

Waves & geosciences: Infrasound and beyond



# Inverse problems for atmospheric dispersion

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ÉCOLE  
**CENTRALE** LYON



Lyon 1

**INSA** INSTITUT NATIONAL  
DES SCIENCES  
APPLIQUÉES  
LYON



# Outline

1. Introduction
2. Phenomenology and modelling of atmospheric dispersion
3. Inverse modelling : problems & approaches
4. Some applied examples



# 1 – Introduction

## LMFA activity

- Environment
- Transports
- Energy & process engineering
- Health

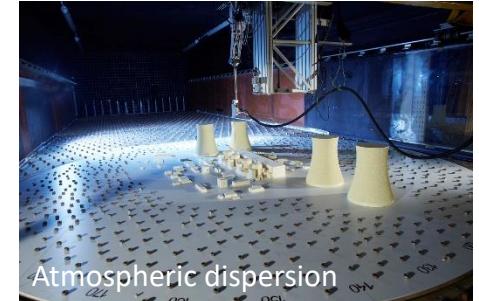
Laboratoire de Mécanique  
des Fluides et d'Acoustique



River hydraulics



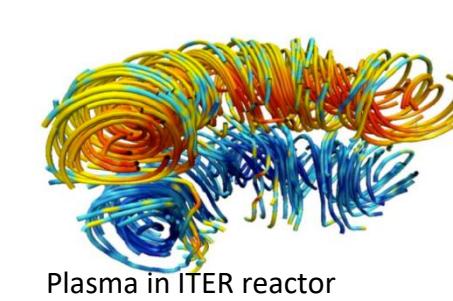
Geophysical flows, mixing  
and stratification



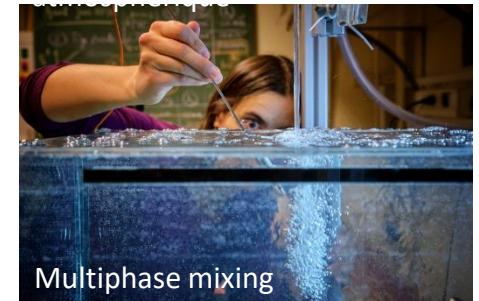
Atmospheric dispersion



Flow in aorta artery



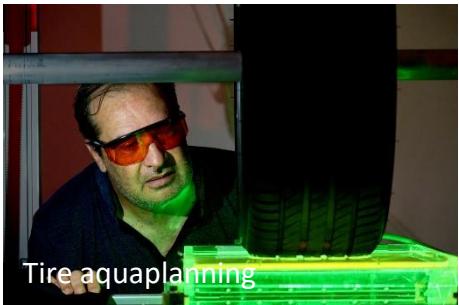
Plasma in ITER reactor



Multiphase mixing



Anechoic chamber



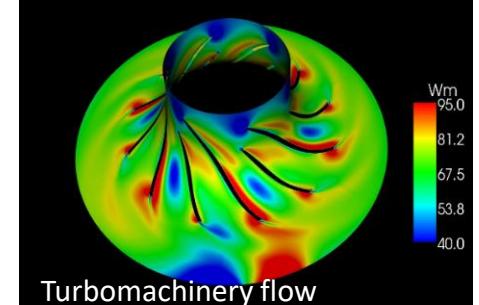
Tire aquaplaning



Aircraft engine



Equipex PHARE

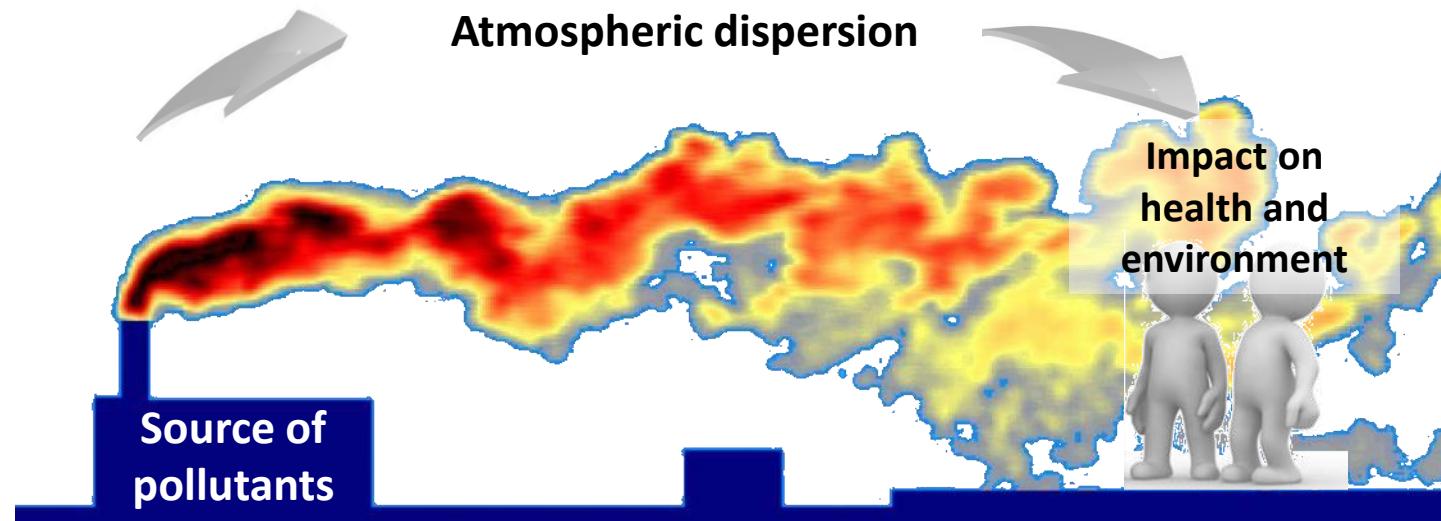


Turbomachinery flow

# 1 – Introduction

## Atmospheric flows research

- Flow and dispersion in the atmospheric boundary layer



# 1 – Introduction

## Atmospheric flows research

- A societal concern

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Auvergne Rhône-Alpes

### Cette pollution de l'air qui tue

Dans un rapport  
l'air dans la  
qui s'améliore  
000 au dioxyde  
d'azote

### Pourquoi les enfants pauvres sont plus touchés par la pollution de l'air

Par Jean-Philippe  
min

Un rapport du Réseau Action Climat et de l'Unicef alerte sur le fait que les enfants issus de classes populaires sont plus touchés par la pollution atmosphérique qu'au sein des classes aisées. Et donc plus vulnérables, également, aux maladies liées à l'exposition aux particules fines.

Par Juliette MITOYEN - 14 oct. 2021 à 09:00 | mis à jour le 14 oct. 2021 à 09:11 - Temps de lecture : 5 min

6 | Vu 2297 fois



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Rhône

### Climat à Lyon en 2050 : des prévisions plus qu'inquiétantes

Quel climat fera-t-il à Lyon dans une trentaine d'années ? Grâce aux prévisions de Météo

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### Réchauffement climatique à Lyon : «Il faut planter plus et mieux»

Damien Lemoine, titulaire d'un doctorat en biologie forestière et maître de conférences à l'université Lyon-1, nous dit tout sur la manière dont un brin d'herbe ou un arbre géant piègent le carbone dans l'air. Mais encore faut-il opter pour le bon végétal.

Par Propos recueillis par Sophie MAJOU - 14 août 2021 à 19:00 | mis à jour le 15 août 2021 à 07:18 - Temps de lecture : 5 min

6 | Vu 6659 fois



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→ Edition Est Lyonnais > Feyzin

Rhône

### Raffinerie de Feyzin: la torchère activée a provoqué un épais dégagement de fumée

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En images

### Exercice de décontamination après un attentat au gaz sarin à Lyon

Ce mercredi à Lyon, un exercice de décontamination s'est déroulé à l'hôpital d'instruction des armées de Lyon, avec la participation d'étudiants de l'hôpital Edouard-Herriot : il s'agissait de prendre en charge des victimes contaminées par du gaz sarin après l'explosion d'un colis piégé déposé dans une poubelle près des urgences de l'hôpital.

Par Le Progrès - 18 avr. 2014 à 11:03 - Temps de lecture : 1 min

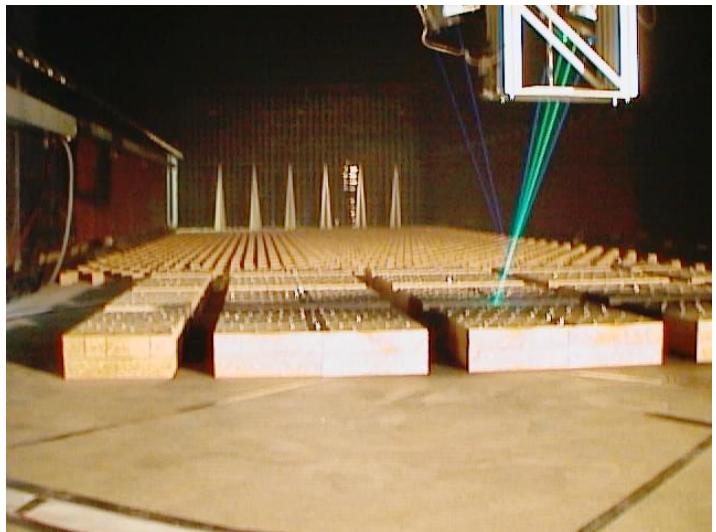
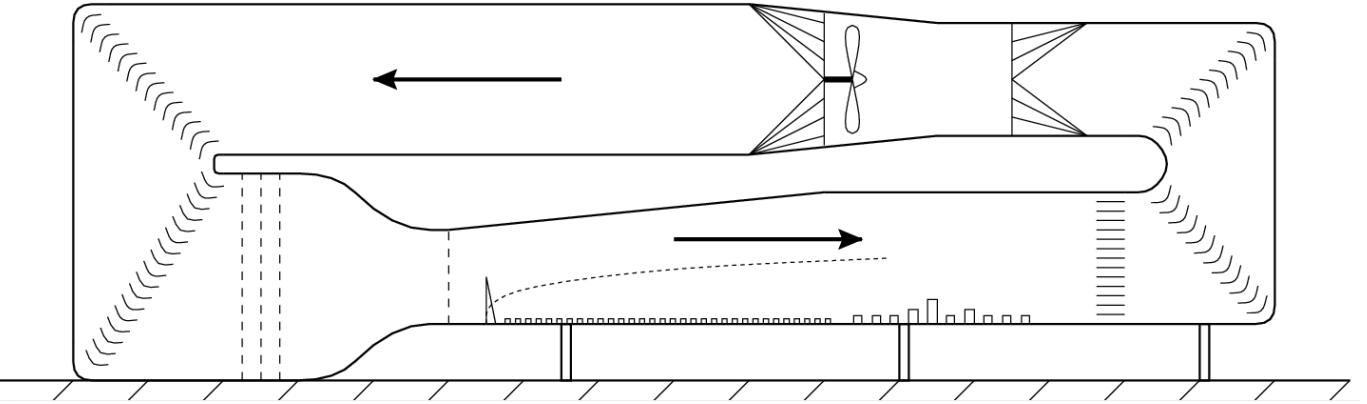
Vu 1601 fois



# 1 – Introduction

## Atmospheric flows research

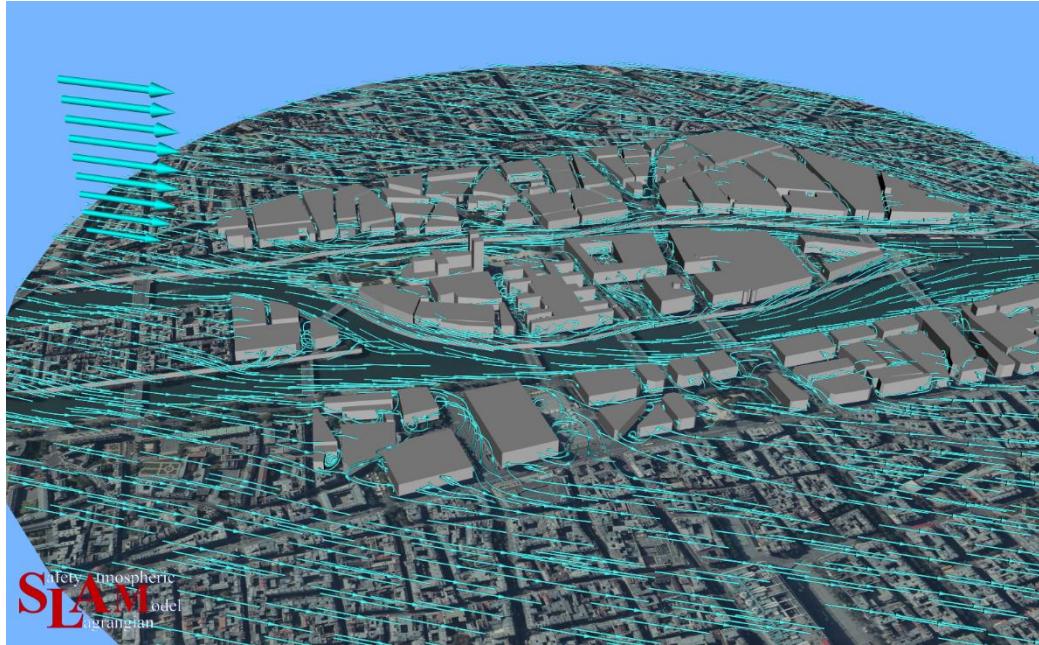
- Approaches :
  - Wind-tunnel experiments



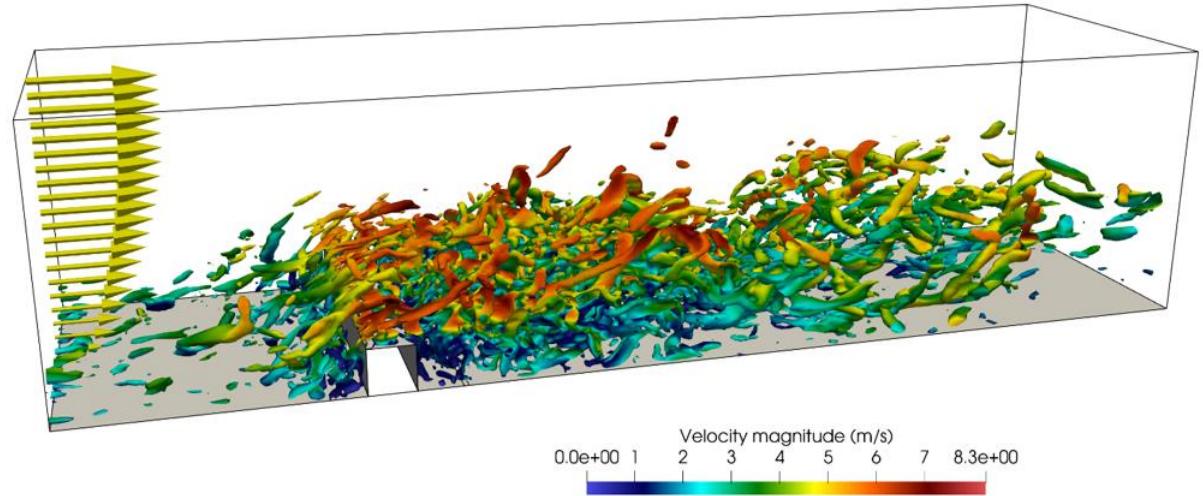
# 1 – Introduction

## Atmospheric flows research

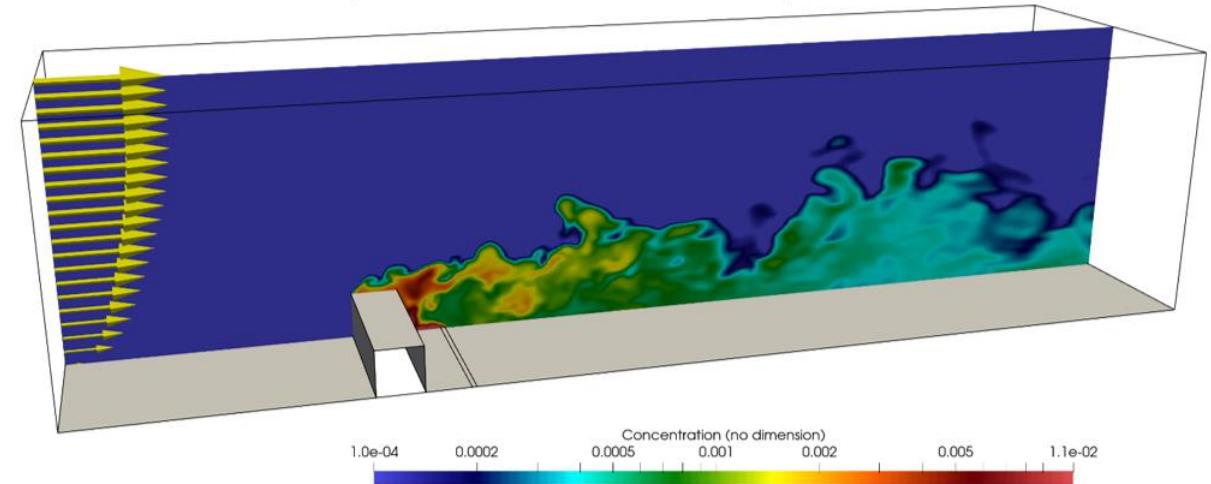
- Approaches :
  - Numerical simulation



a) Instantaneous velocity field, illustrated by isocontour of the  $q$ -criterion



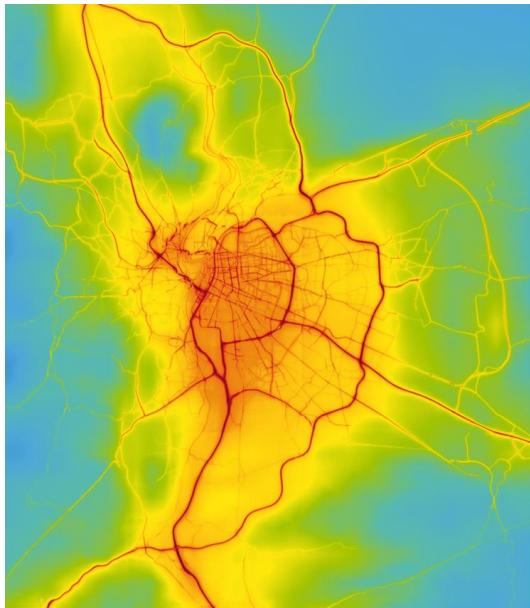
b) Instantaneous concentration field



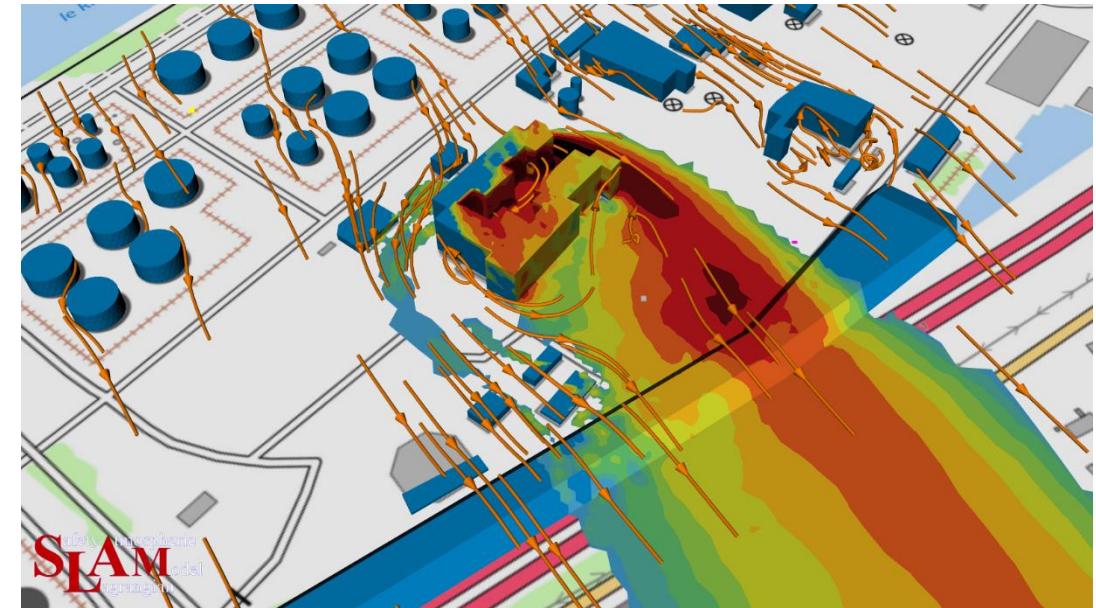
# 1 – Introduction

## Atmospheric flows research

- Approaches :
  - Operational simplified models



SIRANE air quality model

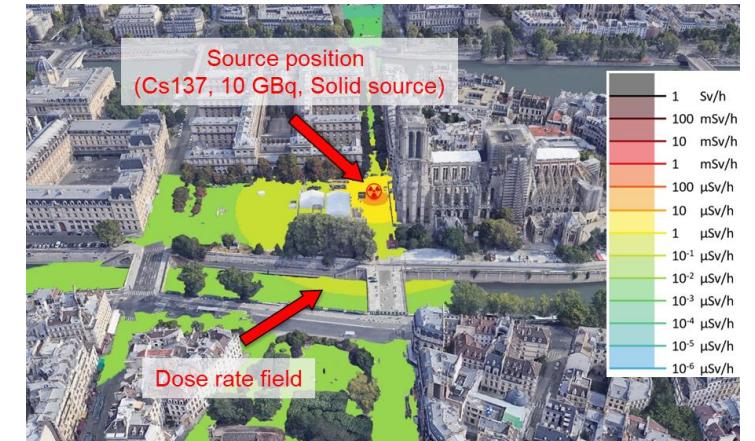
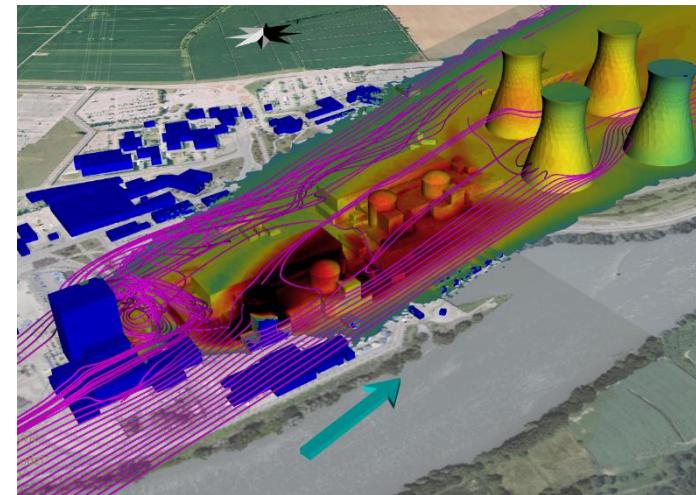
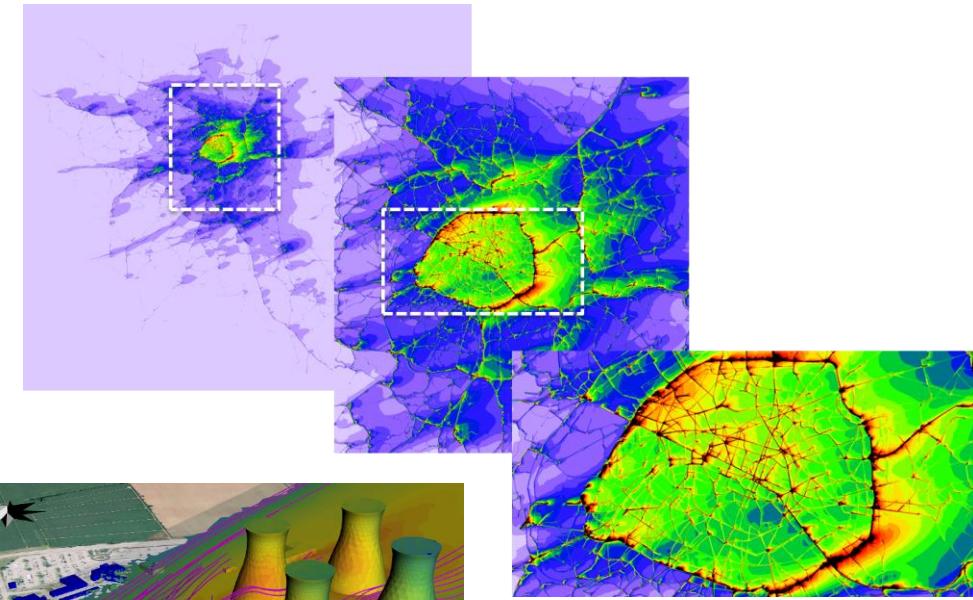


SLAM – Safety Lagrangian Atmospheric Model

# 1 – Introduction

## Atmospheric flows research

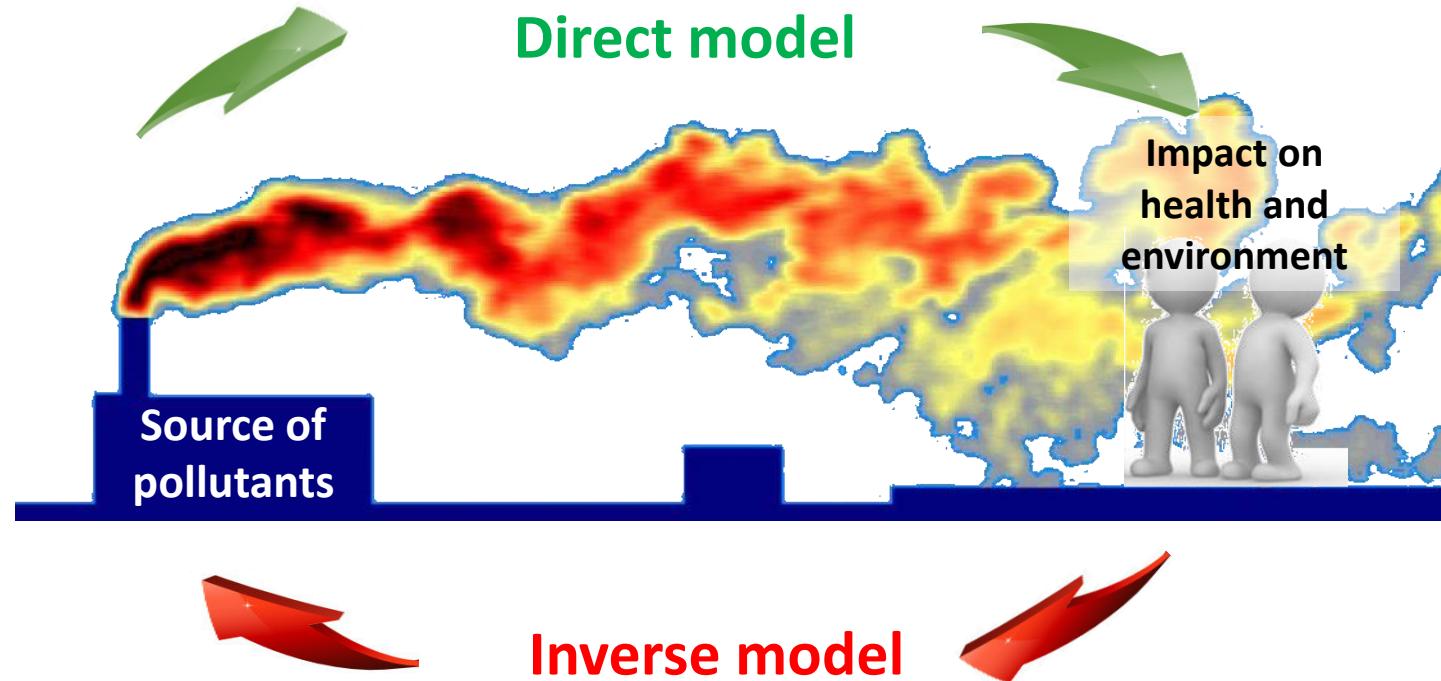
- Domains of application
  - Urban air quality
    - Air quality mapping
    - Population exposure and health effects
  - Industrial risk
    - Environmental impact
    - Risk assessment
    - Crisis management
  - NRBC terrorist attacks
    - Scenarios evaluation
    - Fast response modeling
  - Indoor ventilation



# 1 – Introduction

Why do we need inverse modelling of atmospheric dispersion ?

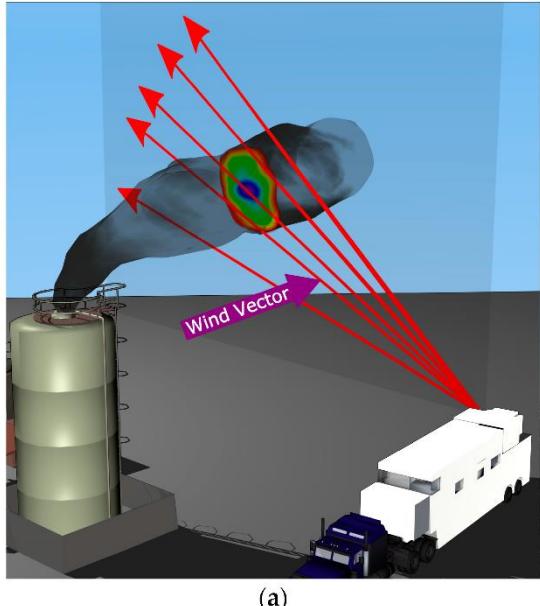
- Direct and inverse model



# 1 – Introduction

Why do we need inverse modelling of atmospheric dispersion ?

- Characterisation of sources of atmospheric pollution
  - Quantification of emissions : third party identification, traffic, industry, etc.



Emission estimation from LIDAR measurements



Traffic air pollution measurements

# 1 – Introduction

Why do we need inverse modelling of atmospheric dispersion ?

- Characterisation of sources of atmospheric pollution
  - Natural emissions (volcanos, bush fires, limnic eruption, etc.)



Eyjafjallajökull volcano  
eruption, 2010



Australia bushfires, 2019-2020



Nyos lake, 1986

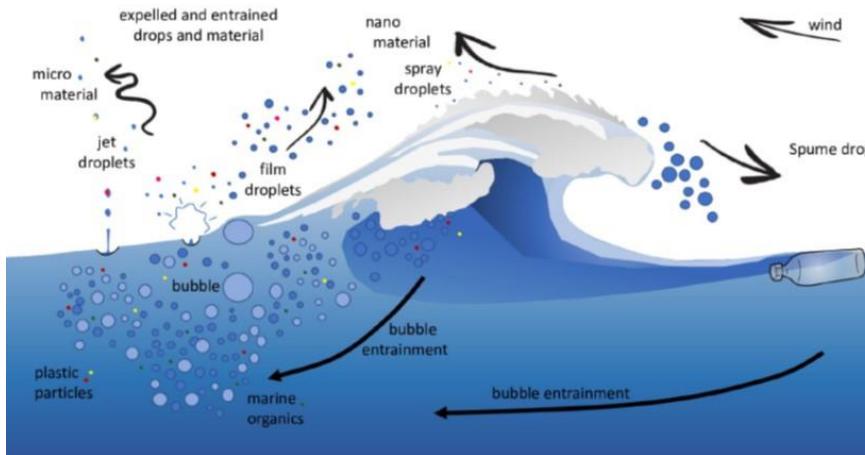
# 1 – Introduction

Why do we need inverse modelling of atmospheric dispersion ?

- Characterisation of sources of atmospheric pollution
  - Diffuse emissions (evaporation, particle entrainment, etc.)



Oil spill



Ocean-atmosphere exchange



Red dust erosion, Gardanne

# 1 – Introduction

Why do we need inverse modelling of atmospheric dispersion ?

- Characterisation of sources of atmospheric pollution
  - Leaks, accident, fires



Leaks on an industrial site



Lubrizol, 2019



Notre-Dame, 2019

# 1 – Introduction

Why do we need inverse modelling of atmospheric dispersion ?

- Characterisation of sources
  - Terrorist attacks



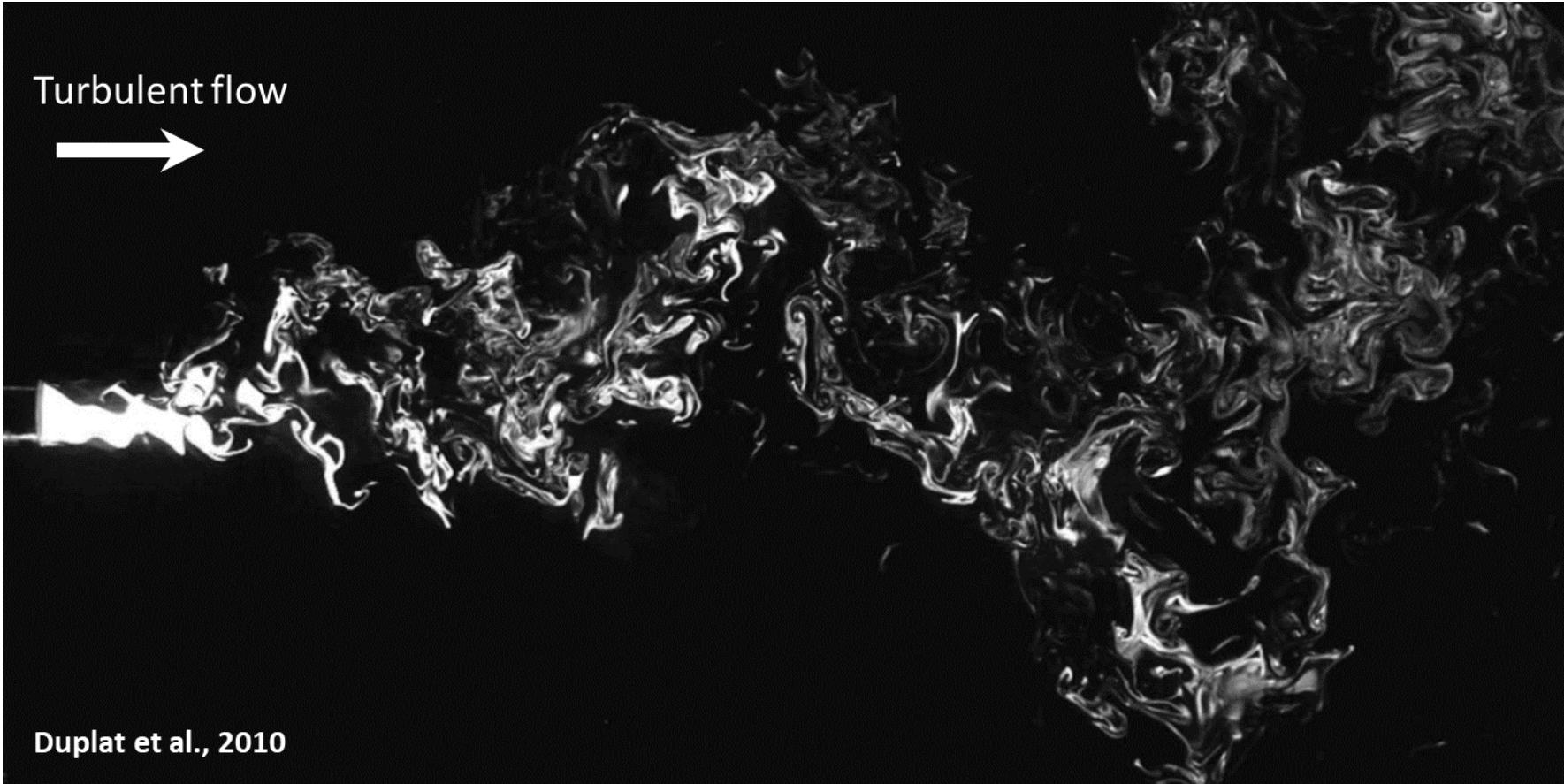
New York, Sept. 11, 2001



Crisis management exercise

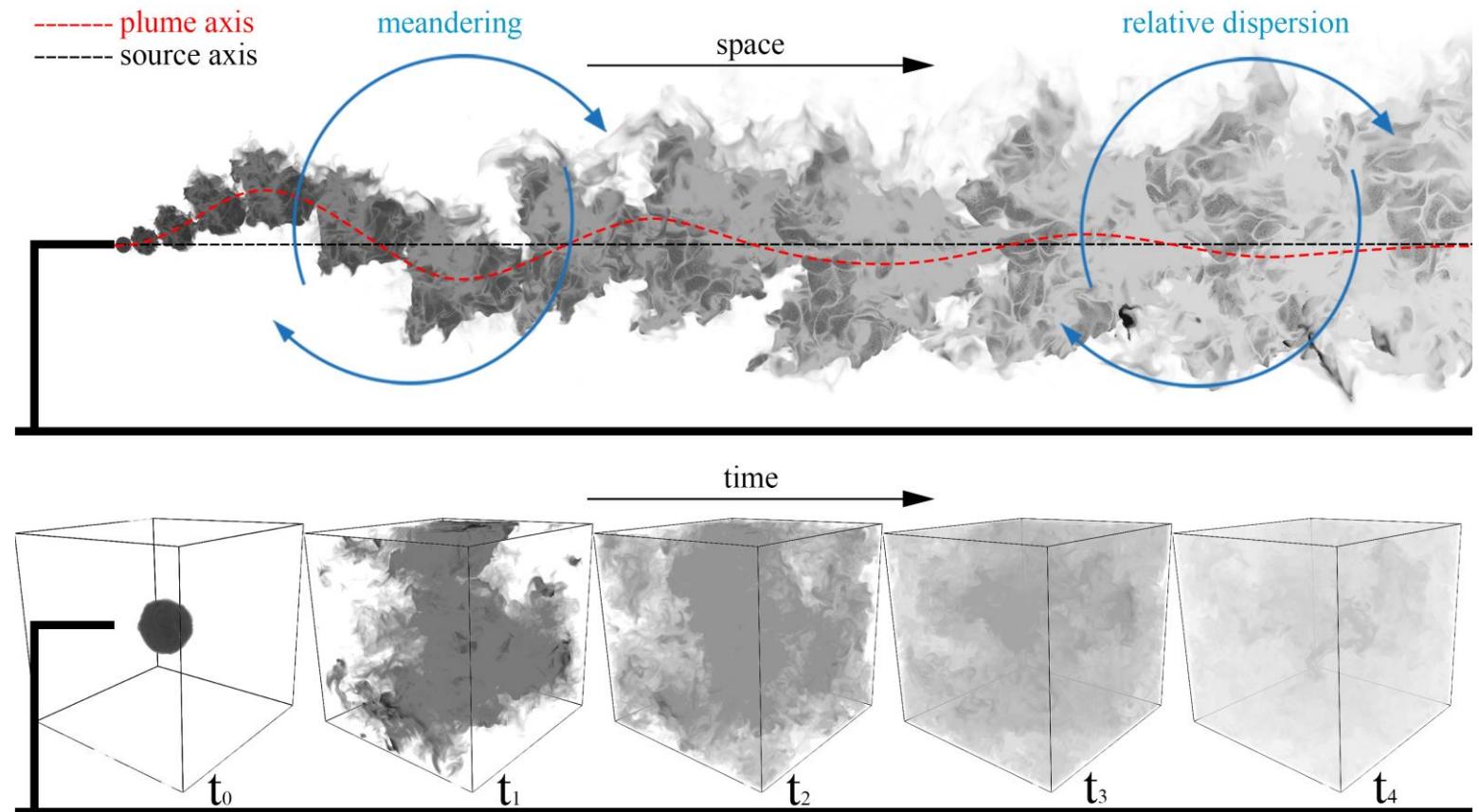
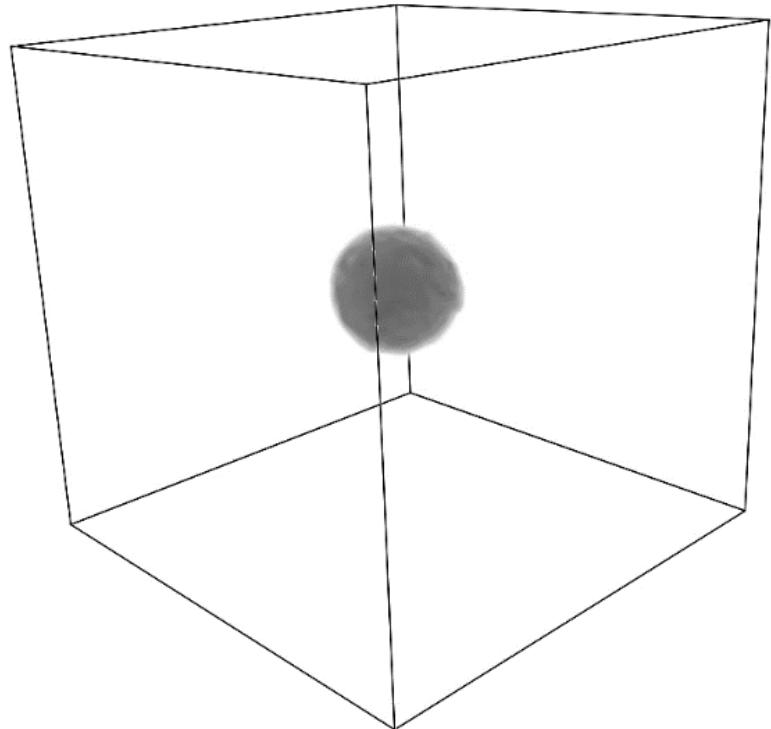
## 2 – Phenomenology and modelling of atmospheric dispersion

### Turbulent dispersion



## 2 – Phenomenology and modelling of atmospheric dispersion

### Turbulent dispersion



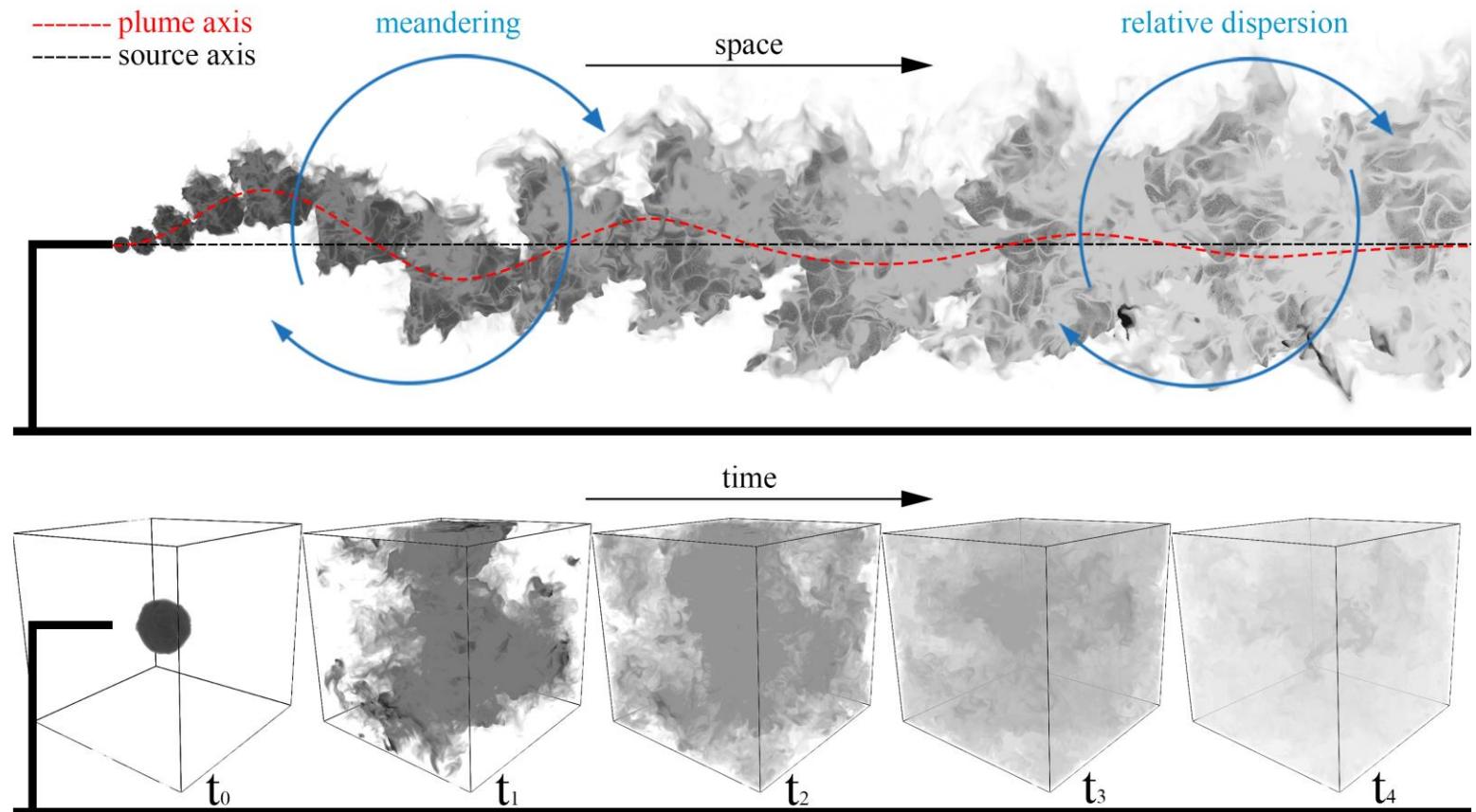
Orsi et al. (2021)

# 2 – Phenomenology and modelling of atmospheric dispersion

## Turbulent dispersion



Raffinerie de Feyzin (source France Inter)

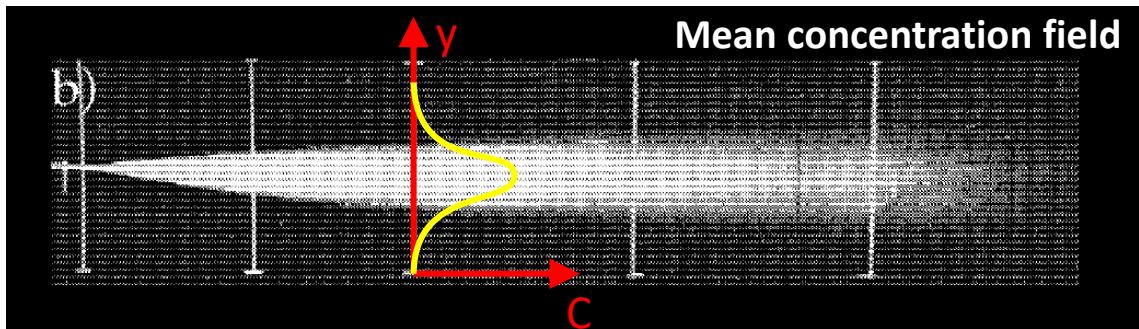
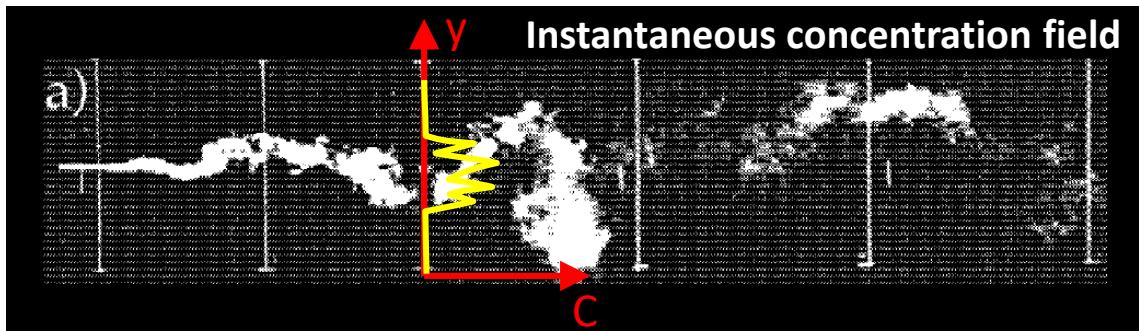


Orsi et al. (2021)

## 2 – Phenomenology and modelling of atmospheric dispersion

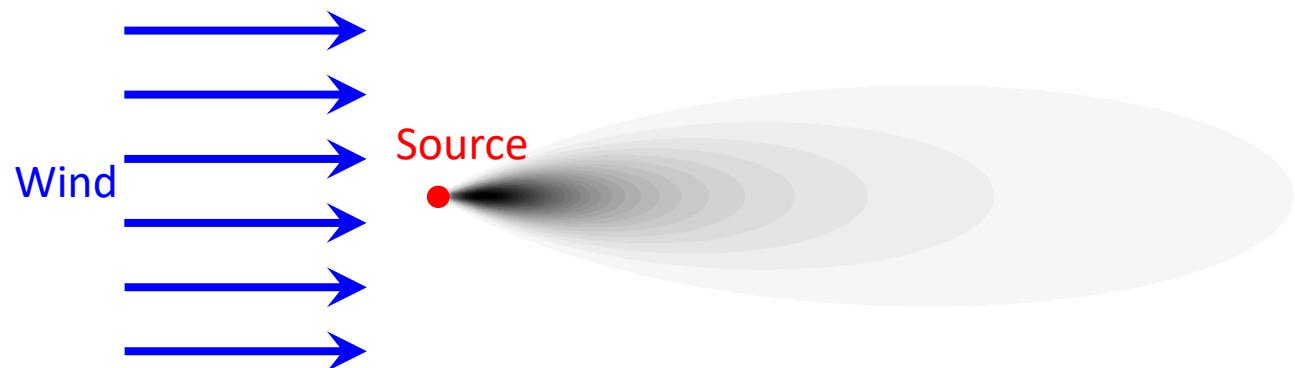
### Turbulent dispersion

- Instantaneous vs mean concentration field



**Gaussian model for the mean concentration**  
Analytical solution of the advection-diffusion equation

$$\bar{c}(x,y,z,t) = \frac{Q}{2\pi U \sigma_y \sigma_z} \exp \left[ -\frac{1}{2} \left( \frac{(y-y_0)^2}{\sigma_y^2} + \frac{(z-z_0)^2}{\sigma_z^2} \right) \right]$$

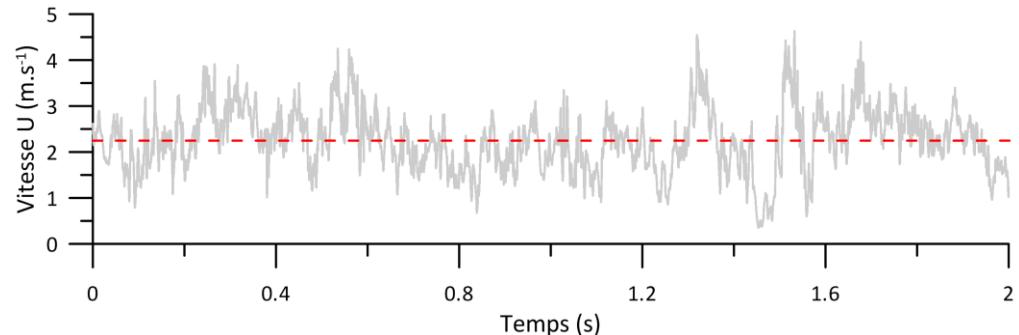


## 2 – Phenomenology and modelling of atmospheric dispersion

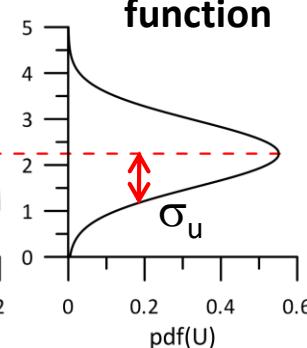
### Turbulent dispersion

- Influence of turbulence on dispersion

Time evolution of the fluctuating velocity



Probability density function



Lagrangian auto-correlation coefficient of turbulence

$$R_{uu}(\tau) = \frac{\int u'(t)u'(t+\tau)dt}{\int u'(t)u'(t)dt}$$

Lagrangian time scale

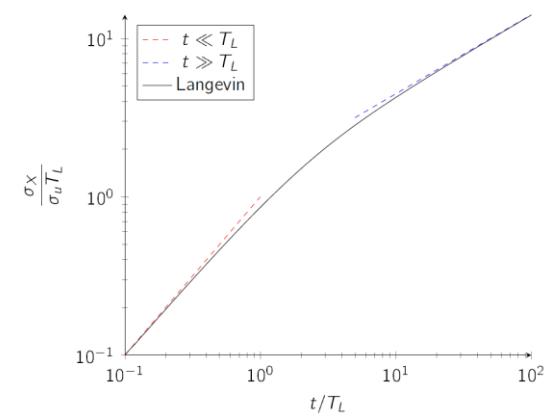
$$T_{L,x} = \int_0^{+\infty} R_{uu}(\tau)d\tau$$

Taylor theory (1921)

$$\sigma_{x_i}^2 = 2\sigma_{u_i}^2 T_{L,i} \left\{ t - T_{L,i} \left[ 1 - \exp\left(-\frac{t}{T_{L,i}}\right) \right] \right\}$$

Asymptotic behaviours

$$\begin{cases} \sigma_{x_i}(t) \simeq \sigma_{u_i} t & \text{for } t \ll T_{L,i} \\ \sigma_{x_i}(t) \simeq \sqrt{2\sigma_{u_i}^2 T_{L,i} t} & \text{for } t \gg T_{L,i} \end{cases}$$



## 2 – Phenomenology and modelling of atmospheric dispersion

### Turbulent dispersion

- Influence of stratification on turbulence and on dispersion

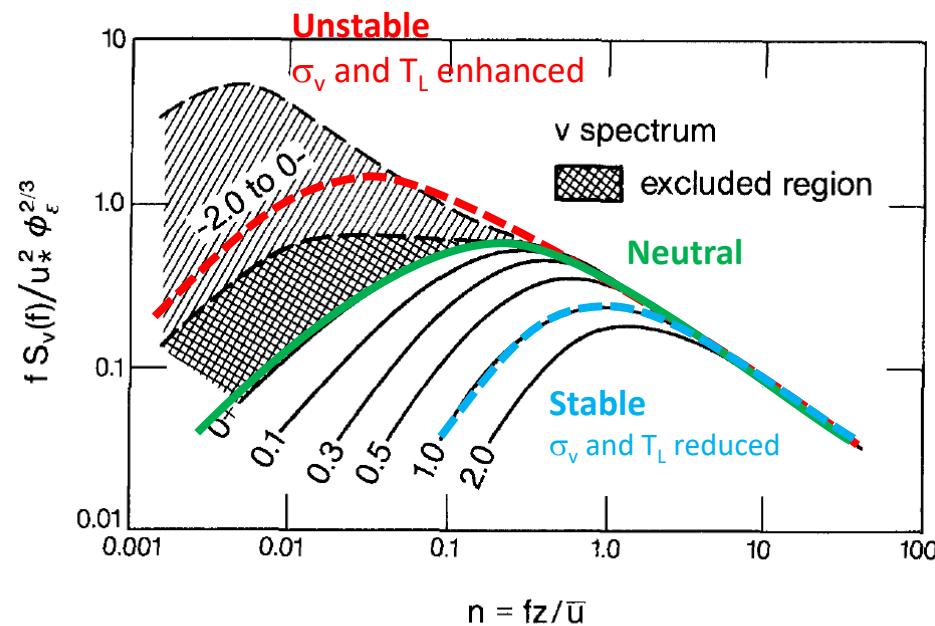


FIG. 2.6. Normalized surface layer  $v$  spectrum shown varying with  $z/L$ .

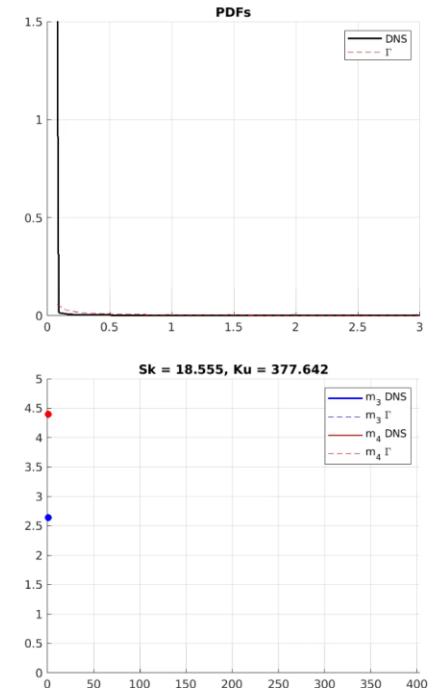
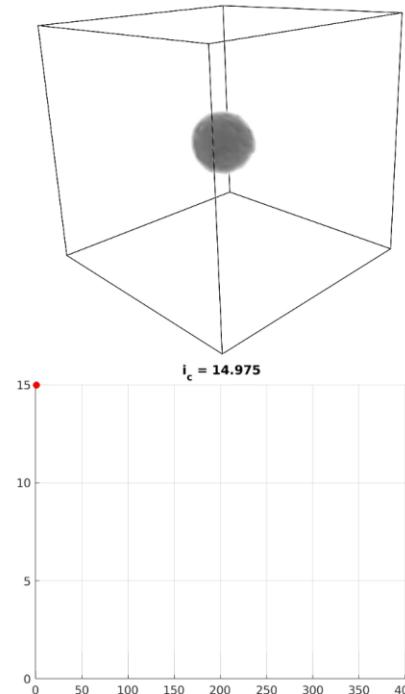
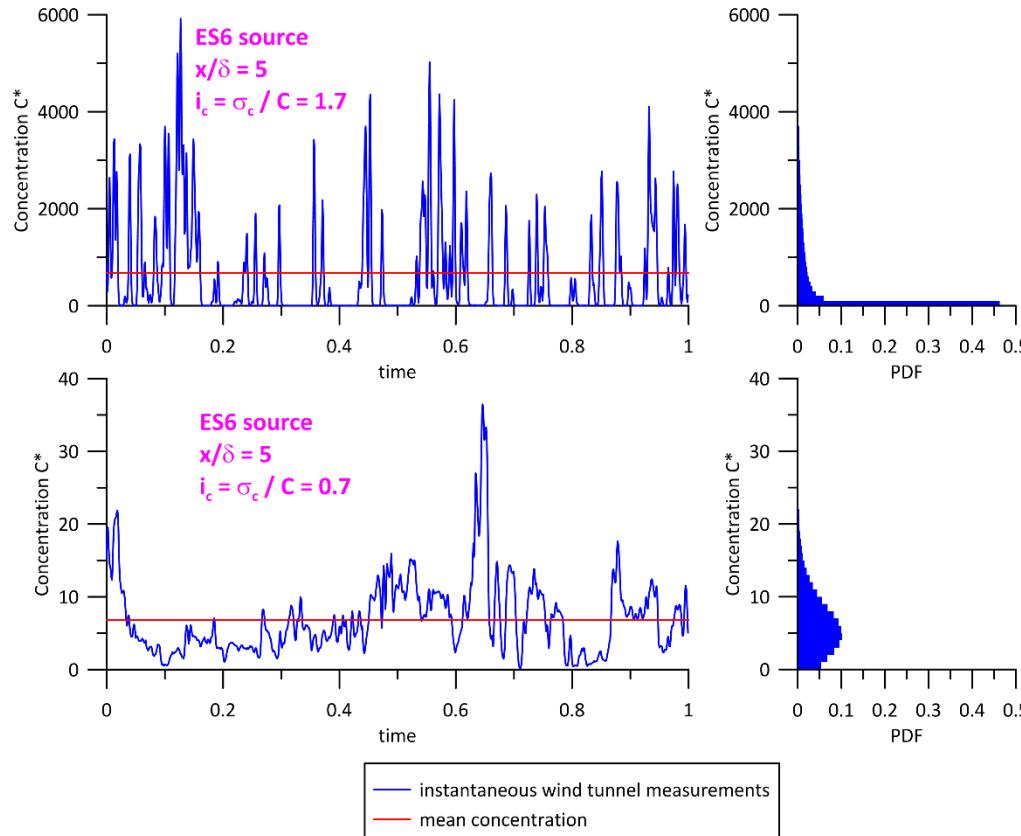
Kaimal and Finnigan (1994)



## 2 – Phenomenology and modelling of atmospheric dispersion

### Turbulent dispersion

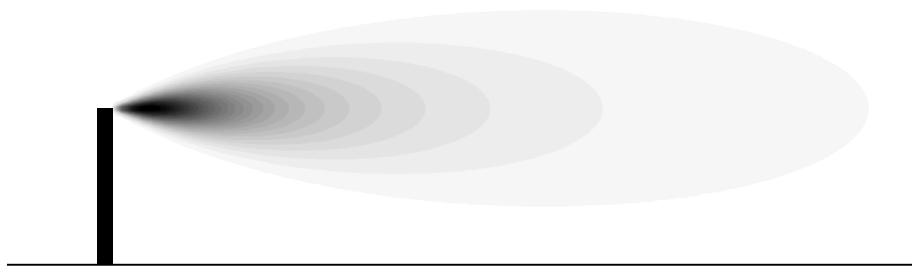
- Turbulent variability of the concentration



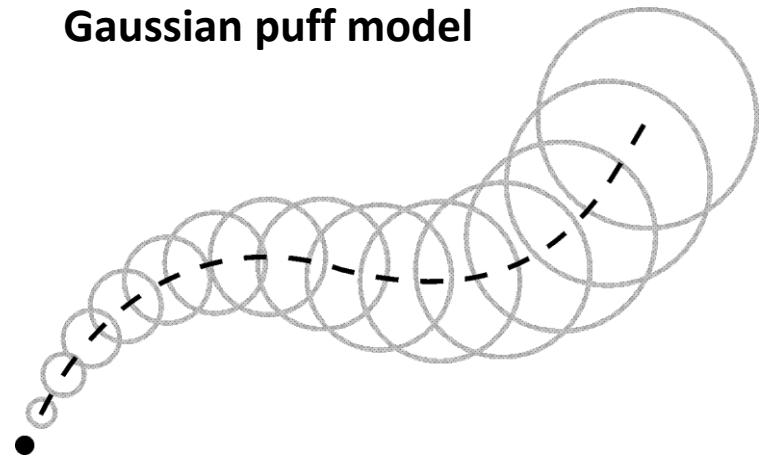
# 2 – Phenomenology and modelling of atmospheric dispersion

## Modelling approaches

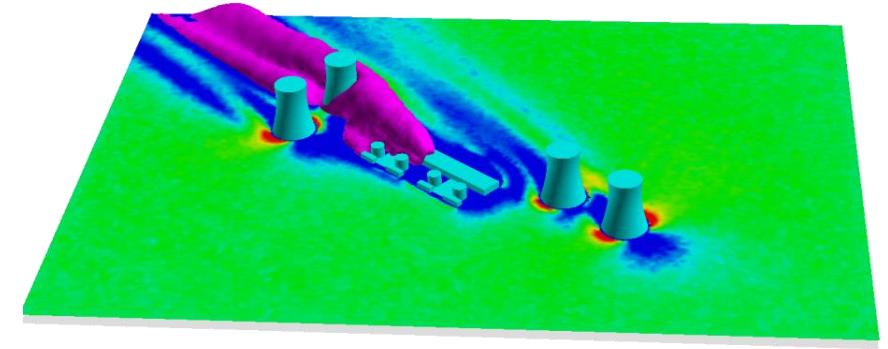
Gaussian plume model



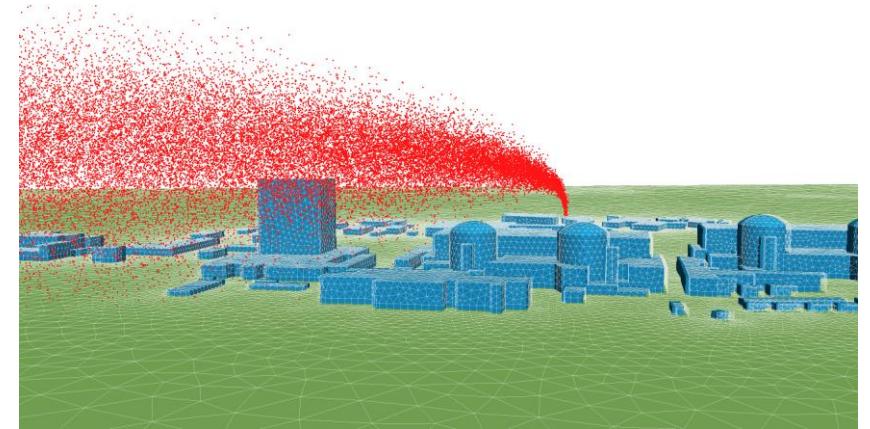
Gaussian puff model



Eulerian CFD model



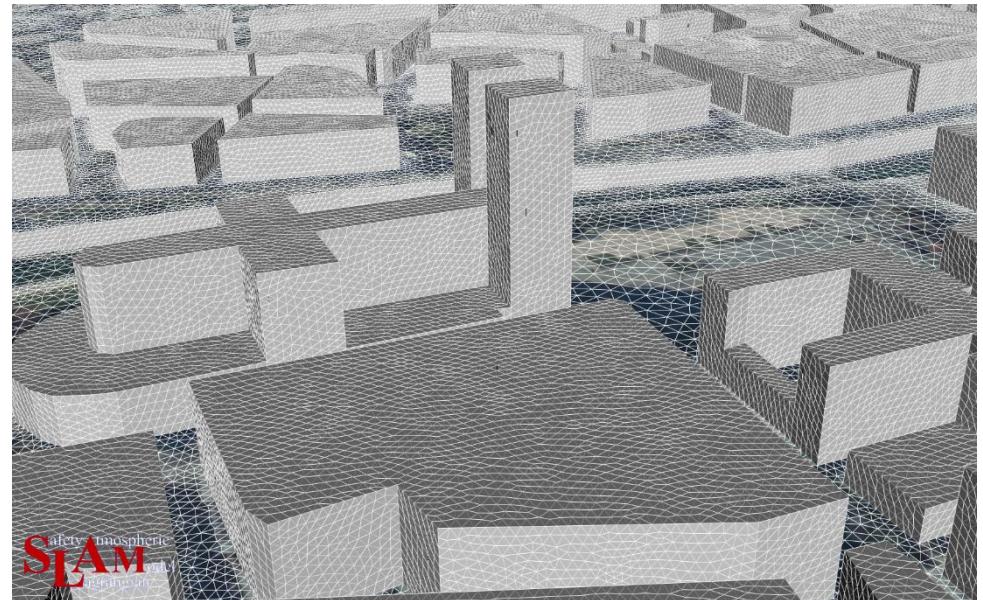
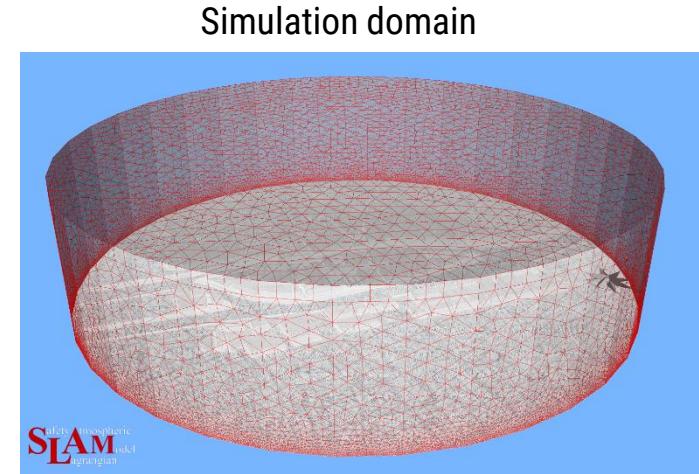
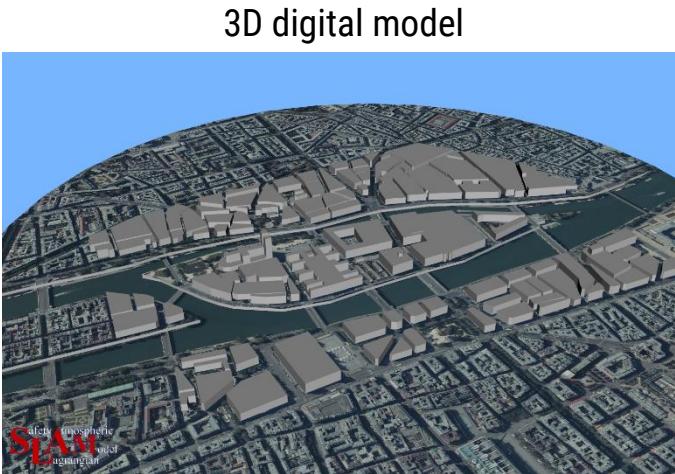
Lagrangian stochastic model



# 2 – Phenomenology and modelling of atmospheric dispersion

## Modelling approaches

- Modelling process



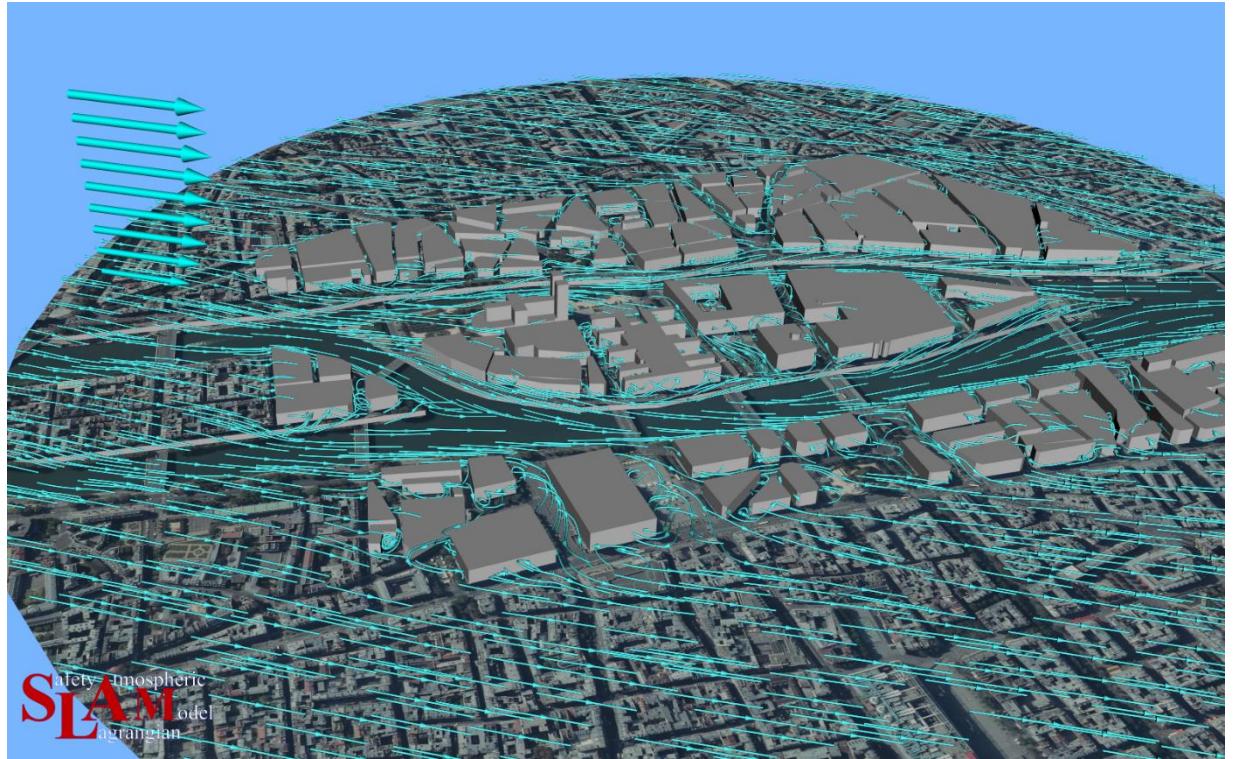
# 2 – Phenomenology and modelling of atmospheric dispersion

## Modelling approaches

- Modelling process



Numerical simulation of the wind in each city street



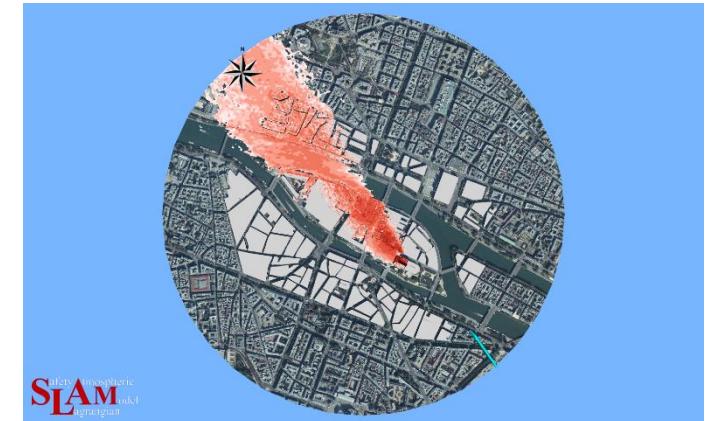
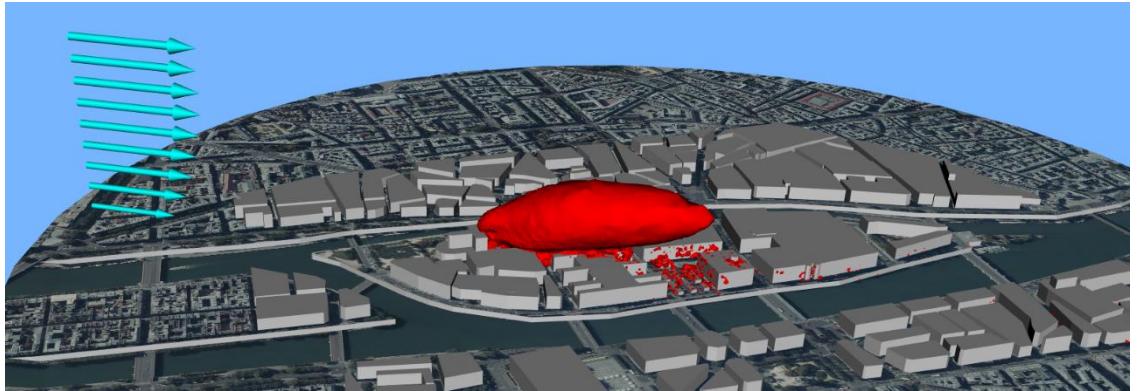
# 2 – Phenomenology and modelling of atmospheric dispersion

## Modelling approaches

- Modelling process



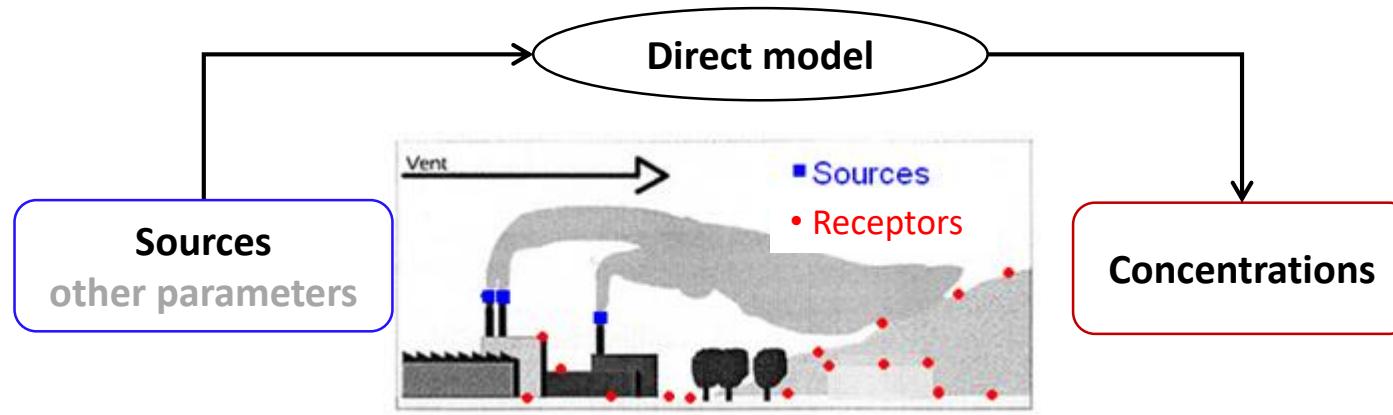
Numerical simulation of atmospheric dispersion



# 3 – Inverse modelling : problems & approaches

## Concept of inverse modelling

- Direct problem & inverse problem



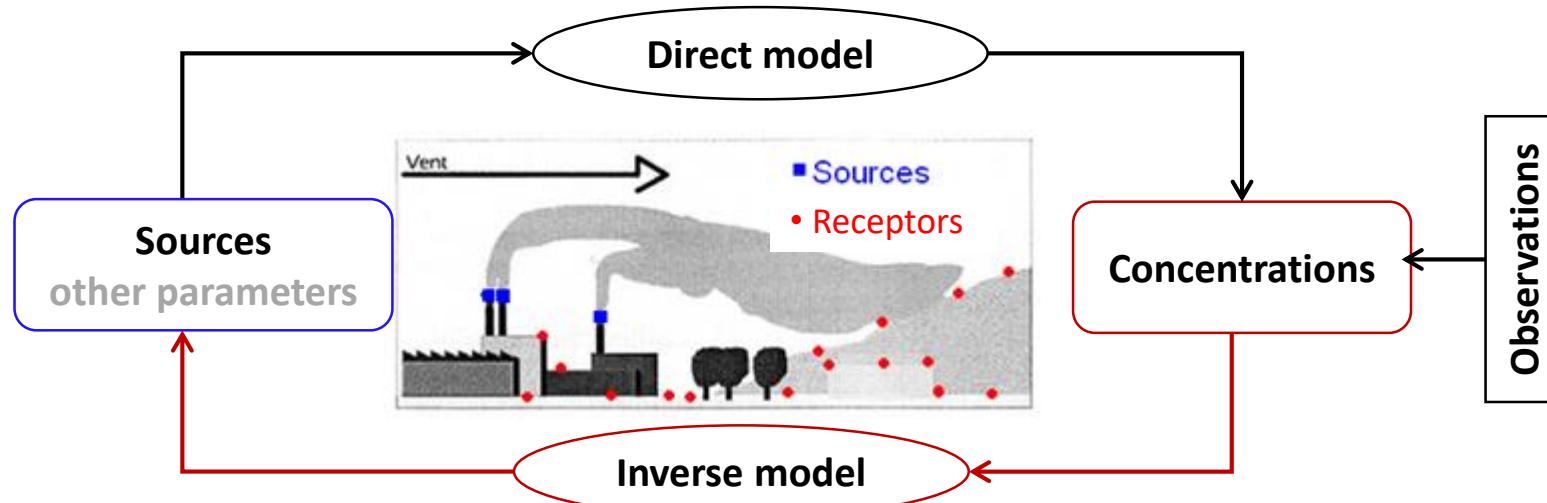
**Direct problem**

Finding  
consequences from  
causes

# 3 – Inverse modelling : problems & approaches

## Concept of inverse modelling

- Direct problem & inverse problem



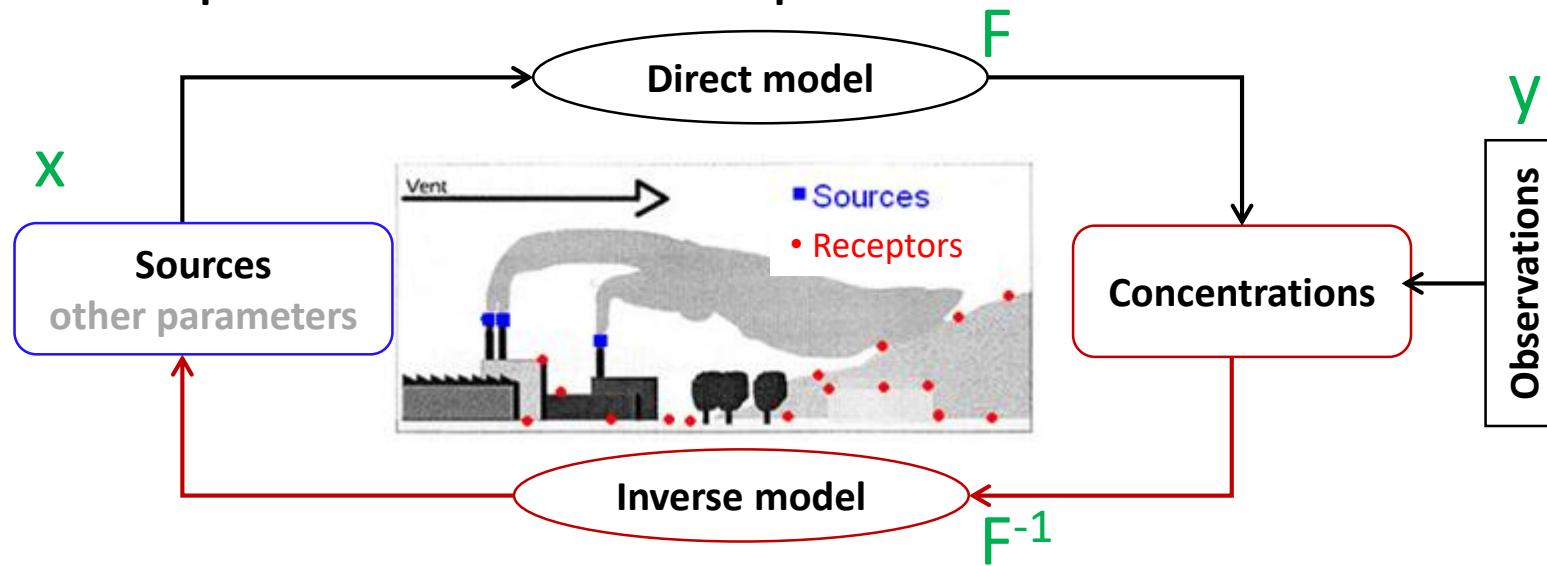
**Inverse problem**

Finding causes from  
consequences

# 3 – Inverse modelling : problems & approaches

## Concept of inverse modelling

- Direct problem & inverse problem



**Direct problem**  
 $F(x) = y$

**Inverse problem**  
 $x = F^{-1}(y)$

- Analogy with 1D equation solving

# 3 – Inverse modelling : problems & approaches

## Concept of inverse modelling

- Inverse modelling is used in many domains

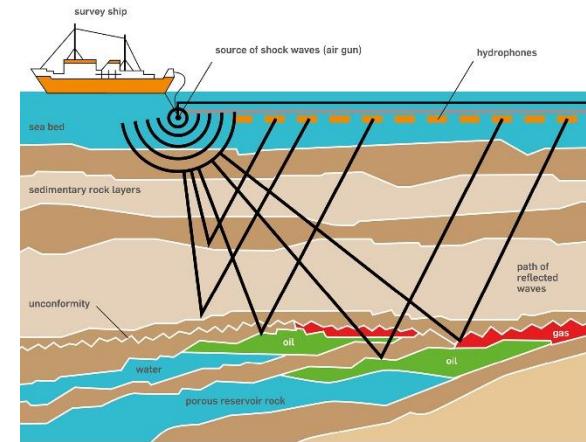
- Astronomy



Discovery of Neptune from the effect on Saturne trajectory  
(Adams, Le Verrier, 1846)

- Geology

Oil fields exploration

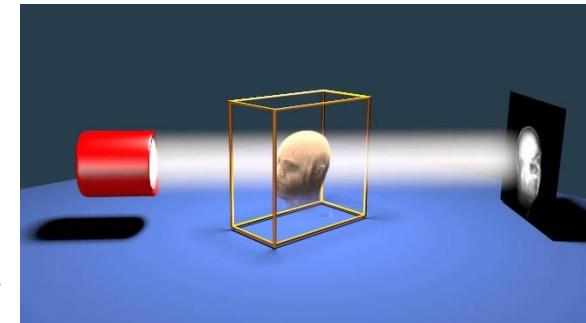


- Acoustics propagation



Acoustic sniper detection

- Medicine



Computed tomography

- Image processing



Image deblurring

- Etc.

Infrasound and beyond  
atmospheric dispersion

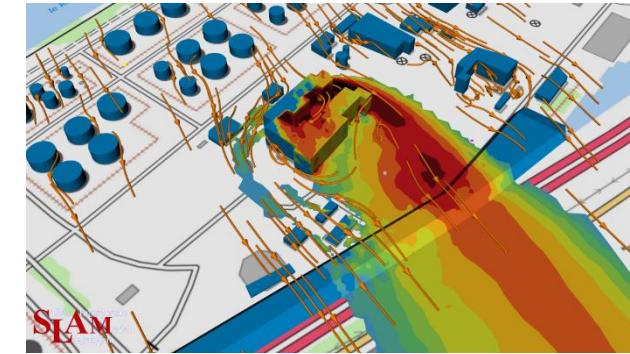


L. Soulhac  
INSA Lyon/LMFA

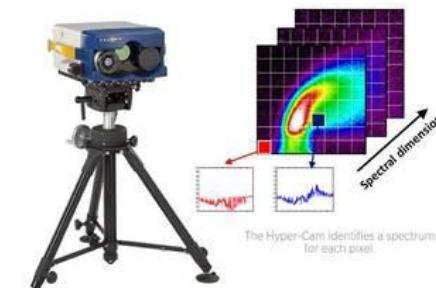
# 3 – Inverse modelling : problems & approaches

## Concept of inverse modelling

- Inverse modelling requires
  - A direct model
    - which have uncertainties
- Measurements
  - which have uncertainties
- An inversion algorithm
  - which has its own limitations



Monitoring station



Hyperspectral camera



μ-sensors

# 3 – Inverse modelling : problems & approaches

## Inverse problems of atmospheric dispersion

- 1 source
  - Point / distributed source (line, area, volume)
  - Continuous / instantaneous / time evolving
  - Fixed / moving
- N sources
  - Point / distributed source (line, area, volume)
  - Continuous / instantaneous / time evolving
  - Fixed / moving

Unknowns of the inverse problem

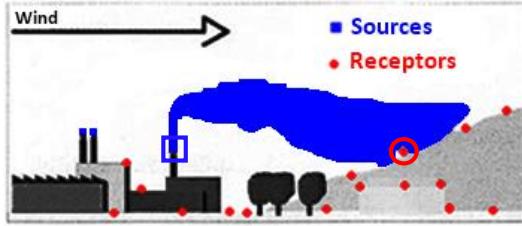
Emission rate	Release conditions	Released species	Meteorol. conditions

→ n unknowns vs m measurements

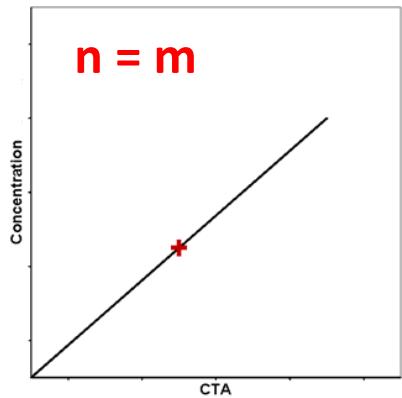


# 3 – Inverse modelling : problems & approaches

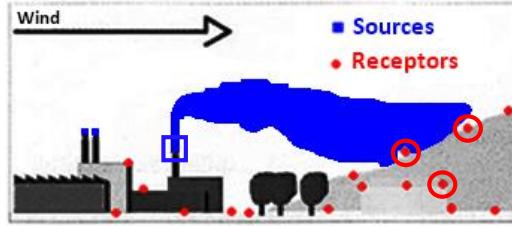
## Inverse problems of atmospheric dispersion



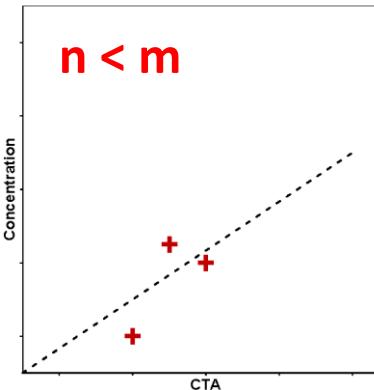
$$C_1^{\text{Obs}} = \text{CTA}_{1 \rightarrow 1} * Q_1$$



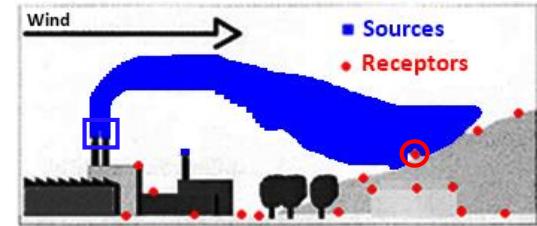
Unique solution



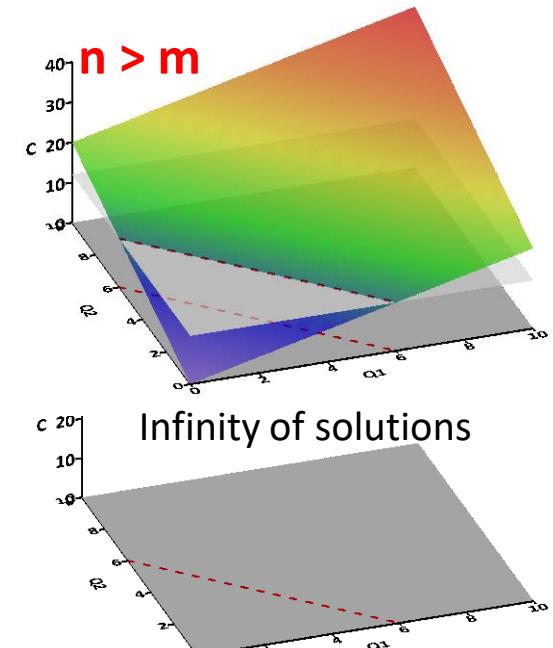
$$\begin{cases} C_1^{\text{Obs}} = \text{CTA}_{1 \rightarrow 1} * Q_1 \\ C_2^{\text{Obs}} = \text{CTA}_{1 \rightarrow 2} * Q_1 \\ C_3^{\text{Obs}} = \text{CTA}_{1 \rightarrow 3} * Q_1 \end{cases}$$



No exact solution



$$C_1^{\text{Obs}} = \text{CTA}_{1 \rightarrow 1} * Q_1 + \text{CTA}_{2 \rightarrow 1} * Q_2$$

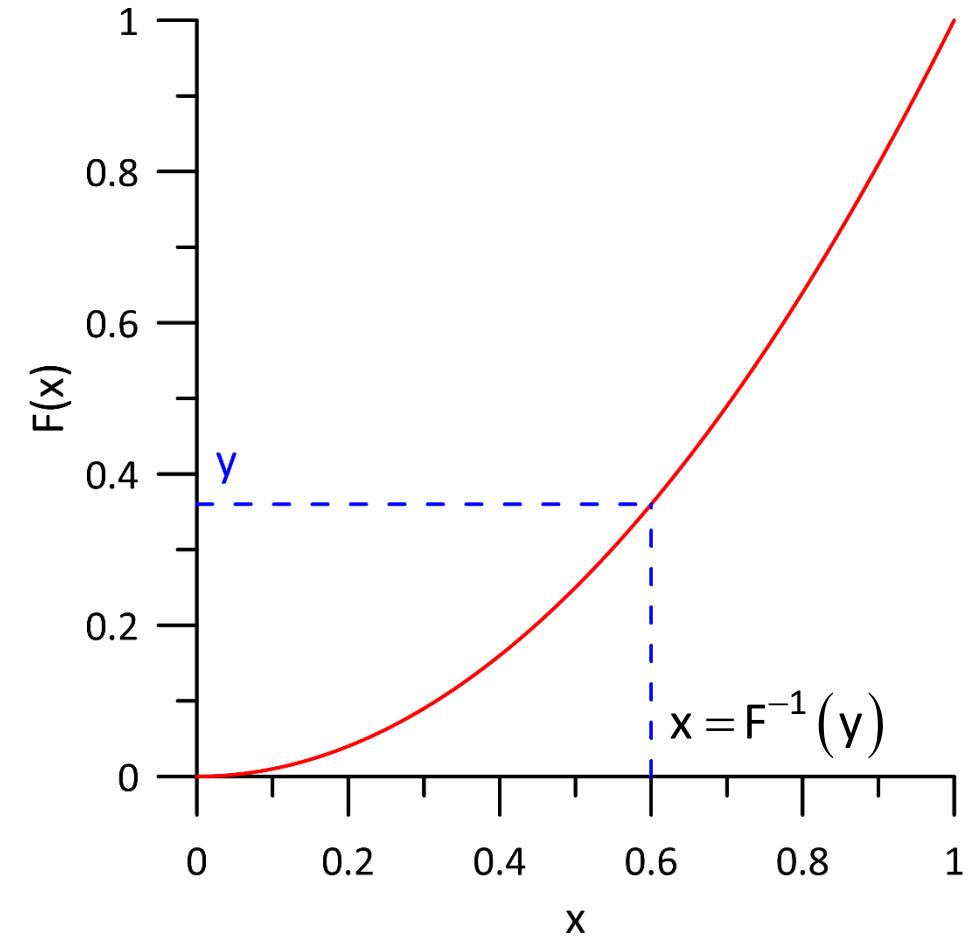


# 3 – Inverse modelling : problems & approaches

## Intuitive introduction to inverse modelling

Minimisation/optimization of a criterion

- Methods for non linear equation solving
  - 1. Guess an initial value of  $x$
  - 2. Calculate  $F(x)$  with the **model F**
  - 3. Compare  $F(x)$  with the **measurement y**
    - Need of a **cost function**
    - e.g. L1 or L2 norm of the difference  $F(x)$  vs  $y$
  - 4. Adjust  $x$ 
    - Systematic testing (**brute force** method)
    - Random testing (**Monte Carlo** method)
    - Dichotomy method
    - Newton-Raphson method
      - requires the inverse of  $F'(x) = \text{adjoint model of } F$



# 3 – Inverse modelling : problems & approaches

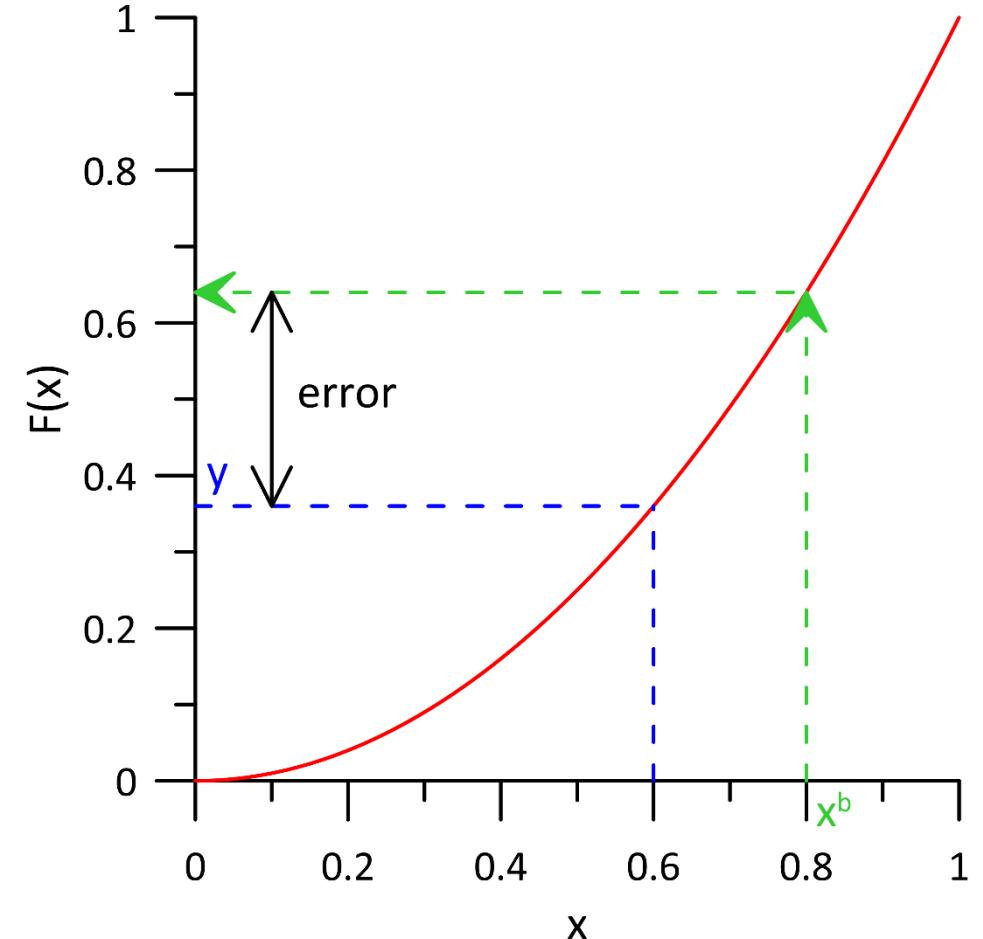
## Intuitive introduction to inverse modelling

- Variational approach

1. Guess an initial value of  $x$
2. Calculate  $F(x)$  with the **model F**
3. Compare  $F(x)$  with the **measurement y**
  - Evaluate of a **cost function** (error)

$$J(x) = \frac{1}{2}(x - x^b)^T B^{-1}(x - x^b) + \frac{1}{2} \sum_{i=0}^m (y_i - H_i F(x))^T R^{-1} (y_i - H_i F(x))$$

4. Adjust  $x$ 
  - Systematic testing (**brute force** method)
  - Optimized Random iterative testing
    - e.g. **Markov Chain Monte Carlo** (MCMC)



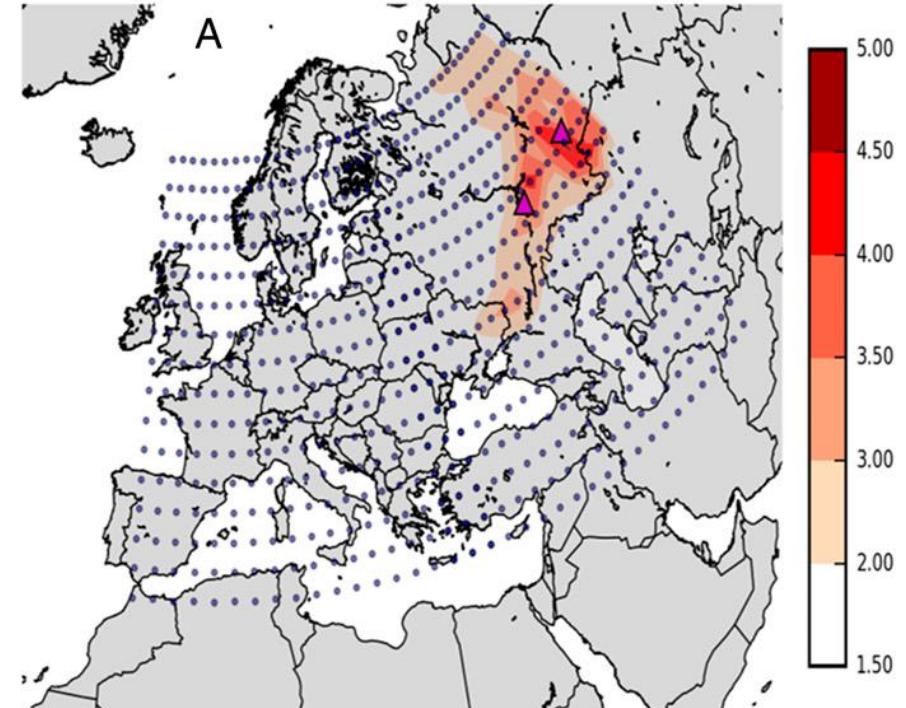
# 3 – Inverse modelling : problems & approaches

## Intuitive introduction to inverse modelling

- Variational approach
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Mayak event of Ruthenium release, 2017  
Saunier et al., IRSN (2019)

# 3 – Inverse modelling : problems & approaches

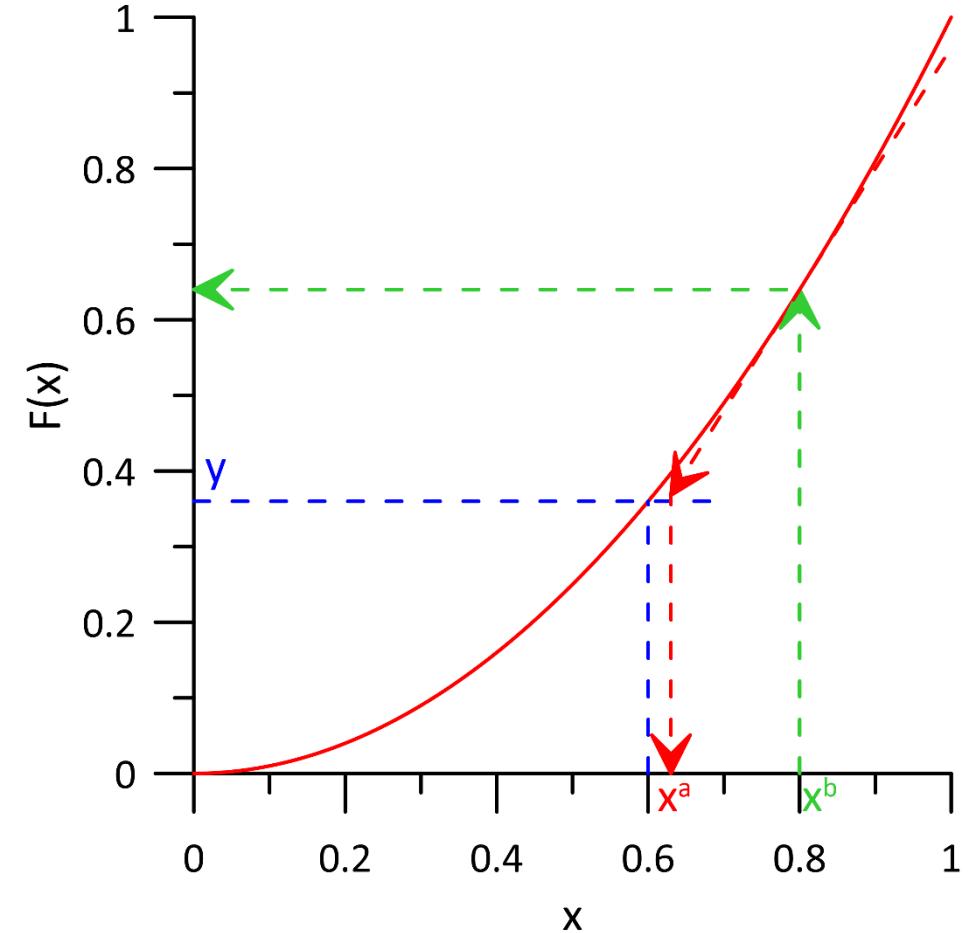
## Intuitive introduction to inverse modelling

- Variational approach with adjoint model

1. Guess an initial value of  $x$
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3. Compare  $F(x)$  with the **measurement y**
  - Evaluate of a **cost function** (error)

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4. Adjust  $x$ 
  - Systematic testing (**brute force** method)
  - Optimized Random iterative testing
    - e.g. **Markov Chain Monte Carlo** (MCMC)
  - Gradient method
    - requires the inverse of  $F'(x) = \text{adjoint model}$  of F



# 3 – Inverse modelling : problems & approaches

## Intuitive introduction to inverse modelling

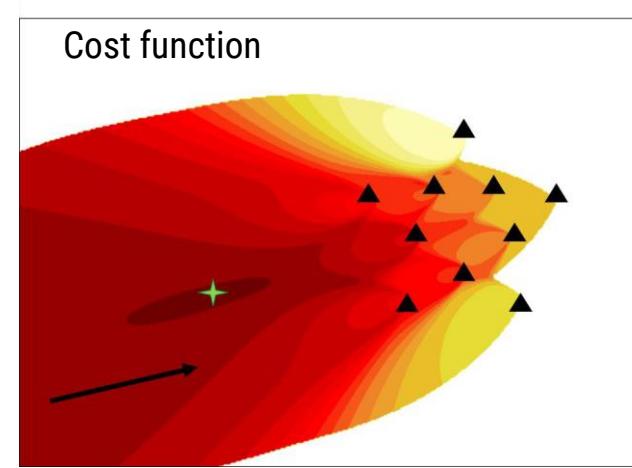
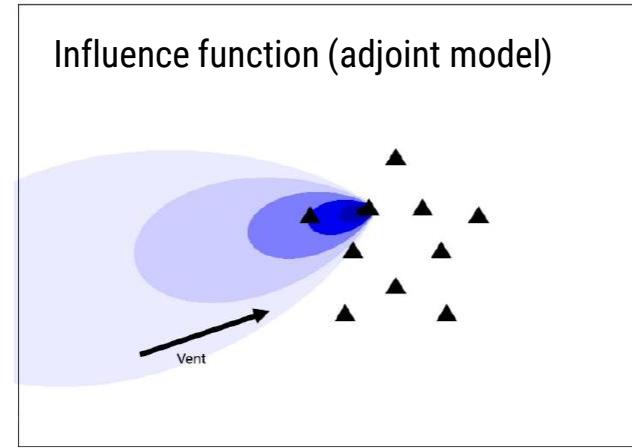
- Variational approach with adjoint model

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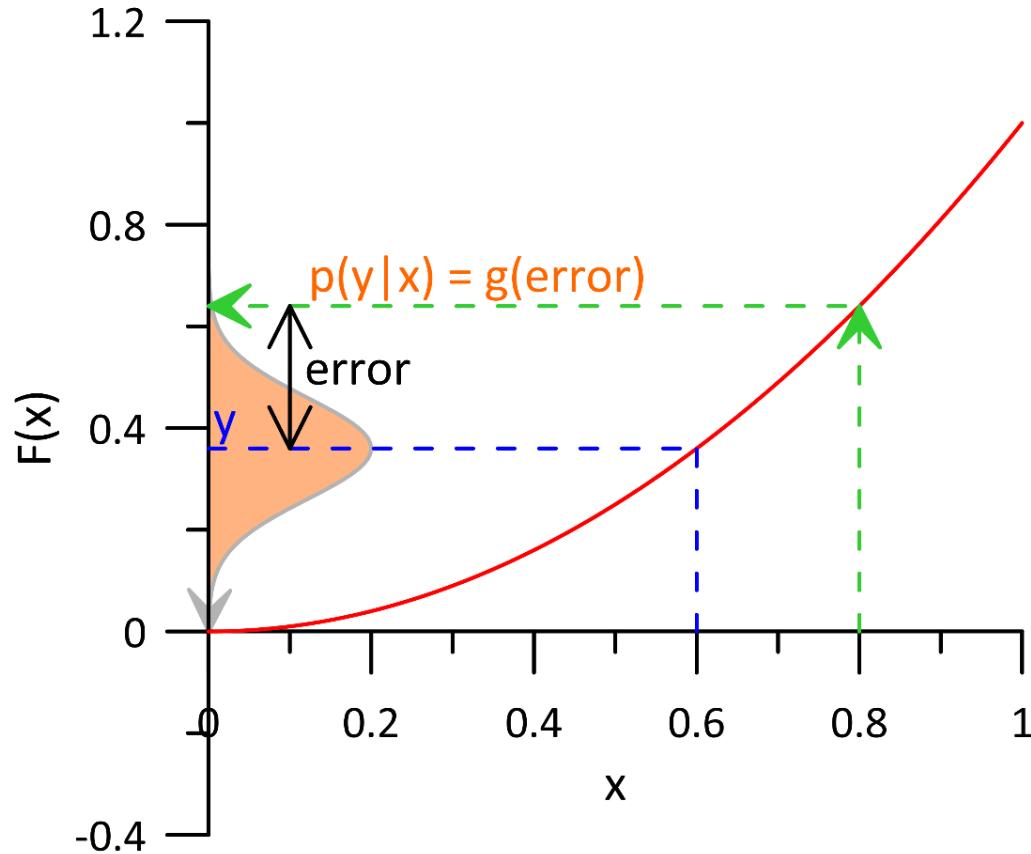
Minimisation/optimization of a criterion



# 3 – Inverse modelling : problems & approaches

## Intuitive introduction to inverse modelling

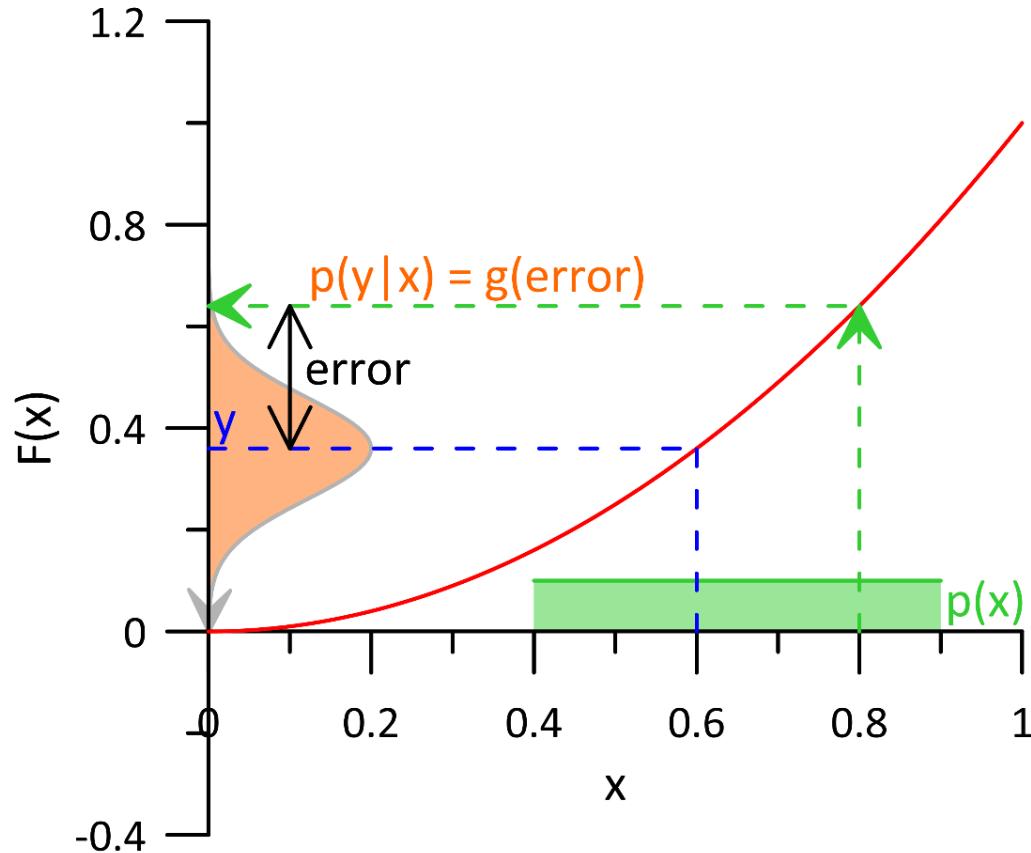
- Bayesian approach
  - 1. Guess some initial values of  $x$
  - 2. Calculate  $F(x)$  with the **model F**
  - 3. Compare  $F(x)$  with the **measurement y**
    - Evaluate of a **likelihood function** from the error which give the probability  $p(y|x)$



# 3 – Inverse modelling : problems & approaches

## Intuitive introduction to inverse modelling

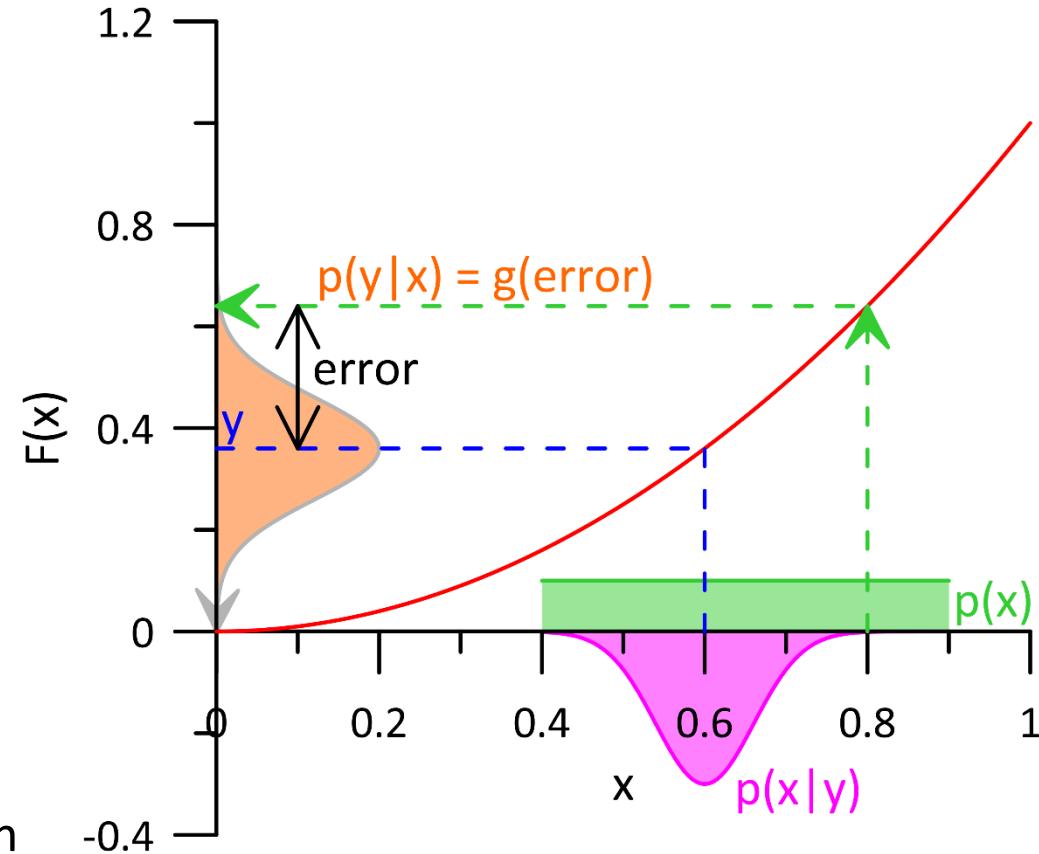
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    - Evaluate of a **likelihood function** from the error which give the probability  $p(y|x)$
  - 4. Assume a prior distribution of  $x$ 
    - $p(x)$  is the prior information about  $x$



# 3 – Inverse modelling : problems & approaches

## Intuitive introduction to inverse modelling

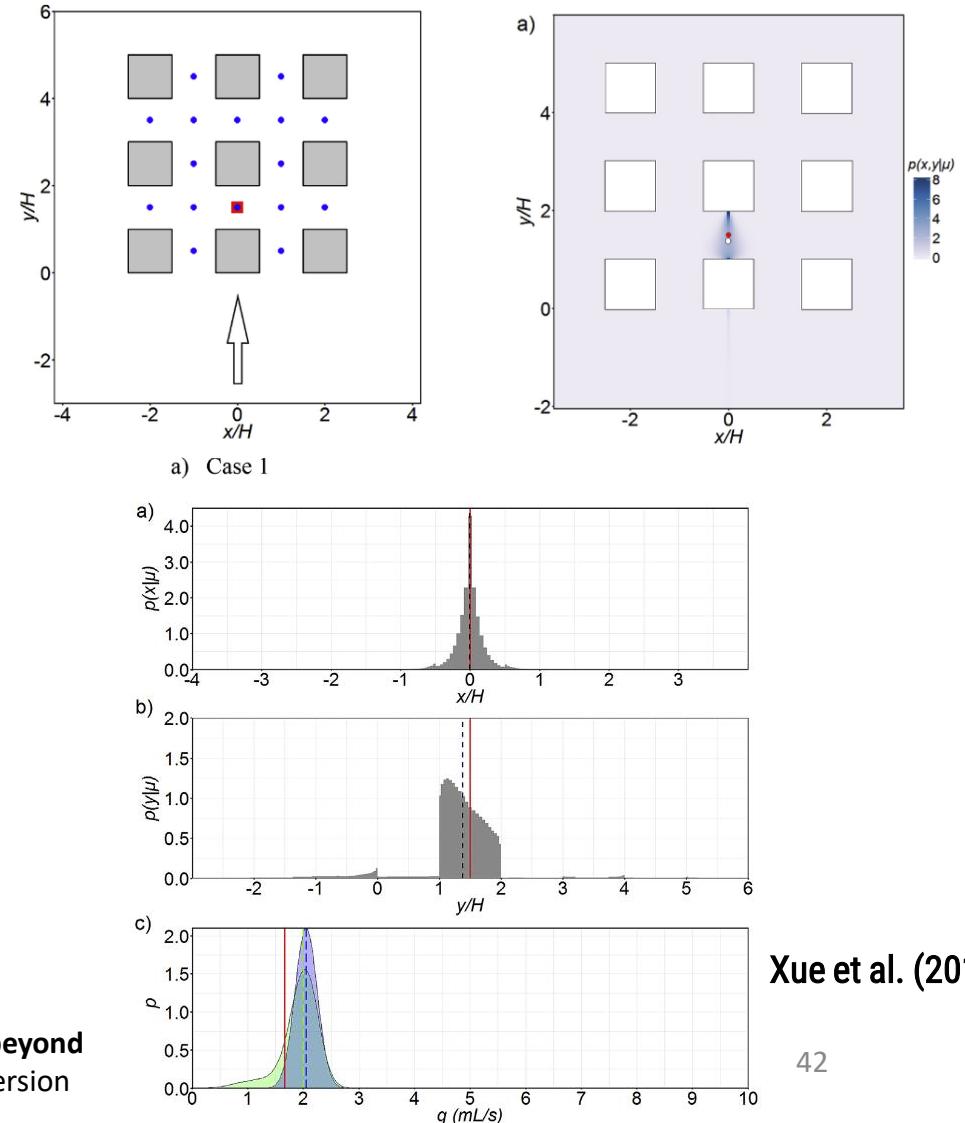
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    - Evaluate of a **likelihood function** from the error which give the probability  $p(y|x)$
  4. Assume a prior distribution of  $x$ 
    - $p(x)$  is the prior information about  $x$
  5. Bayes formula
    - Posterior probability  $p(x|y) = \frac{p(y|x)p(x)}{p(y)}$
    - Marginal probability  $p(y) = \text{normalisation function}$



# 3 – Inverse modelling : problems & approaches

## Intuitive introduction to inverse modelling

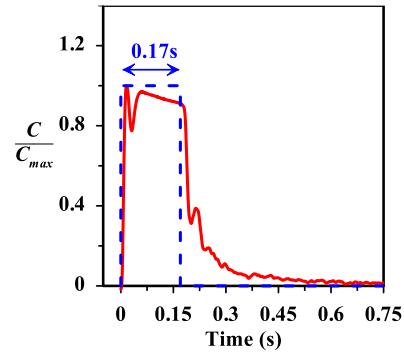
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# 5 – Some applied examples

Estimating accidental pollutant releases in built environment from turbulent concentration signals (Ben Salem et al., 2017)

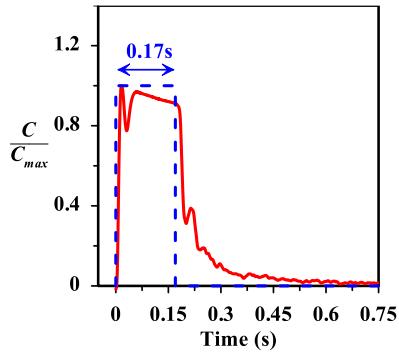
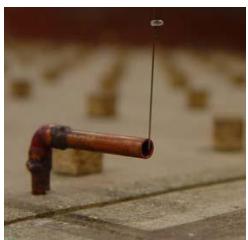
- Wind tunnel experiments of instantaneous releases in a district



# 5 – Some applied examples

Estimating accidental pollutant releases in built environment from turbulent concentration signals (Ben Salem et al., 2017)

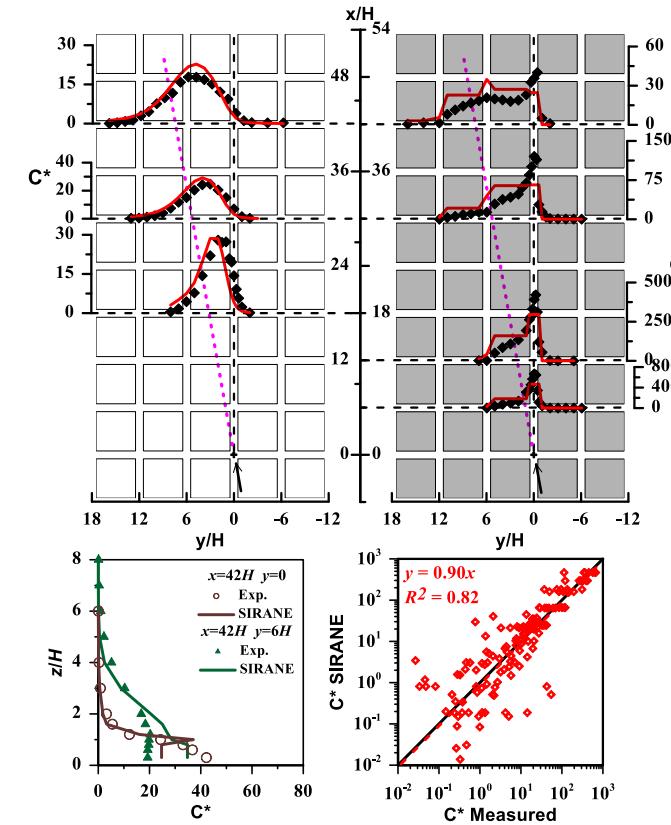
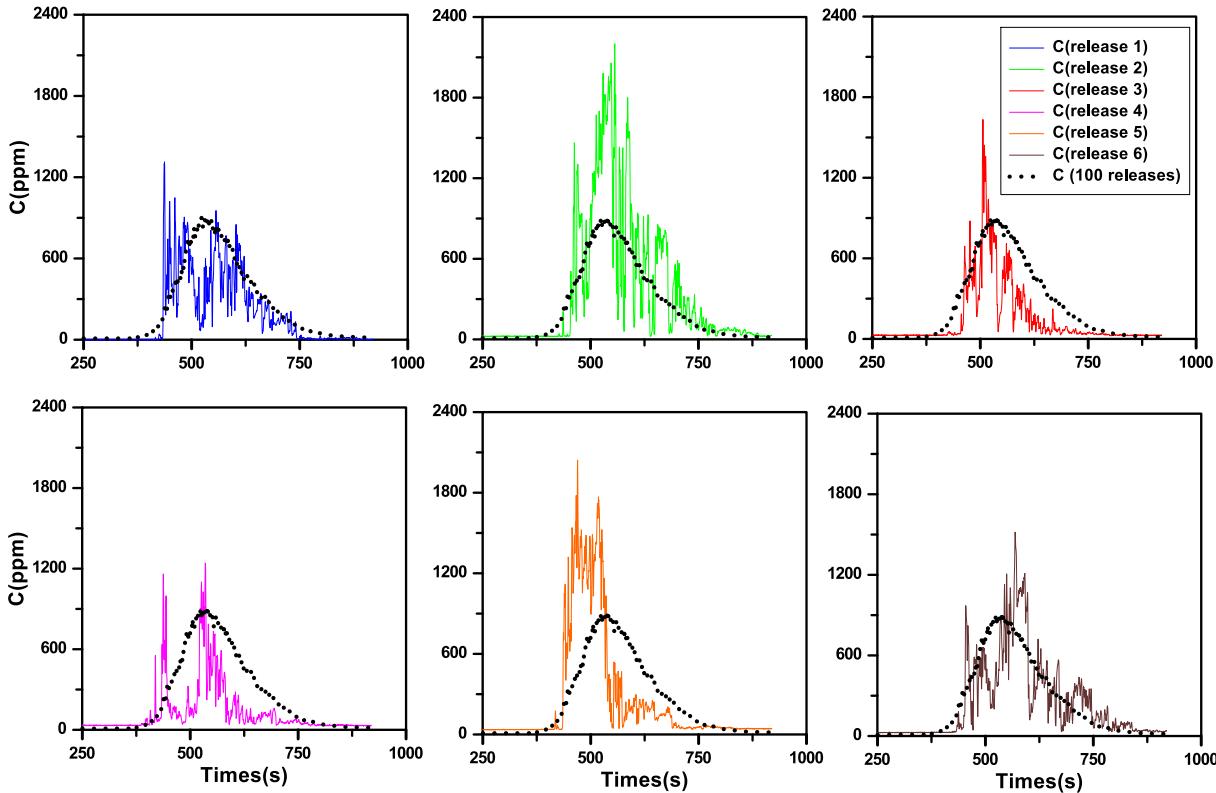
- Wind tunnel experiments



# 5 – Some applied examples

Estimating accidental pollutant releases in built environment from turbulent concentration signals (Ben Salem et al., 2017)

- Wind tunnel experiments of instantaneous releases in a district



Comparison of the direct model for the mean concentration



# 5 – Some applied examples

Estimating accidental pollutant releases in built environment from turbulent concentration signals (Ben Salem et al., 2017)

- Variational approach for inverse modelling

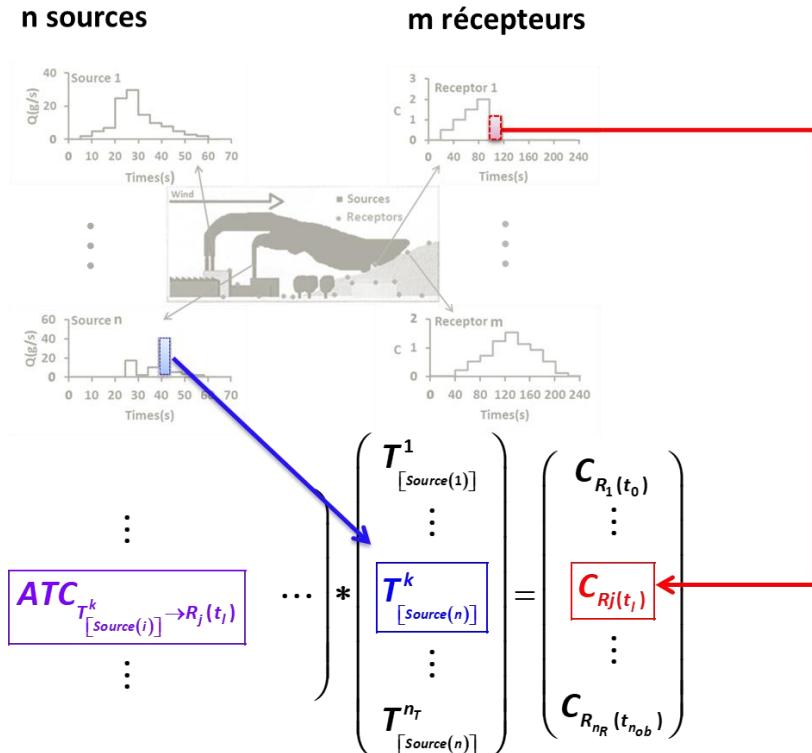
- The position of the source is known
- We assume a linear relation between
  - Concentration C
  - Release rate Q

$$C^{obs}(m) = ATC(m, n) \times Q(n)$$

- We use a cost function with a Tikhonov regularisation term

$$J = \|C^{obs} - CTA \times Q\|_{\mathbb{R}^n_C}^2 + \varepsilon^2 \Gamma(Q)$$

- The idea is to avoid that the model “follows” each fluctuation of the instantaneous measurements
- The optimum solution is given by  $Q = (CTA^t \times CTA + \varepsilon^2 I)^{-1} \times CTA^t \times C^{obs}$



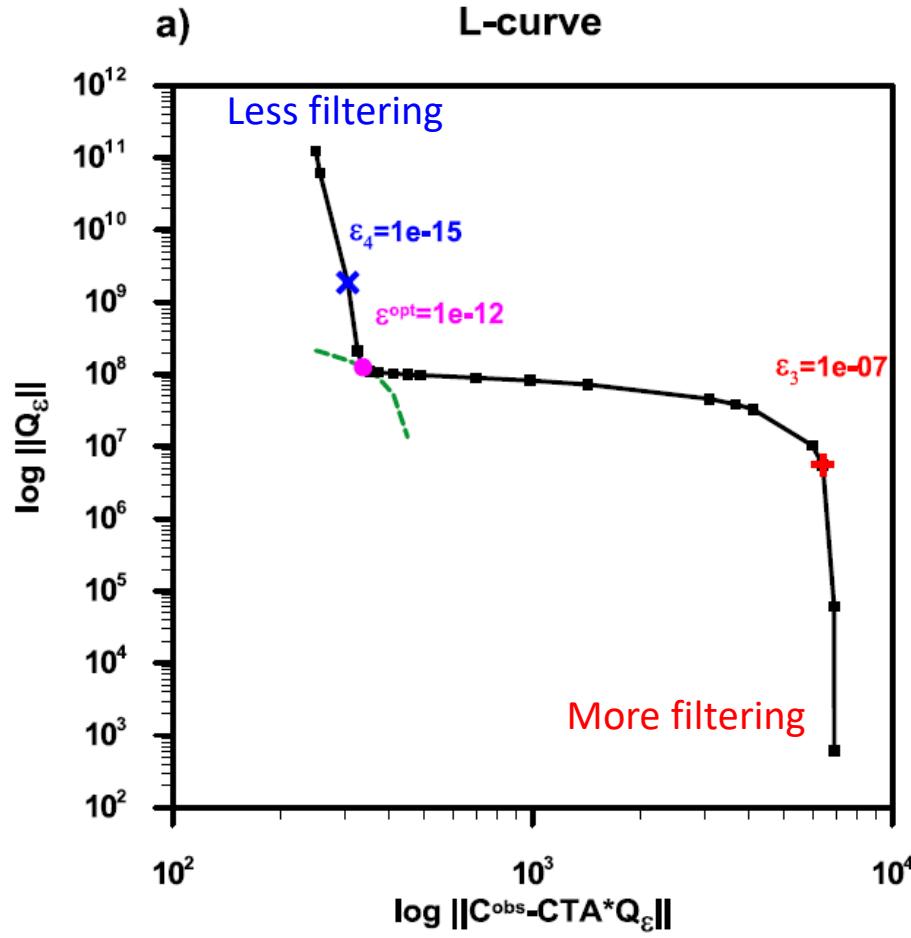
# 5 – Some applied examples

Estimating accidental pollutant releases inbuilt environment from turbulent concentration signals (Ben Salem et al., 2017)

- Regularisation parameter  $\varepsilon$

$$J = \left\| C^{obs} - CTA \times Q \right\|_{\mathbb{R}^{n_C}}^2 + \varepsilon^2 \Gamma(Q)$$

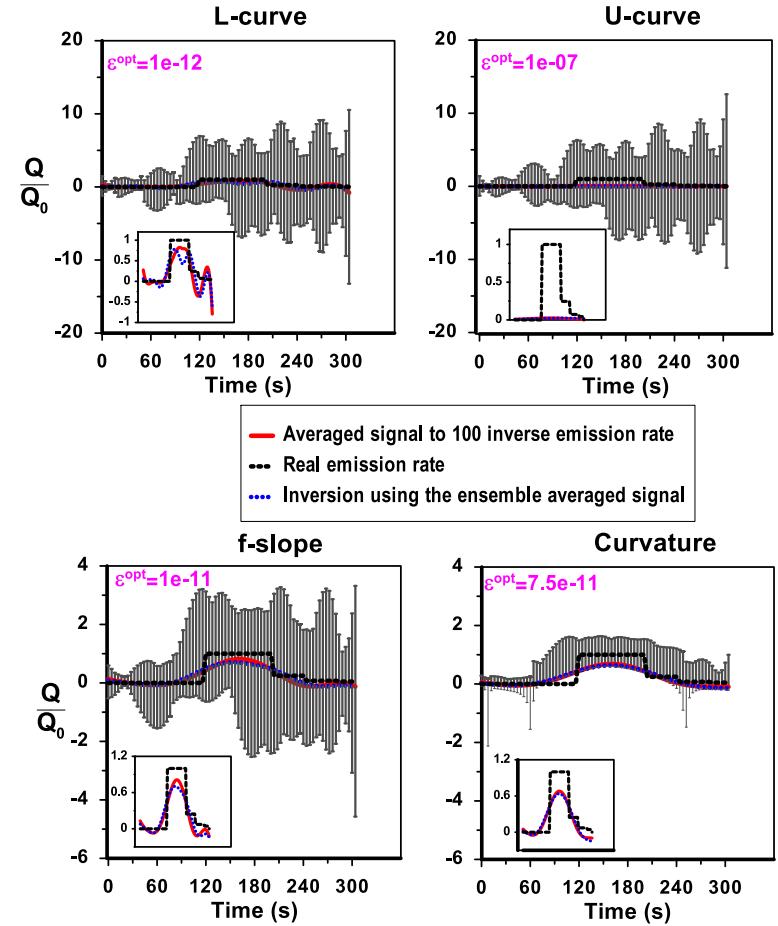
- $\varepsilon$  has to be optimized with specific method
- Example of the L-curve method



# 5 – Some applied examples

Estimating accidental pollutant releases inbuilt environment from turbulent concentration signals (Ben Salem et al., 2017)

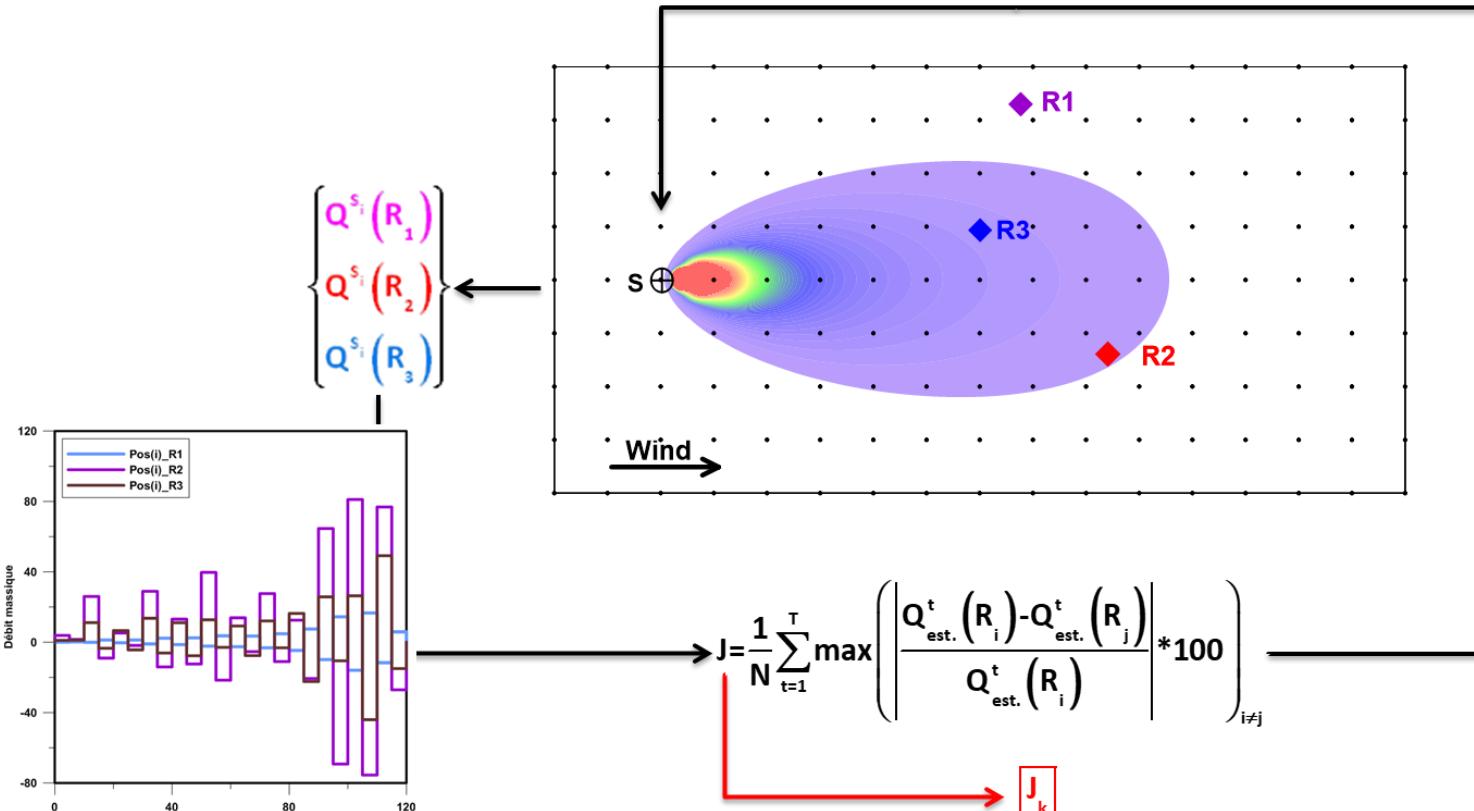
- Results for the time evolution of the emission rate
  - It has to be optimized
  - f-slope approach provides the better results for the inversion from the mean concentration
  - Maximum of curvature method provides the better results for inversion from instantaneous measurements



# 5 – Some applied examples

## Localization of a source in a district (Ben Salem, 2015)

- Brute force approach to characterize the field of the cost function

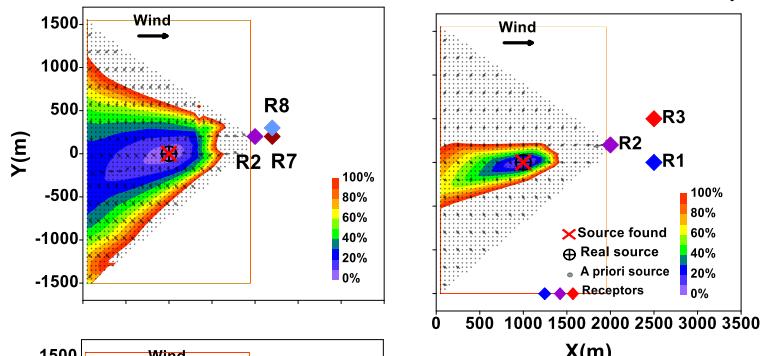


# 5 – Some applied examples

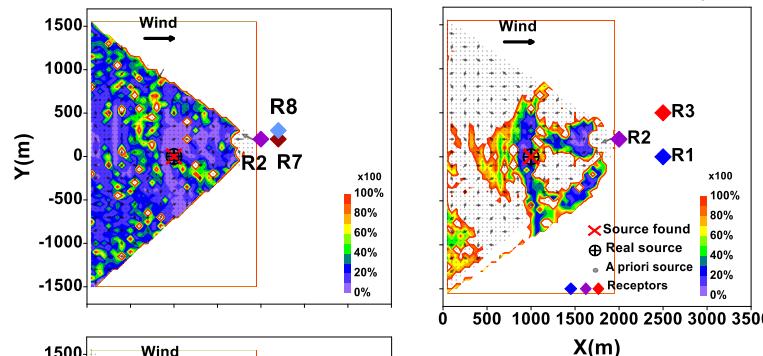
## Localization of a source in a district (Ben Salem, 2015)

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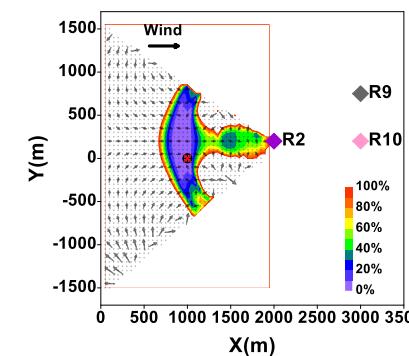
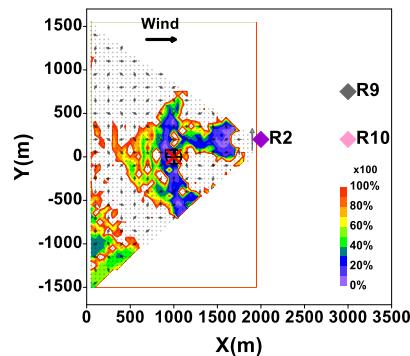
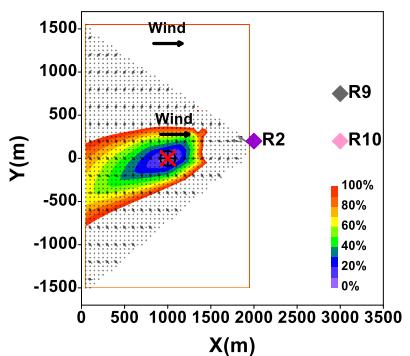
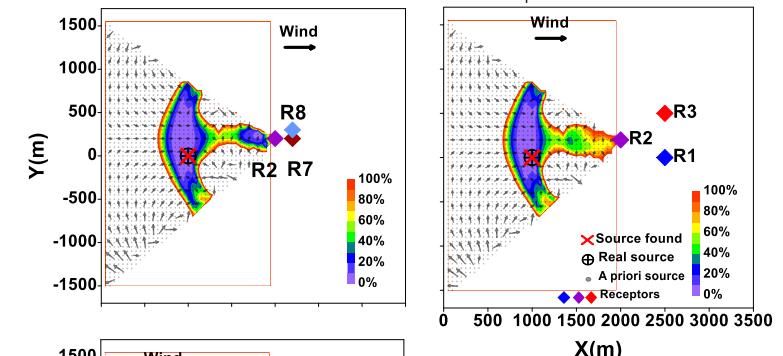
$$CR1(\%) = \max \left( \left| \frac{Qtt(R_i) - Qtt(R_j)}{Qtt(R_i)} \right| * 100 \right)_{i \neq j}$$



$$CR2 = \frac{1}{N} \sum_{k=1}^N \max \left( \frac{Q_{inv}^k(R_i) - Q_{inv}^k(R_j)}{Q_{inv}^k(R_i)} * 100 \right)_{i \neq j}$$



$$CR3 = \frac{\left( \frac{1}{N} \sum_{t=1}^{t_f} \left( \max \left( \left| Q_{inv}^t(R_i) - Q_{inv}^t(R_j) \right|_{i \neq j} \right) * \Delta t \right) \right)}{\min(Qtt(R_i))_{i=1..n_r}} * 100$$



# 5 – Some applied examples

Crisis management tool for CBRN events (H2020 TERRIFIC project,  
<https://www.terrific.eu/> & Nguyen et al, 2021)

- Methodology
  - Inverse modelling of an atmospheric flow/dispersion/radiation simulation system
  - Coupling with real time mobile measurements
  - Minimization of a cost function and optimization of the source position and intensity
- Results
  - Field test case using
    - real radioactive sources
    - drones and robots
    - sensors and cameras
  - Use of wind field database and optimized operational dispersion model to get real time inversion results



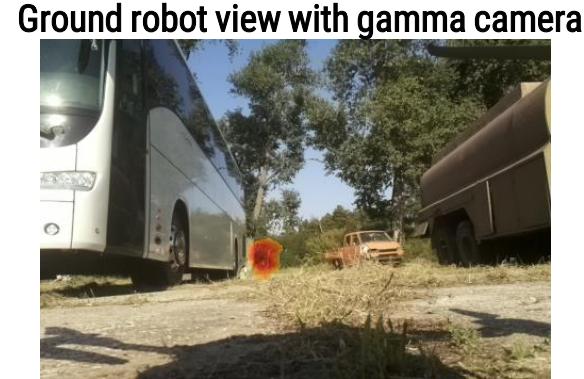
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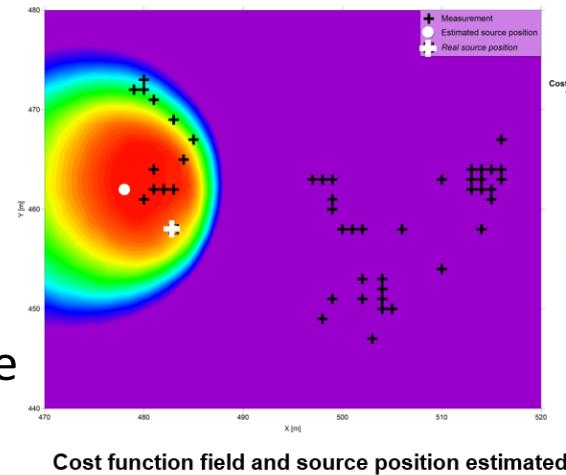
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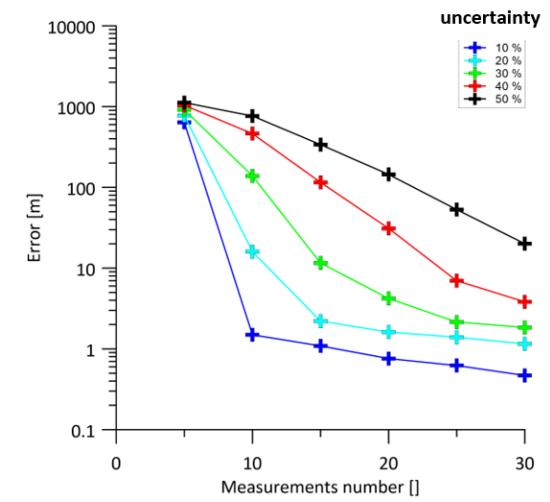
Drone view with gamma camera



Ground robot view with gamma camera



Cost function field and source position estimated



# Thank you !

