Physics of Infrasound Propagation

presented by Roger Waxler

NCPA at UM

March 29, 2022

Acoustic Fluctuations in the Atmosphere

Propagating compression/rarefaction waves

- Coupled fluctuations
  - velocity
  - pressure
  - density
  - temperature
  - entropy

Notation:

- Frequency f in Hz, $\omega = 2\pi f$ in rad/sec
- Sound speed c, 340 m/sec on the ground
- Wavelength $\lambda = \frac{c}{f}$
- Wave number $k = \frac{2\pi}{\lambda} = \frac{\omega}{c}$

Air is compressed and then rebounds

- Propagates outwards at $c \approx 20\sqrt{T}$
- Propagating component has $\lambda \sim \frac{c}{f}$

Other propagating waves

- Buoyancy driven waves
- Planetary waves

A "quiet" atmosphere has

- temperature gradients
- humidity gradients
  - less important for infrasonic frequencies
- prevailing winds
- wind gusts
- turbulence and internal waves

The atmosphere influences the propagation

- temperature gradients refract
- wind shear refracts
- turbulence scatters

Infrasound

Low frequency audible (30 to 1000 Hz).

- Attenuation is significant.
- Sources are often of human or animal origin.

Infrasound: sub-audible frequencies (< 30Hz).

- Attenuation is small $\sim$ (frequency)^2.
- Sources are large or catastrophic.

Working definition: 0.05 Hz to 20 Hz

- between buoyancy waves and audible sound

Wavelengths from $\lambda \sim 7$ km to 17 m

Various Frequency Bands:

Frequency [Hz]

0.001 0.01 0.1 1 10 100 1000

low frequency audible

upper infrasonic

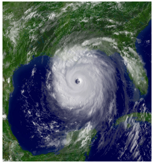
mid infrasonic

deep infrasonic

geo-physical events

environmental / anthropogenic noise

Some sources of infrasound:






Equations of Fluid Mechanics

Air as a gas

- mildly thermally conducting
- mildly viscous
- nearly ideal

Eulerian equations of fluid mechanics:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \qquad \text{(mass conservation)}$$

$$\rho\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla\right)\mathbf{v} + \nabla P + \rho g \hat{\mathbf{z}} = \mu \nabla^2 \mathbf{v} + \nu \nabla \times (\nabla \times \mathbf{v}) \qquad \text{(momentum conservation)}$$

$$P = \rho R T \quad \Longrightarrow \quad \frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dT}{T} \qquad \text{(gas law)}$$

$$dS = c_p \frac{dT}{T} - R \frac{dP}{P} \qquad \text{(3rd law)}$$

$$\rho T\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla\right)S = \kappa \nabla^2 T + \text{viscous terms} \qquad \text{(heat equation)}$$

density ρ, pressure P, temperature T, entropy S, velocity $\mathbf{v}$
bulk and shear viscosities μ and ν, thermal conductivity κ
specific heat c_p, gas constant R, gravitational constant g

ground conditions: $\mathbf{v} = 0$ (lossy; collapses to $\hat{\mathbf{n}} \cdot \mathbf{v} = 0$ in lossless model)

Principle of Solution

Express the state variables as a sum:

- unperturbed (subscript 0) plus perturbed (subscript A)

$$\begin{pmatrix} P \\ \rho \\ T \\ S \\ \mathbf{v} \end{pmatrix} = \begin{pmatrix} P_0 + P_A \\ \rho_0 + \rho_A \\ T_0 + T_A \\ S_0 + S_A \\ \mathbf{v}_0 + \mathbf{v}_A \end{pmatrix}.$$

Determine an appropriate far field approximation for

1) the unperturbed state
2) the system of equations for the perturbation
3) the solution to the system from 2)

Determine an appropriate approximation for the source

Acoustics is sometimes thought of as the linear approximation to fluid mechanics

- Fails for large amplitudes
- Fails for low background density
- Non-linear corrections are frequently required

Linear versus Non-linear Approximations

Formally one can express the equations of fluid mechanics as a functional

$$\begin{aligned}\mathbf{0} &= \mathcal{F}(P, \rho, T, S, \mathbf{v}) = \mathcal{F}(P_0 + P_A, \rho_0 + \rho_A, T_0 + T_A, S_0 + S_A, \mathbf{v}_0 + \mathbf{v}_A) \\ &= \mathcal{F}(P_0, \rho_0, T_0, S_0, \mathbf{v}_0) + \mathcal{F}_A(P_0, \rho_0, T_0, S_0, \mathbf{v}_0; P_A, \rho_A, T_A, S_A, \mathbf{v}_A)\end{aligned}$$

The 0^{th} order terms should satisfy the equations of fluid mechanics

$$\mathbf{0} = \mathcal{F}(P_0, \rho_0, T_0, S_0, \mathbf{v}_0)$$

$\mathcal{F}_A$ is a non-linear mess but has a linear approximation obtained by dropping non-linear terms

$$\mathbf{0} = \mathcal{F}_A(P_0, \rho_0, T_0, S_0, \mathbf{v}_0; P_A, \rho_A, T_A, S_A, \mathbf{v}_A) \approx \mathcal{L}_A(P_0, \rho_0, T_0, S_0, \mathbf{v}_0; \frac{\partial}{\partial t}, \nabla) \begin{pmatrix} P_A \\ \rho_A \\ T_A \\ S_A \\ \mathbf{v}_A \end{pmatrix}$$

Properties Specific to Linear Approximations

In the linear approximation overall amplitude doesn't matter (determined by the source)

$$\mathcal{L}_A \mathcal{A} \begin{pmatrix} P_A \\ \rho_A \\ T_A \\ S_A \\ \mathbf{v}_A \end{pmatrix} = \mathcal{A} \mathcal{L}_A \begin{pmatrix} P_A \\ \rho_A \\ T_A \\ S_A \\ \mathbf{v}_A \end{pmatrix}$$

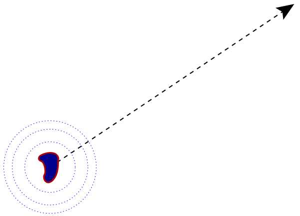
In the linear approximation frequency content doesn't change (principle of superposition)

$$\mathcal{L}_A(\frac{\partial}{\partial t}, \nabla) \int e^{i\omega t} \begin{pmatrix} \hat{P}_A \\ \hat{\rho}_A \\ \hat{T}_A \\ \hat{S}_A \\ \hat{\mathbf{v}}_A \end{pmatrix} d\omega = \int e^{i\omega t} \mathcal{L}_A(i\omega, \nabla) \begin{pmatrix} \hat{P}_A \\ \hat{\rho}_A \\ \hat{T}_A \\ \hat{S}_A \\ \hat{\mathbf{v}}_A \end{pmatrix} d\omega$$

None of this remains true with non-linear terms

- Non-linear equations change with changing amplitude: they don't generally scale homogeneously
- Frequency content is not preserved; how depends on the details

Compact Source in Static Homogeneous Atmosphere



Away from the source region

$$\frac{\partial}{\partial t}\begin{pmatrix}P_0\\ \rho_0\\ T_0\\ S_0\end{pmatrix} = \nabla\begin{pmatrix}P_0\\ \rho_0\\ T_0\\ S_0\end{pmatrix} = 0$$

$$\mathbf{v}_0 = 0$$

Introducing

sound speed $c^2 = \gamma R T_0$

with $\gamma = \frac{c_P}{c_V}$

One has

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)P_A = S(\mathrm{x}, t)$$

$$S = -\sum_{j,k}\frac{\partial^2}{\partial x_j \partial x_k}\left(\rho v_j v_k + (P_A - c^2\rho_A)\delta_{jk}\right) + \text{viscous terms}$$

If $\hat{Q}$ is the Fourier transform of Q then

$$\left(\nabla^2 + \frac{\omega^2}{c^2}\right)\hat{P}_A(\mathrm{x}, \omega) = \hat{S}(\mathrm{x}, \omega)$$

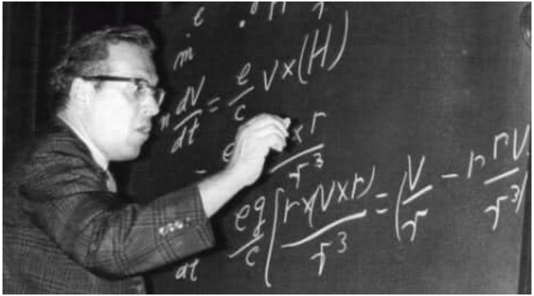
$$\Longrightarrow \hat{P}_A(\mathrm{x}, \omega) = \int \frac{e^{i\frac{\omega}{c}|\mathrm{x}-\mathrm{y}|}}{|\mathrm{x}-\mathrm{y}|}\hat{S}(\mathrm{y}, \omega)\, d^3y$$

$$\longrightarrow \frac{e^{i\frac{\omega}{c}r}}{r}\int e^{i\frac{\omega}{c}\hat{x}\cdot\mathrm{y}}\hat{S}(\mathrm{y}, \omega)\, d^3y = \hat{S}\left(\frac{\omega}{c}\hat{x}, \omega\right)\frac{e^{i\frac{\omega}{c}r}}{r}$$

as $r = |\mathrm{x}| \to \infty$ (due to J. Schwinger).

Outgoing spherical wave; source satisfying the acoustic condition $k = \frac{\omega}{c}$.

Schwinger lecturing on Electrodynamics



Stratified Approximations for the Unperturbed Atmosphere

Assume a vertically stratified medium (subscript H means horizontal):

$$v_{0z} = 0 \quad \text{and} \quad \begin{pmatrix} P_0 \\ \rho_0 \\ T_0 \\ S_0 \\ \mathbf{v}_{0H} \end{pmatrix} \quad \text{depend only on } z.$$

Ignoring heat conduction and viscosity, the equations of motion are satisfied if

$$\frac{dP_0}{dz} = -\rho_0 g \qquad \text{(hydrostatic equ)}$$

$$P_0 = \rho_0 R T_0 \qquad \text{(gas law)}$$

and

$$S_0 - S_0(0) = c_p \ln \frac{T_0}{T_0(0)} - R \ln \frac{P_0}{P_0(0)} \qquad \text{(3rd law)}$$

Note: $\mathbf{v}_{0H}$ and 1 thermodynamic degree of freedom are arbitrary.

Stratified Atmosphere

One generally chooses T_0 as the independent thermodynamic degree of freedom

$$P_0(z) = P_0(0) e^{-g \int_0^z \frac{1}{RT_0}\, dz} \qquad \rho_0(z) = \rho_0(0) \frac{T_0(0)}{T_0(z)} e^{-g \int_0^z \frac{1}{RT_0}\, dz}$$

$$S_0(z) = S_0(0) + c_p \ln \frac{T_0(z)}{T_0(0)} - g \int_0^z \frac{1}{T_0}\, dz$$

Adiabatic case: note that

$$S_0' = c_p \frac{T_0'}{T_0} + g \frac{1}{T_0}$$

so that

$$S_0 = S_0(0) \quad \Longrightarrow \quad T_0(z) = T_0(0) - \frac{g}{c_p} z.$$

$\frac{g}{c_p} \approx 0.01\ ^\circ/\mathrm{m}$ is called the adiabatic lapse rate.

Note: Left on its own, air temperature decreases with altitude.

Infrasound Propagation: Simplest Case Model Equations

Locally stratified approximation

- Deviations from vertical stratification (range dependence) are small on acoustic scales

  $\Leftrightarrow$ $\lambda \frac{\partial}{\partial r}\{c, \rho_0, \mathbf{v}_{0,H}\} \ll \{c, \rho_0, \mathbf{v}_{0,H}\}$

  $\Rightarrow$ Assume vertical stratification and treat range in the atmospheric specifications as a "slow variable"

Treat T_0 (equiv. c) and $\mathbf{v}_{0,H}$ as input to be obtained from the atmospheric sciences

After much algebra one finds the model wave equation in the linear approximation to be

$$\left[\nabla_H^2 + \rho_0 \frac{\partial}{\partial z}\left(\frac{1}{\rho_0}\frac{\partial}{\partial z}\right) - \frac{1}{c^2}\left(\frac{\partial}{\partial t} + \mathbf{v}_{0,H}\cdot\nabla_H\right)^2\right] p_A(\mathrm{x}_H, z, t) = 0\,.$$

Valid when

- Buoyancy is insignificant (nominally frequencies > 0.05)
- Vertical wind shear is small on the scale of a wavelength
- Deviations from stratification (range dependence) are slow on acoustic scales

Note the convective time derivative

Effective Sound Speed Approximation

Consider horizontal plane wave solutions (Fourier transform in horizontal variables)

$$p_{\mathrm{k}_H}(z,t)e^{i\mathrm{k}_H\cdot\mathrm{x}_H} \quad \text{with} \quad \left[\frac{1}{c^2}\left(\frac{\partial}{\partial t}+i\mathbf{v}_0\cdot\mathrm{k}_H\right)^2+\mathrm{k}_H^2-\rho_0\frac{\partial}{\partial z}\frac{1}{\rho_0}\frac{\partial}{\partial z}\right]p_{\mathrm{k}_H}=0.$$

In the frequency domain (temporal Fourier transform) this equation is

$$\left[-\frac{1}{c^2}(\omega-\mathbf{v}_0\cdot\mathrm{k}_H)^2+\mathrm{k}_H^2-\rho_0\frac{\partial}{\partial z}\frac{1}{\rho_0}\frac{\partial}{\partial z}\right]\hat{p}_{\mathrm{k}_H}=0.$$

For shallow angle propagation $k=|\mathrm{k}_H|\sim\frac{\omega}{c}$. For such k_H

$$\frac{\omega}{c}-\frac{\mathbf{v}_0}{c}\cdot\mathrm{k}_H\approx\frac{\omega}{c}\left(1-\frac{\mathbf{v}_0}{c}\cdot\hat{k}_H\right)\approx\frac{\omega}{c+\mathbf{v}_0\cdot\hat{k}_H}$$

Substituting ∇_H for $i\mathrm{k}_H$ results in a Helmholtz-like equation

$$\left[\nabla_H^2+\rho_0\frac{\partial}{\partial z}\frac{1}{\rho_0}\frac{\partial}{\partial z}+\frac{\omega^2}{c_{\mathrm{eff}}^2}\right]\widehat{p}_{\mathrm{k}_H}=0.$$

with

$$c_{\mathrm{eff}}(\hat{k}_H,z)=c(z)+\mathbf{v}_0(z)\cdot\hat{k}_H$$

Attenuation by the Atmosphere: Plane Wave Approximation

without molecular relaxation

$$-i\omega\widehat{\rho}_A + i\rho_0 \mathrm{k}\cdot\widehat{\mathbf{v}}_A = 0$$
$$-i\omega\rho_0\widehat{\mathbf{v}}_A + i\mathrm{k}\widehat{P}_A + \mu k^2\widehat{\mathbf{v}}_A = 0$$
$$-i\omega\rho_0 T_0\widehat{S}_A - \kappa k^2\widehat{T}_A = 0$$
$$\frac{\widehat{T}_A}{T_0} = \frac{\gamma-1}{\gamma}\frac{\widehat{P}_A}{P_0} + \frac{\widehat{S}_A}{c_p}$$
$$\frac{\widehat{\rho}_A}{\rho_0} = \frac{1}{\gamma}\frac{\widehat{P}_A}{P_0} - \frac{\widehat{S}_A}{c_p}$$

$$\Longrightarrow$$

$$-\frac{\omega}{\gamma}\frac{\widehat{P}_A}{P_0} + \omega\frac{\widehat{S}_A}{c_p} + \mathrm{k}\cdot\widehat{\mathbf{v}}_A = 0$$
$$\mathrm{k}\frac{\widehat{P}_A}{\rho_0} - (\omega + i\frac{\mu}{\rho_0}k^2)\widehat{\mathbf{v}}_A = 0$$
$$i\kappa k^2\frac{\gamma-1}{\gamma}\frac{\widehat{P}_A}{P_0} + (i\frac{\kappa}{c_p}k^2 - \rho_0\omega)\widehat{S}_A = 0$$

$$\det\begin{pmatrix} -\frac{\omega}{\gamma P_0} & \mathrm{k} & \frac{\omega}{c_p} \\ \frac{\mathrm{k}}{\rho_0} & -\omega - i\mu\frac{k^2}{\rho_0} & 0 \\ i\kappa\frac{k^2(\gamma-1)}{\gamma P_0} & 0 & -\rho_0\omega + i\kappa\frac{k^2}{c_p}\end{pmatrix} = 0$$

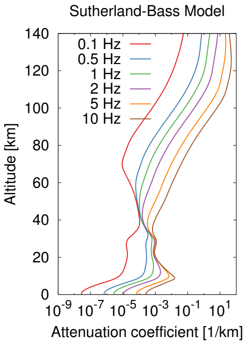
$$k = \frac{\omega}{c} + i\frac{1}{2\rho_0}\frac{\omega^2}{c^2}\Big(\mu + \kappa(\frac{1}{c_v} - \frac{1}{c_p})\Big) = \frac{\omega}{c} + i\alpha$$

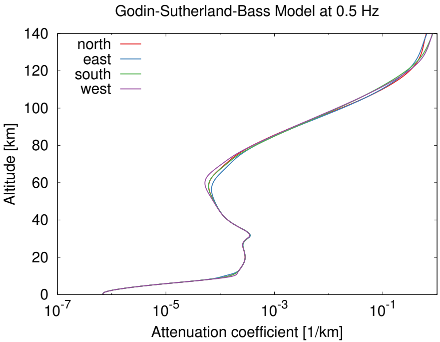
for the acoustic mode to leading order. Kludge: replace $\frac{\omega}{c}$ by $\frac{\omega}{c} + i\alpha$

Atmospheric Attenuation in a Windy Atmosphere

Godin's extension (2015)

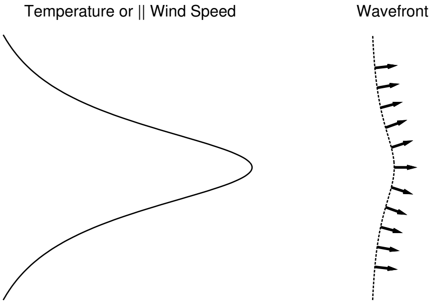
- Godin considers lossy fluid mechanics in the geometric limit
- obtains Sutherland-Bass with Doppler shifted frequency $\omega \mapsto \omega - \mathrm{k} \cdot \mathbf{v}_0$





Increases with frequency $\sim f^2$ and with altitude $\sim e^{\text{const } z}$

Atmospheric Refraction by Temperature and Wind



Sound speed

- $c(z) = \sqrt{(\gamma R)T}$

Sound is refracted by

- temperature gradients
- along-path wind shear
  - due to advection

Cross winds

- out-of-plane advection
- changes apparent bearing

Ducts form between the ground and regions of higher temperature/wind

- these provide channels for efficient long range propagation
- dependence on wind speed makes propagation azimuthally asymmetric

Signals return to the ground from altitudes with $c(z) \geq c(0)$

- approximately (true in the high frequency approximation)

Effective sound speed approximation:

- $c_{\mathrm{eff}} = c + u$
  - u = horizontal wind in the direction of propagation
- Valid for u and elevation angles not very large

Propagation Models

Hierarchies of approximation

- Geometrical versus full wave
  - Geometrical acoustics is a formal expansion in powers of λ
  - Does not capture diffractive effects without modification
- Planar versus 3-d
  - Planar ignores the influence of cross winds
  - Does not capture azimuth deviation and out of plane propagation
- Stratified versus range-dependent
  - Applicability depends on circumstances
- Effective sound speed versus high angle/Mach number
  - Eff. c is typically reasonable for tropospheric propagation
  - Fails for high stratospheric winds and superstratospheric propagation
- Linear versus non-linear

There are a variety of propagation models of varying complexity

- Geometrical acoustics models
- (Bi-)normal mode models
- P. E. models
- Finite Difference Models

Specifications of T_0 and $\mathbf{v}_{0H}$: general comments

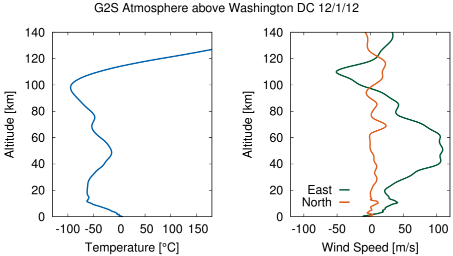
Direct measurements:
- Tethersonde: up to 3 km, FAA restriction to 150 m
- Aircraft data collection: up to $\sim 10km$ Radiosonde: up to 35 to 40 km
- Rocketsonde: up to 90+ km

Remote sensing of wind speed:
- Sodar: acoustic back scatter Doppler shift, from 50 m to 400 m
- Lidar: optical back scatter Doppler shift, up to 100 km
- Lidar: optical emission Doppler shift

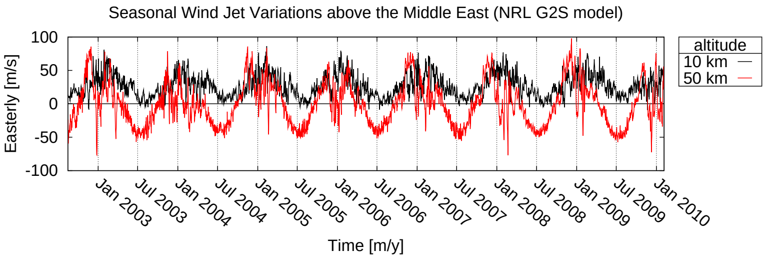
Remote sensing of temperature:
- Optical emission based devices

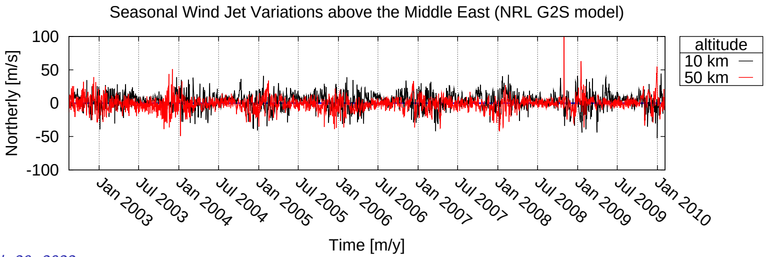
Models and data interpolation:
- WRF, GFS: up to 12 km
- G2S, ECMWF: up to $\sim$ 70 km

Quasi-empirical static model:
- HWM: up to 160 km



Climatology of the Most Significant Wind Jets





Specifications of T_0 and $\mathbf{v}_{0H}$

Climatology of the Northern Hemisphere

Summer environment

- Stable west-flowing stratospheric jet (circumpolar vortex)
- Weak east-flowing jet stream

Winter environment

- Unstable east-flowing stratospheric jet (circumpolar vortex)
  - broken by Sudden Stratospheric Warming (SSW) events
- Strong east-flowing jet stream

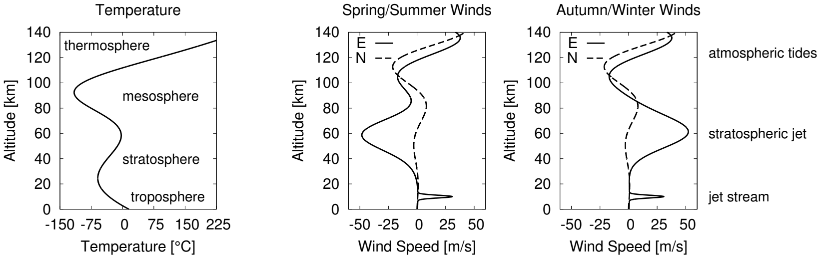
Equinoxes

- several weeks in spring and fall when the circumpolar vortex turns

The southern hemisphere is similar

- The seasons are reversed
- The southern polar vortex is relatively stable

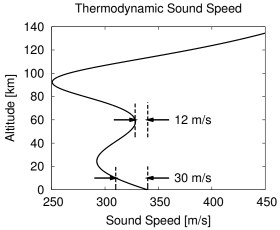
The Qualitative Atmospheric State



The atmosphere tends to be layered

- Temperature layers
  - solar heating/cooling of ground and upper atmosphere
  - chemistry of the ozone layer
- Wind jets generated by global horizontal temperature gradients and the Coriolis force
  - Seasonal, diurnal variations and variations with latitude

The Overriding Significance of the Wind



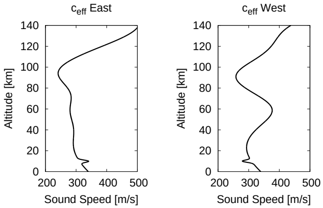
The temperature is insufficient to produce stratospheric ground returns

- exceptions can be found at extreme latitudes
- stratospheric ducting can often be borderline

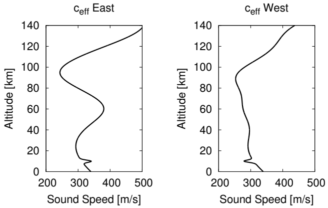
The stratospheric jet is usually sufficient

The jet stream must compensate the steep temperature drop

Spring/Summer



Autumn/Winter

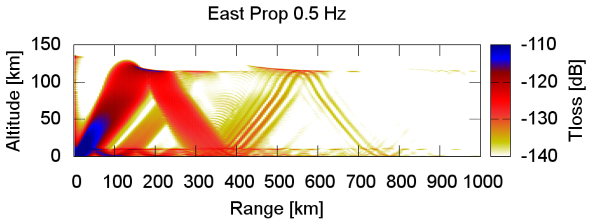


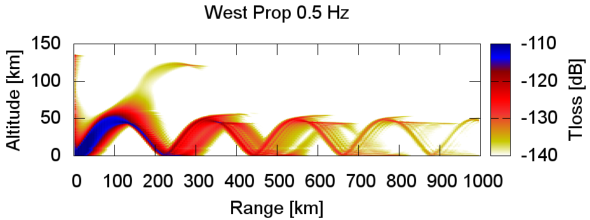
Typical ground return paths:

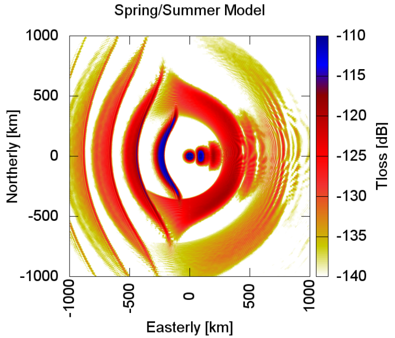
- Tropospheric
  - low altitude wind jets
  - typically easterly
- Stratospheric
  - circumpolar vortex
  - seasonal
- Thermospheric
  - temperature driven
  - lossy, non-linear

Typical Propagation Paths: Spring/Summer model

no attenuation

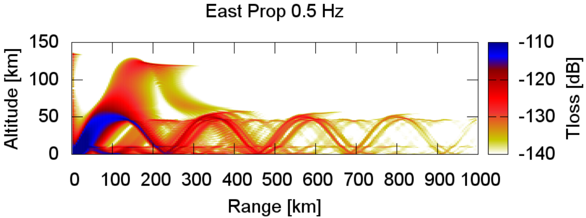


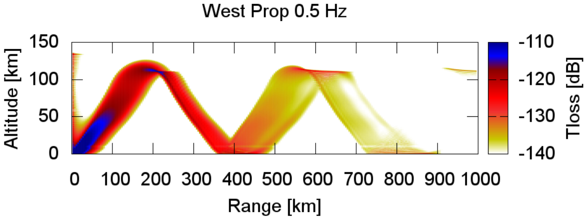


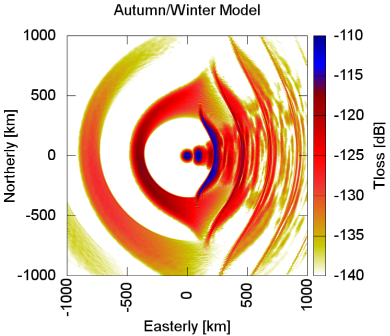


Typical Propagation Paths: Autumn/Winter model

no attenuation







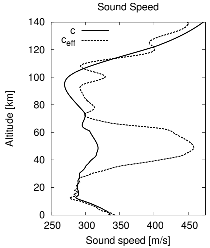
The Buncefield Event: a Rare Clean Example

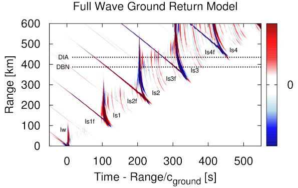
Fuel vapor cloud explosion

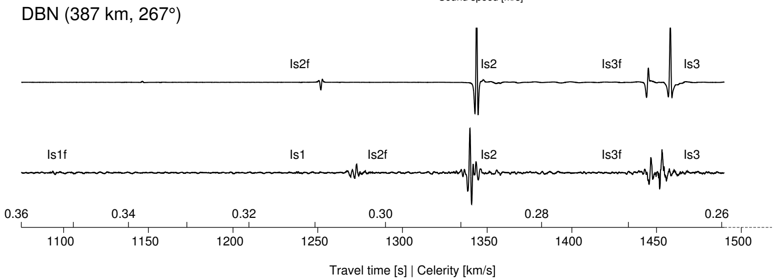
- southern UK on December 11 2005
- large explosive event (no injuries)
- nearly ideal propagation conditions
- detected across Europe

Propagation models worked well

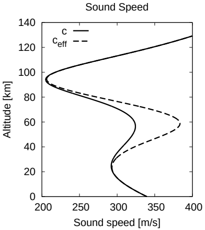
Arrival pairs were predicted

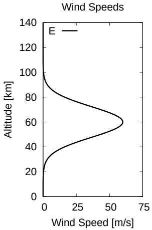


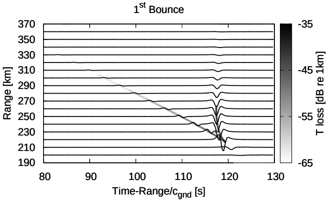


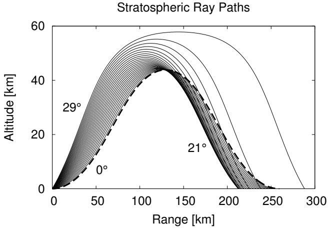


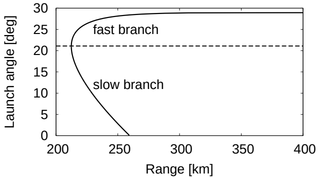
The Stratospheric Pair: 1^{st} Bounce

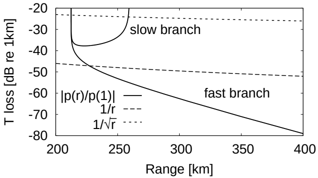




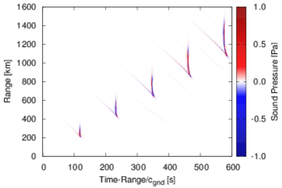


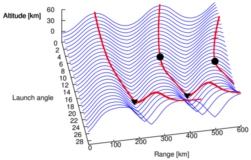


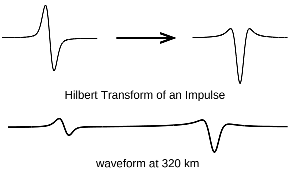




The Stratospheric Pair: n^{th} Bounce and Waveforms



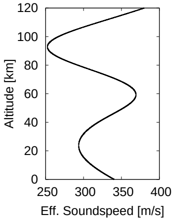


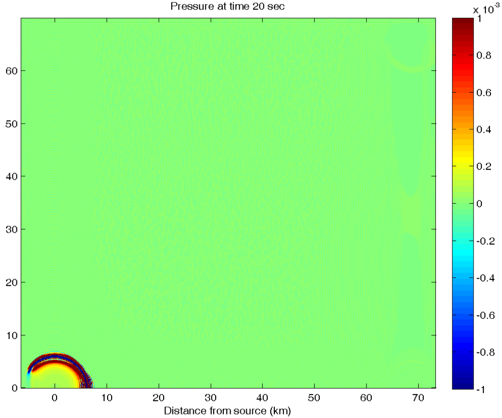


Geometrical acoustics: waveform dispersion only through caustic encounters

- a simple caustic passage changes the phase by $\pi/2$, *ie* by a Hilbert transform
- a double caustic passage changes the phase by π, *ie* by -1
- visible in the Buncefield data
- visible in the Tonga data! (Caustic at the antipodal point)

Stratospheric Duct Alone



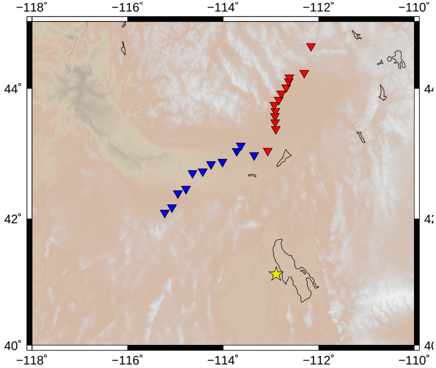


A Less Ideal Stratospheric Duct

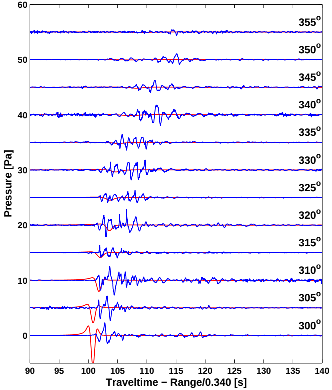
UTTR rocket motor fuel elimination events Aug 2010

A network of arrays deployed in an arc to test the stratospheric models
Model results bear little resemblence to the data

- Assumption: the available atmospheric specifications lack fine structure



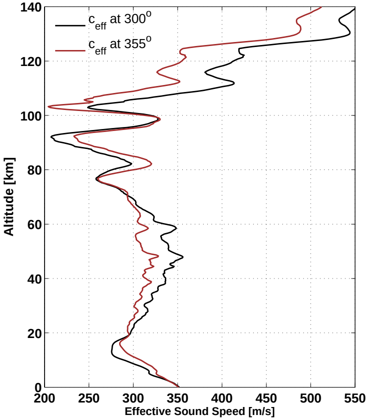
Observations vs G2S Model Predictions

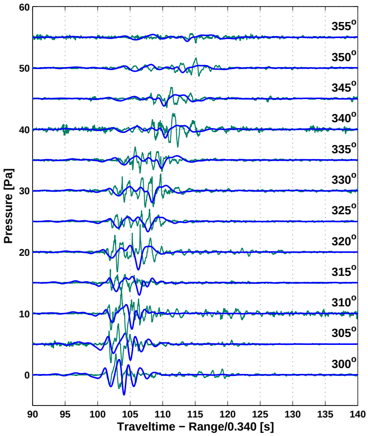


Gravity Wave Perturbations: A UTTR Example

"First Principles" statistical gravity wave model: qualitative improvement

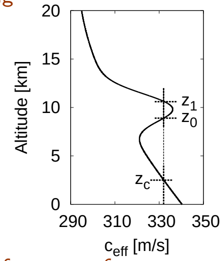
- wrestling with the large parameter space
- a 3d model seems necessary





Tropospheric Ducts

Factors controlling tropospheric ducting



Wind speed versus temperature drop

- c at 10 km is $\approx$ 35 m/s less than c on the ground

Wavelength versus wind jet thickness

- long wavelength components penetrate the jet

Wavelength versus upward refraction

- short wavelength components refract away from the ground

WKB tunnelling factor $e^{-f/F(c)}$, phase speed c, frequency f

Ducting by elevated wind jet

- reduced by tunnelling: high pass filter
- lower phase speeds, more ducting

$$\frac{1}{F(c)} = 2\pi \int_{z_0}^{z_1} \sqrt{\frac{1}{c^2} - \frac{1}{c_{eff}(z)^2}}\, dz$$

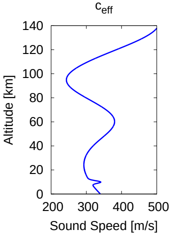
Ground contact

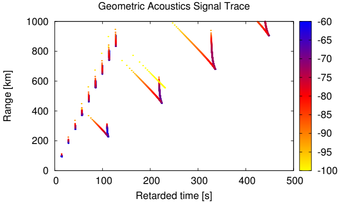
- enhanced by tunnelling: low pass filter
- higher phase speeds, better contact

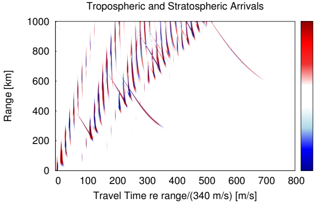
$$\frac{1}{F(c)} = 2\pi \int_{0}^{z_c} \sqrt{\frac{1}{c^2} - \frac{1}{c_{eff}(z)^2}}\, dz$$

Borderline Ducts are Band Pass Filters in both Frequency and Wavenumber.

Stratospheric Arrivals with a Strong Jet Stream
Tropospheric Ducts are Leaky at Infrasonic Frequencies







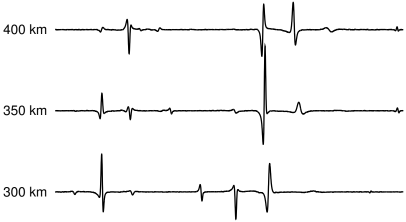
Tropospheric train

- fills the classical shadow
- reverberant signals leak
- geometrical acoustics fails

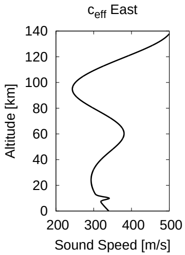
Stratospheric train

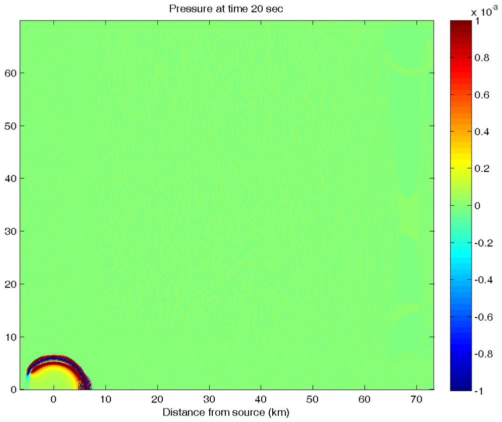
- “slow” arrival with echoes
- “fast” arrival without echoes

Thermospheric late arrival



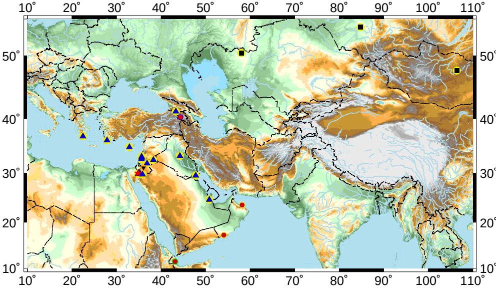
Stratospheric and Tropospheric Duct

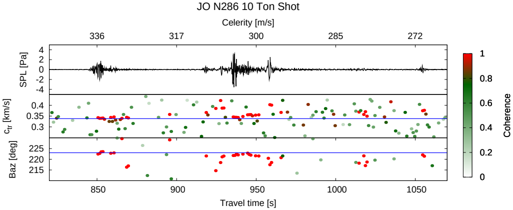


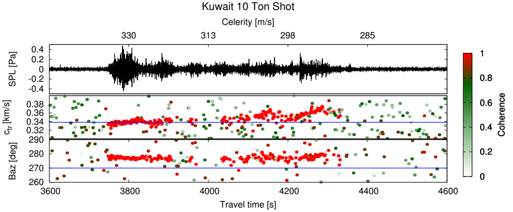


The Sayarim 2011 Ground Truth Experiment

environment with a strong jet stream

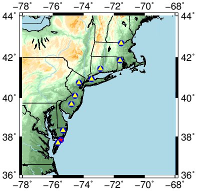


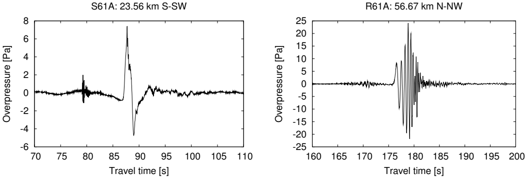


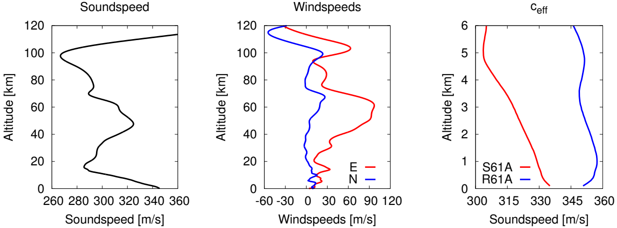


Fee, Waxler, Assink, Gitterman,
Given, Coyne, Mialle, Garces, Drob, Kleinert,
J. Geophys. Res. 118 (2013)

Wallops Island Rocket Explosion Oct 2015







Low altitude duct to the NE

- Nocturnal temperature inversion
- solid jet stream
- low atitude wind jet

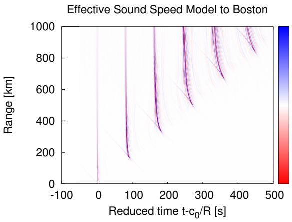
Upward refraction to the south

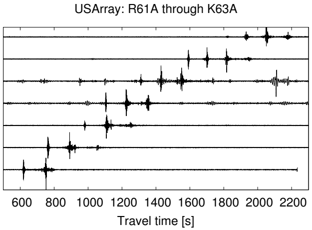
Solid stratospheric duct

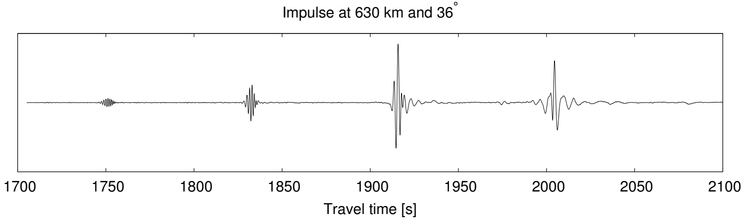
- stratospheric jet E-NE

Wallops Island Antares Rocket Explosion Oct 2015

propagation model output comparison

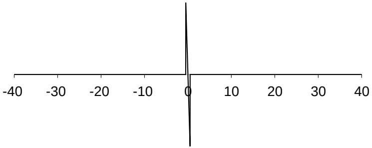




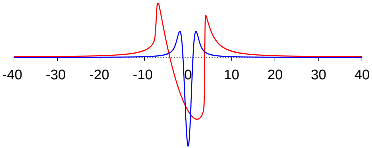


Thermospheric Paths

Initial Waveform



Waveform Comparison at ∼300km
(linear versus non-linear)



Lonzaga, Waxler, Assink, and Talmadge, Geophys. J. Int. 200 (2015)

Very low density

- propagation is very nonlinear
- atmosphere is very attenuating

Nonlinearity

- causes transients to stretch
  - generating low frequencies
- causes wave fronts to steepen
  - generating high frequencies

Attenuation

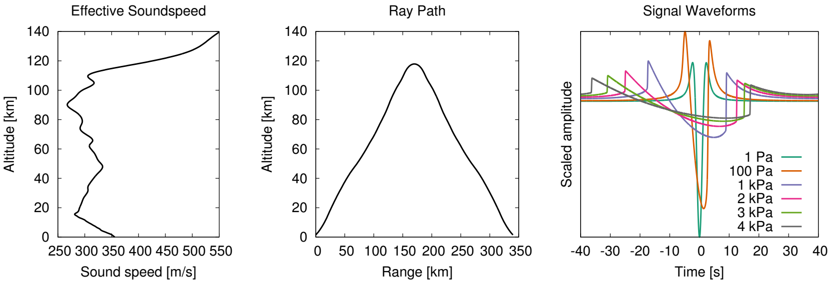
- mollifies shocking
- reduces fine structure
  - in atmosphere
  - in waveform

Period lengthening vs attenuation

- longer periods survive attenuation
- observed signals are low frequency

Modeling Thermospheric Phases

non-linear ray theory



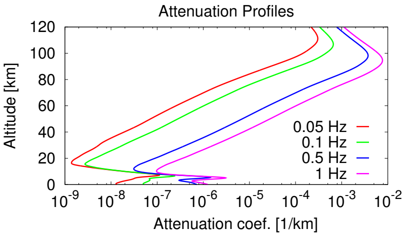
Non-linear ray theory (Robinson, Gainville, Coulouvrat, ...) $P = Ae^{i\phi} + \ldots$

- ϕ is given by the linear eikonal equation
  - provides propagation paths and travel times
- A is given by the 2$^{\text{nd}}$ order non-linear transport equation
  - transport equation is integrated along the propagation path
  - provides signal amplitudes
- Attenuation handled in a split-step approach
  - Sutherland-Bass model is used here

Attenuation vs Non-linear Distortion

Attenuation of sound by the atmosphere

- increases with increasing frequency
- increases with decreasing density
  - ⇒ increases with increasing altitude

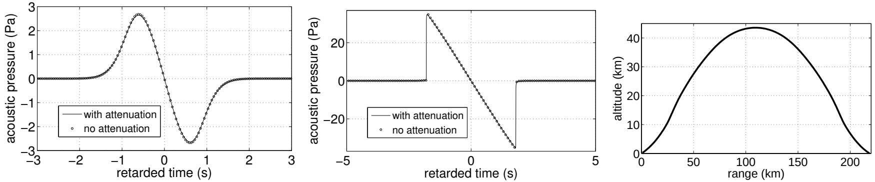


Non-linear distortion of infrasound signals

- is inversely proportional to density
  - non-linear distortion increases dramatically with altitude
- there is an interplay between attenuation and non-linearity
  - harmonic generation ⇔ wave steepening moderated by attenuation
  - attenuation is moderated by period lengthening ⇔ low frequency generation

Attenuation: Stratospheric versus Thermospheric Returns

Stratospheric Returns: 100 Pa @ 1km vs 1500 Pa @ 1km



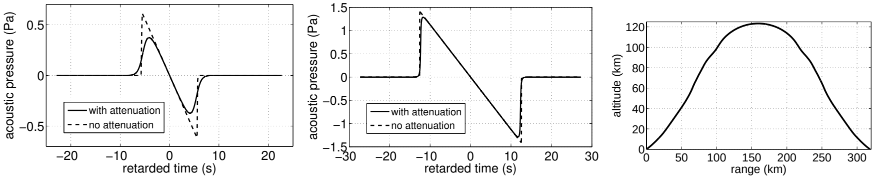
Difficulties with the thermospheric model

- linear propagation models predict no returns but these are regularly observed

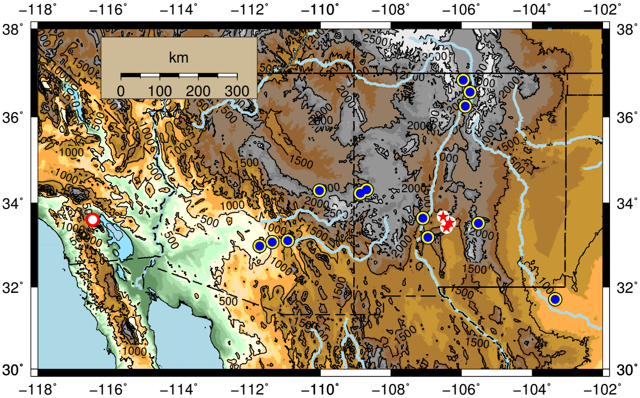
Possible explanation: non-linear propagation

- severe low frequency generation in the thermosphere

Thermospheric Returns: 100 Pa @ 1km vs 500 Pa @ 1km



The Humming Roadrunner Tests



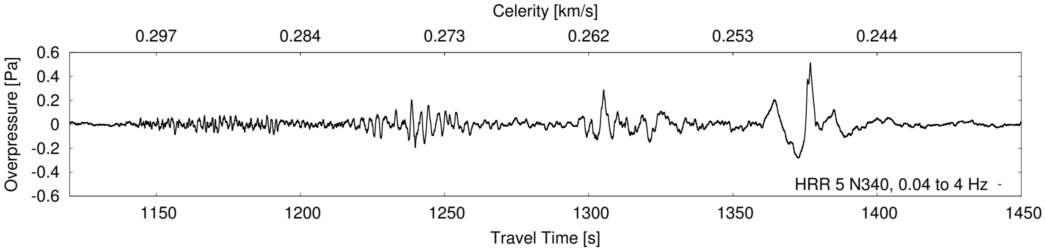
6 large chemical explosions in the western US

- from 10 to 50 tons TNT equivalent

Extensive near field source capture

Far field network deployed to study long range propagation

Humming Roadrunner Example Waveform



Four signal phases observed (from celerity; only the thermospheric is predicted)

- two stratospheric
- one mesospheric
- one thermospheric

Frequency content steadily decreases

- from $\sim$ 0.5 Hz in the stratospheric phases to $\sim$ 0.05 Hz in the thermospheric

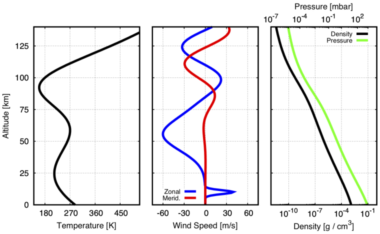
Waveform complexity steadily decreases

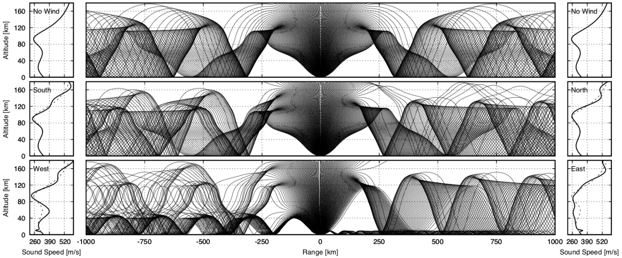
- from highly dispersed stratospheric phases to a simple thermospheric U-phase

Thermospheric Return Paradigms

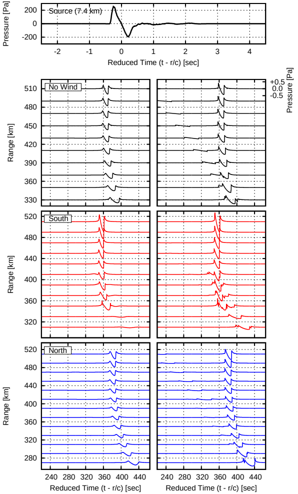
Is there any systematic behavior to expect similar to the stratospheric pair?

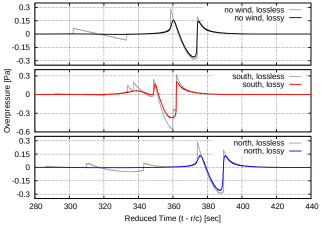
- Naively one expects either a U or an N
- One observes U's, N's, and what appear to be hybrids (see, eg., Vergoz et al on the Antares Rocket explosion)
- The atmospheric tides are responsible for the variability





Thermospheric Return Paradigms





Profile as in last slide:

No wind

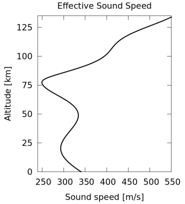
- a pair is predicted
- the fast arrival attenuates

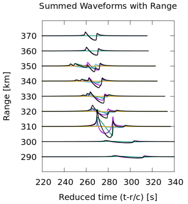
With the wind

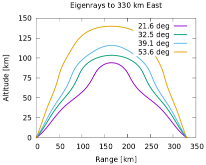
- more complex structure
- also largely attenuated
- a U is predicted

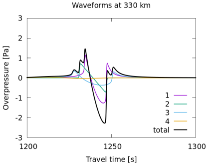
It is common to find coincident arrivals

- leads to observed complex structure



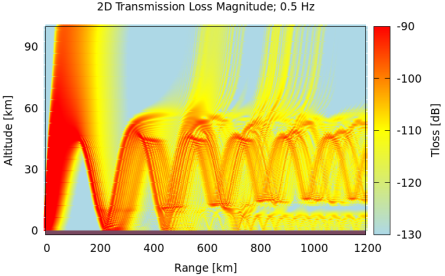


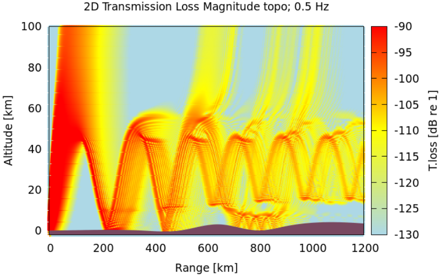




Not Discussed: range dependence and topography

Plots from the NCPAprop manual





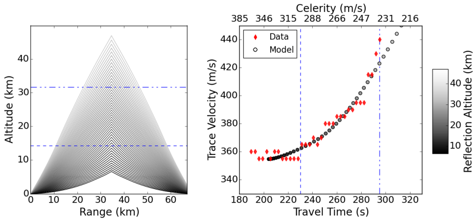
Jet stream (or stratospheric jet) changes in direction along propagation path

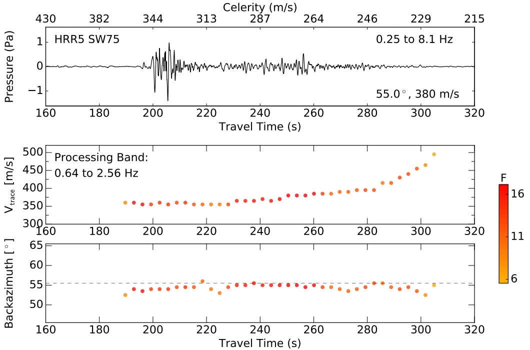
Mountain generated winds (Föhn effect)

Topography is intrinsically range dependent

Not Discussed: backscatter off of fine structure (Kulichkov and Chunchuzov)

Data analysis from HRR (David Green et al)





More generally: the inadequacy of atmospheric specification