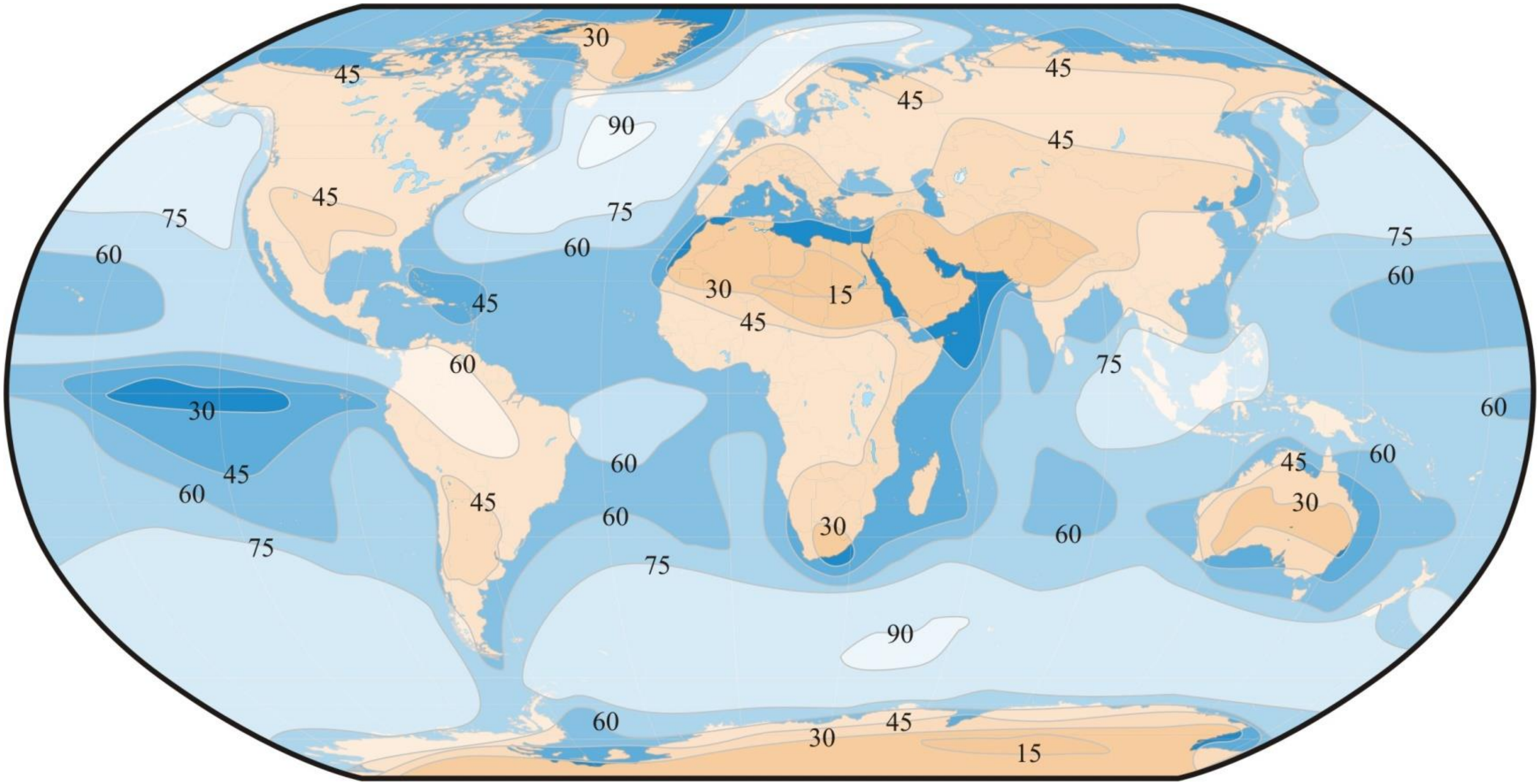
We can identify three types of atmospheric scattering, depending on the relative size of wavelength λ and dimension d of 'particles':

1) **Rayleigh scattering ($\lambda > d$):** primarily caused by oxygen and nitrogen molecules. The effective diameters are at least 0.1 times smaller than the affected wavelengths.

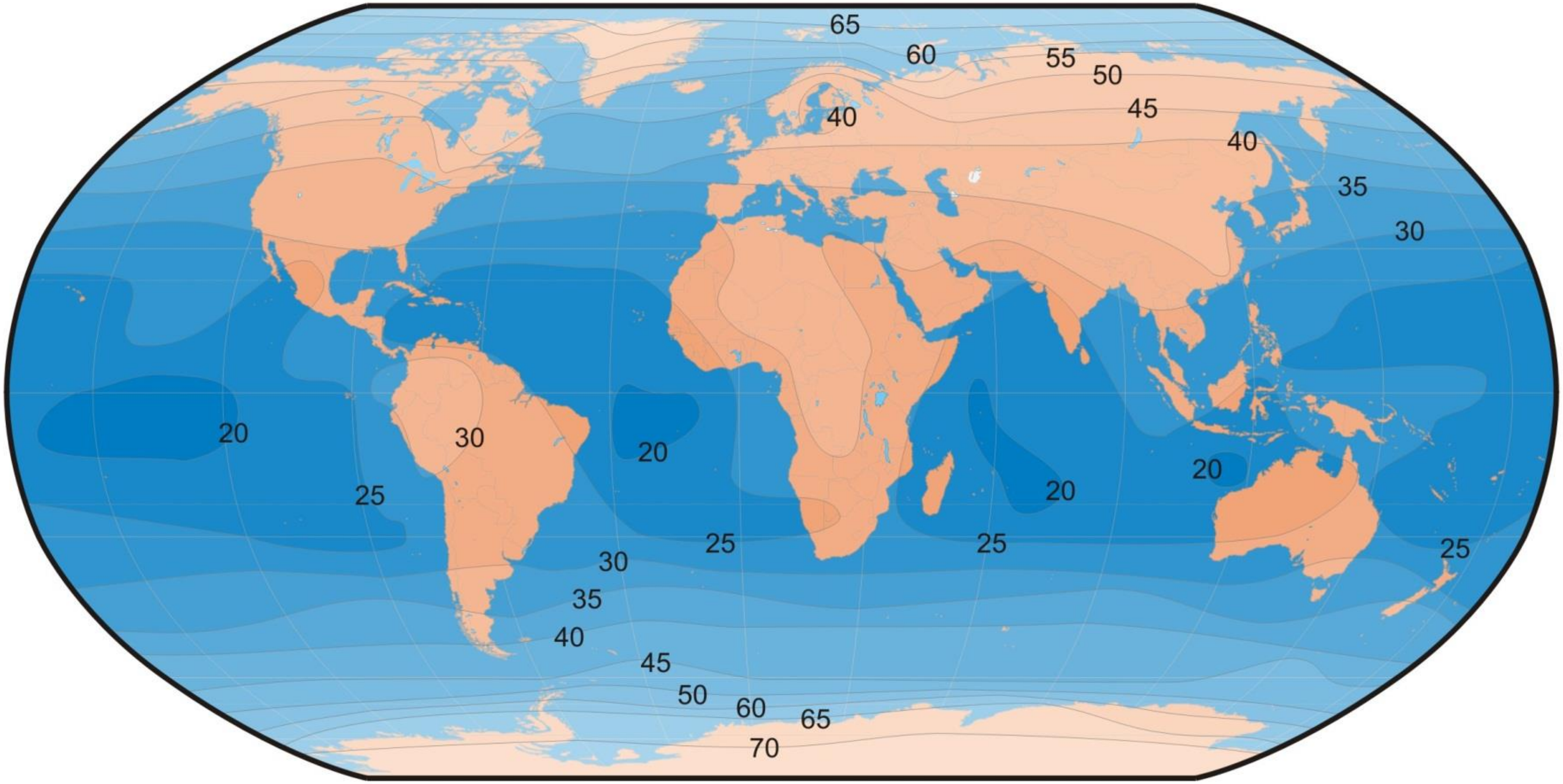
1) **Mie scattering ($\lambda \sim d$):** occurs when there are sufficient particles in the atmosphere that have mean diameters from 0.1 to 10 times larger than the wavelength under consideration.

1) **Nonselective scattering ($\lambda < d$):** due to suspended aerosols with diameters at least 10 times larger than the wavelengths (particles of smoke, water droplets, ice crystals in the clouds and fog). Nonselective scattering has impacts on almost all spectral bands.

Annual average percentage cloud cover



Albedo of Earth surface (%)



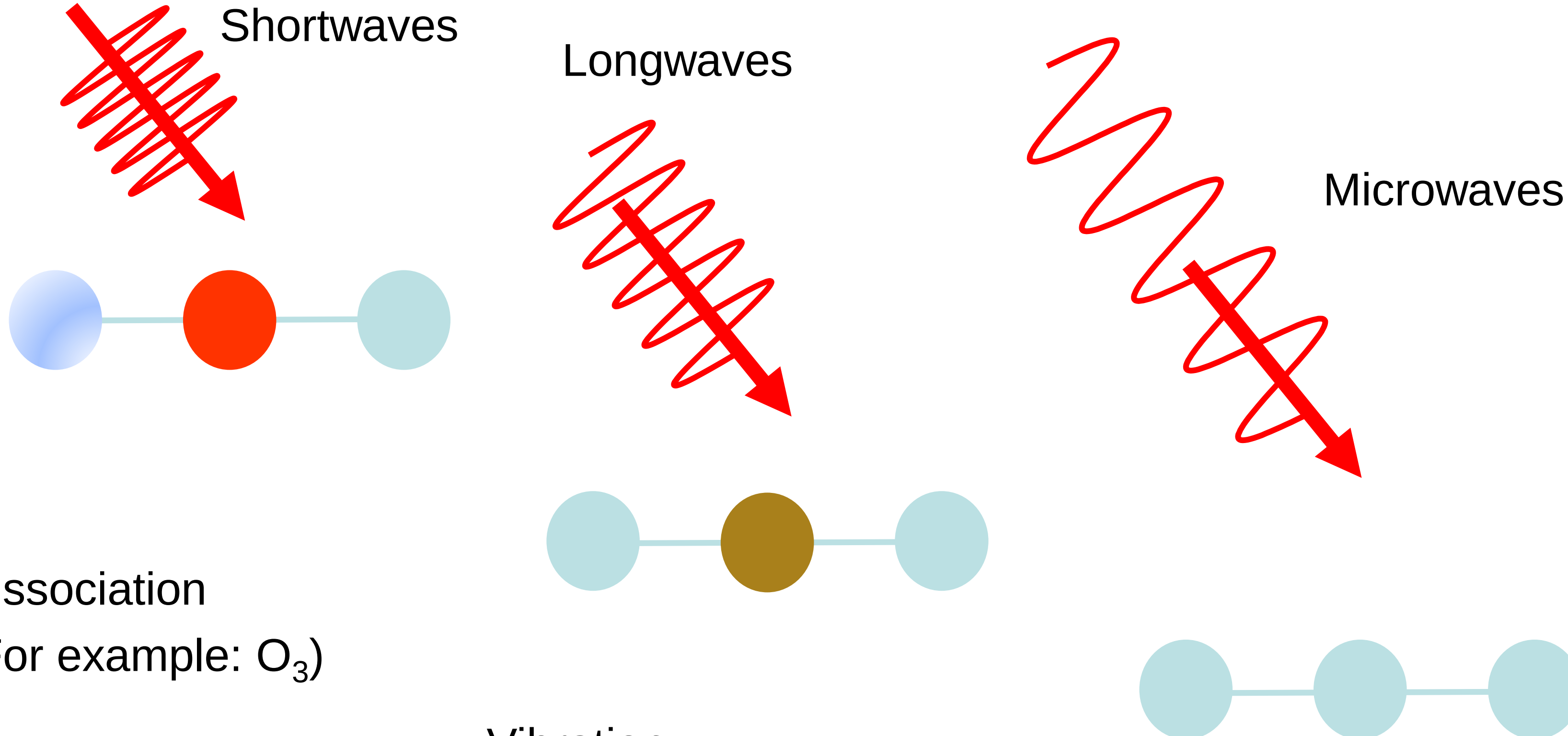
Albedo and emissivity

Surface	Albedo (short waves)	Emissivity (long waves)
Land	0.05 – 0.45	0.90 – 0.98
Desert	0.20 – 0.45	0.84 – 0.91
Field	0.16 – 0.26	0.90 – 0.95
Forest	0.05 – 0.20	0.97 – 0.99
Water	0.03 – 1.00	0.92 – 0.97
Snow	0.40 – 0.95	0.82 – 0.99
Ice	0.20 – 0.45	0.92 – 0.97

Clouds	Albedo
low	0.60 – 0.70
Mid-level	0.40 – 0.60
high (cirrus)	0.18 – 0.24
Cumulus	0.65 – 0.75

https://earthobservatory.nasa.gov/images/84499/measuring-earths-albedo

Radiation interaction with atmosphere: absorption



Dissociation

(For example: O_3)

Vibration

(For example: CO_2)

Rotation

(For example: H_2O)

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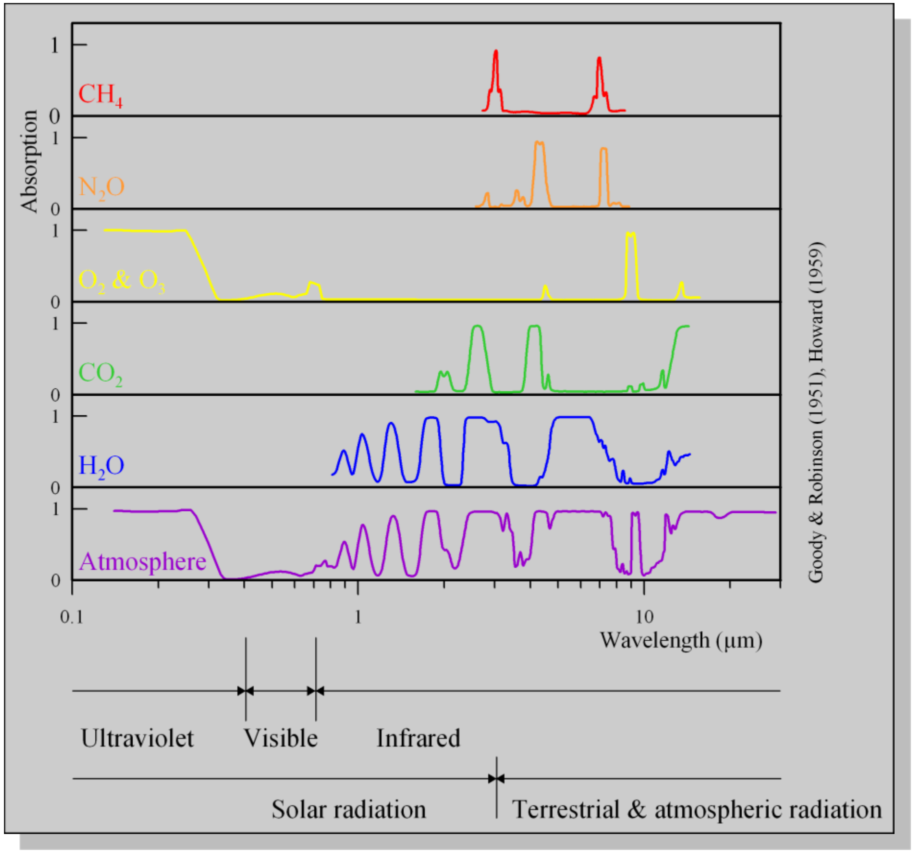
Atmosphere – energy balance

Absorption due to atmosphere and clouds

The absorption coefficient depends on the wave length.

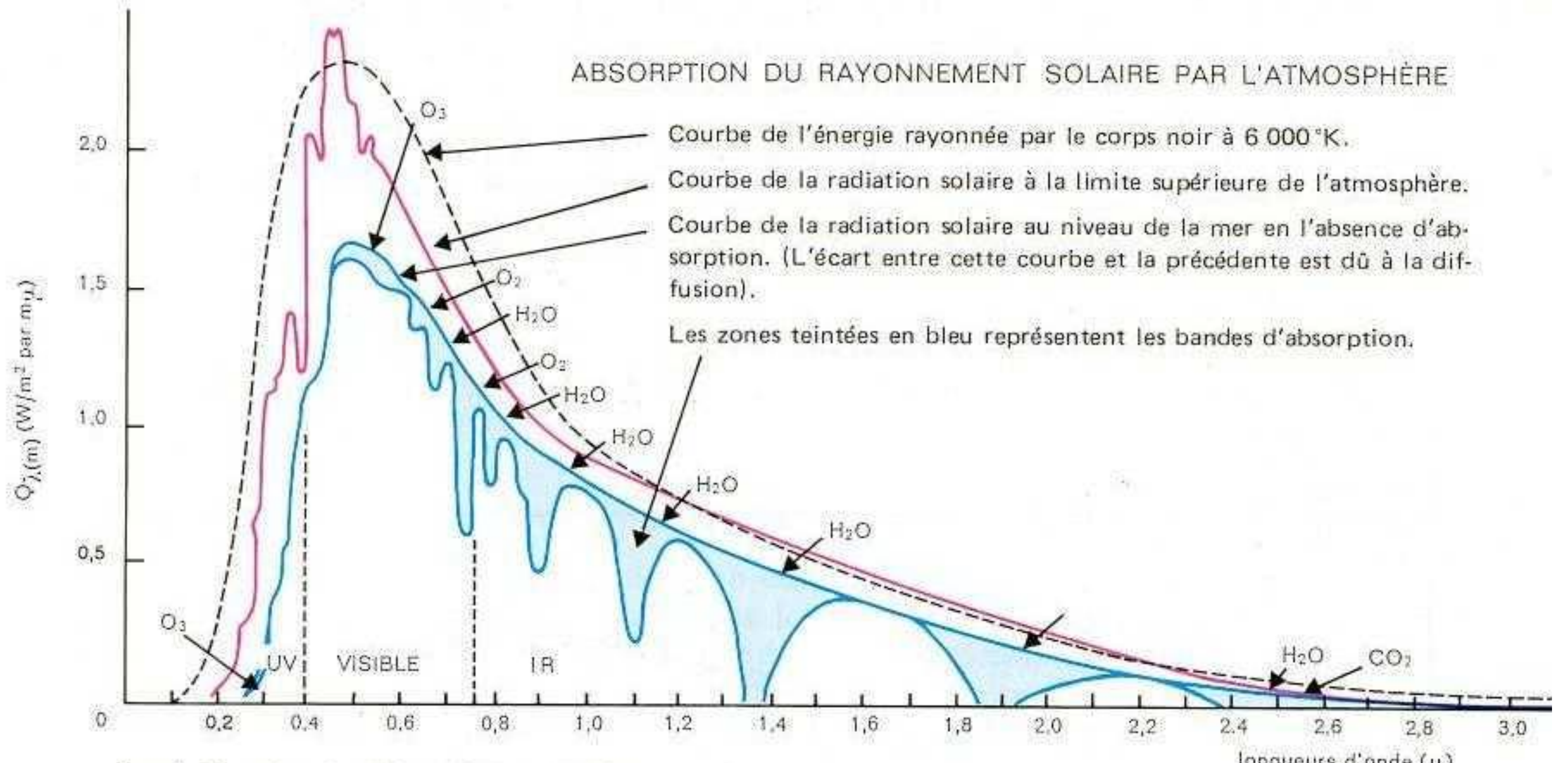
The atmosphere essentialy absorbs long wave radiation



Atmosphere – energy balance

Energy balance of the atmosphere

Absorption of the incident solar flux

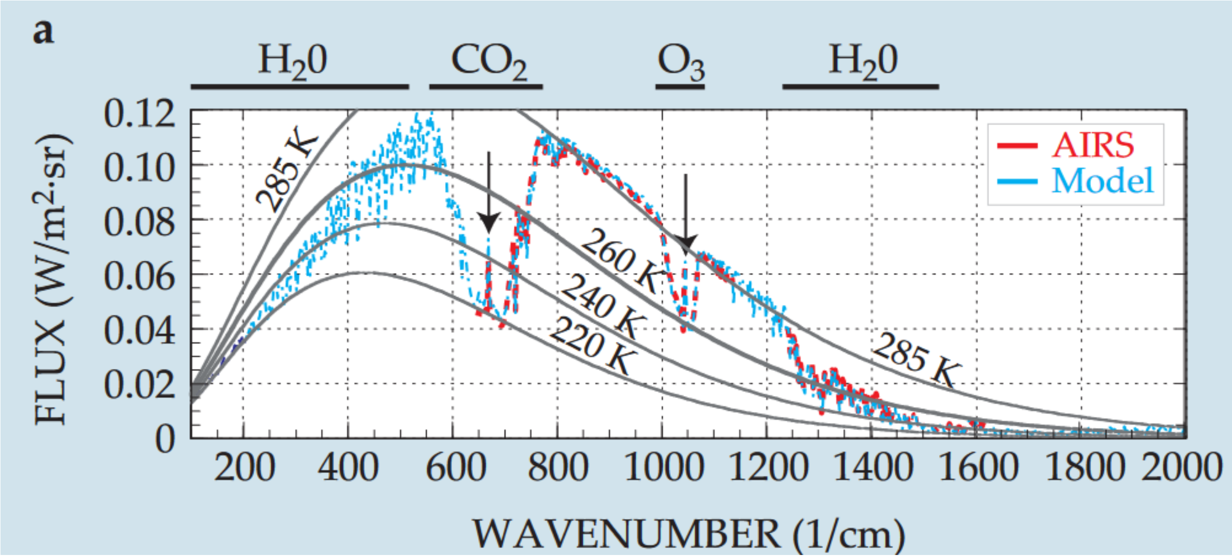


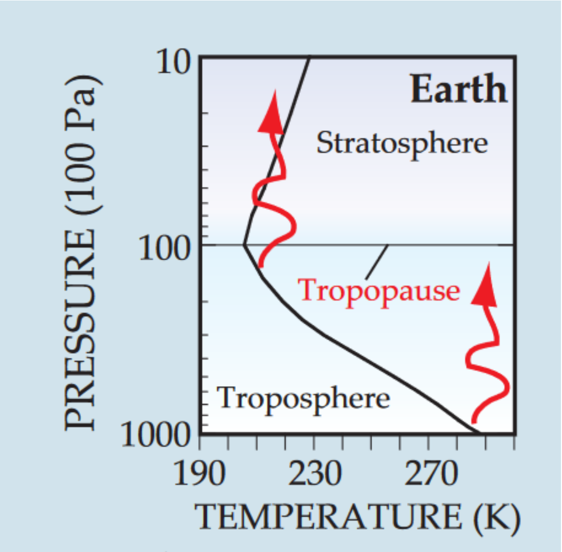
d'après Handbook of Geophysics (1960).

Atmosphere – energy balance

Spectrum of the long-wave radiation emitted by the Earth and the atmosphere.

Average emission temperature seen by space is 255 K.





Pierrehumbert, Physics Today (2011)

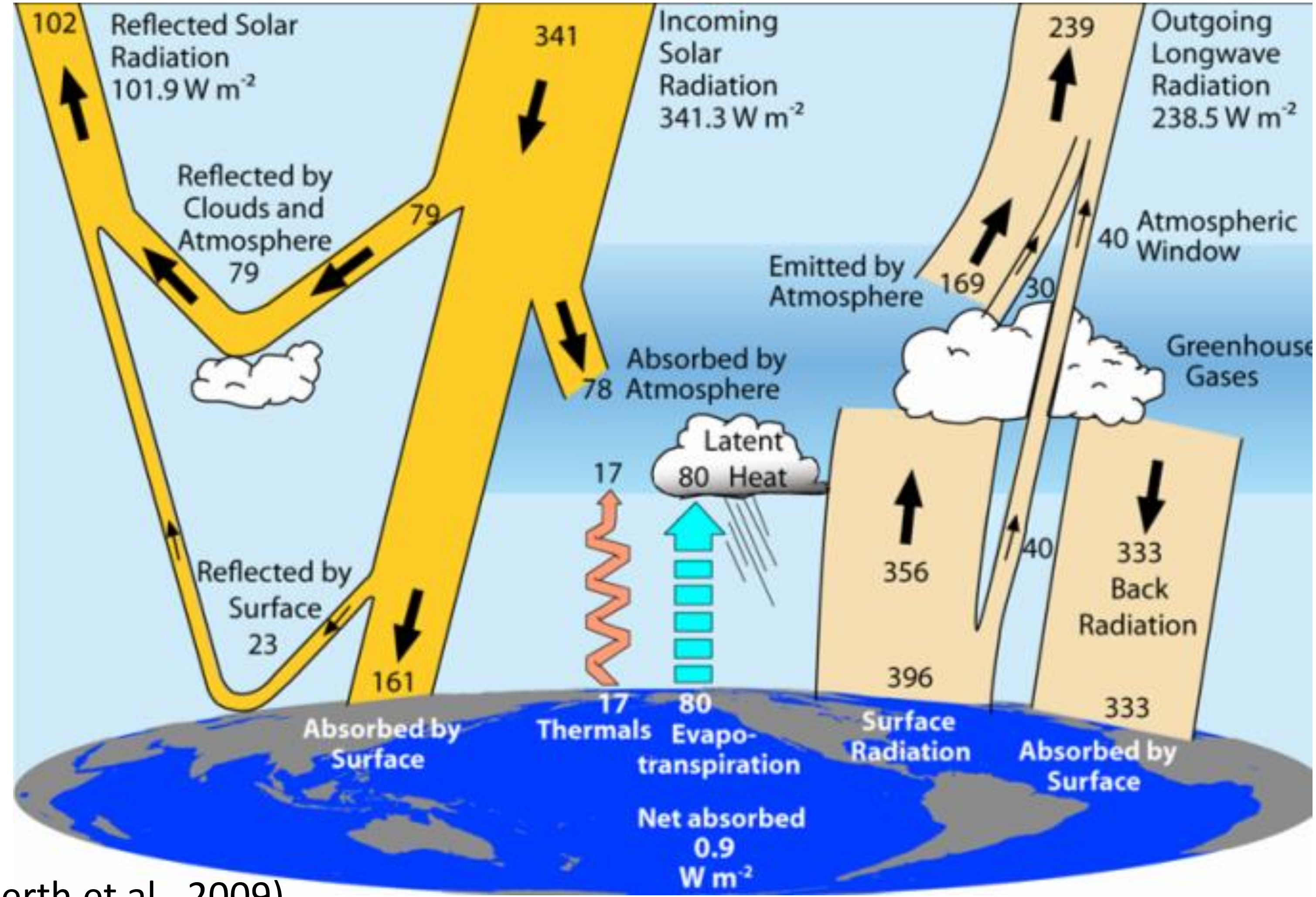
Greenhouse effect

Absorber	Just the absorber (%)	Evrything but the absorber (%)	Range of contribution(%)
Water vapour	62	61	39-62
Clouds	36	85	15-36
Water vapour and clouds	81	33	67-85
CO2	25	86	14-25
Methane	1.6	99.3	0.7 – 1.6

The first two columns of numbers give the approximate percentage of the present green house effect that would remain if either just the absorber or evrything but the absorber were present, with temperature fixed; the third columun summarizes the range of the contribution of the absorber (Vallis, Climate & the oceans, 2012).

Water vapour is the main GHG, BUT its concentration adjusts depending on temperature over 'short' time scales!

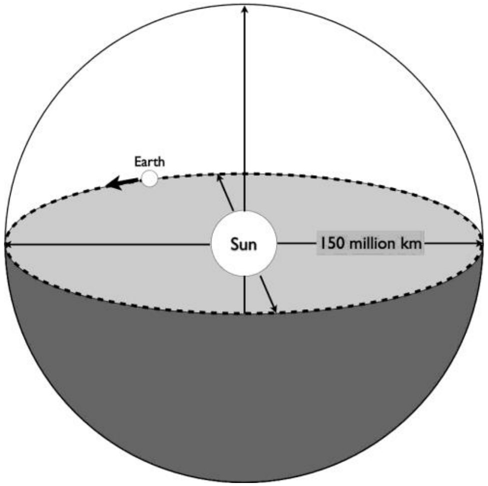
Global Energy Flows (W m^{-2})



(Trenberth et al., 2009)

A radiation budget

Incident solar flux



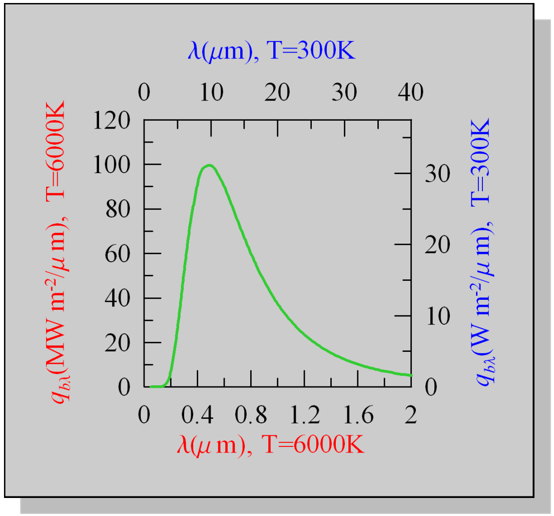
- Temperature of the Sun surface = 5796 K

- Stefan-Boltzmann law

$$\Rightarrow E_S = \sigma T_S^4 = 6{,}4.10^7\ W\ m^{-2}$$

- Wien's law $\Rightarrow \lambda_{\text{max}} = 0{,}5\,\mu m$

  Solar radiation is within the visible range, also referred to as « short waves » (SW)

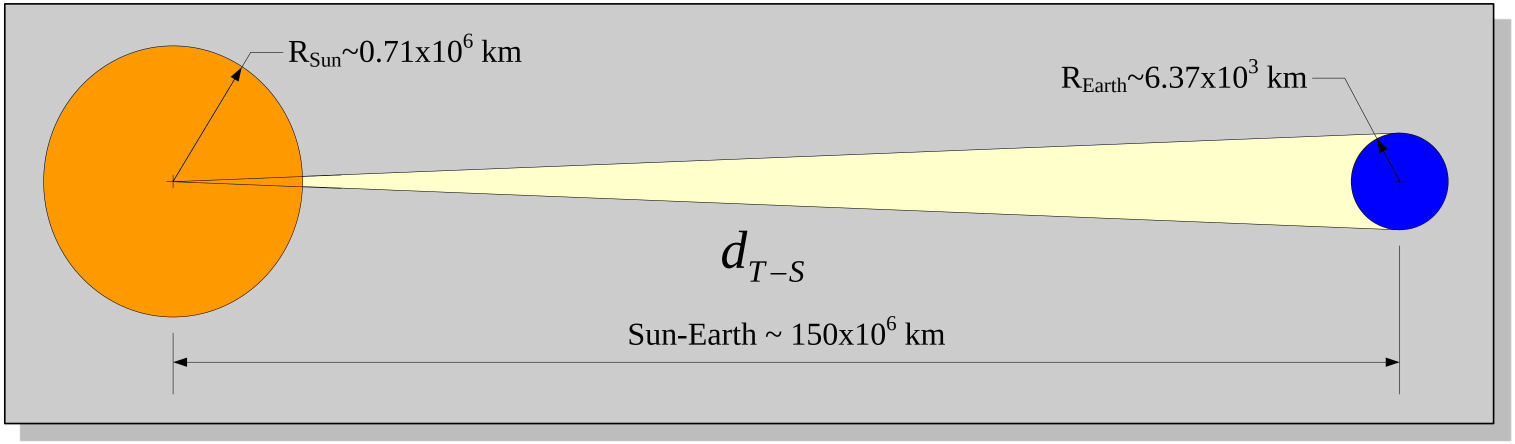


A radiation budget

Incident solar flux

- **Solar constant S:** average solar flux [W.m^{-2}] attaining the Earth

$$E_S \cdot 4\pi R_S^2 = S \cdot 4\pi d_{T-S}^2$$



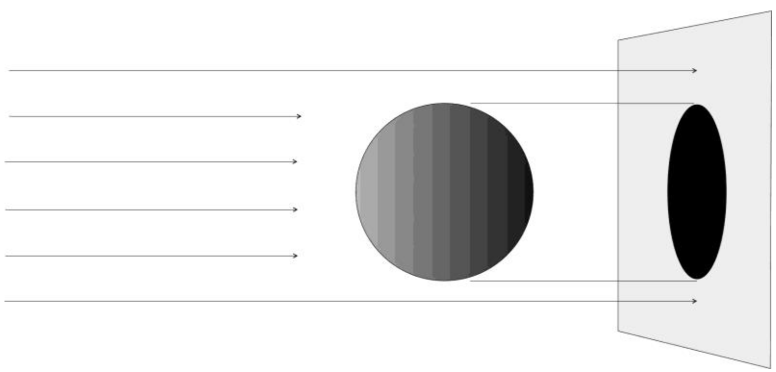
A radiation budget

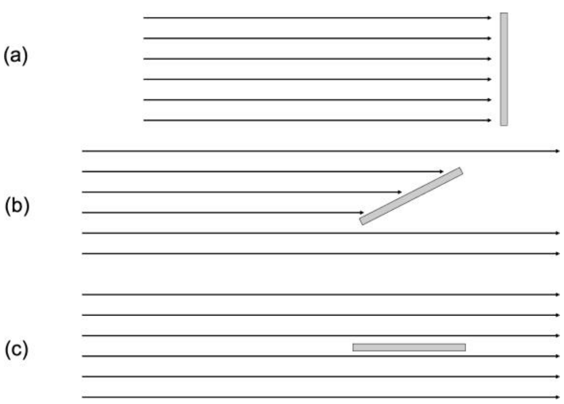
Incident Solar Flux

- **Solar constant :** average solar flux [W.m^{-2}] attaining the Earth

$$S = E_S \frac{R_S^2}{d_{T-S}^2} \approx 1365 W.m^{-2}$$

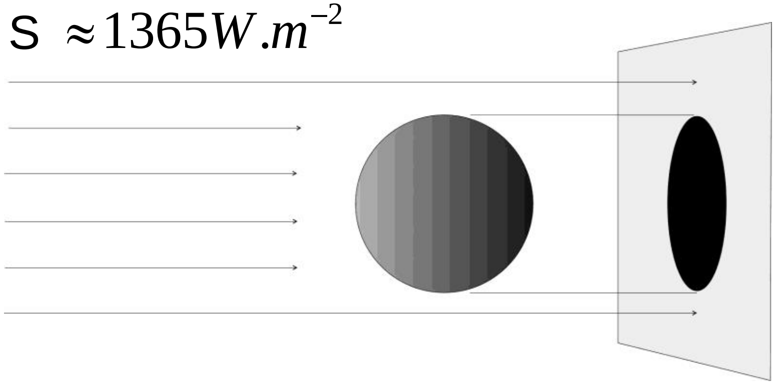
$\Rightarrow$ variations of the solar flux at a given point of the surface depends on the local orientation of the Earth surface



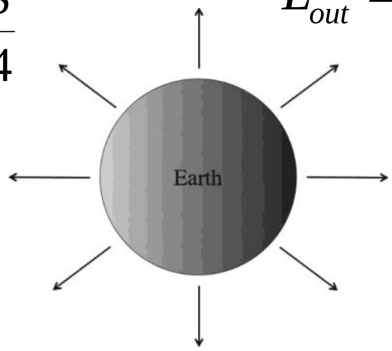


A radiation budget

S $\approx 1365 W.m^{-2}$



$\alpha \frac{S}{4}$ $L_{out} = \sigma T_e^4$



$$S(1-\alpha)\cdot \pi R^2 = \sigma T_e^4 \cdot 4\pi R^2 \quad \Rightarrow \quad E_{in} = \frac{S(1-\alpha)}{4} = \sigma T_e^4 = L_{out}$$

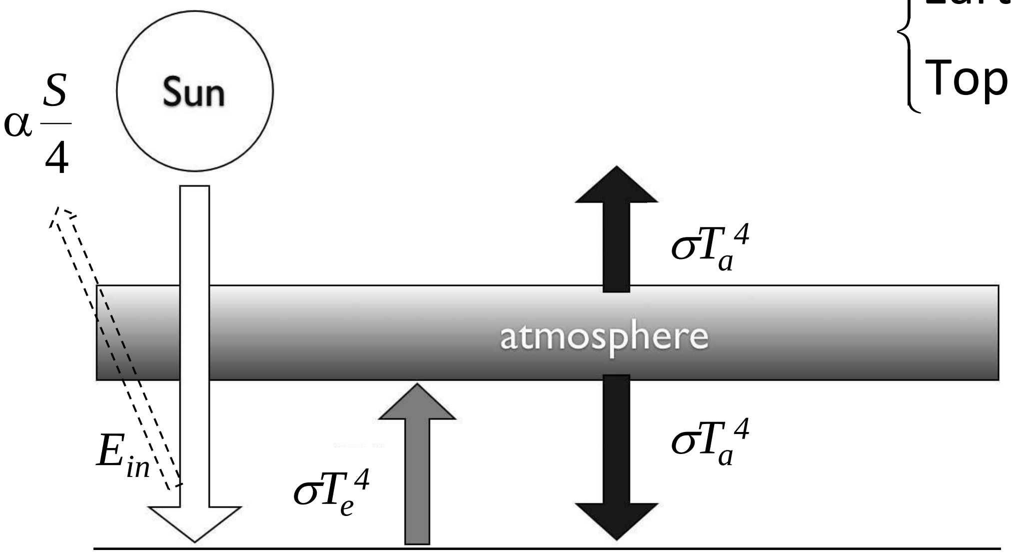
with $\alpha = 0.3$, we get $E_{in} = \dfrac{S(1-\alpha)}{4} \simeq 238 \;\; W/m^2$ and $T_e = \sqrt[4]{\dfrac{E_{in}}{\sigma}} = 255 \;\; K$

Average temperature at the Earth surface is $T_e = 288\ K\ (15°C)$.

The green house effect

To explain the averaged earth surface temperature T = 288 K (15°C) we need to introduce the atmosphere and its green-house effect.

The atmospehre is opaque to infrared photons emitted by the surface, that are therefore absorbed. **We model the atmosphere as a black body**, so it emits photons based on its temeperature. These photons are emitted downward and upward, toward space.



$$\begin{cases} \text{Earth surface} & E_{in} + \sigma T_a^4 = \sigma T_e^4 \\ \text{Top of atmopshere} & E_{in} = \sigma T_a^4 \end{cases}$$

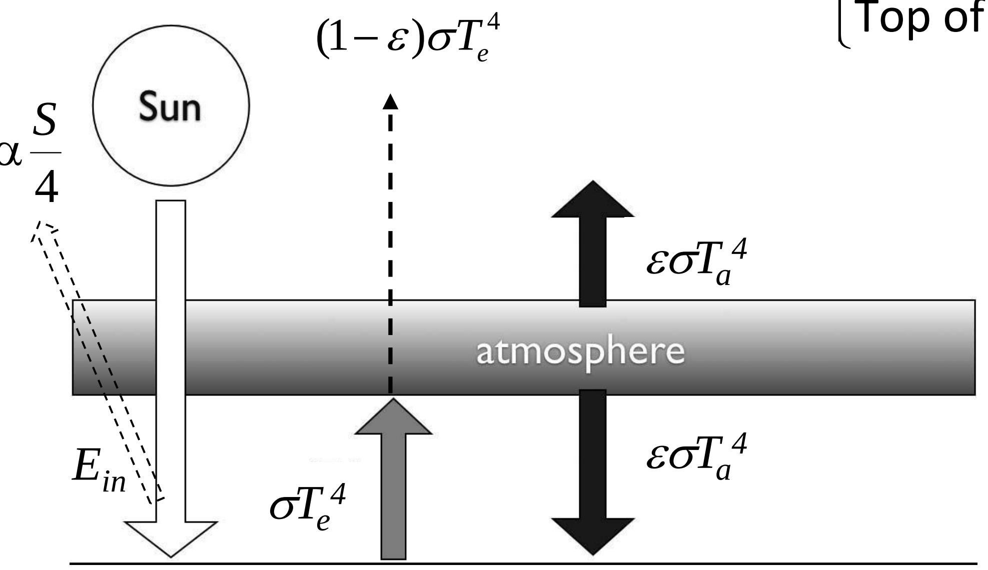
$$T_e = \sqrt[4]{\frac{2E_{in}}{\sigma}} = 303 \ K$$

with $E_{in} = \dfrac{S(1-\alpha)}{4}$

A leaky blanket

We assume that the atmosphere absorbs just a fraction ε (emissivity) of the surface radiation that escapes to space.

$$\begin{cases} \text{Earth surface} & E_{in} + \varepsilon\sigma T_a^4 = \sigma T_e^4 \\ \text{Top of atmopshere} & E_{in} = \varepsilon\sigma T_a^4 + (1-\varepsilon)\sigma T_e^4 \end{cases}$$



Combining the two we have

$$E_{in} = \frac{(2-\varepsilon)}{2}\sigma T_e^4$$

and therefore, for $\varepsilon = 3/4$

$$T_e = \sqrt[4]{\frac{2E_{in}}{(2-\varepsilon)\sigma}} = 287 \;\; K$$

Multiple layer model

A first estimate of the vertical structure of the atmosphere can be retrieved assuming local radiative equilibrium, by means of a multi-layer model for the IR radiation and considering an optical depth, defined as

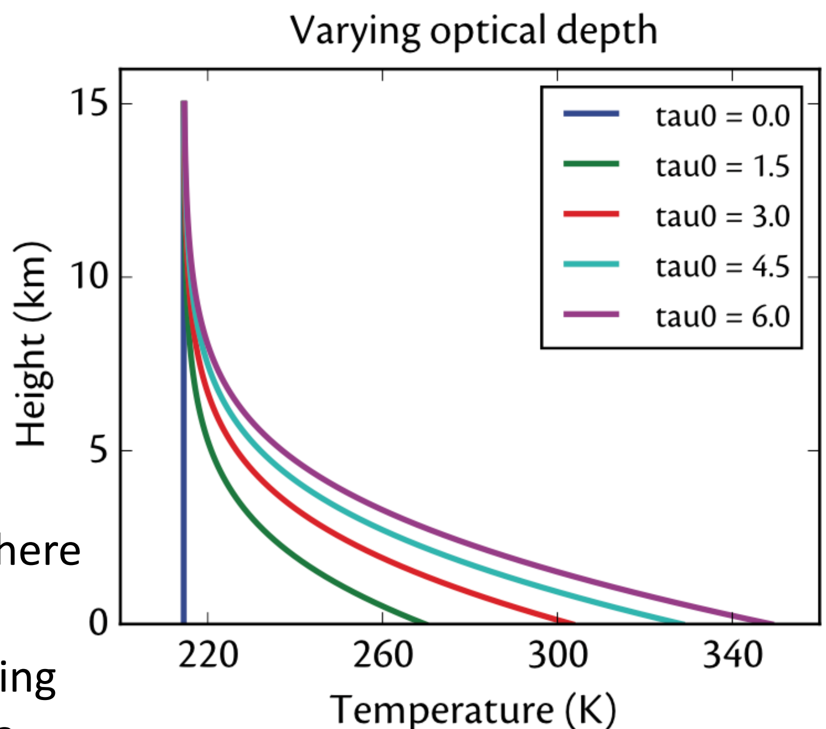
$$\tau(z) = \int_z^{\infty} \varepsilon(z)dz$$

varying with height, with a model of the form

$$\tau(z) = \tau_0 e^{-z/H_a}$$

Dominated by **water vapour** in the lower troposphere and carbon dioxide in the upper troposhere.

Curves show equilibrium temperature with a varying optical depth (reference τ_0 = 8/3), H_a = 2 km and a net incoming solar radiation of 239 W m^{-2}.



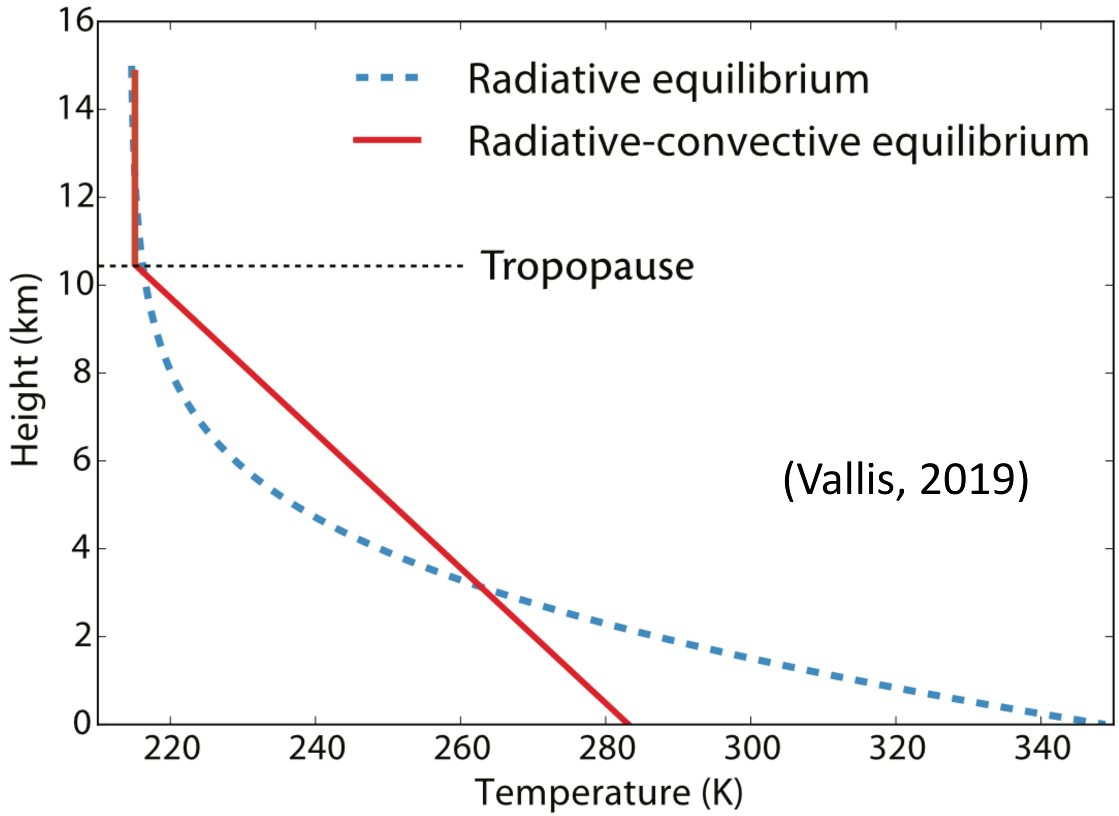
(Vallis, 2019)

Multiple layer model

The model provides a **good estimate for the height of the tropopause**. The results however, provide a too high estimate of the Earth surface temperature and a therefore a too steep temperature gradient. The profile has to be adjusted considering the role of air convection.

Radiative equilibrium temperature with $\tau_0 = 8/3$, $H_a = 2$ km.

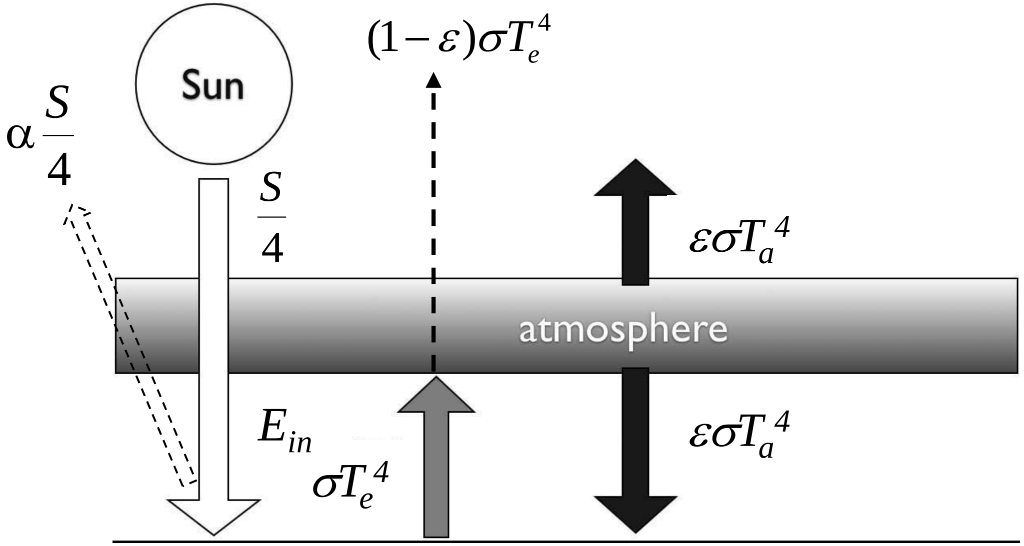
The dashed line shows a schematic adjusted temperature with a lapse rate of 6.5 K km^{-1} up to a tropopause (about 11 km) and a radiative equilibrium temperature in the stratosphere.



Perturbation of the equilibrium

What are the possible causes of an imbalance of the radiative budget ?

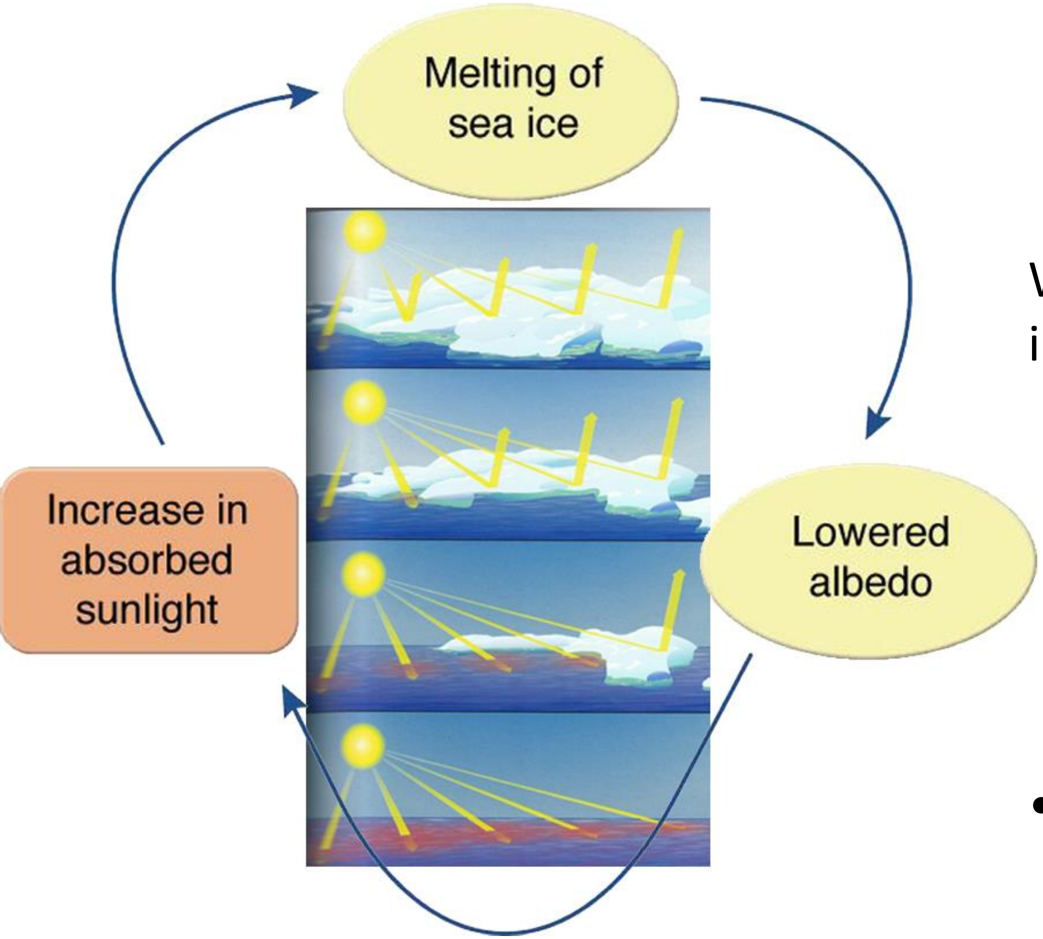
$$E_{in} = \frac{S(1-\alpha)}{4} = \frac{(2-\varepsilon)}{2}\sigma T_e^4 = L_{out}$$



- Changes in albedo α
- Changes in emissivity ε

$\Rightarrow$ role of 'feedbacks'

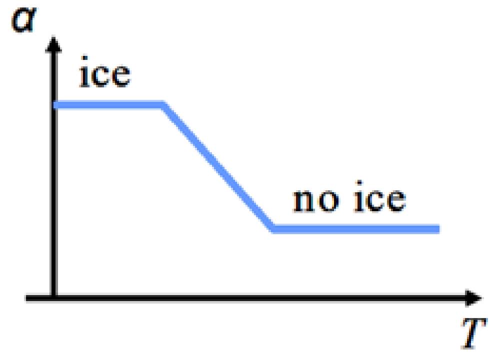
Albedo varying with T_e?



Ice – Albeedo Feedback

- Positive
- Slow

We expect albedo to be reduced by increasing temperature



- When T_e increases α decreases and E_{in} will increase
- When T_e decreases α increases and E_{in} will decrease

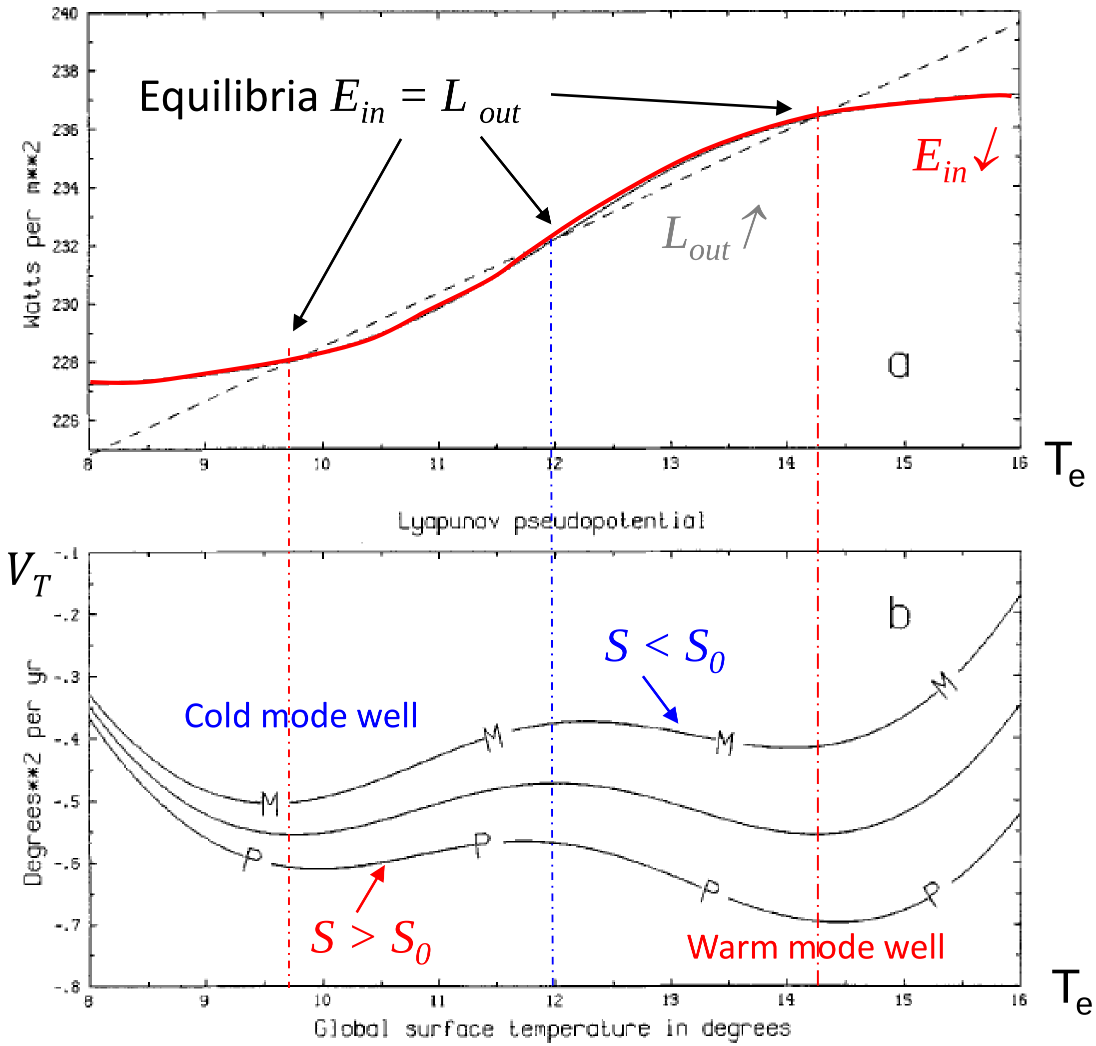
Albedo varying with T_e?

We expect albedo to be reduced by increasing temperature. Therefore E_{in} will increase with T_e.

→ Multiple equilibria

Given by the minima of the Lyapunov potential V_T, *defined such that*

$$\frac{dT}{dt} = -\frac{dV_T}{dT}$$



ÉCOLE CENTRALE LYON

Tellus (1982) *34*, 10–16

Stochastic resonance in climatic change

By ROBERTO BENZI, *Istituto di Fisica dell'Atmosfera, C.N.R., Piazza Luigi Sturzo 31, 00144, Roma, Italy,*
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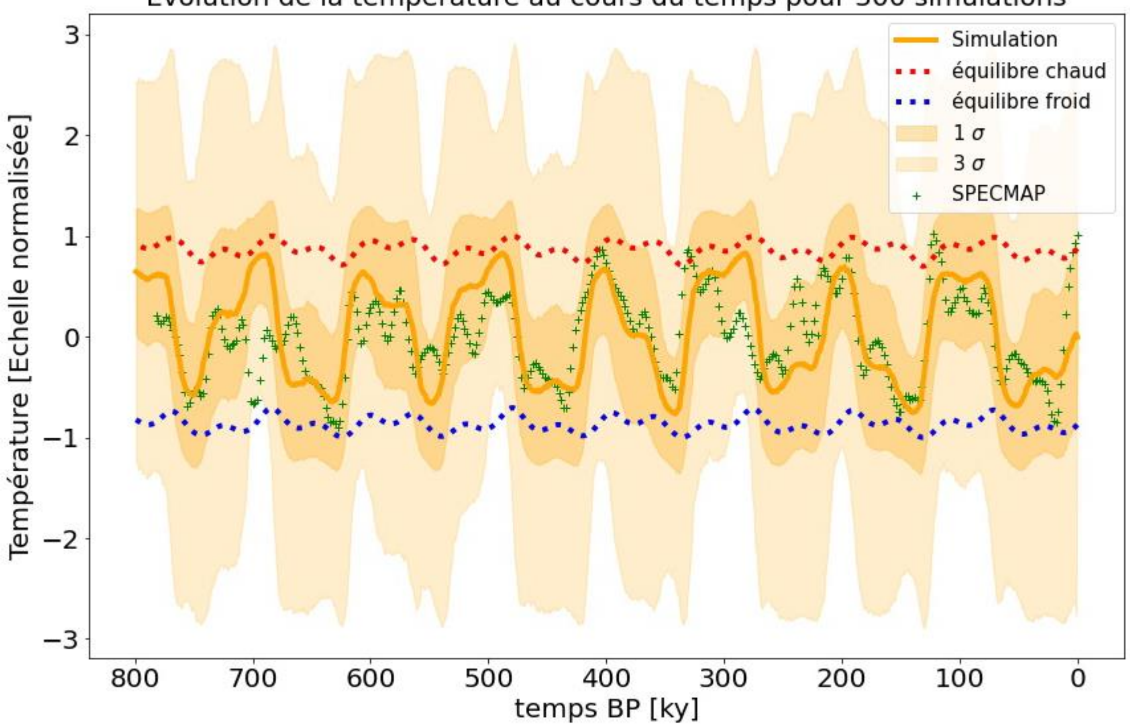


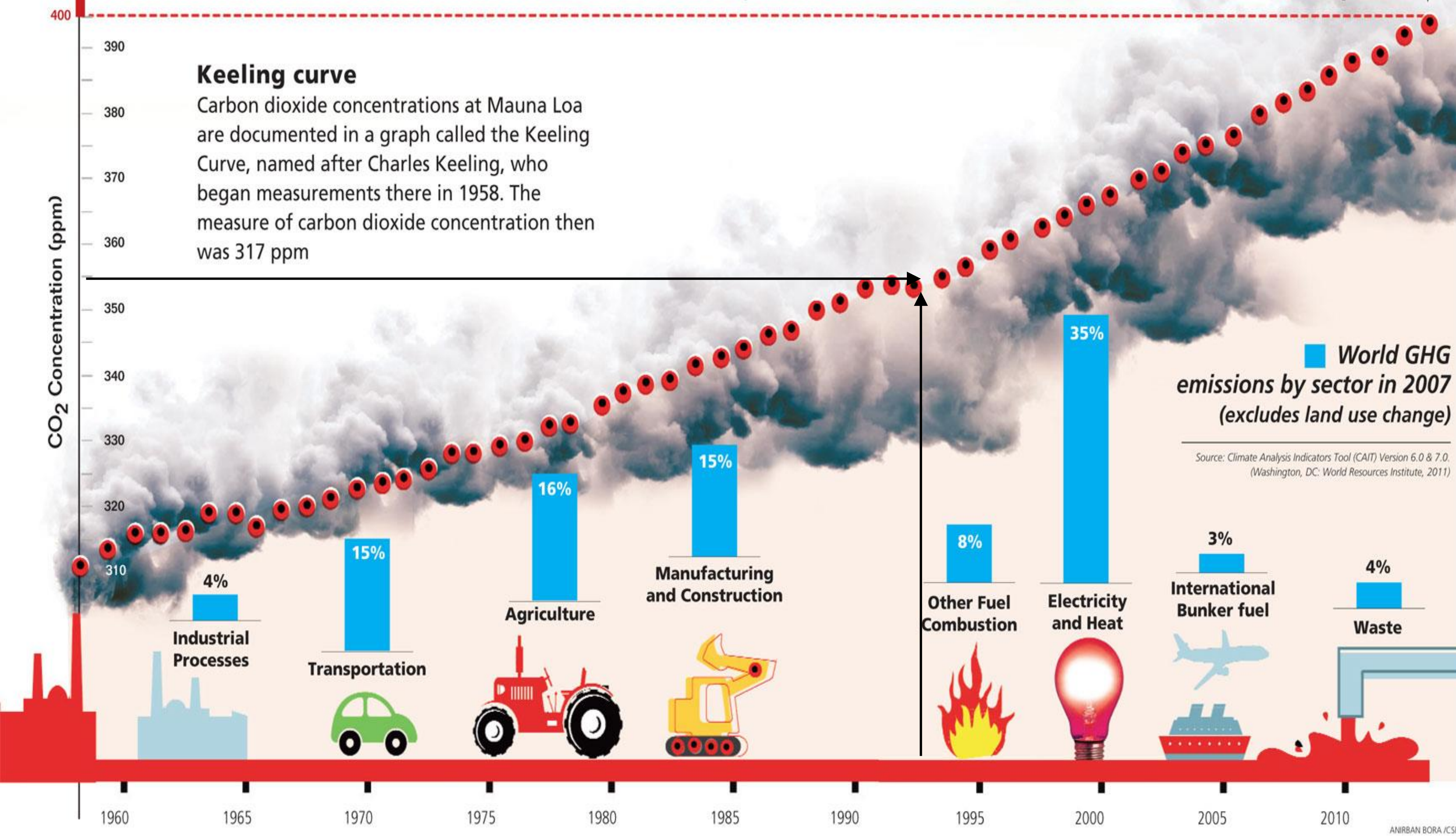
A **first** in human history

Carbon dioxide levels have crossed 400 ppm. This means the impacts of climate change will be even more pronounced—droughts, floods and sea level rise, for instance. If the world does not act to limit carbon dioxide emissions, climate change will cause devastation worldwide, and more so in South Asia. The poor will end up with a raw deal

BREACHES
400
PPM ON
MAY 9, 2013

Source: keelingcurve.ucsd.edu

Carbon dioxide concentration at Mauna Loa Observatory, Hawaii



- For the past several years, CO_2 concentrations have hovered close to 390 ppm. The 400 ppm daily average is a first in human history
- In the last 50 ppm increase, the Arctic melted. Imagine what another 50 ppm increase will do
- Diversity of even common species found in most parts of the world will be badly hit. Animal species in particular may decline more as a result of loss of food from plants
- Some environmentalists are of the view that to return to the 350 ppm level, use of conventional energy sources has to be re-examined. But this is easier said than done. Any alternative measure will require finance and technological assistance from industrialised countries to developing countries. Even though institutions for financial transfer exist under UNFCCC, there is no money and the industrialised world has not provided exclusive climate finance to developing countries yet

Change in radiative budget by adding GHG

$$E_{in} = \frac{(2-\varepsilon)}{2}\sigma T_e^{\,4} = \varepsilon_b \sigma T_e^{\,4}$$

The effective emissivity of the climate system is

$$\varepsilon_b = \frac{(2-\varepsilon)}{2}$$

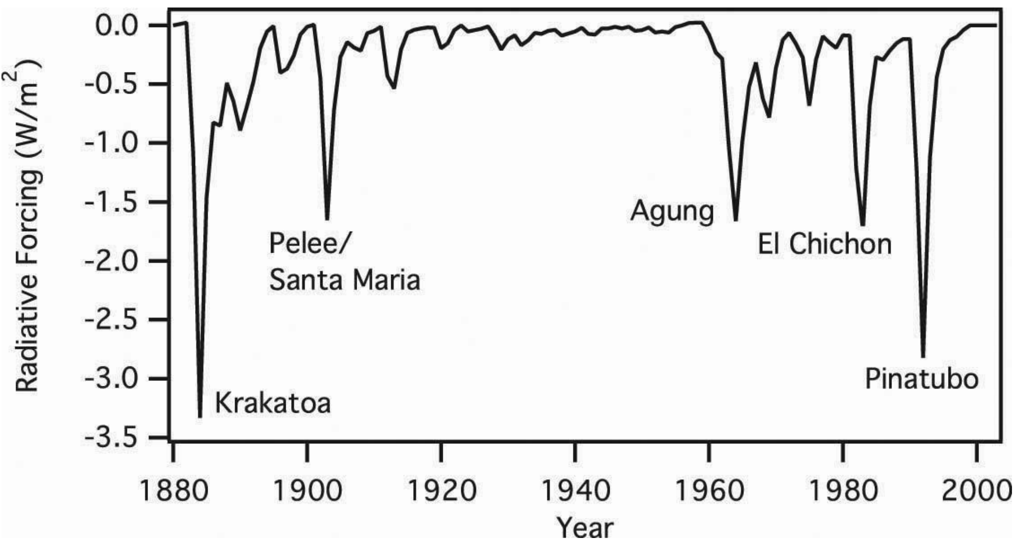
If we model the atmospheric emissivity as a logarithmic function of CO_2 concentration (which is itself an approximation),

$$\varepsilon = k\ log[CO_2]$$

Increasing the $[CO_2]$ increases ε (i.e. the atmosphere becomes more opaque to long-wave radiation). This in turn results in a decreased effective emissivity of the climate system: the atmosphere emits more strongly, but it also absorbs the surface emission more strongly, leading to a net decrease of TOA emission.

To re-establish a balance, T_e has to increase. According to this balance, by **doubling** $\boldsymbol{[CO_2]}$ we have that $\boldsymbol{\Delta Te \approx 1.2\ K}$

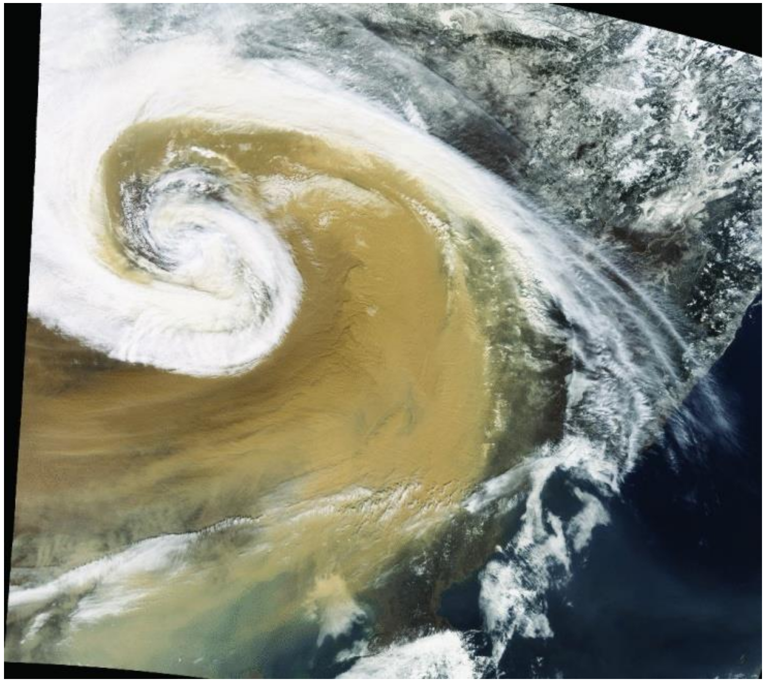
Radiative Forcing due to aerosols



Suspended aerosols modifiy albedo α and therefore

$$E_{in} = \frac{S(1-\alpha)}{4}$$

Average direct radiative effect = = -0.5 W/m$_2$



Forcing caused by volcanic eruptions

Sulfate aerosols absrob incoming radiation BUT are also **higly reflective**

Radiative Forcing due to aerosols

Volcanoes

The 1815 Eruption of Mount Tambora (on the island of Sumbawa, Indonesia) was one of the most powerful eruptions recorded in history. The eruption of the volcano reached a climax on 10 April 1815 and was followed by increased steaming and small phreatic eruptions (over 3 years).

Radiative Forcing due to aerosols


Chichester Canal, J.M.W. Turner

The year without summer

1816


Weymouth Bay
by John RA Constable


Two men at see C.D. Friedrich

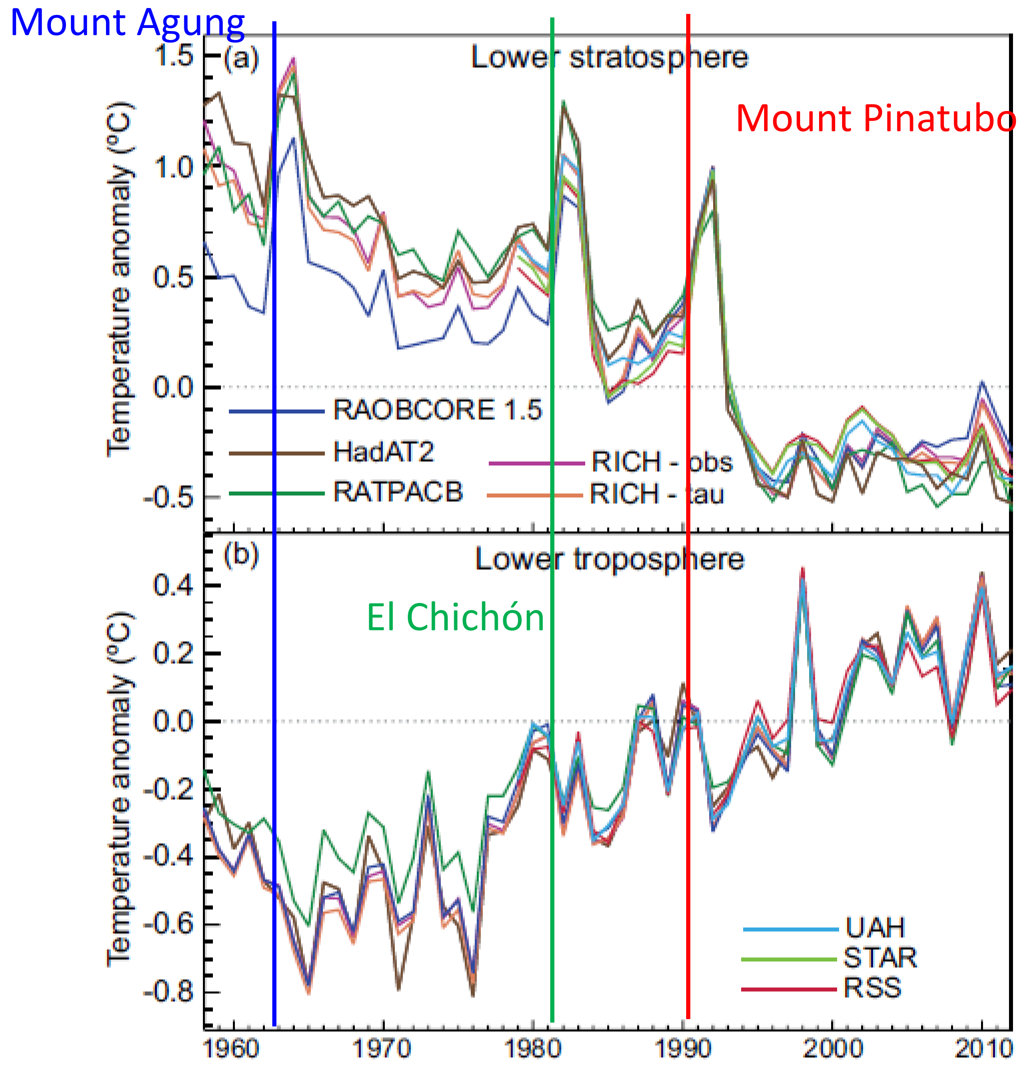


Figure 2.24 | Global annual average lower stratospheric (top) and lower tropospheric (bottom) temperature anomalies relative to a 1981–2010 climatology from different data sets. STAR does not produce a lower tropospheric temperature product. Note that the *y*-axis resolution differs between the two panels.

Vertical temperature profile and CO_2 concentration

VOL. 24, NO. 3 JOURNAL OF THE ATMOSPHERIC SCIENCES MAY 1967

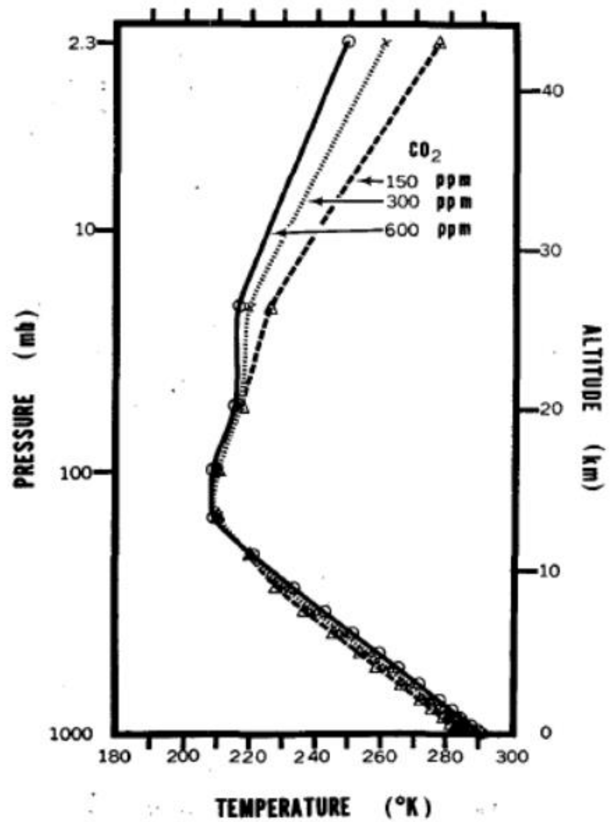


FIG. 16. Vertical distributions of temperature in radiative convective equilibrium for various values of CO_2 content.

Equilibrium of the Atmosphere with a Given Distribution of Relative Humidity

SYUKURO MANABE AND RICHARD T. WETHERALD

Geophysical Fluid Dynamics Laboratory, ESSA, Washington, D. C.

(Manuscript received 2 November 1966)

Therefore, a series of radiative convective equilibrium computations were performed. Fig. 16 shows the vertical distributions of equilibrium temperature corresponding to the three different CO_2, i.e., 150, 300, and 600 ppm contents by volume. In this figure, the following features are noteworthy:

1) The larger the mixing ratio of carbon dioxide, the warmer is the equilibrium temperature of the earth's surface and troposphere.
2) The larger the mixing ratio of carbon dioxide, the colder is the equilibrium temperature of the stratosphere.
3) Relatively speaking, the dependence of the equilibrium temperature of the stratosphere on CO_2 content is much larger than that of tropospheric temperature.

Ill. Niklas Elmehed © Nobel Prize Outreach

Syukuro Manabe

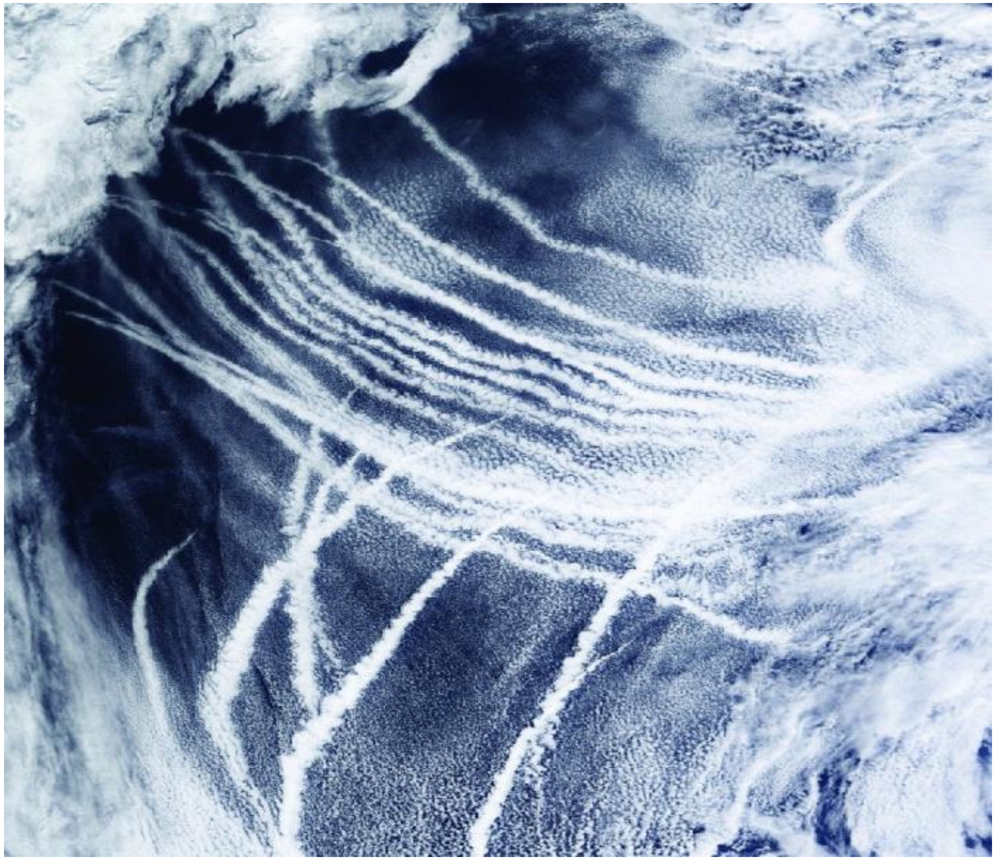
Radiative Forcing due to aerosols

Indirect radiative effect = -0.7 W/m_2

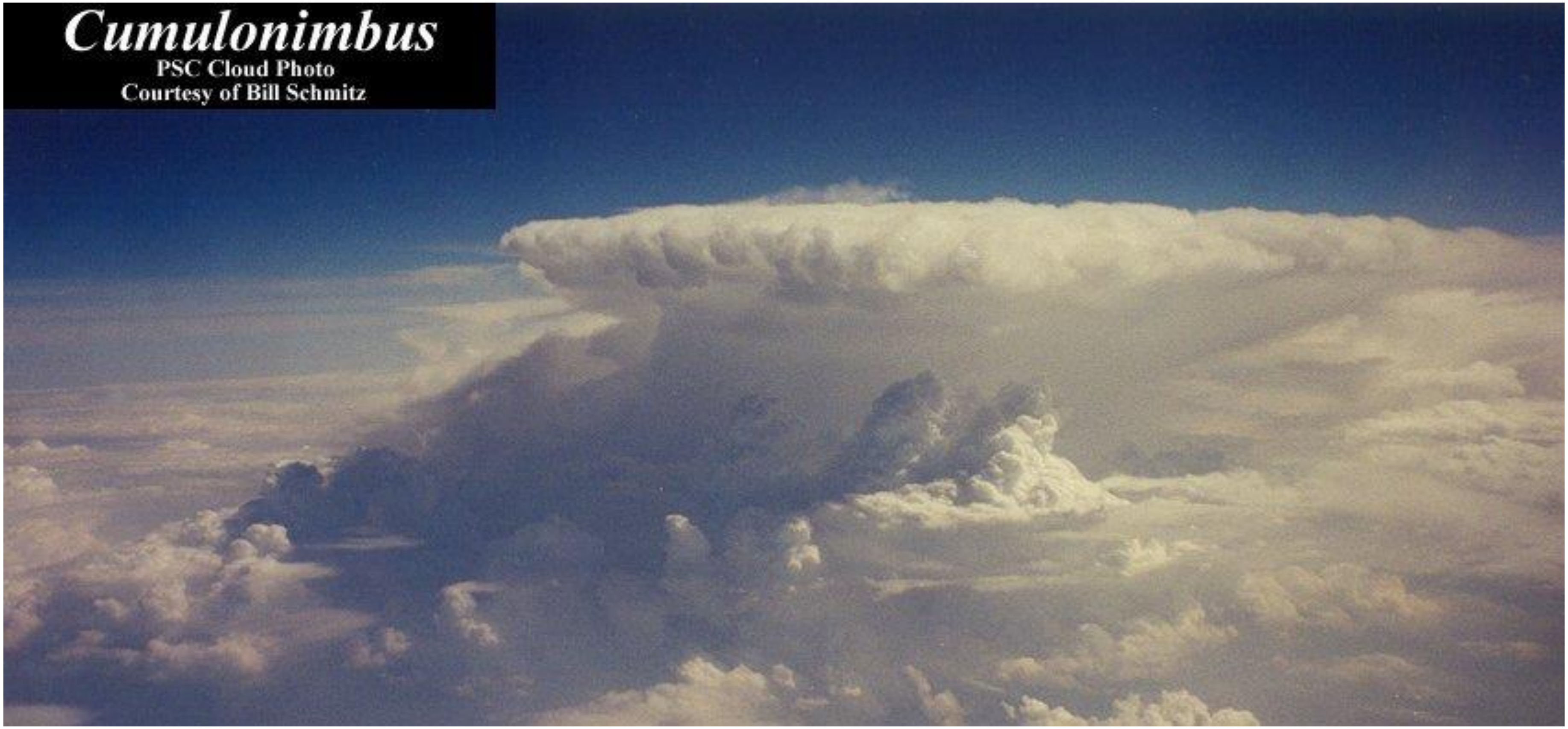
Aerosols – Condensation nuclei that trigger condesation

Ex: Clouds formations in ships tracks

→ Aerosols Feedbacks

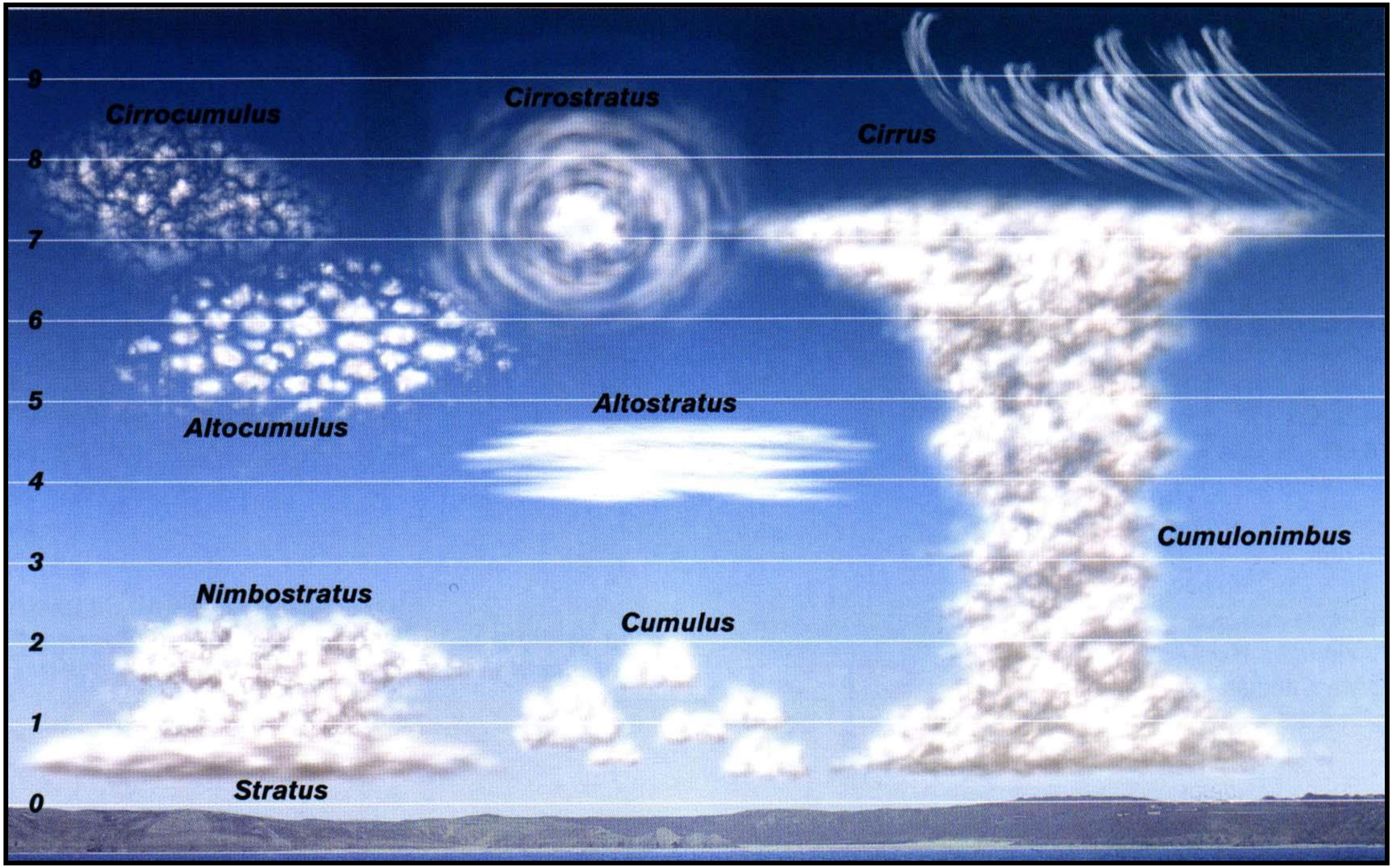


Cloud formation



Cloud formation

Cloud formation



Clouds feedbacks (albedo & emissivity)

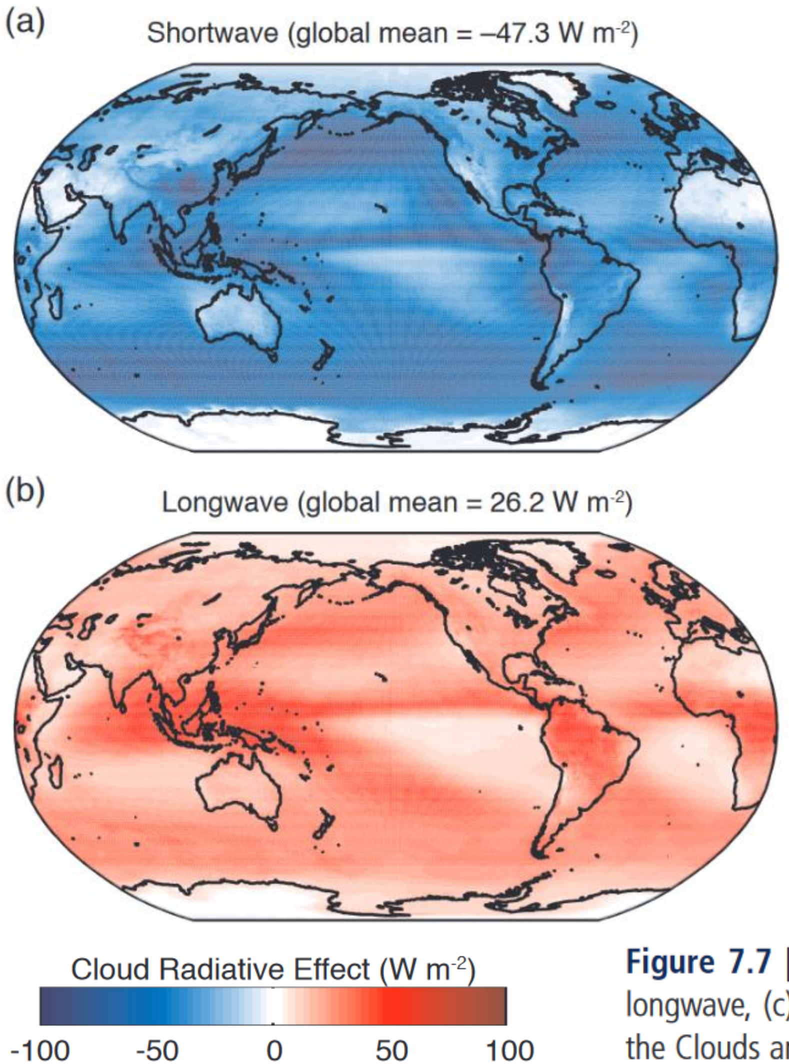
Clouds:

- **Positive** and **negative** feedback
- Fast

High level clouds (cirrus) trap outgoing longwave radiation

Low level clouds (Cumulus) reflect incoming shortwave radiation

$R_{cloudy} - R_{clear}$



Clouds

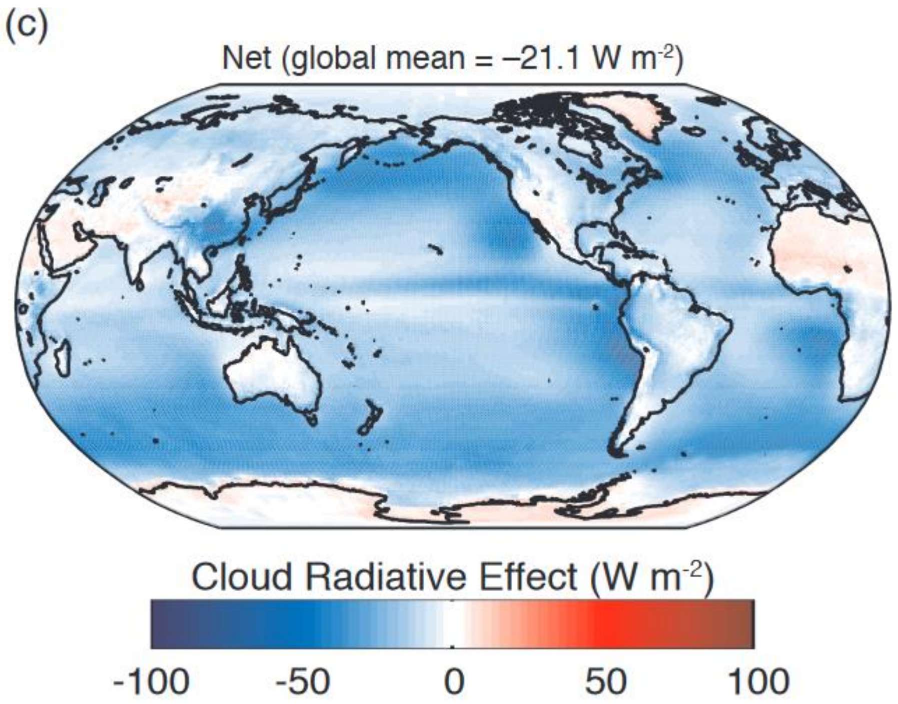
Effect of clouds inferred from satellite data by comparing upwelling radiation in cloudy and non-cloudy conditions (Ramanathan et al., 1989).

- **Enhancement of the planetary albedo:** short-wave cloud radiative effect (SWCRE) of approximately **–50 W m$^{–2}$**
- **Contribution to the greenhouse effect:** mean longwave effect (LWCRE) of approximately **+30 W m$^{–2}$**

Boucher et al. 2013

Figure 7.7 | Distribution of annual-mean top of the atmosphere (a) shortwave, (b) longwave, (c) net cloud radiative effects averaged over the period 2001–2011 from the Clouds and the Earth's Radiant Energy System (CERES) Energy Balanced and Filled

Clouds



Boucher et al. 2013

Net <u>cooling effect</u> of clouds in current climate.

BUT!

"Owing to the large magnitudes of SWCRE and LWCRE, clouds have the potential to cause significant climate feedback.

The sign of this feedback on climate change cannot be determined from the sign of CRE in the current climate, **<u>but depends instead on how climate-sensitive the properties are that govern the LWCRE and SWCRE</u>**".

https://www.climat-en-questions.fr/auteur/olivier-boucher

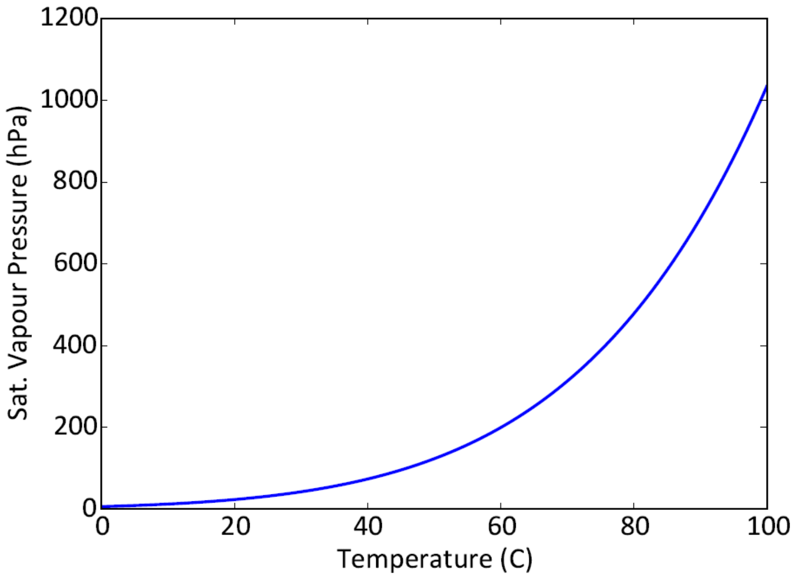
https://www.ipcc.ch/site/assets/uploads/2018/02/WG1AR5_Chapter07_FINAL-1.pdf

Water vapour feedbacks (emissivity)

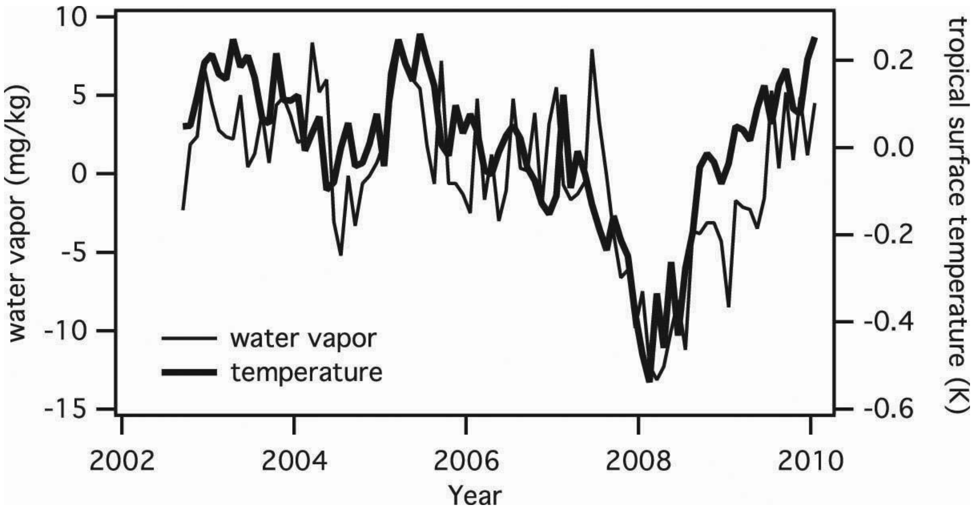
Water-vapour feedback

- Positive
- Fast

A warming of the surface increases atmospheric humidity and, because the water vapor is itself a greenhouse gas, this leads to additional warming



Time series of tropical water vapour and tropical surface temperature. Plotted values are monthly average anomalies, calculated relative to the entire time series and average over 30° N – 30° S.

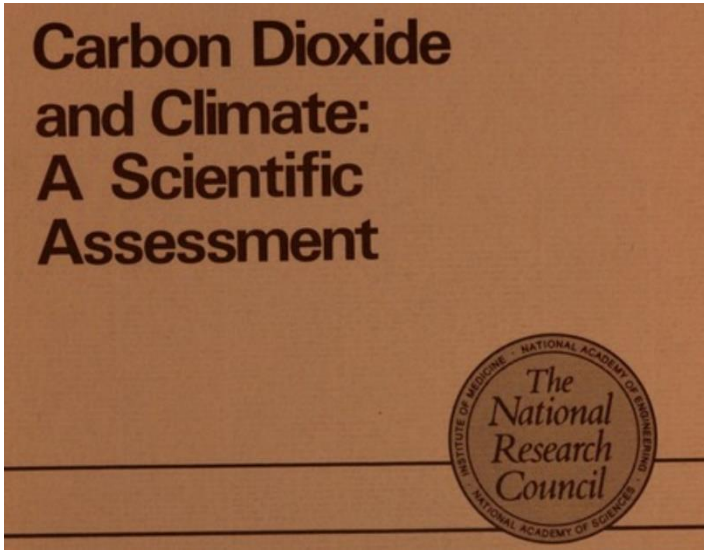


Most important feedback in the system

Saturation water vapour increases exponentially with temperature

Climate Sensitivity

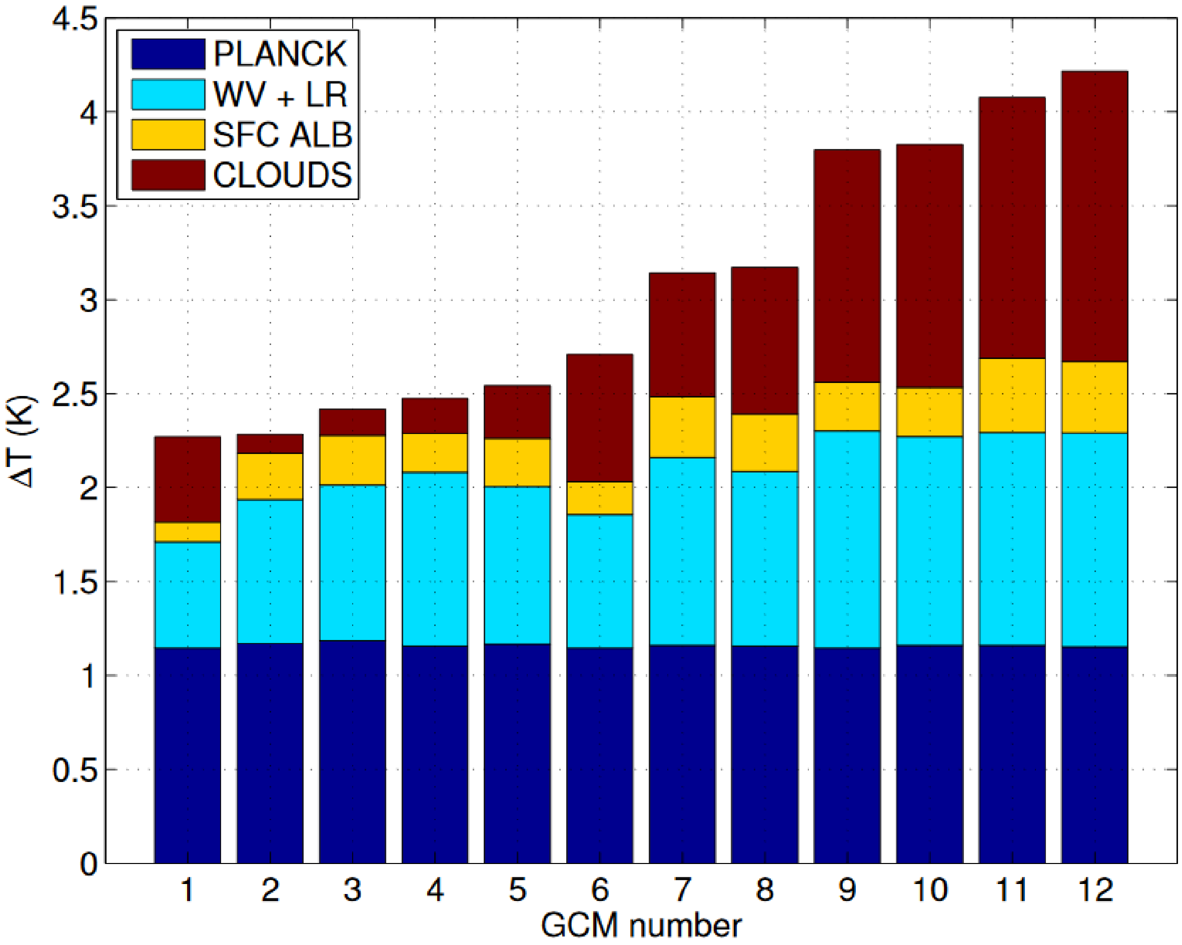
Measure of how much the Earth's **climate** will cool or warm after a change in the **climate** system, for instance, how much it will warm for doubling in carbon dioxide concentrations



Charney Report (1979) : “We estimate the most probable warming fo a doubling of CO2 to be near 3°C with a probable error of 1.5 °C”

Jule Gregory Charney

Climate Sensitivity – the role of feedbaks

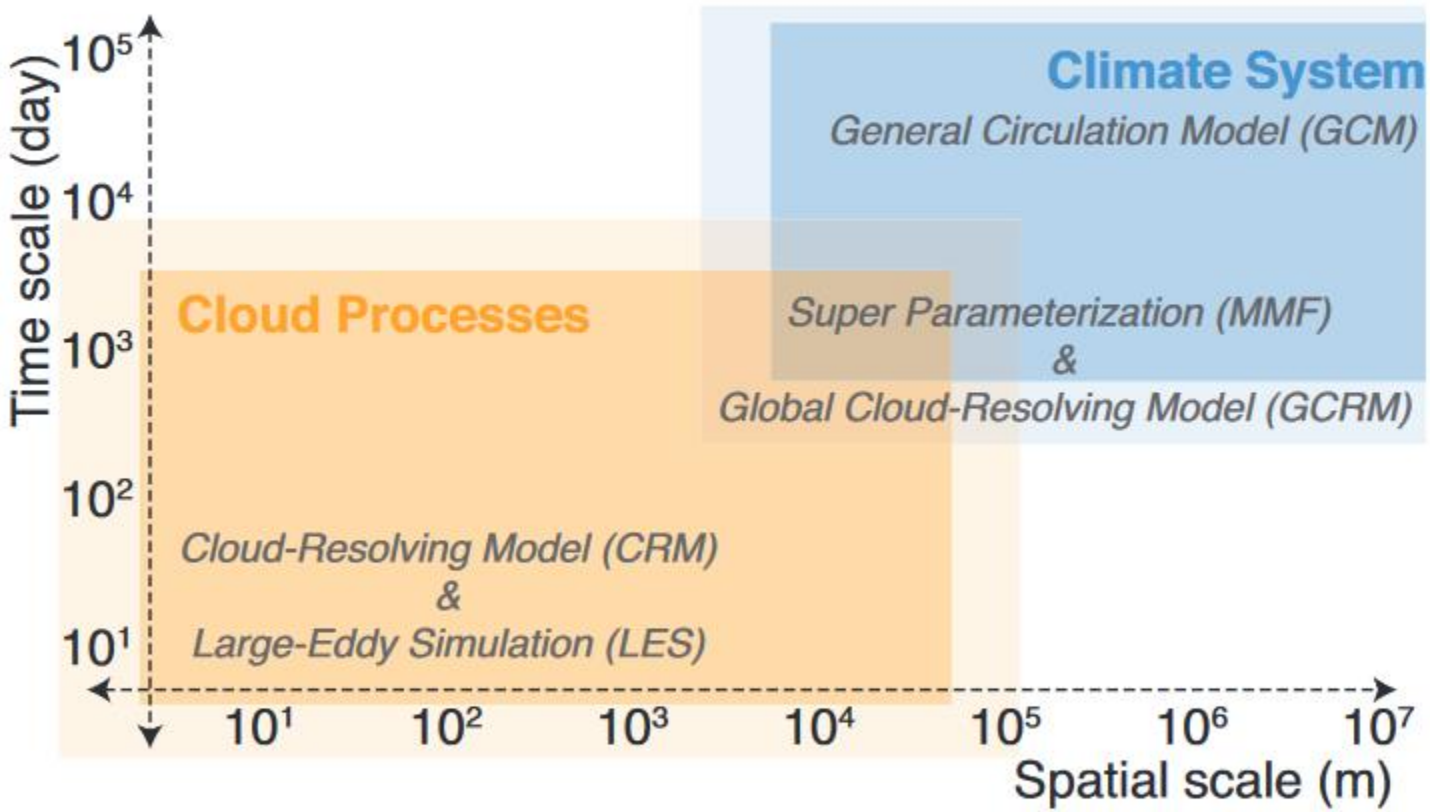


Bony & Dufresne (2008)

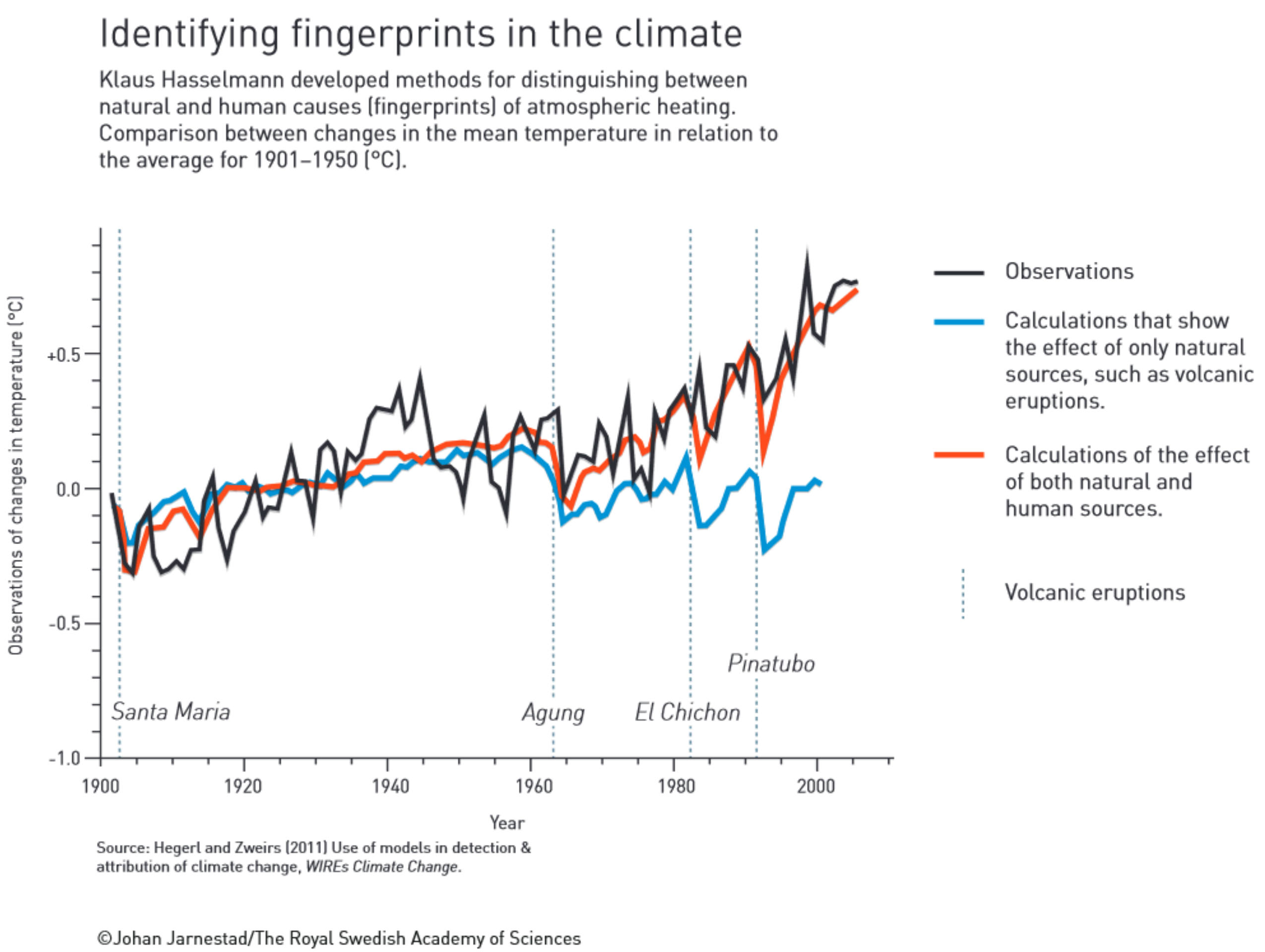
Response to a doubling of CO_2 concentrations as given by twelve different GCM

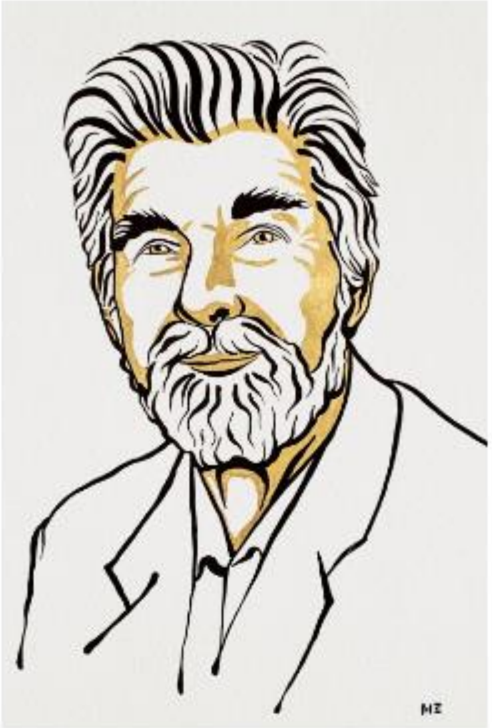
https://www.carbonbrief.org/explainer-how-scientists-estimate-climate-sensitivity

Cloud and climate system
Typical time & lenght scales



Global circulation models

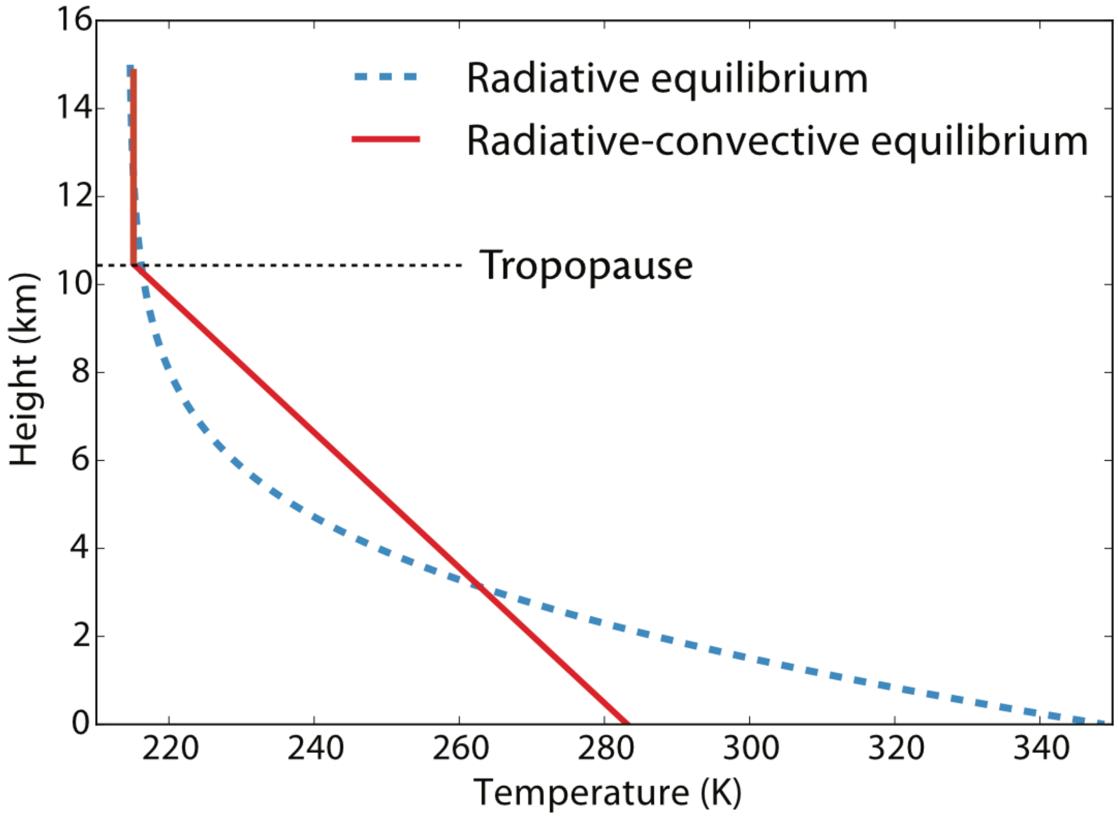




Ill. Niklas Elmehed © Nobel Prize Outreach

Klaus Hasselmann

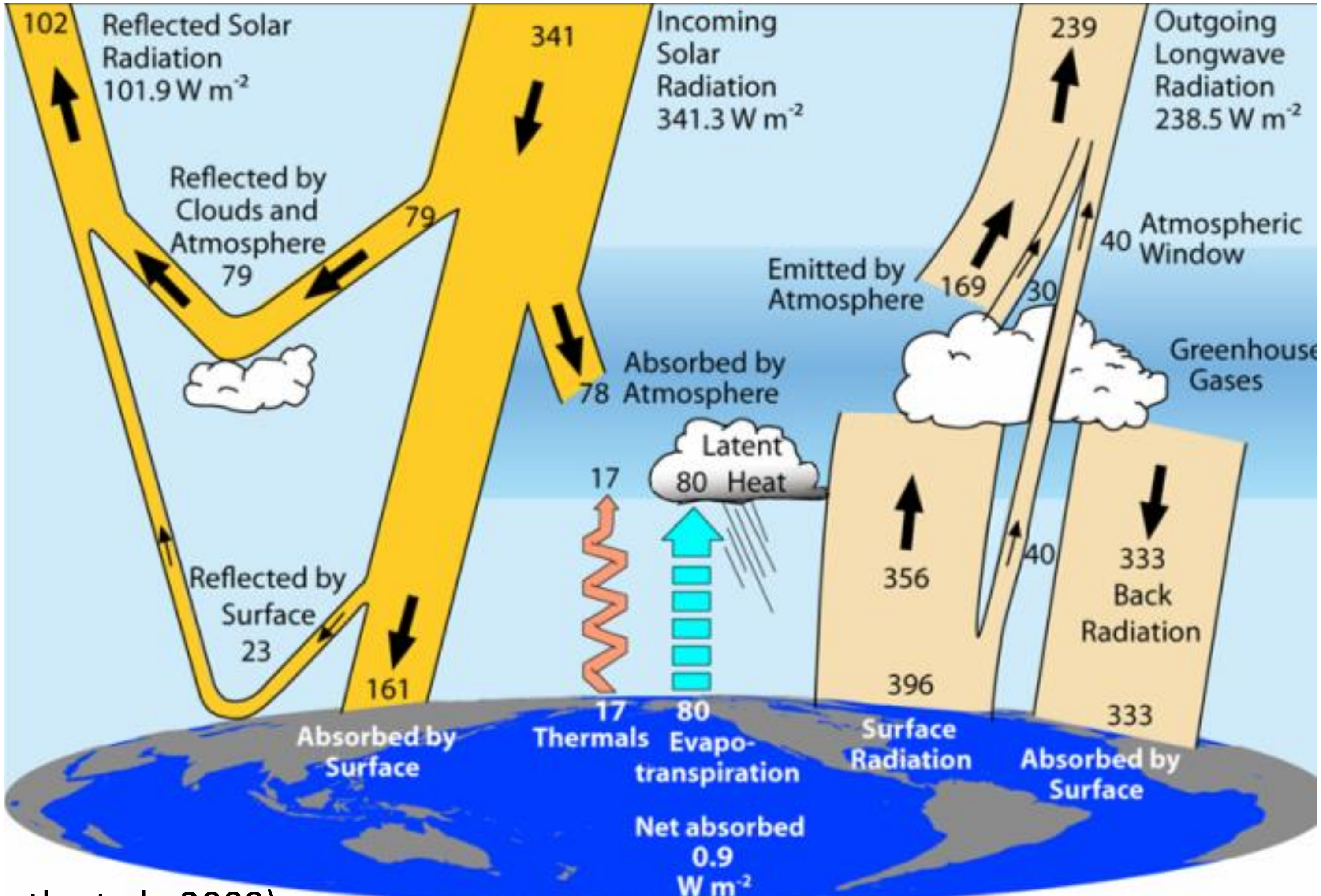
Multiple layer model



(Vallis, 2019)

1. The ‘radiative equilibrium’ profile is **unstbable**.
2. It has to be adjusted to take into account the role of **convection**.

Global Energy Flows (W m^{-2})



(Trenberth et al., 2009)

Radiative balance at the Earth surface

$$\boxed{R_N = K_\downarrow - K_\uparrow + L_\downarrow - L_\uparrow}$$

R_N	$K_\downarrow$	$K_\uparrow$	$L_\downarrow$	$L_\uparrow$
Net radiative flux	Incident solar flux	Reflected solar flux	Incident atmospheric *flux*	Flux from the Earth surface

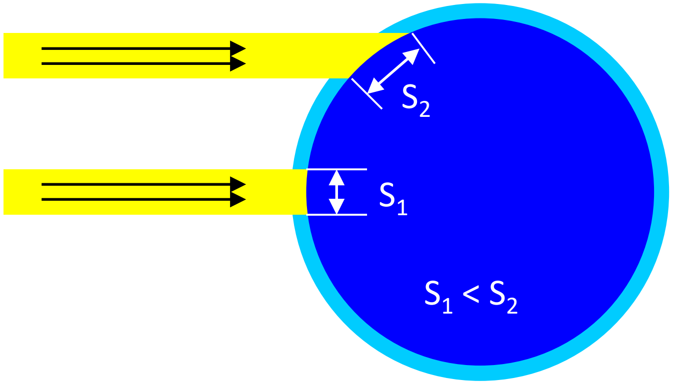
- The net radiation ($W.m^{-2}$) is the power "availible" at the surface in order to:
  - Heat the surface Q_G
  - Heat the air by conduction/convection (sensible heat flux) $\boldsymbol{H_0 = 17\ W\ m^{-2}}$
  - Induce evaporation (latent heat flux) $\boldsymbol{Q_E = 80\ W\ m^{-2}}$

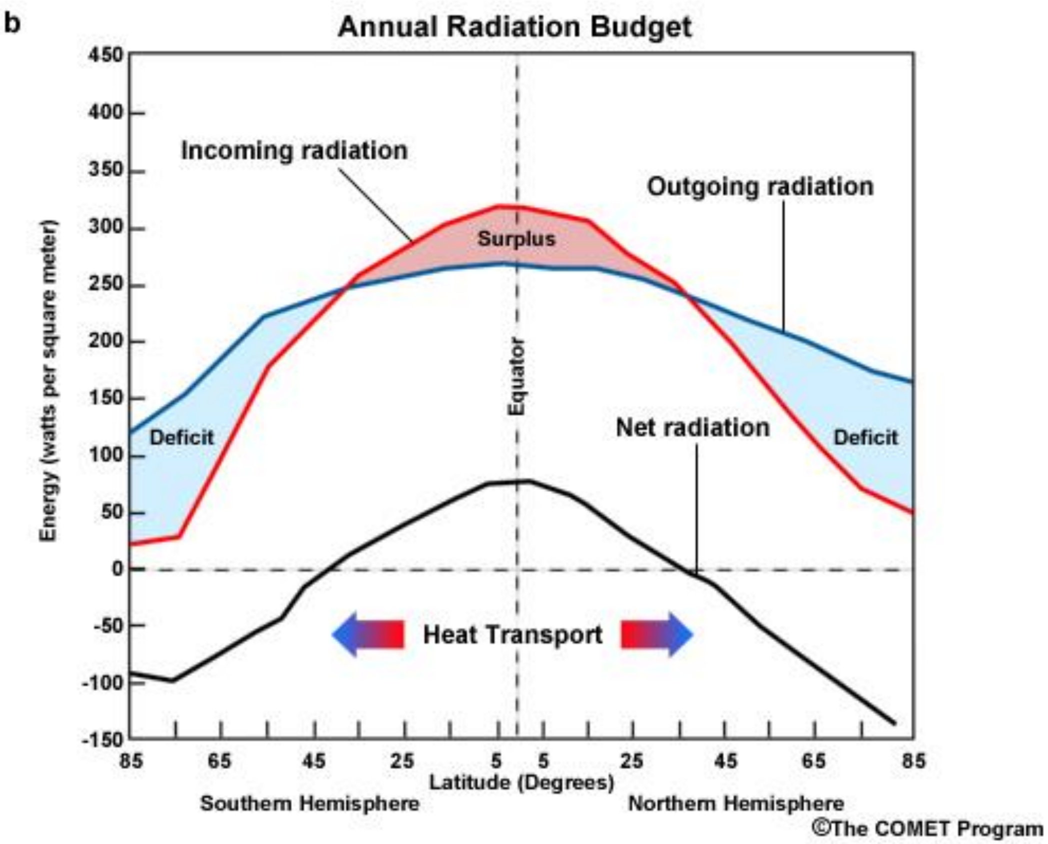
$$R_N = Q_G + H_O + Q_E$$

R_N	Q_G	H_O	Q_E
Net radiative flux	Flux toward the ground	Sensible heat flux	Latent heat flux

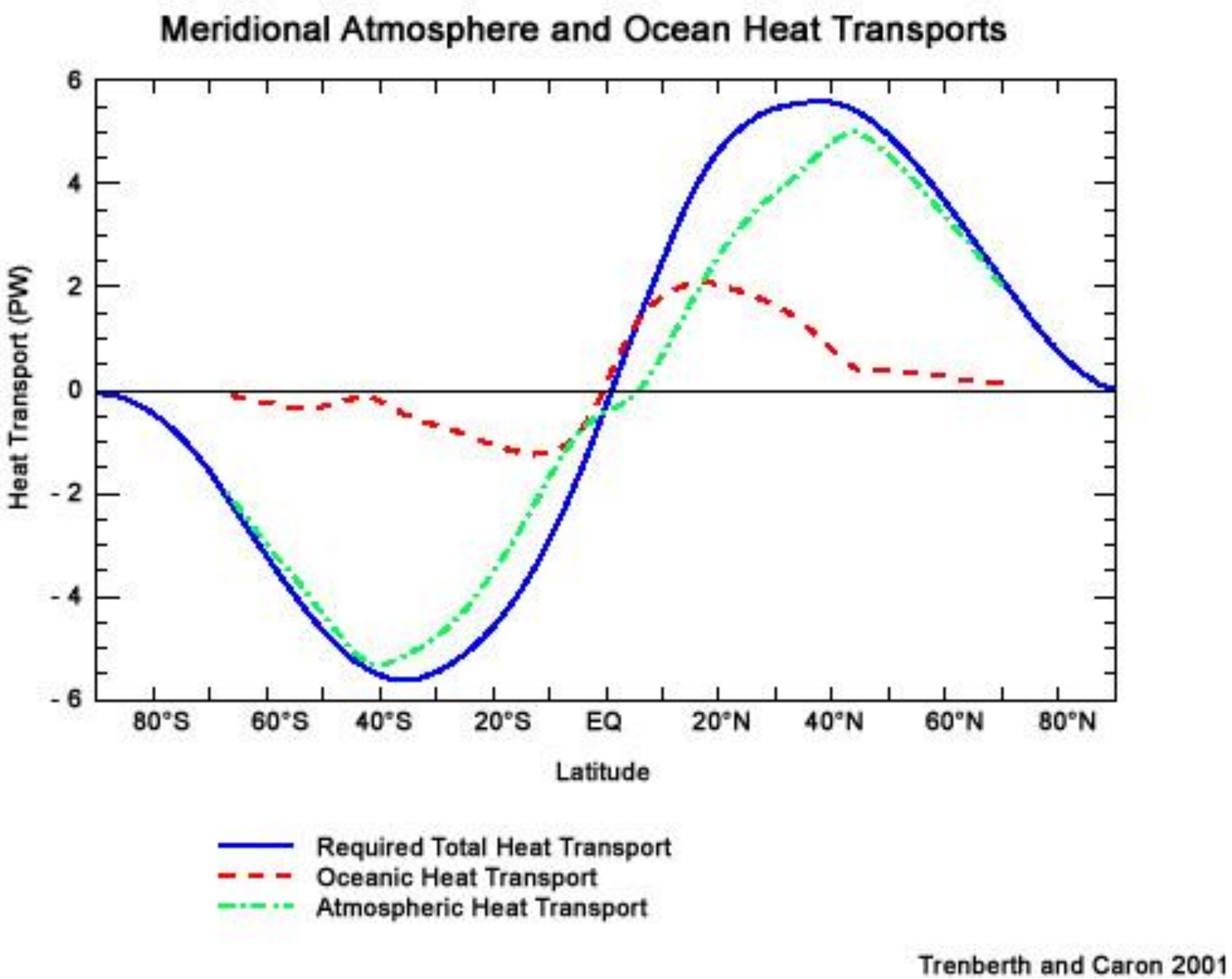
- Most of the solar and infrared energy input is used to evaporate water! Atmospheric warming occurs ‘later’ when vapour condenses

Radiation budget

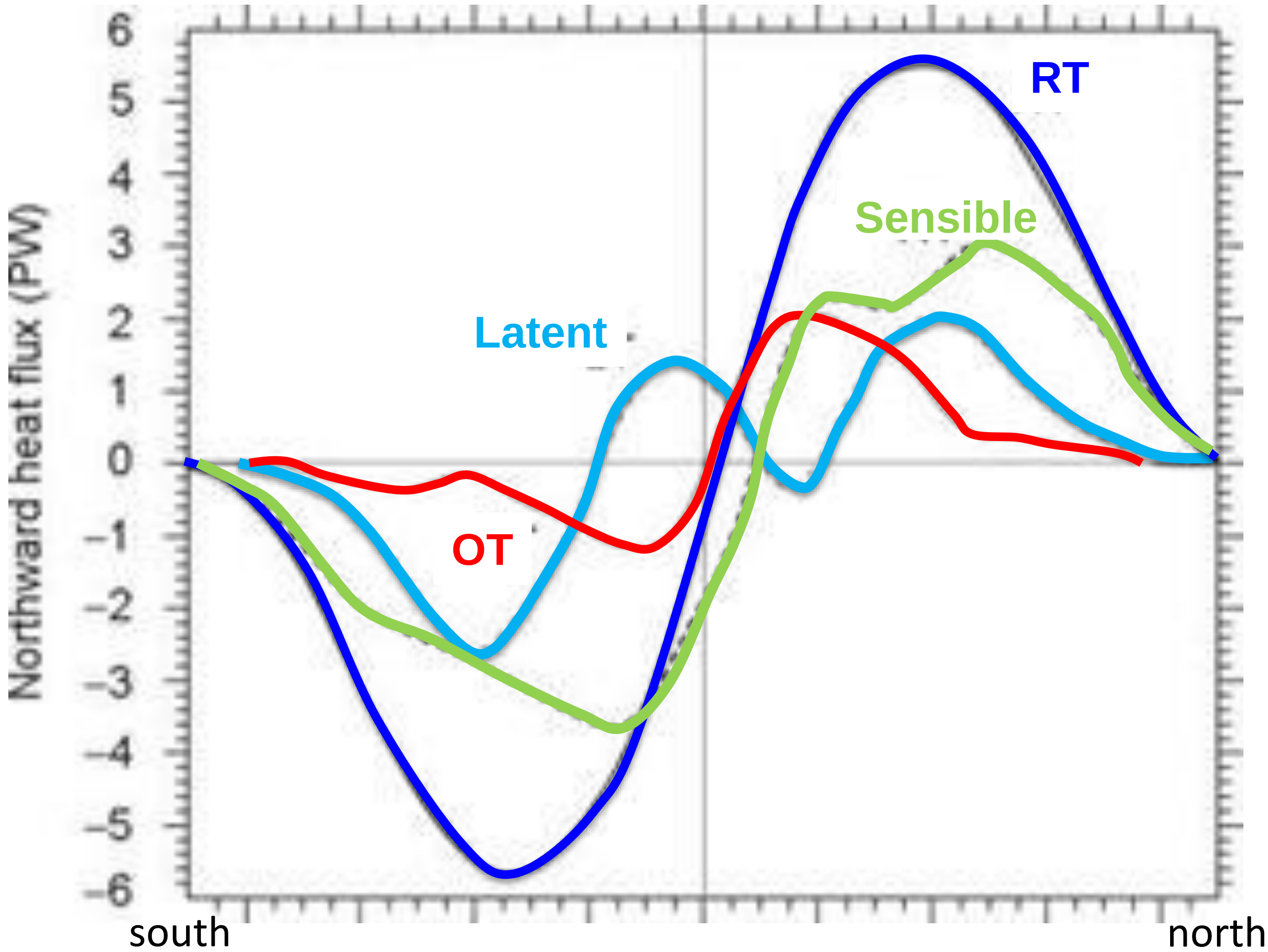




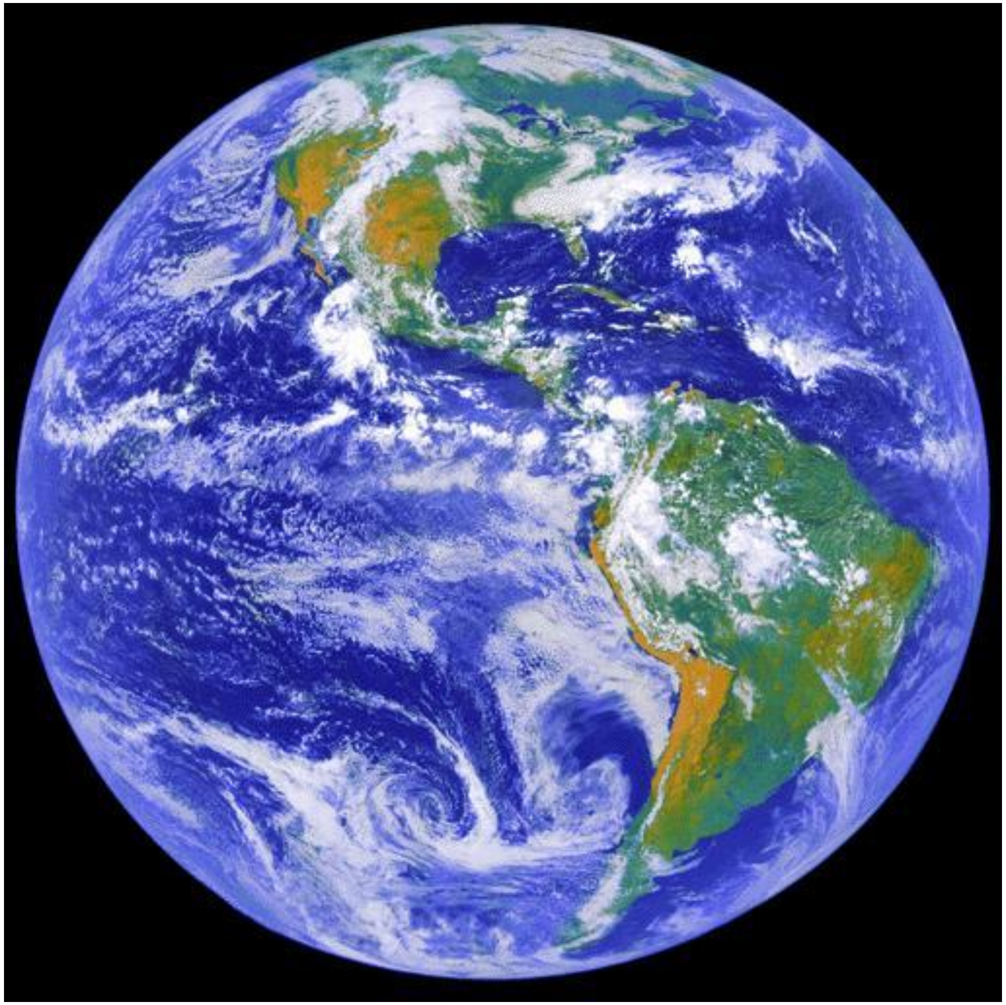
Poleward heat transfer



Poleward heat transfer



Atmosphere Dynamics



NASA Goddard Space Flight Centre

Driven by:

- **Buoyancy effects**

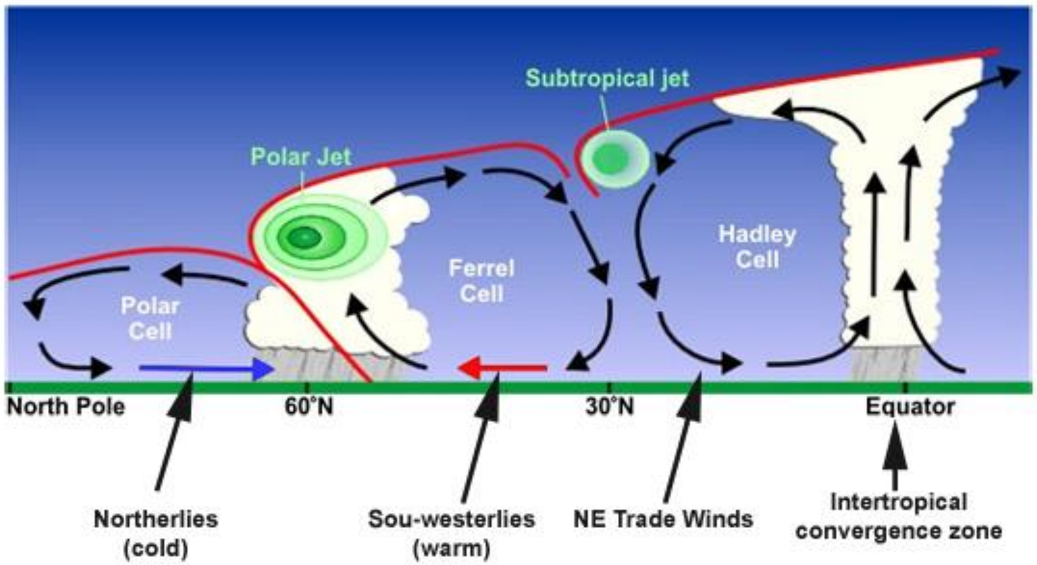
  solar radiation, convection, stratification

- **Rotational effects**

  Coriolis

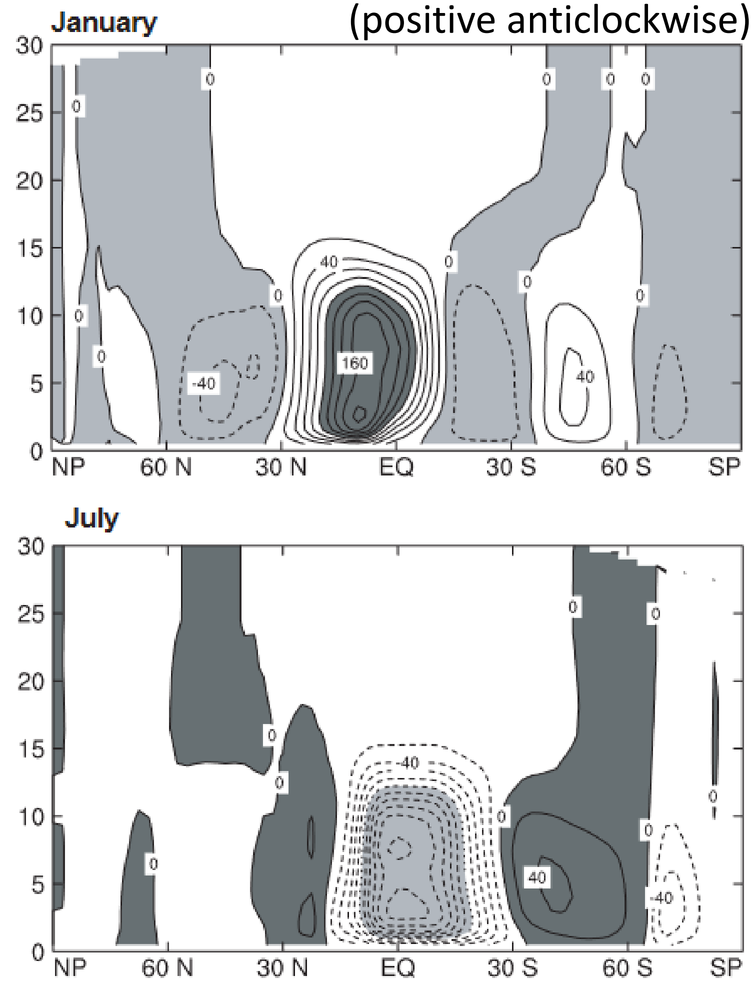
Hadley Cells

Sketch of the circulation



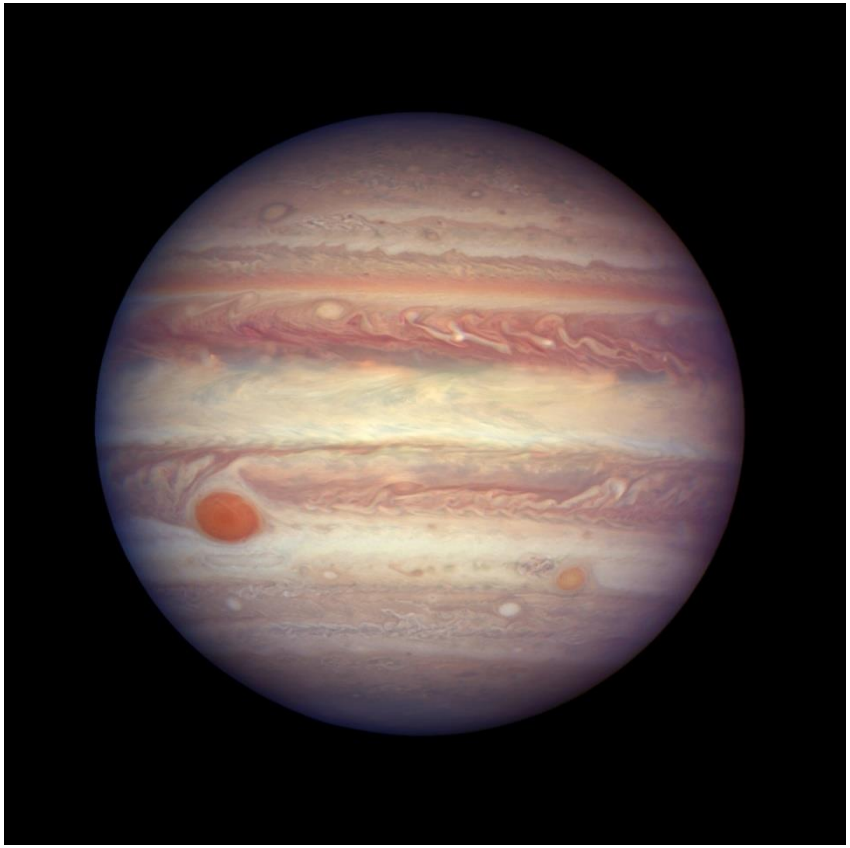
1. Sun warms air over the equator
2. Air rises, drawing in warm moist air from both sides
3. Moist rising air cools, forming clouds

Estimates of the stream function (positive anticlockwise)



(Randall, 2012)

Cells and zonal winds in Jupiter

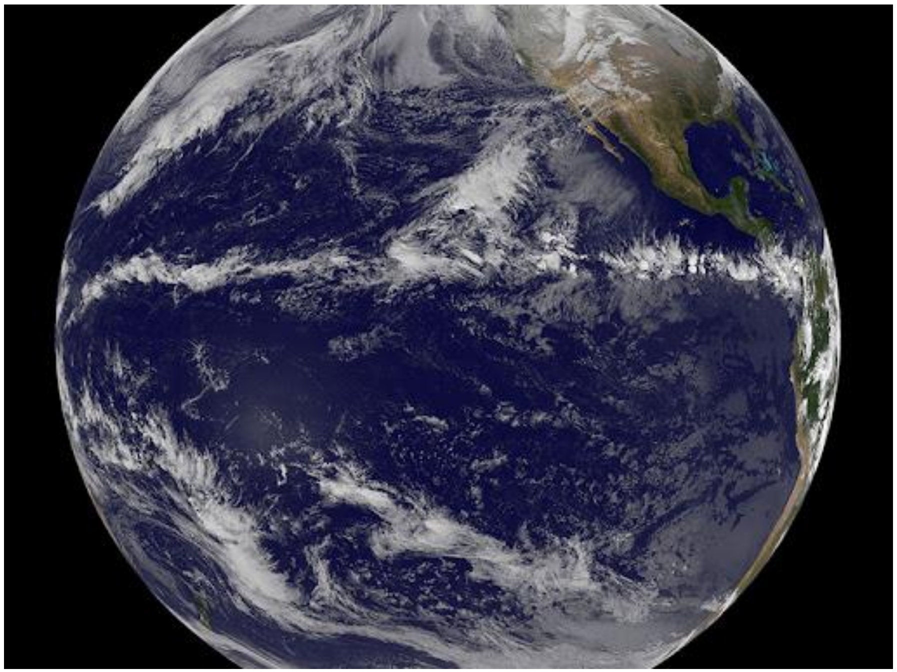


Jupiter:

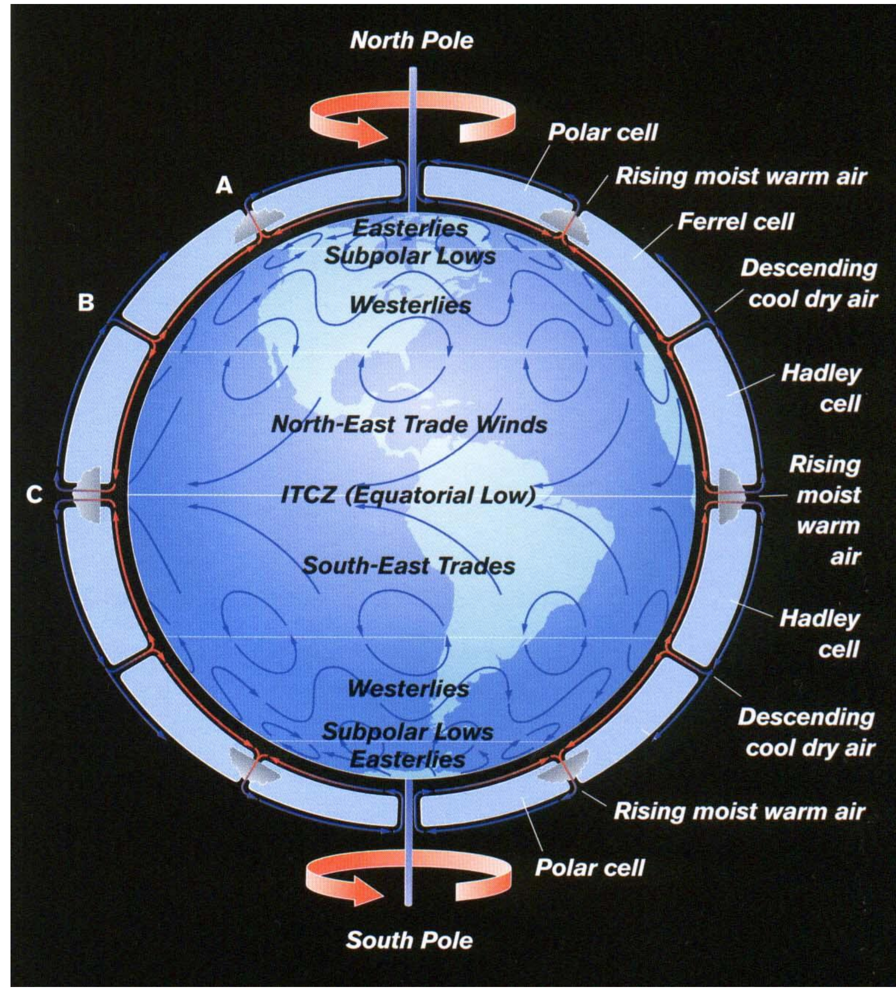
- Length of the day is approx. 10h
- Radius is 69 911 km (Earth radius is 6 371 km)

Image obtained from the Cassini probe in autumn 2000

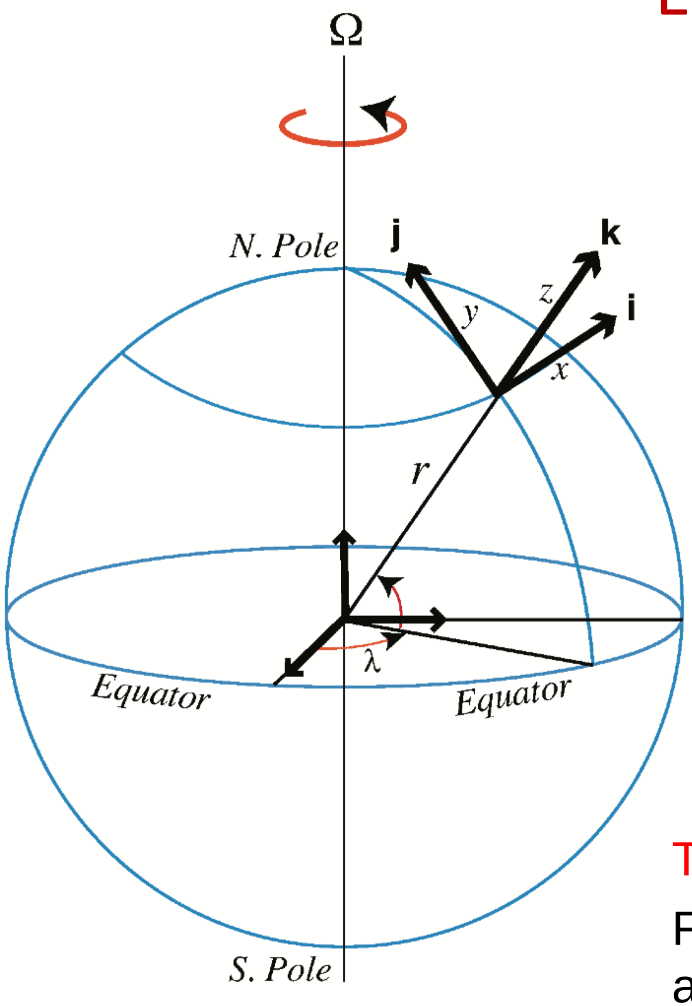
Atmospheric patterns

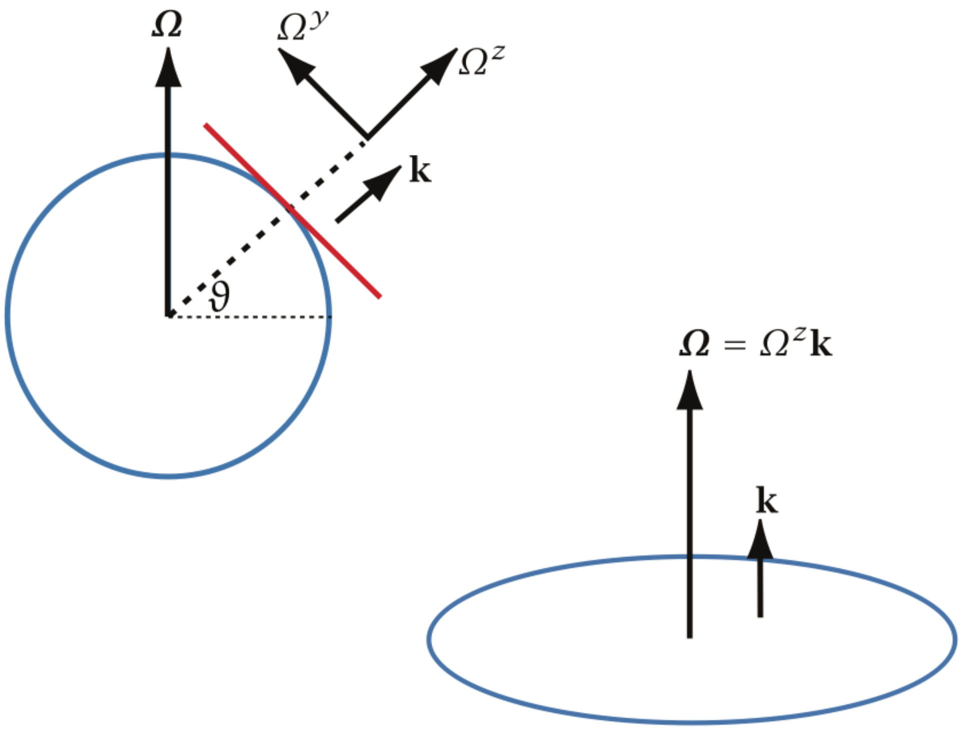


NASA Goddard Space Flight Centre



Effect of rotation





The *f*-plane approximation

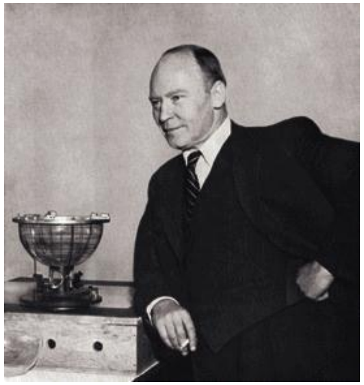
Flow occurs in the tangent plane, with a constant angular velocity:

$$f_0 = 2\Omega \sin \vartheta$$

Geostrophic Flows

Scale analysis of the Navier-Stokes equation (horizontal component)

$$\frac{D}{Dt}\vec{u} = -\frac{1}{\rho}\nabla p - f\,\hat{\vec{k}}\times\vec{u} + \nu\nabla^2\vec{u}$$



Carl-Gustaf Arvid Rossby (1898 – 1957)

Rossby number

Inertial and Coriolis effects:

$$\frac{\vec{u}_r\cdot\nabla_r\vec{u}_r}{\omega\times\vec{u}_r} \sim \frac{\mathrm{U}^2/\mathrm{L}}{\Omega \mathrm{U}} \sim \frac{\mathrm{U}}{\Omega \mathrm{L}}$$

Rossby number small $\Rightarrow$ Coriolis effects important

Vagn Walfrid Ekman (1874 – 1954)

Ekman number

Viscous and Coriolis effects:

$$\frac{\nu\,\nabla_r^2\vec{u}_r}{\omega\times\vec{u}_r} \sim \frac{\nu \mathrm{U}/\mathrm{L}^2}{\Omega \mathrm{U}} \sim \frac{\nu}{\Omega \mathrm{L}^2}$$

Ekman number small $\Rightarrow$ Coriolis effects important

Geostrophic Flows

Scale analysis of the Navier-Stokes equation (horizontal component)

Typical scales

$\nu \sim 10^{-5}\left[\mathrm{m}^2 \cdot \mathrm{s}^{-1}\right]$

$\rho \sim 1\left[\mathrm{kg} \cdot \mathrm{m}^{-3}\right]$

$f \sim 10^{-4}\left[\mathrm{rad} \cdot \mathrm{s}^{-1}\right]$

$\mathrm{U} \sim 10\left[\mathrm{m} \cdot \mathrm{s}^{-1}\right]$

$\mathrm{L} \sim 10^{6}\left[\mathrm{m}\right]$

$$\underbrace{\frac{D}{Dt}\vec{u}}_{\frac{\mathrm{U}^2}{L}\sim 10^{-4}} = \underbrace{-\frac{1}{\rho}\nabla p}_{?} - \underbrace{f\,\hat{\vec{k}} \times \vec{u}}_{f\mathrm{U}\sim 10^{-3}} + \underbrace{\nu\nabla^2\vec{u}}_{\frac{\nu\mathrm{U}}{\mathrm{L}^2}\sim 10^{-16}}$$

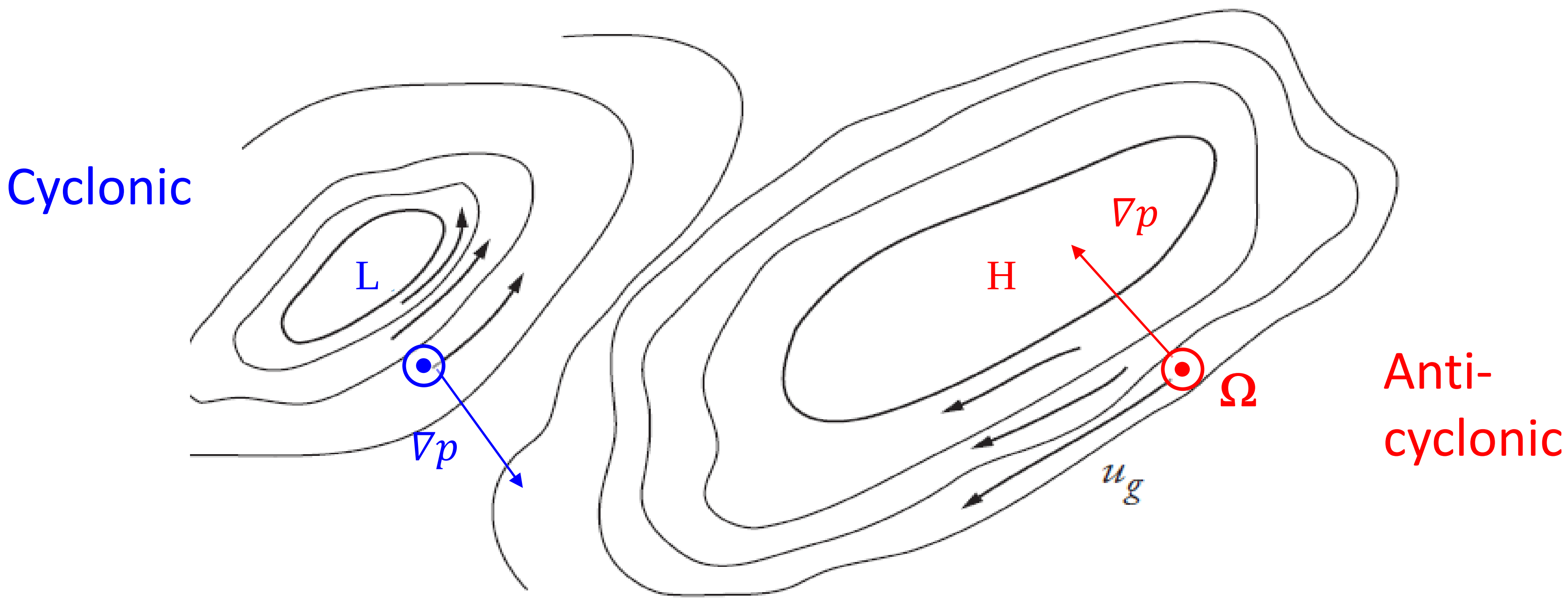
Rossby and Ekman numbers are small, so that the horizontal ‘geostrophic’ flow $\boldsymbol{u_g}$ is a balance between the Coriolis term and the pressure gradient

$$\boxed{f\,\hat{\vec{k}} \times \vec{u}_g = -\frac{1}{\rho}\nabla p}$$

Geostrophic Flows

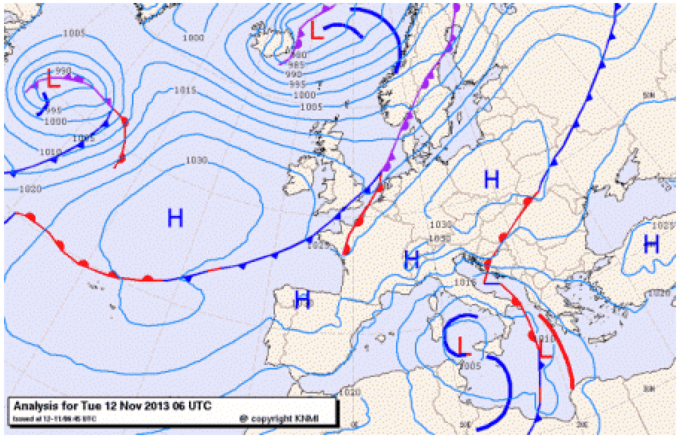
$$\boxed{\vec{u}_g = -\frac{1}{f\rho}\hat{\vec{k}} \times \nabla p}$$

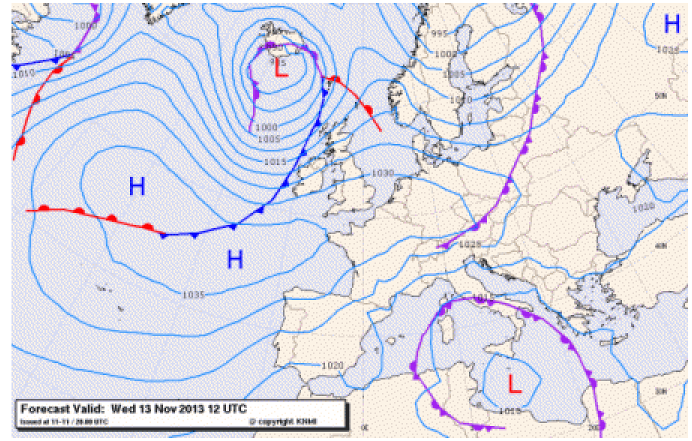
Geostrophic wind: mid-latitude synoptic circulation



Geostrophic Flows

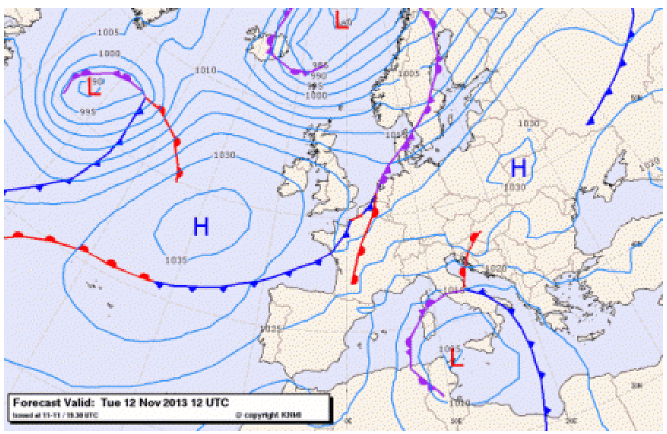
Weather systems

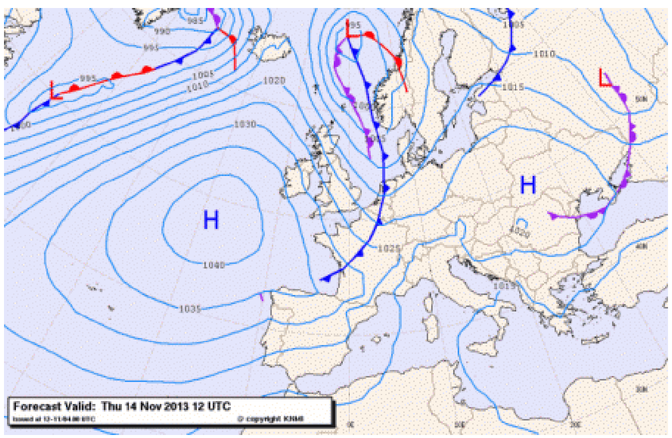




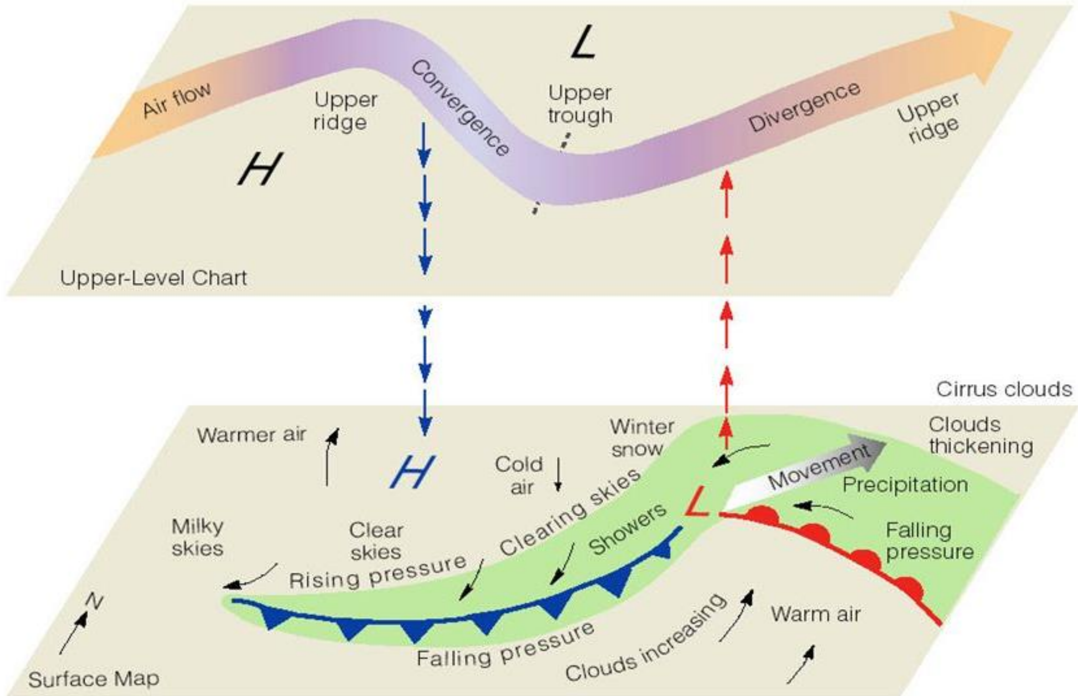
Note:

The fronts show the direction of air movement



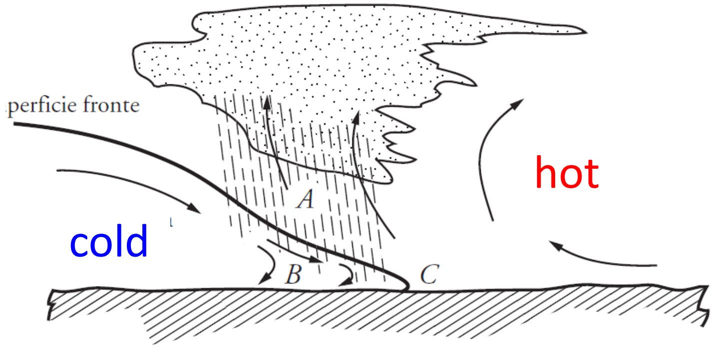


Fronts at mid latitudes

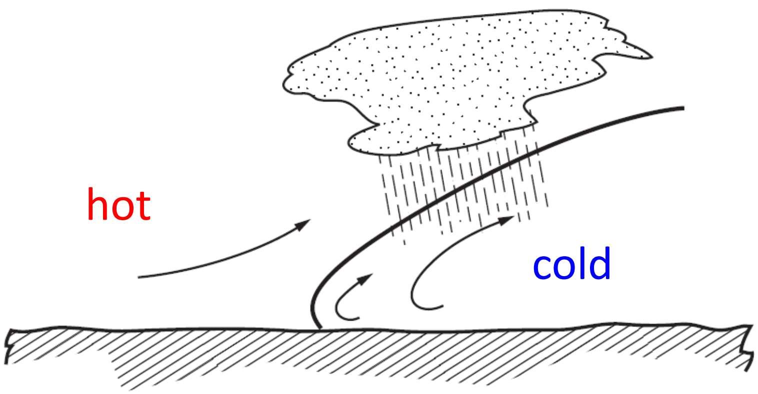


https://www.youtube.com/watch?v=dwIQds-4I7I

Cold front



Warm front



Taylor-Proudman theorem

Steady Geostrophic flow:

$$f\,\hat{\vec{k}}\times\vec{u}=-\frac{1}{\rho}\nabla p$$

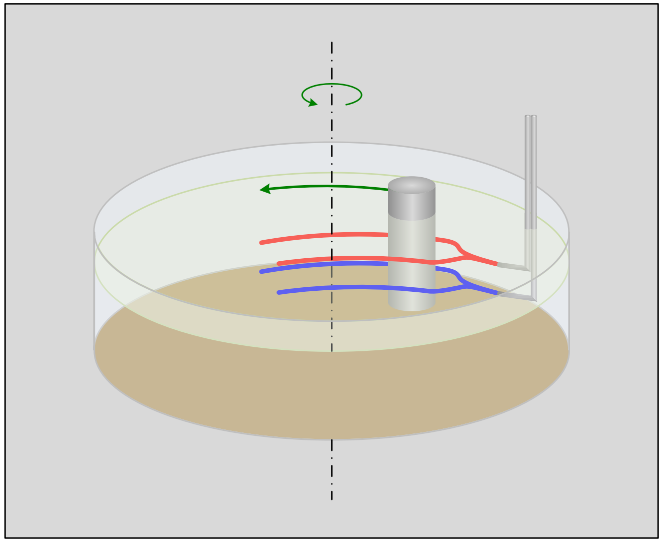
Taking the curl:

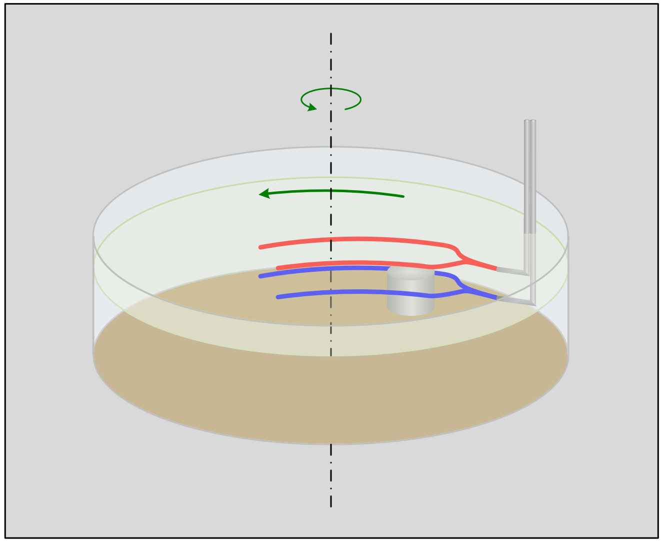
$$\nabla\times\left(f\,\hat{\vec{k}}\times\vec{u}\right)=-\frac{1}{\rho}\nabla\times(\nabla p)$$

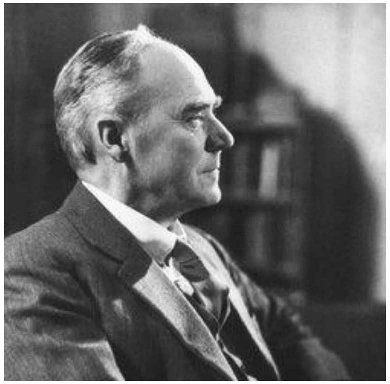
$$f\left(\hat{\vec{k}}\cdot\nabla\right)\vec{u}=\vec{0}\quad\text{or:}\quad\frac{\partial\vec{u}}{\partial z}=\vec{0}$$

$\Rightarrow$ Geostrophic flow is 2D

Taylor-Proudman columns







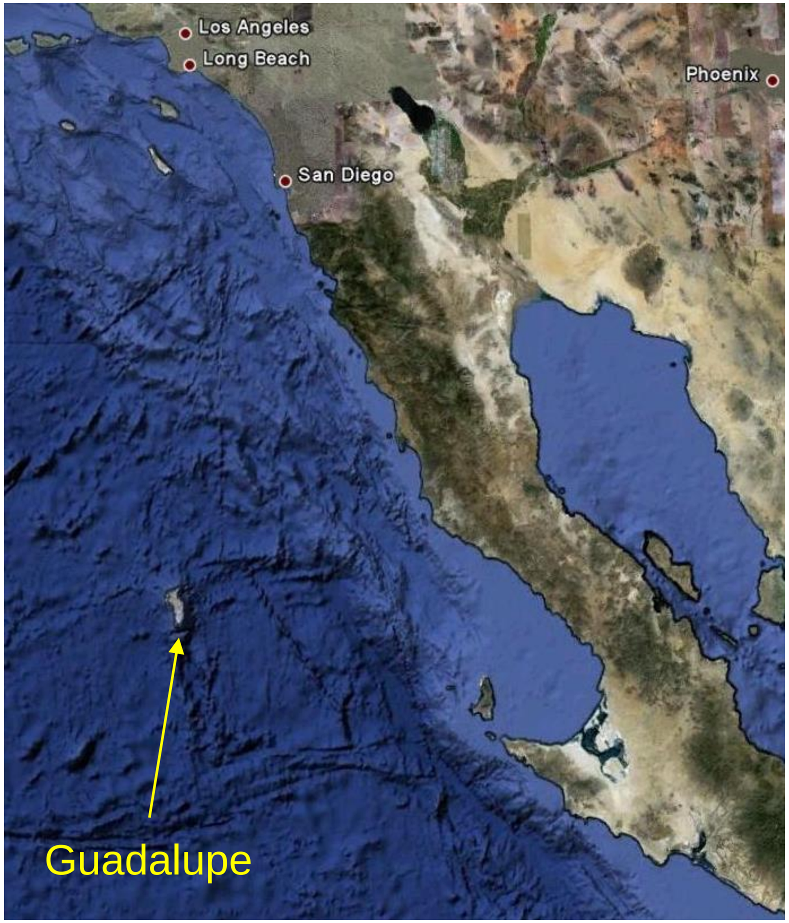
Sir Geoffrey Ingram Taylor (1886-1975)

Joseph Proudman (1888-1975)

Taylor-Proudman columns

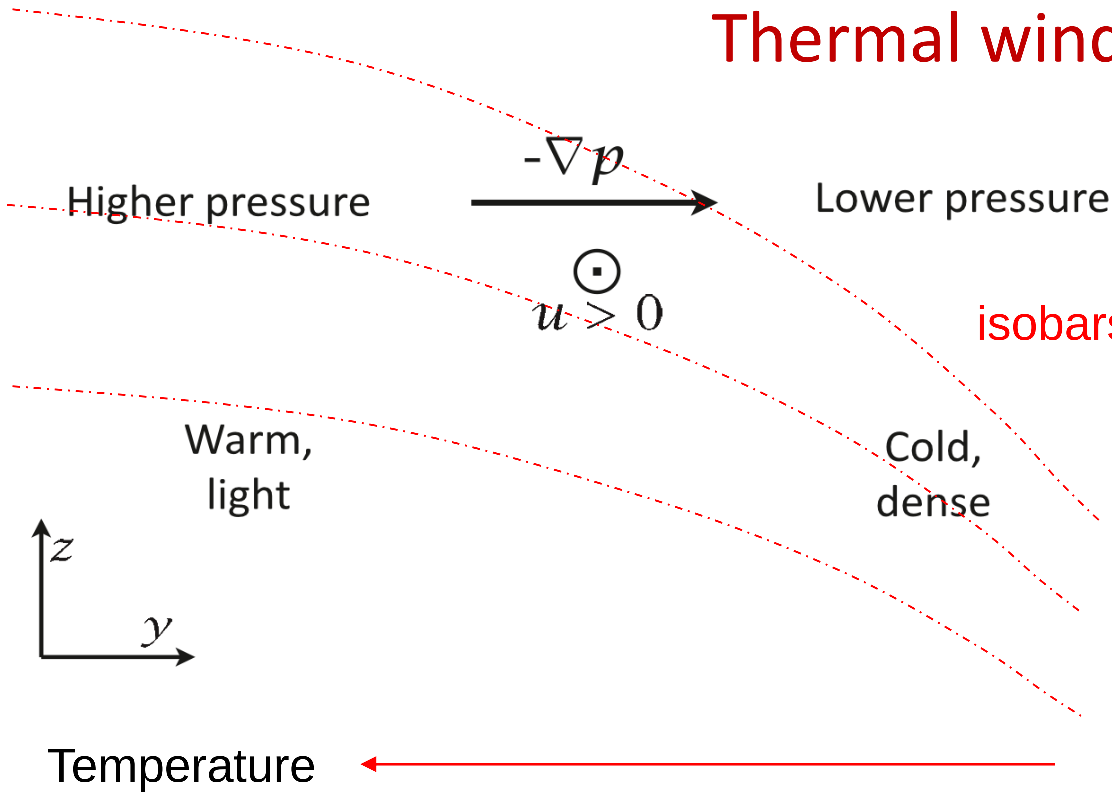
MIT : Atmosphere, Ocean and Climate Dynamics
http://www.youtube.com/watch?v=UKA8RoZrCdg

Taylor-Proudman columns

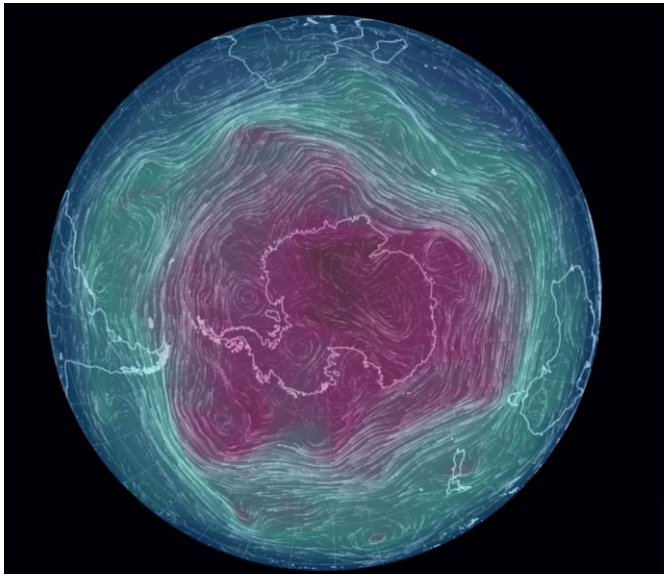


Google Earth

Thermal wind



$$\boxed{f\,\hat{\vec{k}} \times \vec{u}_g = -\frac{1}{\rho}\nabla p}$$

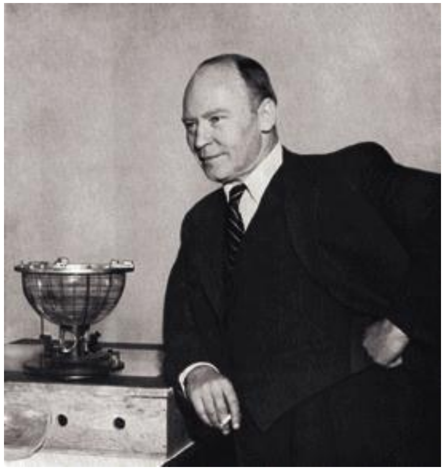


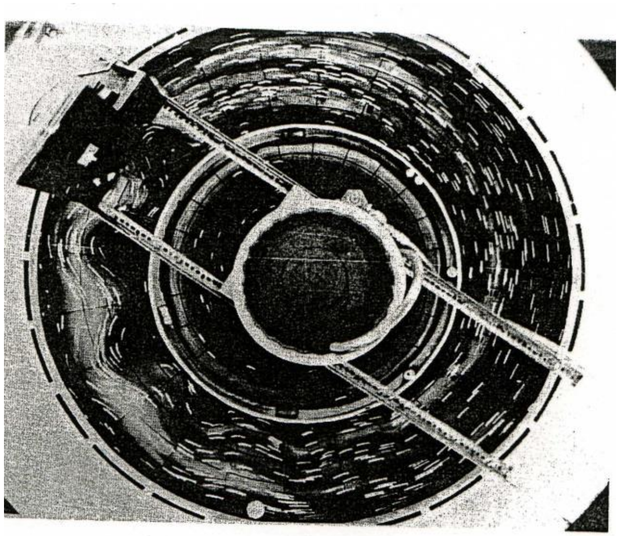
Rossby waves

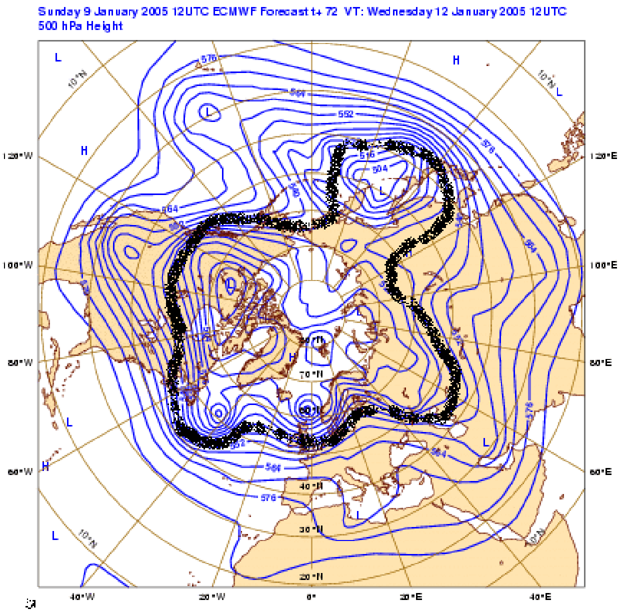
Occur when geostrophic flow is deflected in such a way that it violates the Taylor-Proudman theory.

Can be caused by:

- Changes in angular velocity
- Changes in depth
- ….







Rossby waves

Geostrophic flow

$$\frac{\partial \vec{u}}{\partial t} + 2\vec{\Omega} \times \vec{u} = -\frac{1}{\rho} \nabla p$$

Continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Flow on the $xy-$ plane

$$\begin{cases} \dfrac{\partial u}{\partial t} - 2\Omega v = -\dfrac{1}{\rho} \dfrac{\partial p}{\partial x} \\ \dfrac{\partial v}{\partial t} + 2\Omega u = -\dfrac{1}{\rho} \dfrac{\partial p}{\partial \partial y} \\ \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0 \end{cases}$$

The β-plane approximation:

Ω varies with latitude $2\Omega = f_0 + \beta y$

where $\beta = 2\Omega \cos \vartheta / R \quad (R$ is the radius of the Earth)

Rossby waves

Cross differentiate

$$\begin{cases} \dfrac{\partial^2 u}{\partial t \partial y} - \beta v = -\dfrac{1}{\rho}\dfrac{\partial^2 p}{\partial x \partial y} \\ \dfrac{\partial^2 v}{\partial t \partial x} + \left(f_0 + \beta y \right)\dfrac{\partial u}{\partial x} = -\dfrac{1}{\rho}\dfrac{\partial^2 p}{\partial x \partial y} \end{cases}$$

Look for solutions independent of y

$$\Rightarrow \frac{\partial u}{\partial y} = \frac{\partial v}{\partial y} = 0$$

$\Rightarrow$ from continuity then $\dfrac{\partial u}{\partial x} = 0$

$$\Rightarrow \quad \frac{\partial^2 v}{\partial t \partial x} + \beta v = 0$$

Assume a wave-type solution:

$$v = v_0 \exp\left\{ i\left(\omega t - kx \right) \right\}$$

From which:

$$\left[\omega k + \beta \right] v_0 = 0$$

Dispersion relationship:

$$\omega(k) = -\frac{\beta}{k}$$

Phase velocity:

$$c_p = \frac{\omega}{k} = -\frac{\omega^2}{\beta}$$

Group velocity:

$$c_g = \frac{d\omega}{dk} = \frac{\omega^2}{\beta} = -c_p$$

Rossby waves

Geostrophic flow

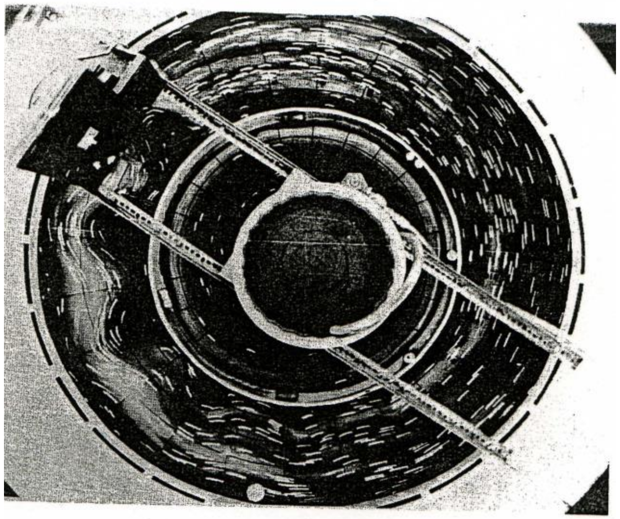
$$\frac{\partial \vec{u}}{\partial t} + 2\vec{\Omega}\times\vec{u} = -\frac{1}{\rho}\nabla p \qquad (1)$$

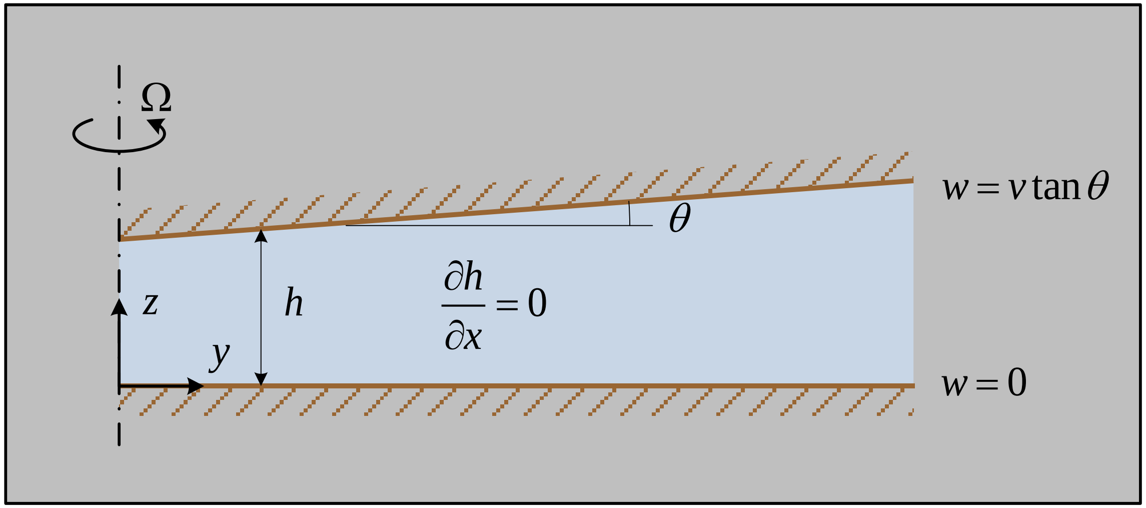
Uniform flow

$$\frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = 0 \text{ but } \frac{\partial w}{\partial z} \neq 0 \qquad (2)$$

Continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (3)$$

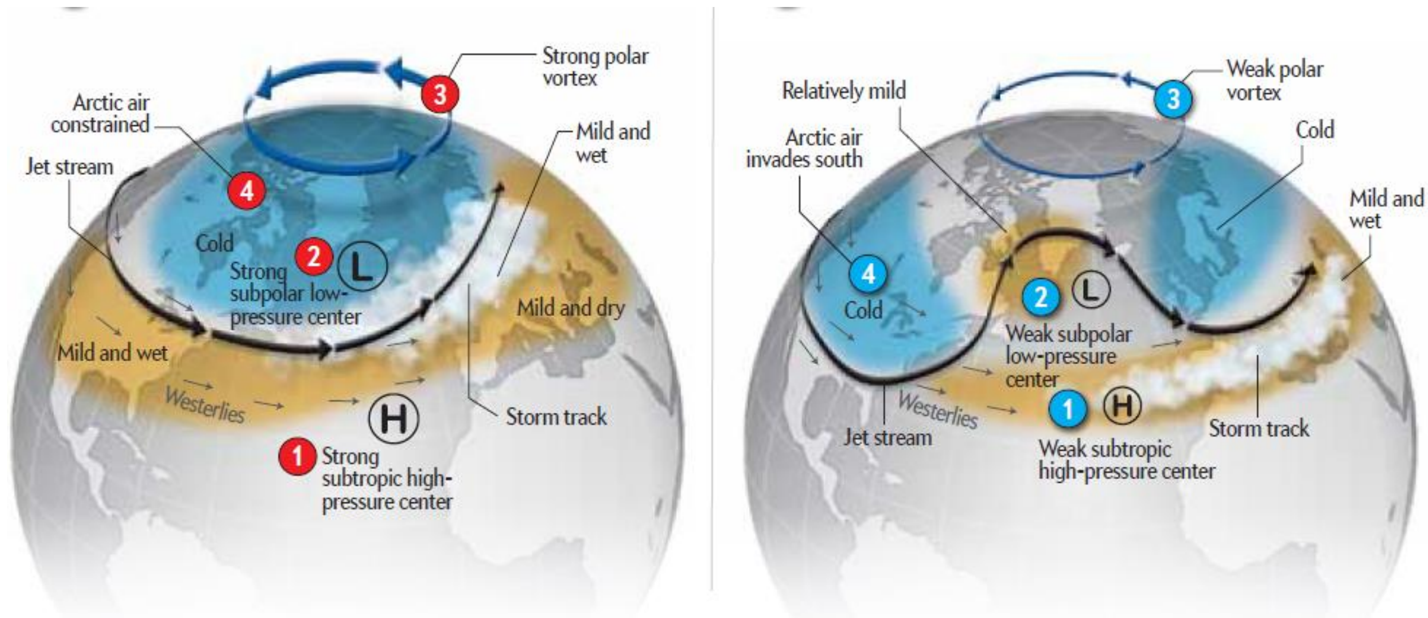




Laboratory experiments in rotating tank

Northern Atlantic Oscillation (NAO)

NAO is related to the pressure difference between Island (low) and Azores (high)

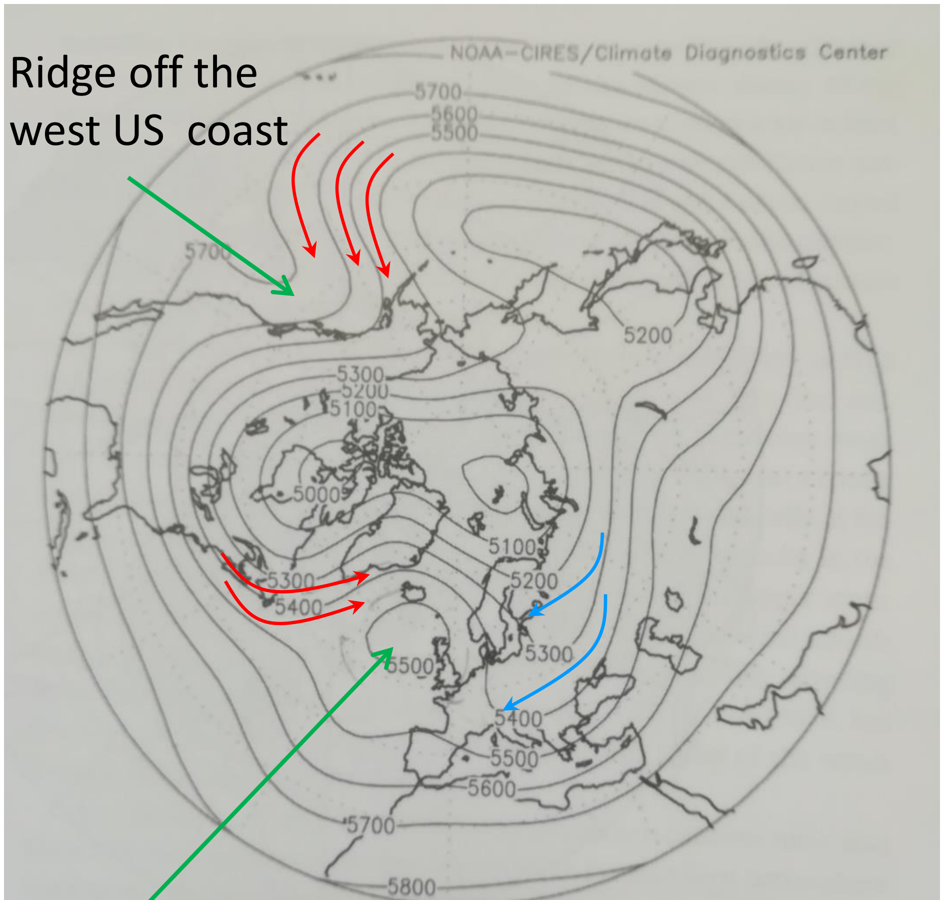


- **Positive NAO**: strong polar vortex that constrains cold artic air to the north and allows warm air from southern latitudes to reach far north in Europe
- **Negative NAO**: weak polar vortex, more sinusoidal jet stream that allow cold air to invade northern Europe

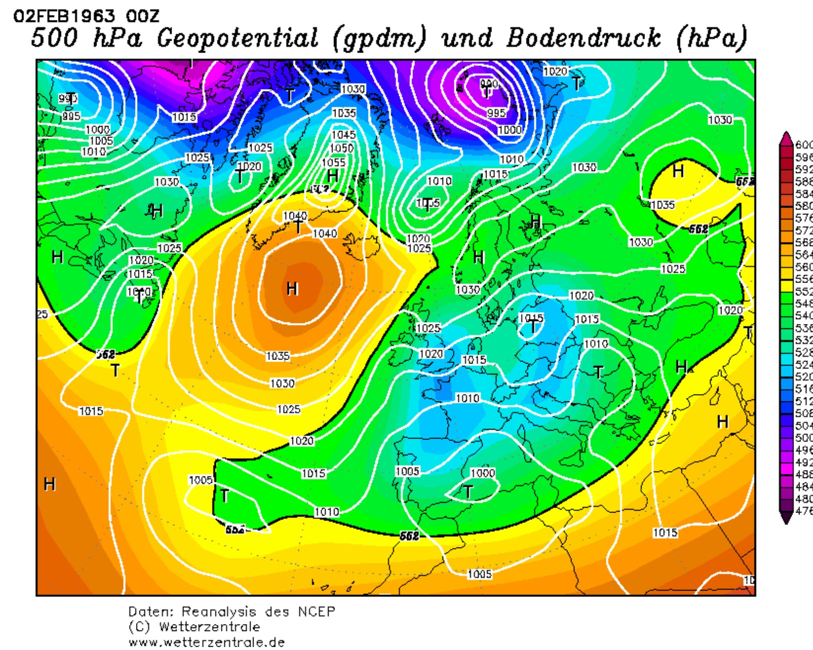
Polar modes

Blocking (Winter 1963) – The Big Freeze (-20°in Scotland)

Ridge off the west US coast



Anticyclonic cell over Iceland



Thank you for your attention