ENSTA – MF 208

Aéroacoustique et propagation en écoulement

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http://acoustique.ec-lyon.fr

I – Introduction : quelques applications et enjeux, organisation du cours, bibliographie

First jet flight with fare-paying passengers

- **De Havilland DH 106 Comet 1 G-ALYP**
  2 May 1952 leaving London to Johannesburg

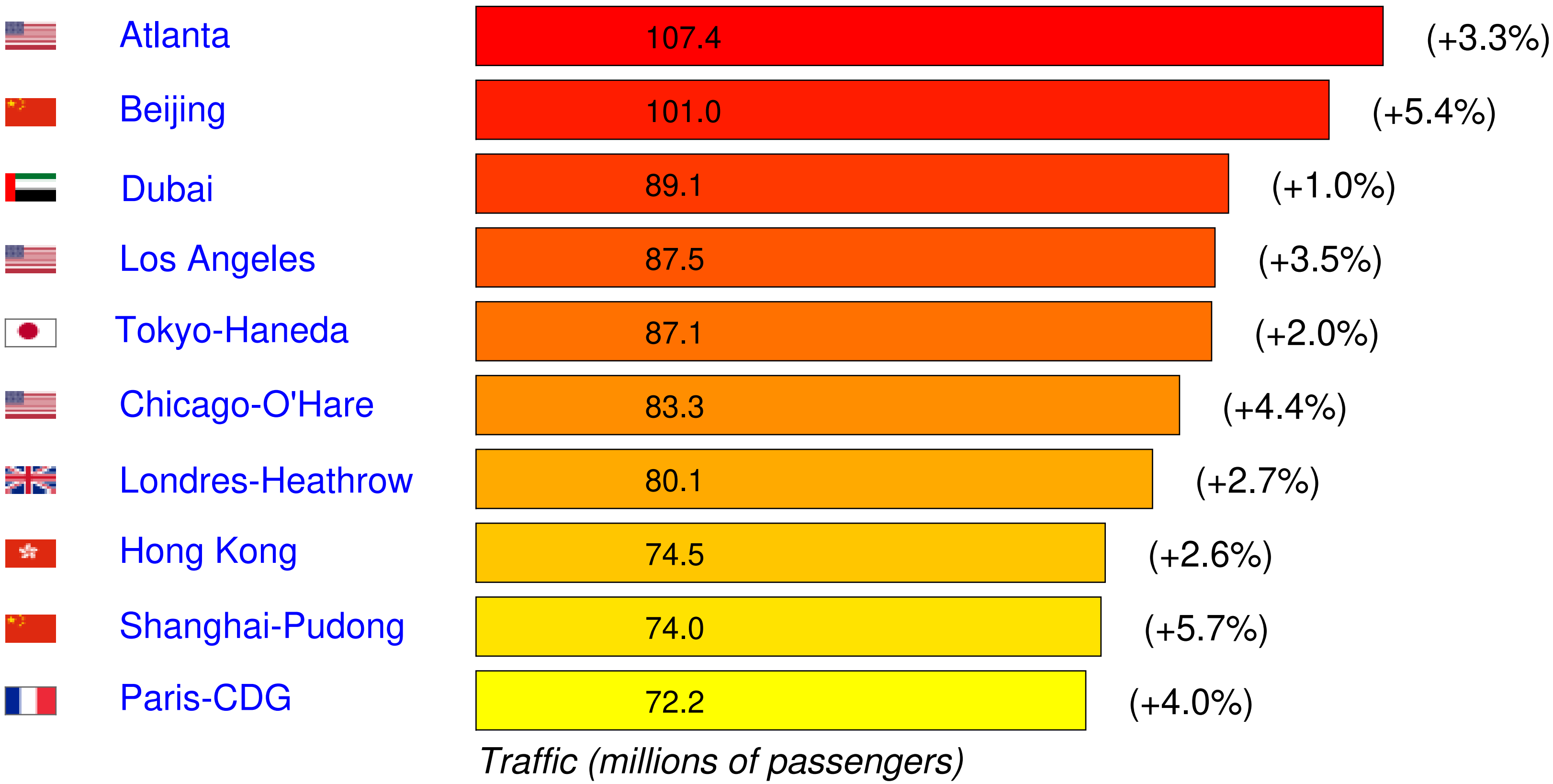
Heathrow – British Overseas Airways Corporation (BOAC)
http://www.bamuseum.com/

- **By way of Rome, Beirut, Khartoum, Entebbe & Livingston**

5 stops, 36 passengers, return fare 315 £, 6663-mile journey of 23 hours 33′ (about 33 hours with a piston-engine)

BOAC - Handley Page Hermes 4 G-ALDI

- **2018 Top 10 world airports** (Airports Council International)



Long-term traffic forecasts predict that the number of passengers will double in the next 15 years

AirTraffic Worldwide 2008
http://radar.zhaw.ch/

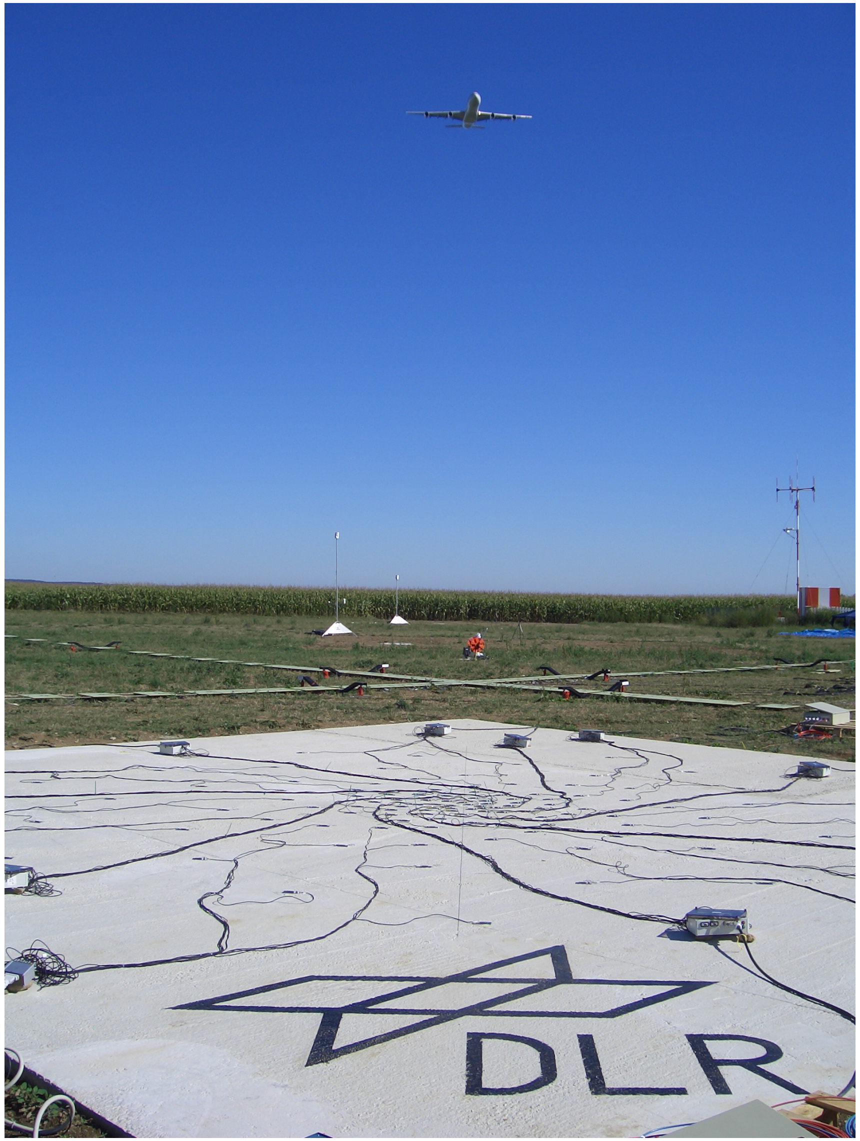
- **Roissy-CDG versus Orly**

1974, 23 km from Paris

$\sim$ 1952 for civil, 14 km from Paris

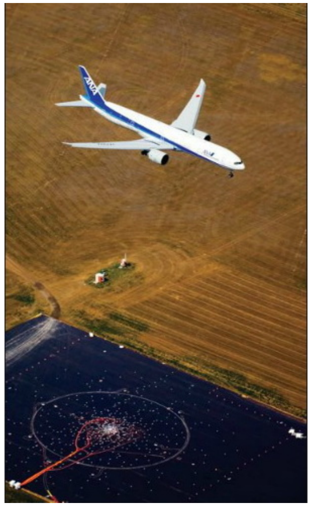
- Aircraft noise is a major inhibitor of the growth of air transport (airports in key locations are operating at full capacity)
- Traffic growth must at least be compensated for by improving aircraft noise emission at source or the way they are operated

Noise source localization by microphone array



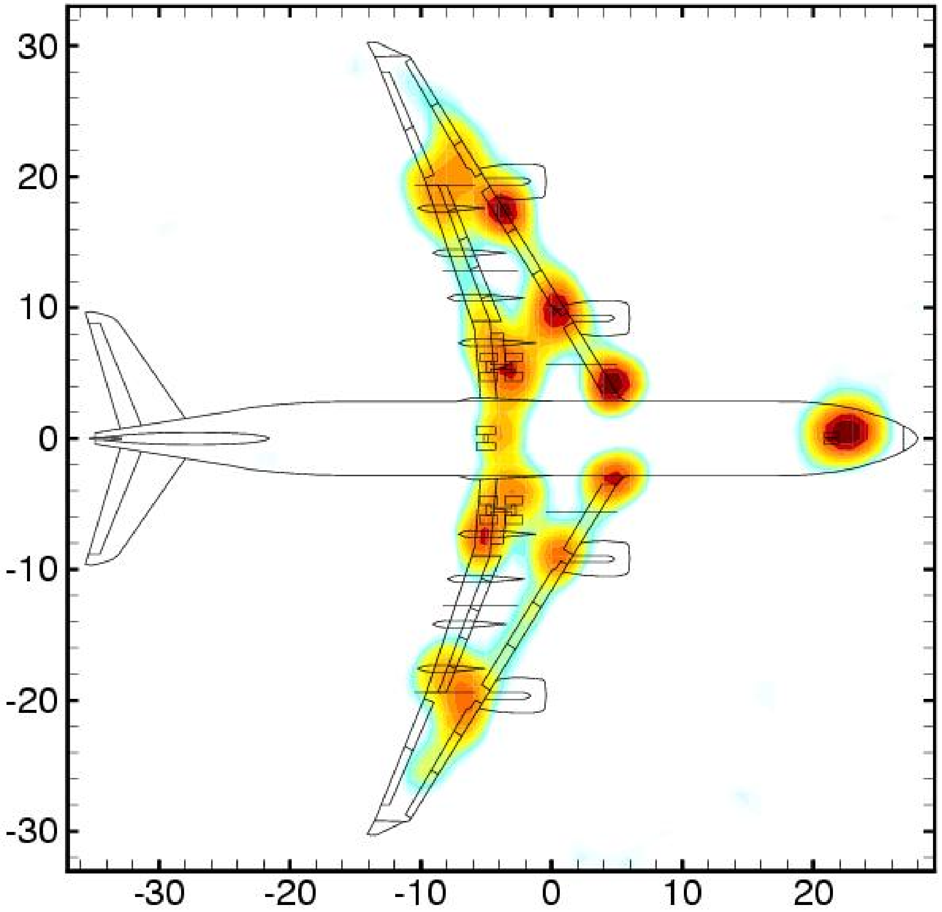
A340 flight tests Cross-shaped array for lower frequency range (0.2 to 5 kHz, size 32 m, ONERA) and multi-arm spiral array for upper frequency range (2 to 8 kHz, size 4 m, DLR), 196 microphones.

Courtesy of Henri Siller, DLR, Berlin.
See AIAA Papers 2005-3007, 2005-2981 & 2006-2463



A flyover noise test of Boeing's 787 aircraft used a ground aray consisting of 614 microphones

Noise source localization by microphone array



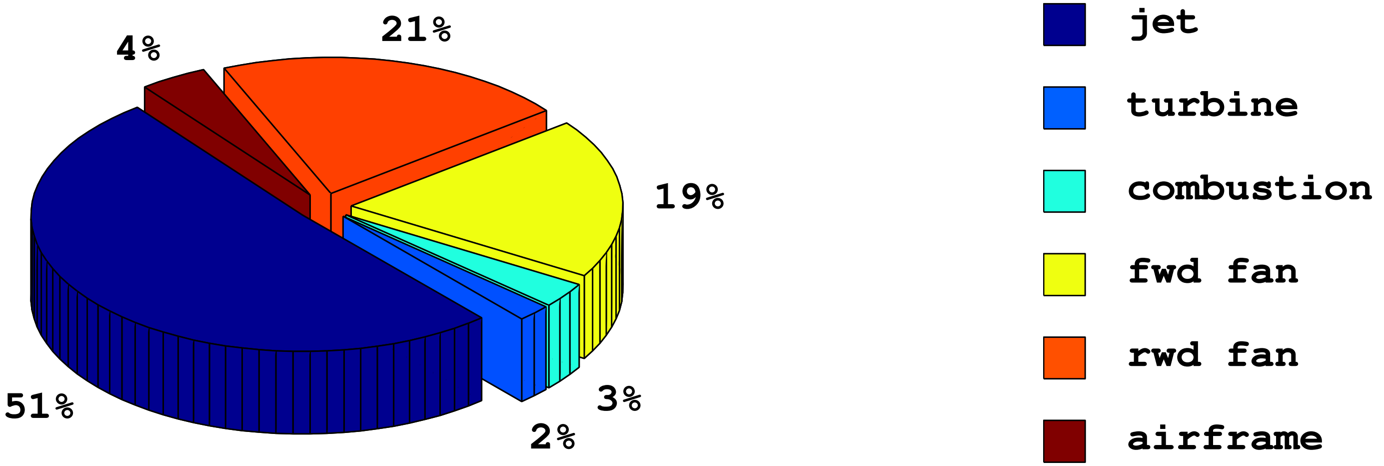
A340 flight tests

Landing approach configuration - noise sources at 5 kHz one-third octave band for an emission angle of 90 deg

$\rightsquigarrow$ landing gears, slat horns and also high-lift devices

- **Jet noise during take-off** remains the major component of total aircraft community noise (between a third and half of the energy)

Typical noise source breakdown

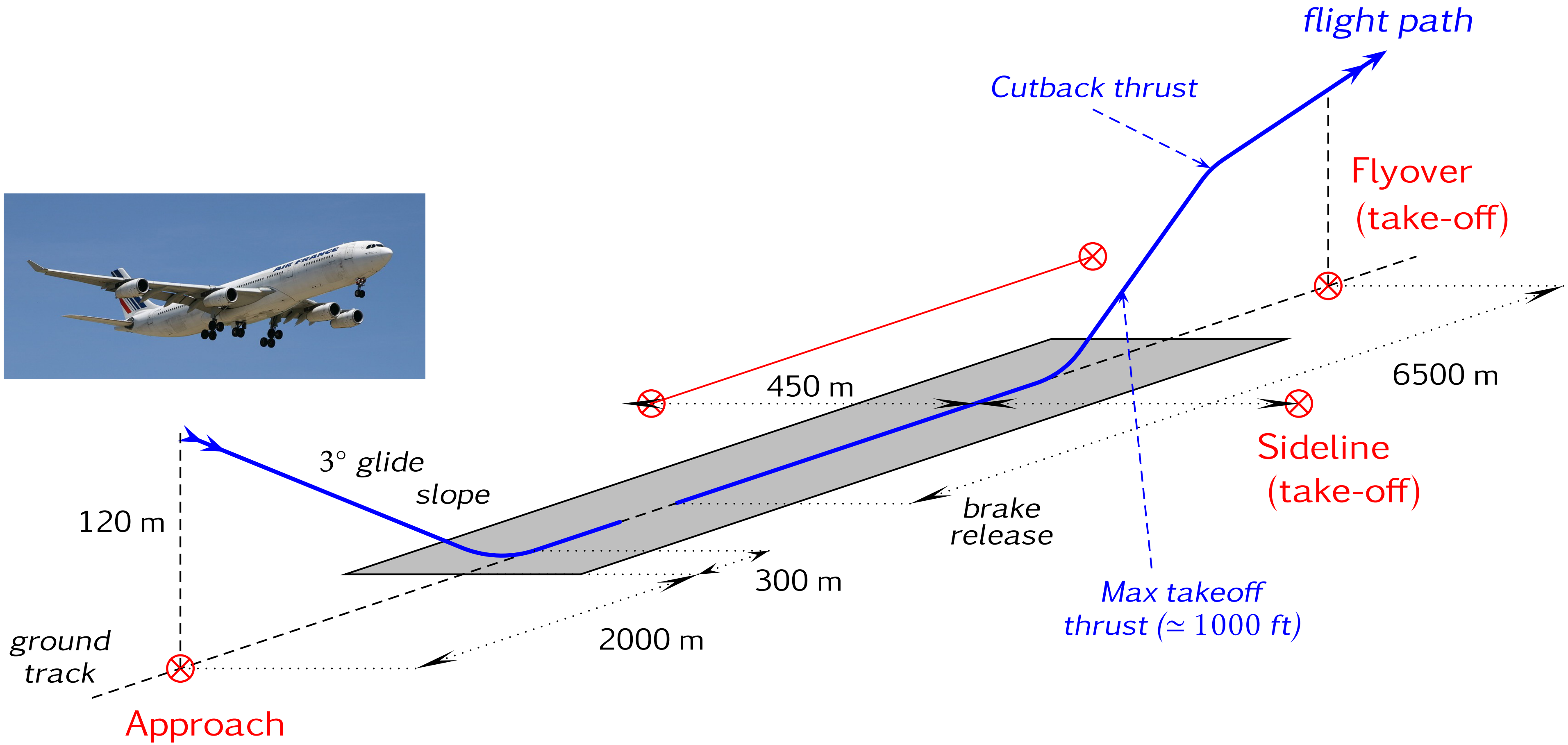


typical long range aircraft with 4 current engines at the sideline point

(only certification point in which engines always operating at full power)

Huber & Illa, AIAA Paper 2007-3728

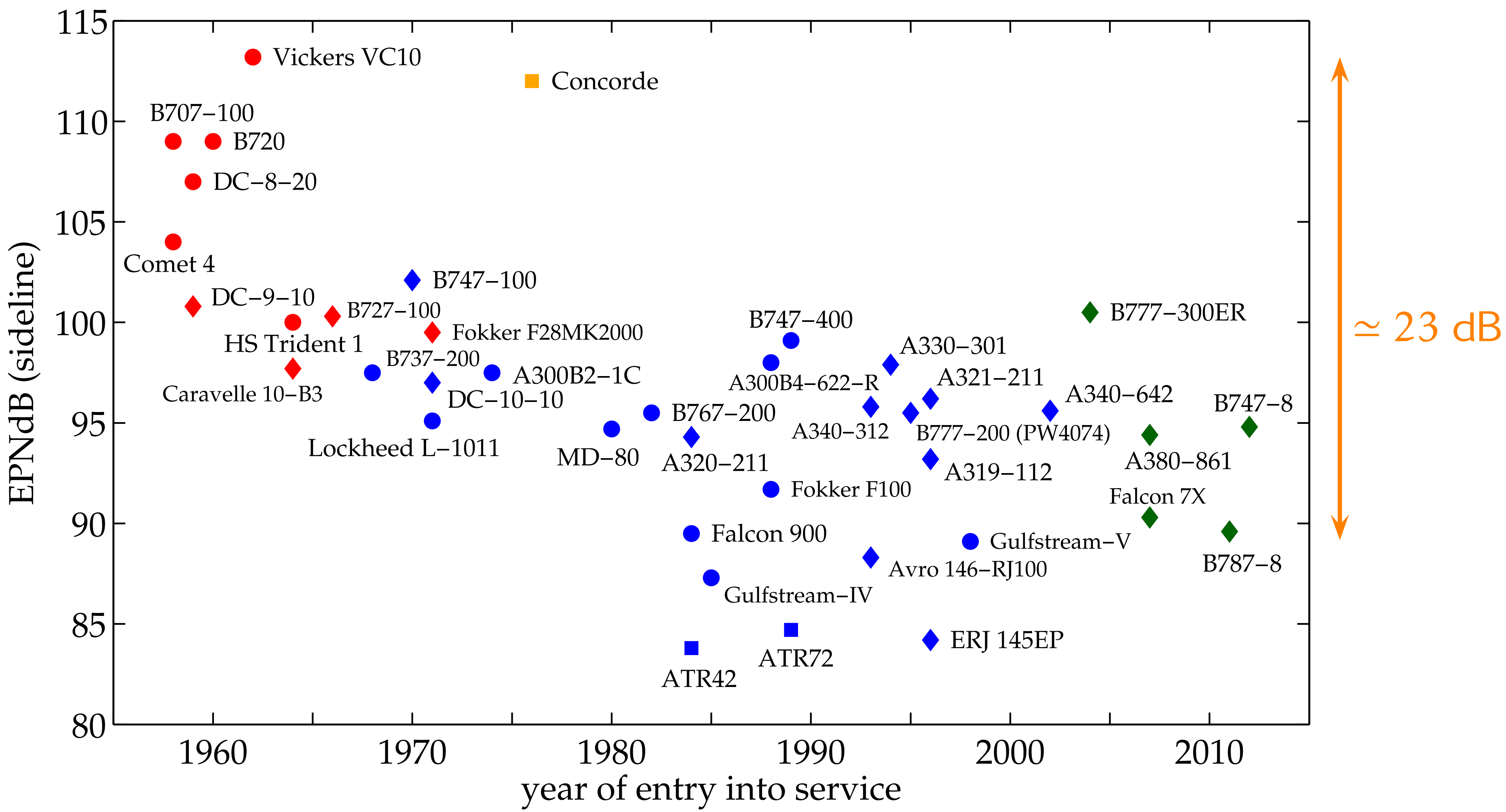
- **Three certification points**



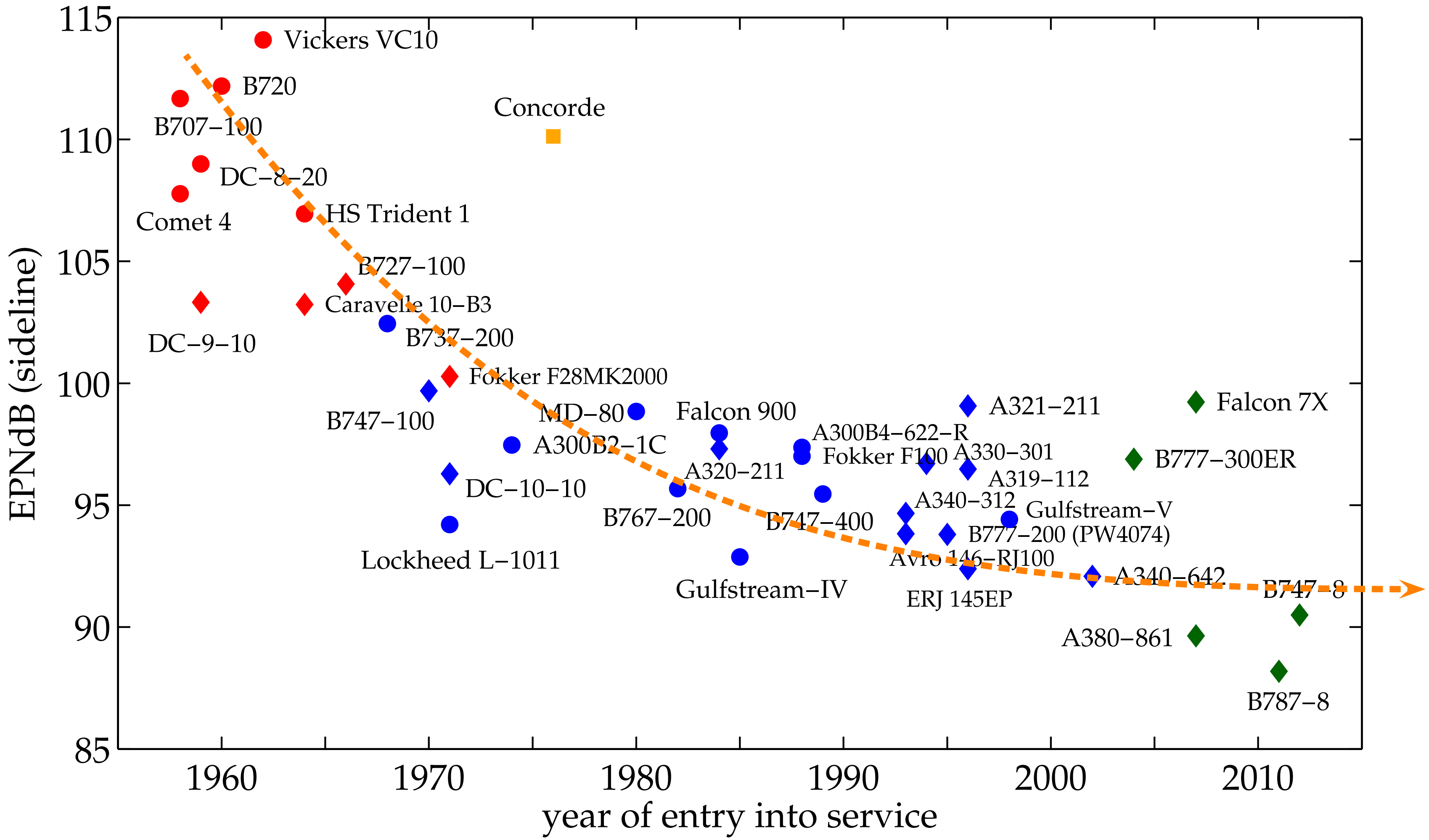
sideline point : the only point in which the engines are always operating at full power

Sideline noise level

(Federal Aviation Administration FAA-AC-36-1H & FAA-AC-36-2H)



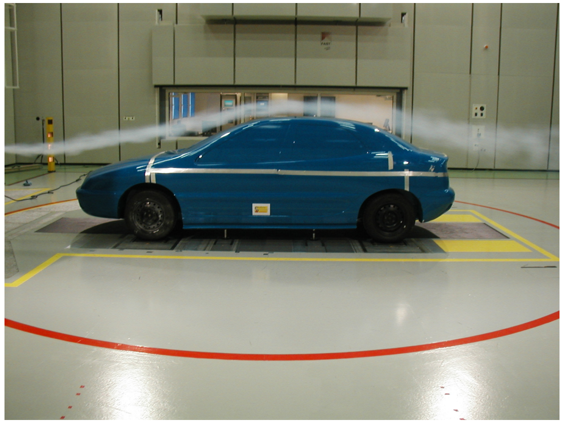
- **Sideline noise level** (normalized by thrust, $T = 10^5$ lbs)
  (Federal Aviation Administration FAA-AC-36-1H & FAA-AC-36-2H)



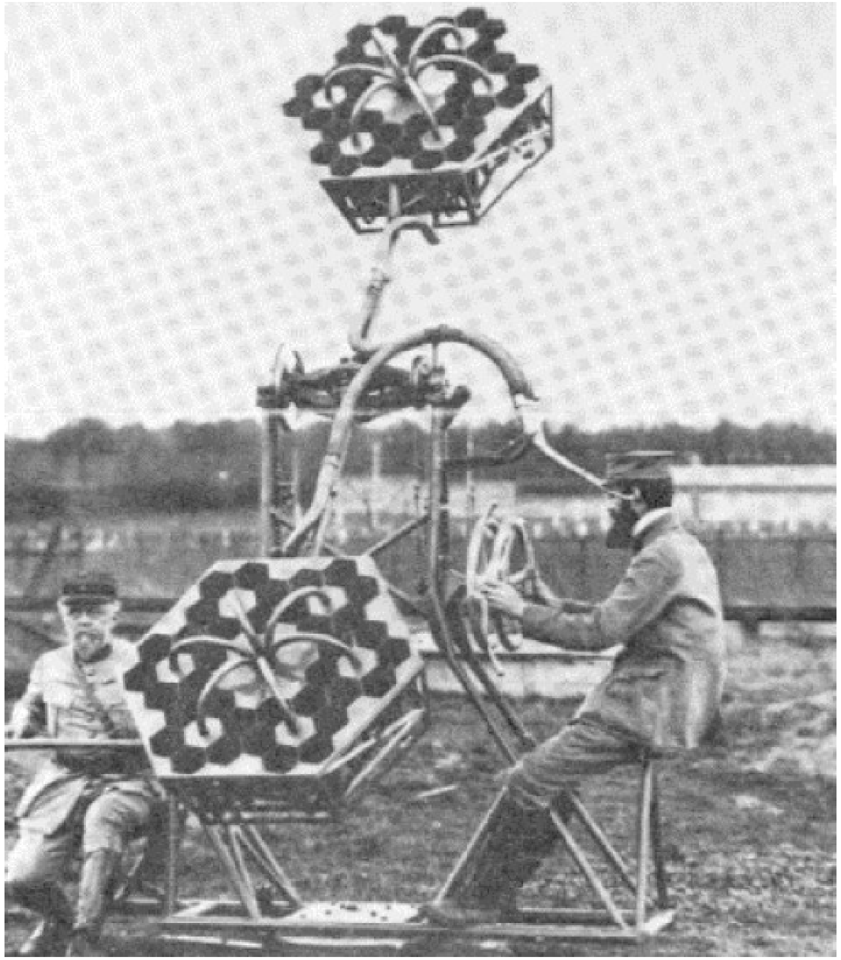
Urban noise, comfort & perception

Characterization of tramway noise using a microphone antenna (IFSTTAR/LAE)

Michelin & S2A wind tunnel



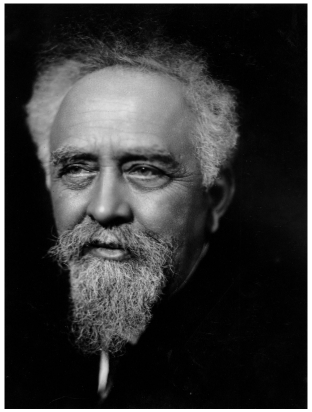
Detection and localization



Johnson & Dudgeon, *Array Signal Processing* (1993)

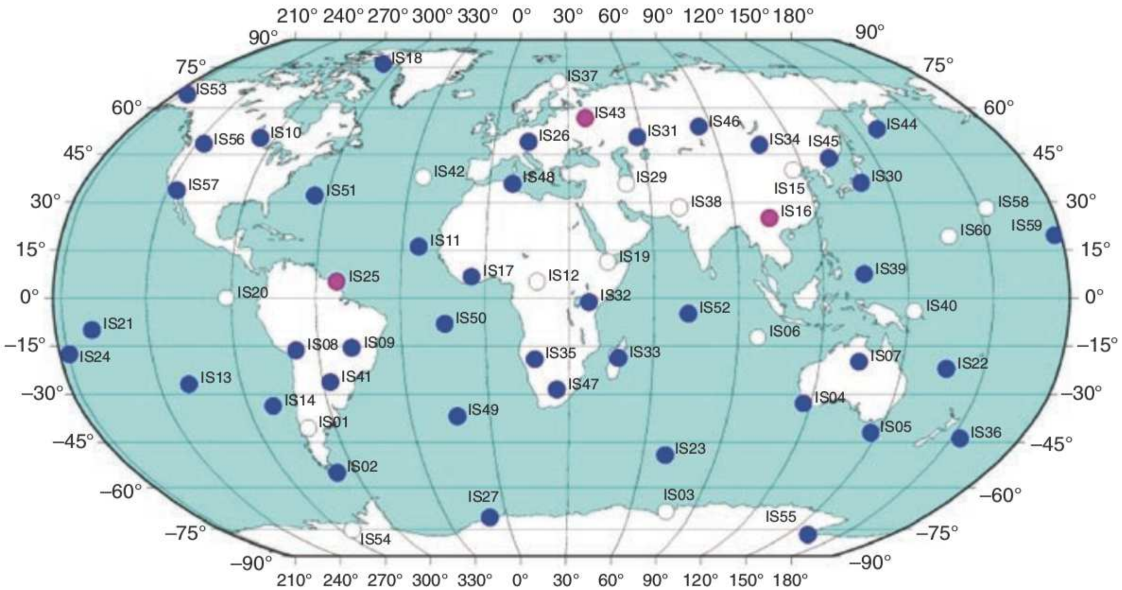
Acoustic array used by the French in World War I to detect enemy aircraft

developed by the Sergeant Jean Perrin (right), who received the Nobel Prize in Physics (1926)



(1870-1942)

- **Worldwide infrasound monitoring network** developed to verify compliance with the Comprehensive Nuclear-Test-Ban Treaty (CTBT)



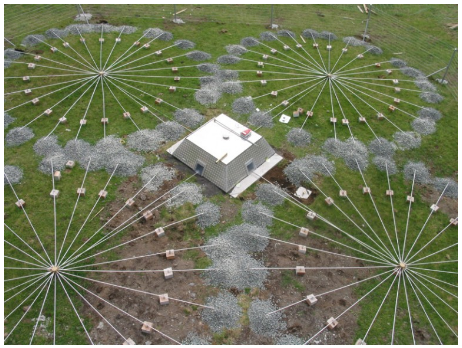
Headquarter : Vienna, Austria

60 stations with 4 to 8 micro-barometers over an area of 1 - 9 km^2

• Certified and sending data to the International Data Centre (IDC)
• under construction, ∘ planned

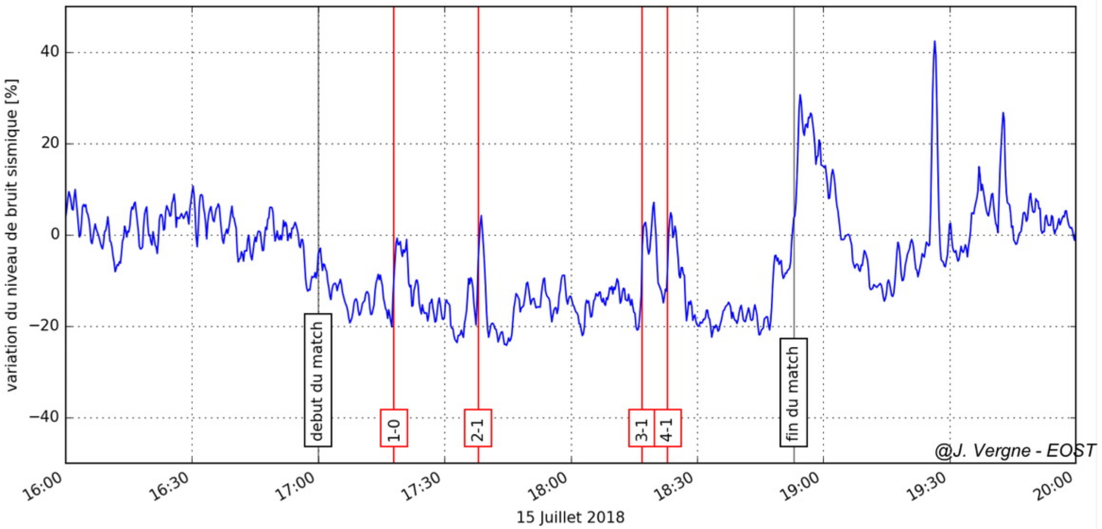
(Christie & Campus, 2010)
`http://www-dase.cea.fr`



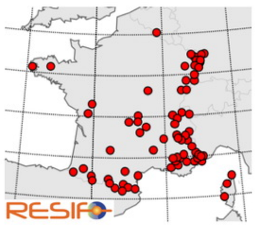


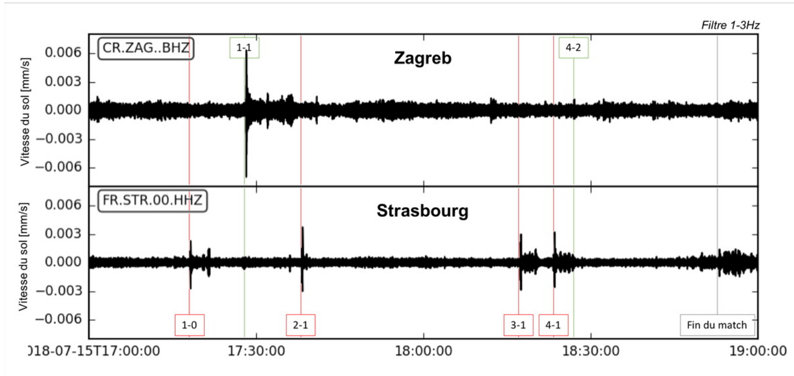

Capteurs sismiques le 15 juillet 2018 !

Coupe du monde de football, France – Croatie

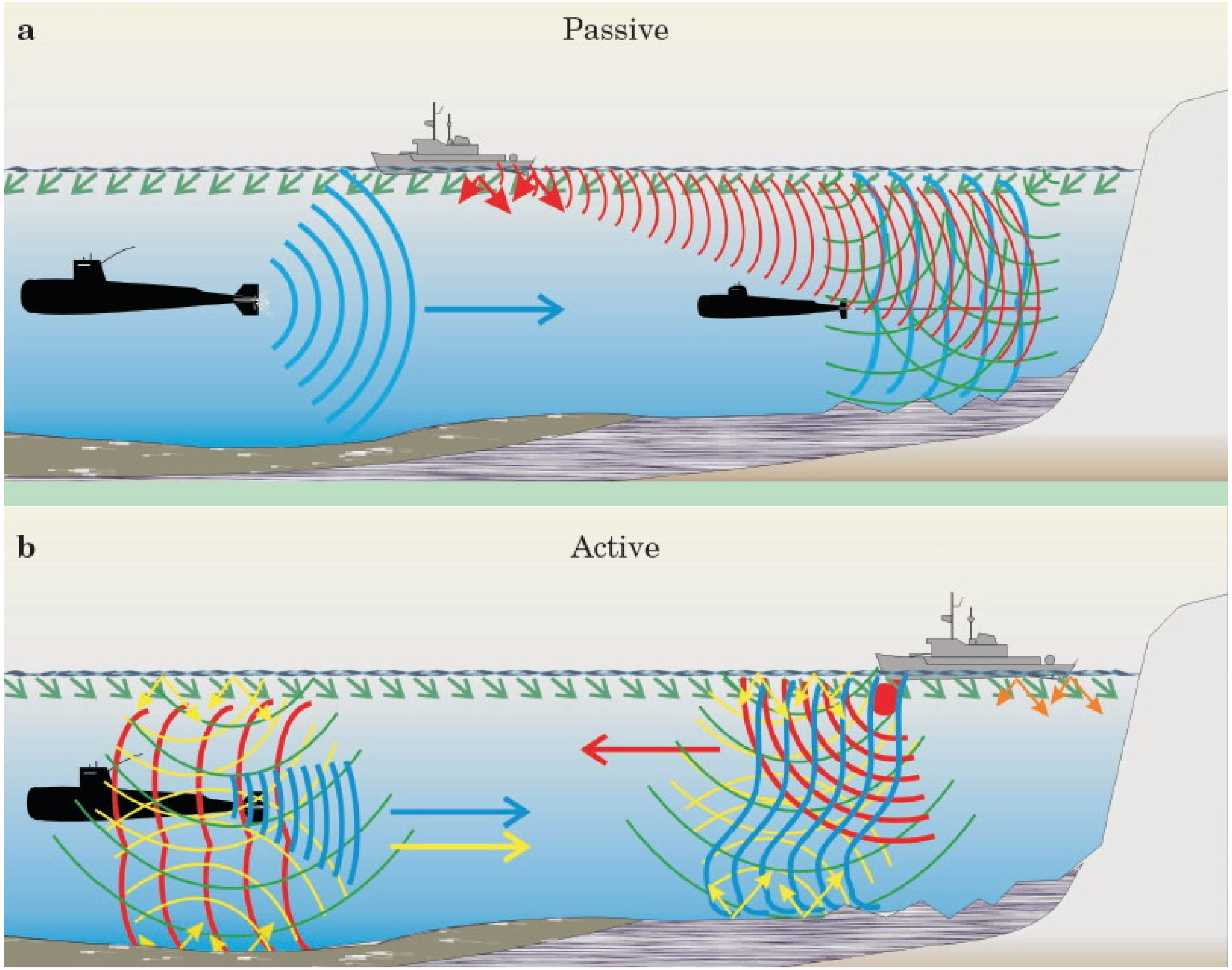


Energie sismique enregistrée le 15 Juillet 2018 (relativement à celle du 8 juillet) dans la bande de fréquence 2-3Hz à 74 stations du réseau sismologique français (cf. carte). On observe 1) une baisse de l'énergie sismique pendant toute la durée du match (diminution globale de l'activité humaine en France) et 2) des pics d'énergie à chaque but de l'équipe de France et au coup de sifflet final, probablement liés aux mouvements de la foule (sauts) dans les différents lieux de rassemblement.





Antisubmarine Warfare (ASW)



Physics Today, **57**(10), 2004

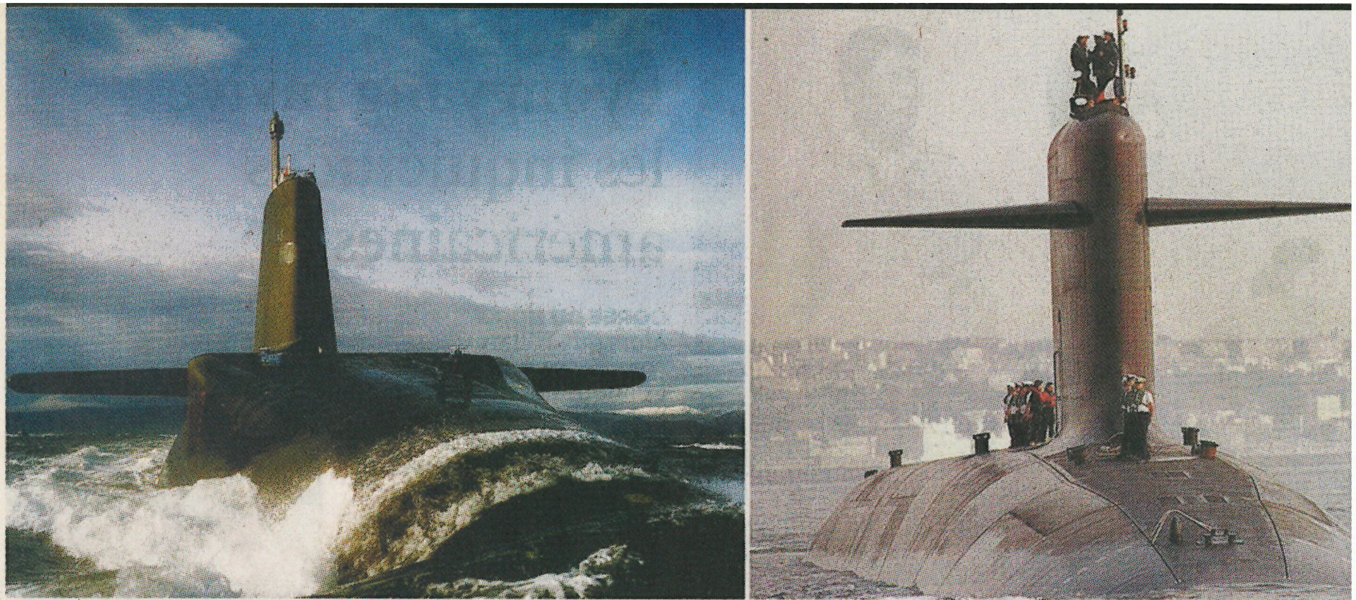
GE sonar, operational in 1996, *Seawolf*

Range (max. active) ~ 35 NM, Range (max. passive) ~ 139 NM

($ 16 million)

Antisubmarine Warfare (ASW)

(Le Figaro, 17 fév. 2009)



Les sous-marins nucléaires lanceurs d'engins, *HMS-Vanguard* et le *Triomphant*, étaient en « *patrouille de routine* » dans l'océan Atlantique lors de l'accident. *HO/AFP*

L'incroyable collision entre deux sous-marins nucléaires

DÉFENSE
Les submersibles français et britannique le « Triomphant » et le « HMS-Vanguard » se sont heurtés au début du mois au large des côtes britanniques.

tuaient des missions séparées lorsqu'ils se sont percutés. « *Il y avait un risque de fuite radioactive. Pire, nous aurions pu perdre l'équipage et les têtes nucléaires* », a affirmé, sous couvert d'anonymat, un responsable britannique cité par *The Sun*.

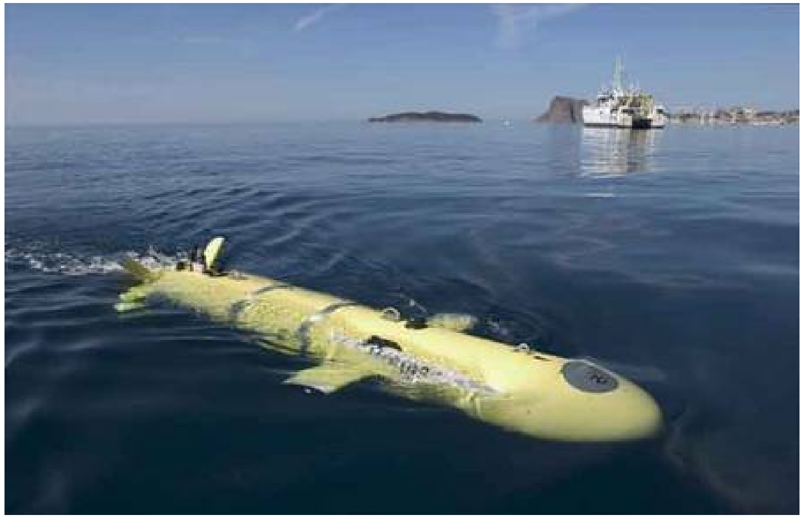
versations, par lâcher du lest. « *Deux sous-marins nucléaires lanceurs d'engins (SNLE), l'un français et l'autre britannique, conduisaient il y a quelques jours des patrouilles nationales de routine. Ils sont entrés en contact brièvement à très basse intensité alors*

Une collision entre submersibles est considérée par les spécialistes comme rarissime avec un taux de probabilité d'une chance sur un million. Elle est d'autant plus surprenante que les deux sous-marins sont réputés pour leur capacité à échapper à d'éven-

Submarine versus Autonomous Underwater Vehicule (AUV)

Alvis submersible (limited to depth of 4500 m)

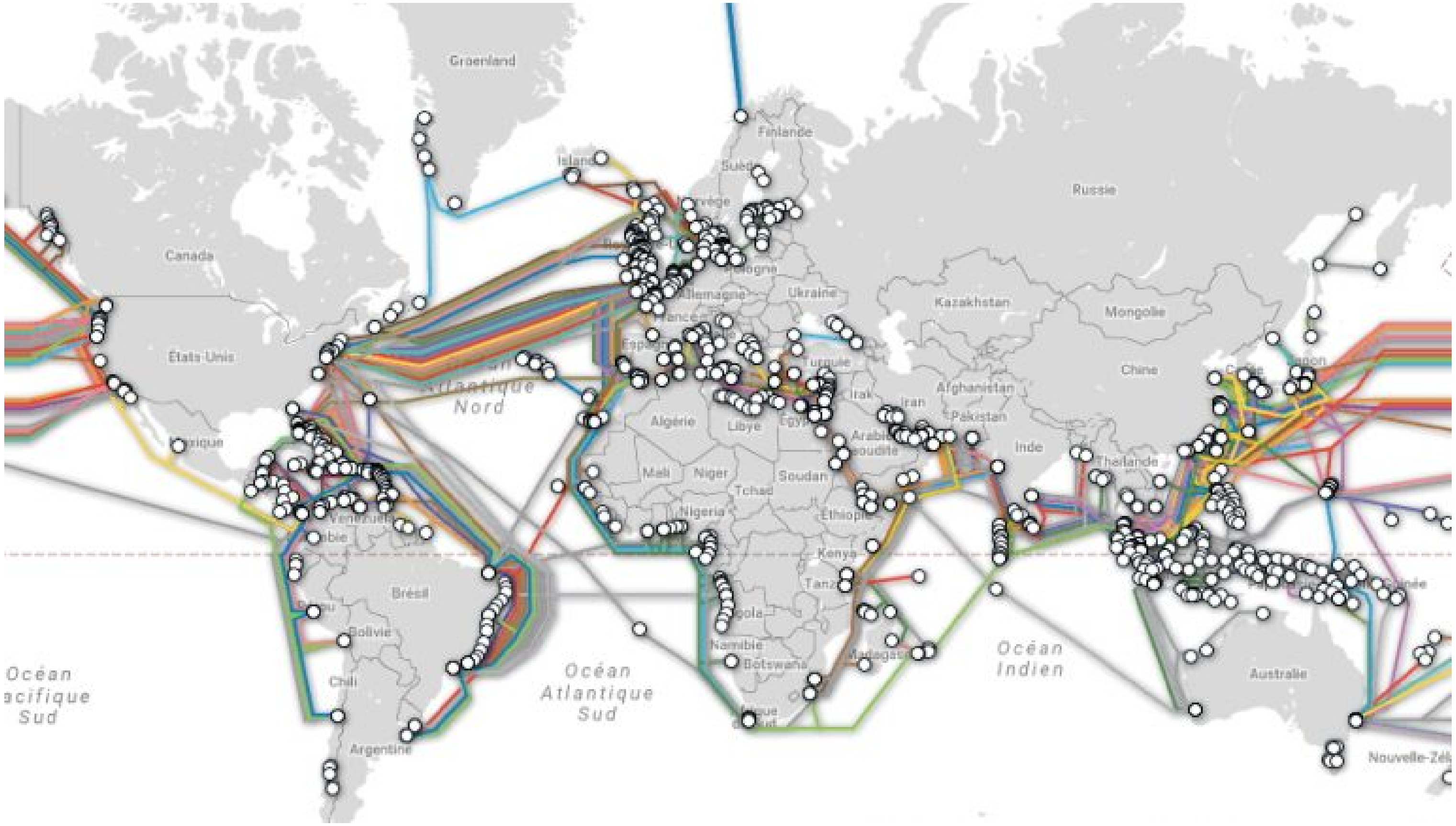
Only 8 submarines are currently able to dive up to 2000 m (Oct. 2015)



Aster^{x}

4.5 m in length
designed to travel to depths of 3000 m
autonomy 100 km
5 knots (9.3 km/h)

- **Internet cables**
  900 000 km of optical fiber, 99% intercontinental internet trafic

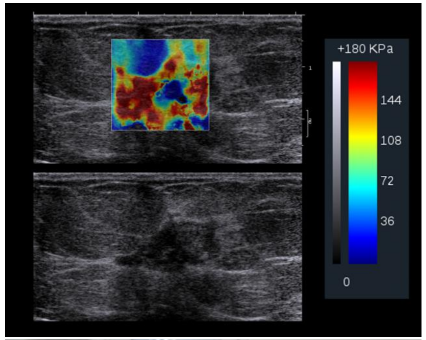


Marea (2017) - cable stretching 4,000 miles between the US and Spain (highest-capacity subsea cable, 160 TBps)

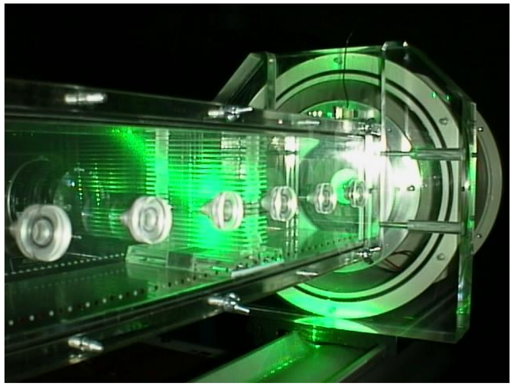
https://www.submarinecablemap.com/

Applications dans de nombreux autres domaines

- production et transport de l'énergie
- acoustique architecturale
- ultrasons
- imagerie médicale
- thermoacoustique
- ...



www.supersonicimagine.fr



Applications dans de nombreux autres domaines

2 ■ Grand Lyon — Vendredi 11 avril 2014

SANTÉ Les équipes de l'hôpital Herriot ont réalisé une première mondiale

Le cancer de la prostate soigné par les ultrasons

Caroline Girardon

Cette fois, plus besoin d'avoir recours à la radiothérapie ou à l'ablation de l'organe touché. Pour la première fois au monde, les équipes de l'hôpital Edouard-Herriot ont mis au point Focal One, un nouveau procédé pour soigner les patients atteints d'un cancer de la prostate à « faible risque » ou « risque intermédiaire ».

Pas de cicatrice

« On propose de traiter localement le cancer grâce à des ultrasons focalisés, précise le professeur Albert Gelet, du service d'urologie. On vise uniquement les tumeurs, ce qui permet d'épargner le reste de la prostate. Car, lorsque l'on traite l'organe dans son ensemble, il y a des risques élevés d'incontinence urinaire ou de séquelles sexuelles. » « En ciblant le traitement, on diminue de dix le risque d'effets secondaire »,

HCL

Lors d'une intervention.

poursuit Sébastien Crouzet, chirurgien urologue. L'opération est similaire à une péridurale : seul le bas du corps du patient est endormi. Une sonde, introduite dans le rectum, permet ensuite de visualiser en trois dimensions l'état de la tumeur suspectée et d'intervenir. « Normalement, une séance de 20 minutes à 2 heures, selon les cas, suffit pour traiter le patient qui n'a aucune cicatrice à la sortie », explique Sébastien Crouzet. Ce traitement, en cours d'évaluation, a été expérimenté sur une soixantaine de malades à Lyon. Et les médecins envisagent de le tester sur d'autres pathologies. « On a déjà traité une vingtaine de patients qui avaient un cancer du foie, indique un membre de l'équipe. On envisage de traiter de la même façon les tumeurs du poumon. » ■

Testé ailleurs

L'appareil, développé il y a 3 ans à Lyon, est actuellement testé à Nantes, Bordeaux, Toulouse, Lille et Paris mais aussi en Pologne, en Suisse et en Allemagne.

Aéroacoustique

C'est la discipline consacrée à l'étude du **bruit d'origine aérodynamique**, à sa **propagation** généralement **en milieu non homogène**, à son rayonnement et aux moyens de réduire ce bruit par contrôle actif du champ sonore ou de la source de bruit elle-même lorsque cela est possible.

On considère ici la génération et la propagation dans les gaz et dans les fluides lourds, comme pour la propagation acoustique sous-marine par exemple.

L'étude des instabilités thermo - acoustiques, l'étude des fluctuations de pression pariétale, qui constituent le champ excitateur pour le rayonnement des structures ou bien encore la modélisation des conditions aux limites pour aborder des problèmes de propagation ou de contrôle font partie intégrante des domaines d'activités actuels de cette thématique.

- **Cours organisé en deux grandes parties**

I – Introduction (presque la fin !)

II – Propagation acoustique

- en milieu inhomogène $\rho_0(\boldsymbol{x}), T_0(\boldsymbol{x})$
- avec un écoulement $\boldsymbol{u}_0(\boldsymbol{x}), \rho_0(\boldsymbol{x}), T_0(\boldsymbol{x})$

  - équations d'Euler linéarisées
  - acoustique géométrique (rayons, approximation HF)
  - ~~approximations paraboliques (MF / BF)~~

III – Aéroacoustique

- sources élémentaires
- théorie de Lighthill & bruit de jet
- couplages (bruit de cavité, thermoacoustique)

Travaux dirigés & contrôle des connaissances

1. Supports de cours, articles complémentaires, forum, sujets de travaux dirigés :
   https://ecampus.paris-saclay.fr/course/view.php?id=18628
2. Évaluation avec deux contributions : un commentaire d'article par écrit, et un mini-projet basé sur la résolution des équations des rayons en 2-D sous Matlab pour traiter un problème au choix dans une liste.
3. **Commentaire d'article.**
   Vous trouverez sur la plateforme *moodle* quelques articles liés aux différentes parties du cours. Il est demandé d'un choisir un (vous pouvez aussi en proposer un à faire valider), et de faire un commentaire de cet article sur au moins 2 pages (lien avec le cours, reproduire des figures et les commenter, refaire un calcul d'une section particulière, ...)
4. **Mini-projet (sujet avec sujets d'exercices / travaux dirigés)**.
   On rédigera un document de 4 à 6 pages figures incluses (police et figures de taille raisonnable), **en prenant soin de commenter** les figures de propagation obtenues, de justifier le choix des paramètres numériques, d'évaluer les erreurs numériques, ...

Ces deux contributions sont à déposer sur *moodle* sous la forme d'un fichier pdf nommé avec votre nom au plus tard le vendredi 8 mai 2020.

Textbooks

Crighton, D.G., 1975, Basic principles of aerodynamic noise generation, *Prog. Aerospace Sci.*, **16**(1), 31-96.

Crighton, D.G., Dowling, A.P., Ffowcs Williams, J.E., Heckl, M. & Leppington, F.G., 1992, *Modern methods in analytical acoustics*, Springer-Verlag, London.

Dowling, A.P. & Ffowcs Williams, J.E., 1983, *Sound and sources of sound*, Ellis Horwood Limited, England.

Goldstein, M.E., 1976, *Aeroacoustics*, McGraw-Hill, New York.

Howe, M.S., 1998, *Acoustics of fluid-structure interactions*, Cambridge University Press, Cambridge.

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Lighthill, J., 1978, *Waves in fluids*, Cambridge University Press, Cambridge.

Morse, P.M. & Ingard, K.U., 1986, *Theoretical acoustics*, Princeton University Press, Princeton, New Jersey.

Ockendon, H. & Ockendon, J. R., 2000, *Waves and compressible flow*, Springer-Verlag, New York, New-York.

Pierce, A.D., 1994, *Acoustics*, Acoustical Society of America, third edition.

Rayleigh, J. W. S., 1877, *The theory of sound*, Dover Publications, New York, 2nd edition (1945), New-York.

Temkin, S., 2001, *Elements of acoustics*, Acoustical Society of America through the American Institute of Physics.

Whitham, G.B., 1974, *Linear and nonlinear waves*, Wiley-Interscience, New-York.

- Introduction
- **Propagation en écoulement**
  - Équations de la mécanique des fluides (rappels)
  - Équations d’Euler linéarisées
  - Propagation en milieu homogène et au repos
  - Propagation en milieu non homogène et au repos
  - Propagation en écoulement
  - Acoustique géométrique : équations des rayons
- Introduction to aeroacoustics
- Advances in aeroacoustics research

Conservation de la masse

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0 \qquad \text{ou encore} \qquad \underbrace{\frac{\partial \rho}{\partial t} + \boldsymbol{u} \cdot \nabla \rho}_{\equiv D\rho/Dt} + \rho \nabla \cdot \boldsymbol{u} = 0$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \boldsymbol{u} \cdot \nabla = \frac{\partial}{\partial t} + u_j \frac{\partial}{\partial x_j}$$

dérivée particulaire

Autre forme : $$\nabla \cdot \boldsymbol{u} = -\frac{1}{\rho}\frac{D\rho}{Dt}$$

Le taux de dilatation d'une particule fluide est égal à la décroissance du taux de masse volumique de cette particule matérielle

Conservation de la quantité de mouvement : Navier-Stokes

$$\rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u}\right) = -\nabla p + \nabla\cdot\overline{\overline{\boldsymbol{\tau}}} + \rho\boldsymbol{g}$$

D'autres formes sont possibles en notant que :

$$\rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u}\right) = \frac{D}{Dt}(\rho\boldsymbol{u}) \qquad \text{et} \qquad \boldsymbol{u}\cdot\nabla\boldsymbol{u} = \nabla\left(\frac{\boldsymbol{u}^2}{2}\right) + \boldsymbol{\omega}\times\boldsymbol{u}$$

- Tenseur des contraintes visqueuses pour un fluide newtonien :
  μ viscosité dynamique et $\nu = \mu/\rho$ viscosité cinématique

$$\overline{\overline{\boldsymbol{\tau}}} = \mu\left(\nabla\boldsymbol{u} + \nabla\boldsymbol{u}^t\right) - \frac{2}{3}\mu(\nabla\cdot\boldsymbol{u})\overline{\overline{\boldsymbol{I}}}$$

ou sous forme indicielle :

$$\tau_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\mu\frac{\partial u_k}{\partial x_k}\delta_{ij}$$

Remarks about the viscous stress tensor

Shear and bulk viscosities

$$\overline{\overline{\tau}} = \mu\left[\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^t\right] + \lambda_2 (\nabla \cdot \boldsymbol{u})\,\overline{\overline{\boldsymbol{I}}}$$
$$= \mu\left[\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^t - \frac{2}{3}(\nabla \cdot \boldsymbol{u})\,\overline{\overline{\boldsymbol{I}}}\right] + \left(\lambda_2 + \frac{2}{3}\mu\right)(\nabla \cdot \boldsymbol{u})\,\overline{\overline{\boldsymbol{I}}}$$

λ_2 second viscosity

$\mu_b \equiv \lambda_2 + (2/3)\mu$ bulk viscosity

The bulk viscosity can be deduced experimentally from absorption of sound :
$\mu_b \equiv 0$ for monoatomic gases (Stokes's hypothesis, 1845),
$\mu_b = \mu_b(T)$ otherwise.

Ref. Karim & Rosenhead (1952), Landau & Lifchitz (1989), Pierce (1991)

Remarks about the viscous stress tensor

Decomposition into a solenoidal part (zero divergence) and an irrotational part (zero curl)

$$
\begin{aligned}
\nabla \cdot \overline{\overline{\boldsymbol{\tau}}} &= \nabla \cdot \left[\mu \left(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^t \right) - \frac{2}{3} \mu \left(\nabla \cdot \boldsymbol{u} \right) \overline{\overline{\boldsymbol{I}}} \right] \\
&= \mu \nabla^2 \boldsymbol{u} + \frac{1}{3} \mu \nabla \left(\nabla \cdot \boldsymbol{u} \right) \\
&= -\mu \nabla \times \left(\nabla \times \boldsymbol{u} \right) + \frac{4}{3} \mu \nabla \left(\nabla \cdot \boldsymbol{u} \right)
\end{aligned}
$$

by using $\nabla \times (\nabla \times \boldsymbol{u}) = \nabla \times \boldsymbol{\omega} = \nabla(\nabla \cdot \boldsymbol{u}) - \nabla^2 \boldsymbol{u}$

Helmholtz equation, $\nabla \times (\nabla \cdot \overline{\overline{\boldsymbol{\tau}}}) = -\mu \nabla \times (\nabla \times \boldsymbol{u}) = -\mu \nabla (\nabla \cdot \boldsymbol{\omega}) + \mu \nabla^2 \boldsymbol{\omega}$

Conservation de l'énergie, formulée pour l'entropie s

$$\rho\left(\frac{\partial s}{\partial t} + \boldsymbol{u}\cdot\nabla s\right) = -\frac{1}{T}\nabla\cdot\boldsymbol{q} + \frac{q_e}{T} + \frac{\Phi}{T}$$

- $\boldsymbol{q}$ flux de chaleur
  loi de Fourier $\boldsymbol{q} = -\lambda\nabla T$ $\quad\lambda$ coefficient de conductivité thermique
- q_e apport extérieur de chaleur massique
- Φ fonction de dissipation par les contraintes visqueuses

$$\Phi = \overline{\overline{\boldsymbol{\tau}}} : \nabla\boldsymbol{u} = \tau_{ij}\frac{\partial u_i}{\partial x_j} \geq 0$$

Relations thermodynamiques

$$de = Tds - pd\left(\frac{1}{\rho}\right) \qquad dh = Tds + \frac{1}{\rho}dp$$

Equation of state (EOS)

$\rightsquigarrow$ state fixed by any two independent thermodynamic variables

$s = s(\rho, p)$ or $p = (\rho, s)$

- **Ideal (perfect) gas** (other choice : van der Walls)

$$p = \rho r T$$

$$de = c_v dT \qquad dh = c_p dT$$

$$ds = c_v d\left[\ln\left(\frac{p}{\rho^\gamma}\right)\right] \quad dp = d\left[\rho^\gamma \exp\left(\frac{s}{c_v}\right)\right]$$

- **Liquid (water)** : Tammann (1895)

$$p = (\gamma - 1)\rho e - \gamma p_{\mathrm{eos}}$$

air : $\gamma = 1.4$, $p_{\mathrm{eos}} = 0$

water : $\gamma = 6 - 7.15$, $p_{\mathrm{eos}} = 3 \times 10^8$

Vitesse du son

Avec $p = p(\rho, s)$

$$dp = \left.\frac{\partial p}{\partial \rho}\right|_s d\rho + \left.\frac{\partial p}{\partial s}\right|_\rho ds = c^2 d\rho + \frac{p}{c_v} ds \quad \text{pour un gaz parfait}$$

Vitesse du son pour un gaz parfait, $c^2 = \gamma r T = \gamma p/\rho$

Cas général :

$$c^2 = \partial p/\partial \rho\big|_s = 1/(\rho \chi_s) \qquad \chi_s \text{ compressibilité isentropique}$$

Fluide parfait $(\mu = 0)$ et non conducteur $(\lambda = 0)$

$$\frac{Ds}{Dt} = 0$$

conservation de l'entropie
le long des lignes de courant

Properties of air and water at atmospheric pressure

fluid (20° C)	ρ kg.m^{-3}	ν m^2.s^{-1}	c_p J.kg^{-1}.K^{-1}	$r = c_p - c_v$ J.kg^{-1}.K^{-1}	λ J.m^{-1}.s^{-1}.K^{-1}	σ	γ
air	1.20	1.5×10^{-5}	1005	287.06	2.54×10^{-2}	0.71	1.400
water	1000.5	1.0×10^{-6}	4.182	461.50	5.97×10^{-1}	7.02	1.329

air	$c = \sqrt{\gamma r T} \simeq 343.2$ m.s^{-1} at 20° C
water	$c \simeq 1447 + 4.0\Delta T + (1.6 \times 10^{-6} p)$
seawater	$c \simeq 1490 + 3.6\Delta T + (1.6 \times 10^{-6} p) + 1.3\Delta S$

$\Delta T = T - 283.6$ $\quad p$ absolute pressure in pascals

$\Delta S = S - 35\text{‰}_0$ S salinity

- **Linéarisation des équations d'Euler**
  autour d'un écoulement de base (ou moyen) indépendant du temps $\boldsymbol{u}_0(\boldsymbol{x}), \rho_0(\boldsymbol{x}), p_0(\boldsymbol{x}), s_0(\boldsymbol{x})$ et incompressible $\nabla \cdot \boldsymbol{u}_0 = 0$

$$\rho = \rho_0 + \rho' \qquad \boldsymbol{u} = \boldsymbol{u}_0 + \boldsymbol{u}' \qquad p = p_0 + p' \qquad s = s_0 + s'$$

acoustique ≡ petites perturbations

$$\rho'/\rho_0 \ll 1 \qquad u'/c_0 \ll 1 \qquad \ldots$$

Fluctuations acoustiques

$$p' = A\cos(\omega t) \quad A = 90\ \text{Pa} \quad f = 1\ \text{kHz} \qquad \lambda_a = \frac{c_0}{f} = \frac{340}{1000} = 0.34\ \text{m}$$

Niveau sonore :

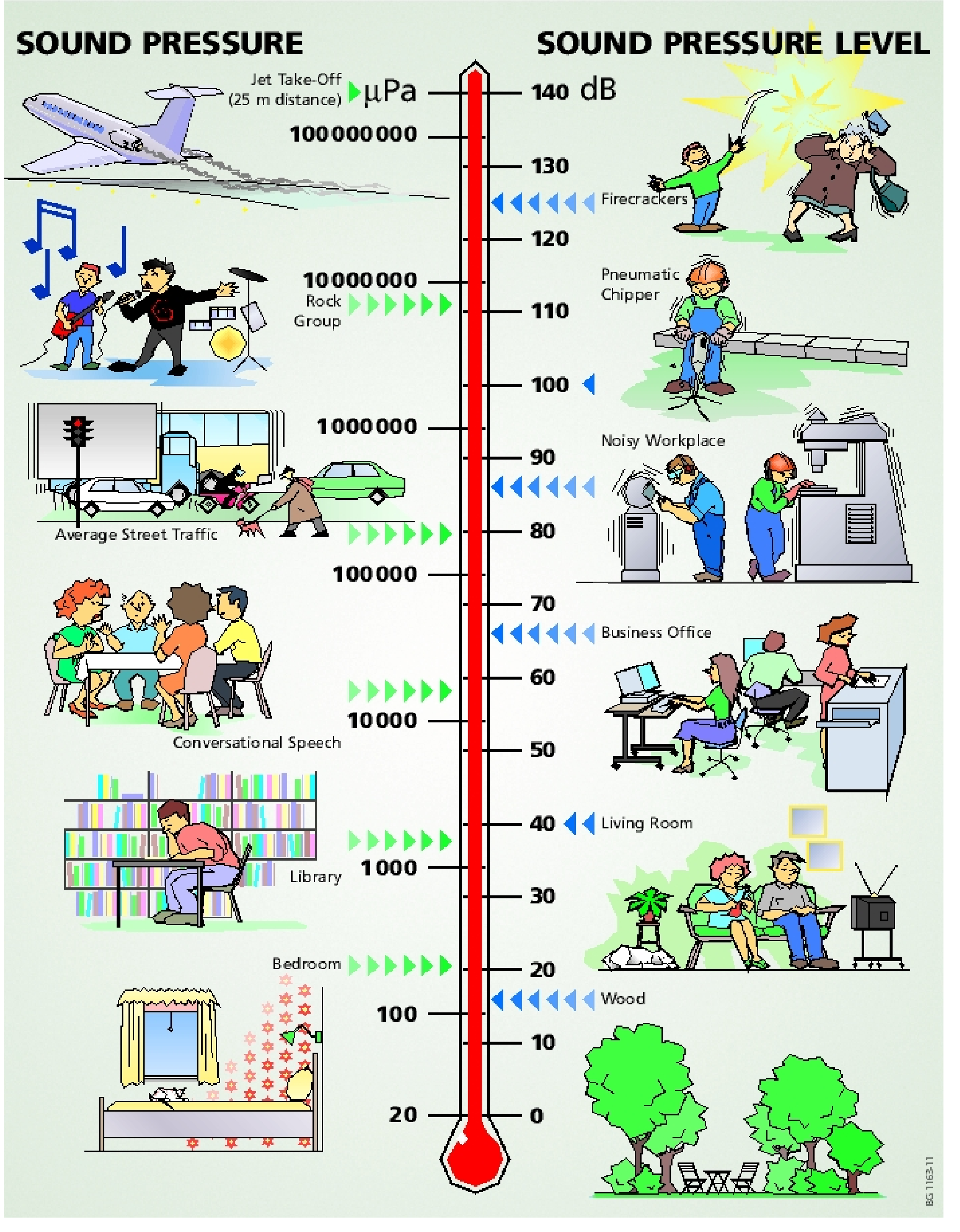
$$\text{SPL} = 10\log_{10}\left(\frac{\overline{p'^2}}{p_{\text{ref}}^2}\right) = 10\log_{10}\left(\frac{90^2/2}{(2\times 10^{-5})^2}\right) \simeq 130\ \text{dB}$$

(seuil de la douleur)

$$u' = p'/(\rho_0 c_0) \simeq 0.22\ \text{m.s}^{-1} \qquad \frac{p'}{p_0} \simeq 9\times 10^{-4} \qquad \frac{u'}{c_0} \simeq 6\times 10^{-4}$$

Les fluctuations acoustiques sont généralement des petites perturbations : acoustique linéaire

$p_{\text{ref}} = 2\times 10^{-5}$ Pa dans l'air et $p_{\text{ref}} = 10^{-6}$ Pa dans l'eau


SOUND PRESSURE
SOUND PRESSURE LEVEL
Jet Take-Off
(25 m distance)
µPa
140 dB
100000000
130
Firecrackers
120
10000000
Rock
Group
Pneumatic
Chipper
110
100
1000000
Noisy Workplace
90
Average Street Traffic
80
100000
70
Business Office
60
10000
Conversational Speech
50
40
Living Room
Library
1000
30
Bedroom
20
Wood
100
10
20
0
BG 1163-11

Conservation de la masse

$$\frac{\partial(\rho_0+\rho')}{\partial t}+(\boldsymbol{u}_0+\boldsymbol{u}')\cdot\nabla(\rho_0+\rho')+(\rho_0+\rho')\nabla\cdot(\boldsymbol{u}_0+\boldsymbol{u}')=0$$

ordre 0

$$\underline{\frac{\partial\rho_0}{\partial t}}+\boldsymbol{u}_0\cdot\nabla\rho_0+\underline{\rho_0\nabla\cdot\boldsymbol{u}_0}=0$$

équation vérifiée par le champ de base

ordre 1

$$\frac{\partial\rho'}{\partial t}+\boldsymbol{u}_0\cdot\nabla\rho'+\boldsymbol{u}'\cdot\nabla\rho_0+\rho_0\nabla\cdot\boldsymbol{u}'+\underline{\rho'\nabla\cdot\boldsymbol{u}_0}=0$$

équation sur les perturbations au premier ordre

ordre 2

$$\boldsymbol{u}'\cdot\nabla\rho'+\rho'\nabla\cdot\boldsymbol{u}'=0$$

Équations d'Euler linéarisées

ordre 0	$\rho_0 \boldsymbol{u}_0 \cdot \nabla \boldsymbol{u}_0 = -\nabla p_0 + \rho_0 \boldsymbol{g}$
ordre 1	$\rho_0 \left(\dfrac{\partial \boldsymbol{u}'}{\partial t} + \boldsymbol{u}_0 \cdot \nabla \boldsymbol{u}' + \boldsymbol{u}' \cdot \nabla \boldsymbol{u}_0 \right) + \rho' \boldsymbol{u}_0 \cdot \nabla \boldsymbol{u}_0 = -\nabla p' + \rho' \boldsymbol{g}$
ordre 2	$\rho' \dfrac{\partial \boldsymbol{u}'}{\partial t} + \rho_0 \boldsymbol{u}' \cdot \nabla \boldsymbol{u}' + \rho' (\boldsymbol{u}_0 \cdot \nabla \boldsymbol{u}' + \boldsymbol{u}' \cdot \nabla \boldsymbol{u}_0) = 0$
ordre 3	$\rho' \boldsymbol{u}' \cdot \nabla \boldsymbol{u}' = 0$

Autre forme pour l'ordre 1 : (gravité $\boldsymbol{g}$ éliminée avec l'ordre 0)

$$\rho_0 \left(\frac{\partial \boldsymbol{u}'}{\partial t} + \boldsymbol{u}_0 \cdot \nabla \boldsymbol{u}' + \boldsymbol{u}' \cdot \nabla \boldsymbol{u}_0 \right) + \nabla p' - \frac{\rho'}{\rho_0} \nabla p_0 = 0$$

Équation de conservation de l'entropie

ordre 0	$\boldsymbol{u}_0 \cdot \nabla s_0 = 0$
ordre 1	$\dfrac{\partial s'}{\partial t} + \boldsymbol{u}_0 \cdot \nabla s' + \boldsymbol{u}' \cdot \nabla s_0 = 0$
ordre 2	$\boldsymbol{u}' \cdot \nabla s' = 0$

Loi d'état $p = p(\rho, s)$

- Développement limité de $p(\rho, s)$ autour de (ρ_0, s_0)

$$p(\rho, s) \simeq p_0 + (\rho - \rho_0) \left.\frac{\partial p}{\partial \rho}\right|_{s,0} + (s - s_0) \left.\frac{\partial p}{\partial s}\right|_{\rho,0} + \ldots$$

À l'ordre 1, pour un gaz parfait : $p' = c_0^2 \rho' + \dfrac{p_0}{c_v} s'$

- On peut aussi repartir de $p = p(\rho, s)$ et utiliser la conservation de l'entropie $ds/dt = 0$

$$\frac{dp}{dt} = \left.\frac{\partial p}{\partial \rho}\right|_s \frac{d\rho}{dt} + \left.\frac{\partial p}{\partial s}\right|_\rho \frac{ds}{dt} = c^2 \frac{d\rho}{dt}$$

$\rightsquigarrow$ au premier ordre :

$$\frac{\partial p'}{\partial t} + \boldsymbol{u}_0 \cdot \nabla p' + \boldsymbol{u}' \cdot \nabla p_0 = c_0^2 \left(\frac{\partial \rho'}{\partial t} + \boldsymbol{u}_0 \cdot \nabla \rho' + \boldsymbol{u}' \cdot \nabla \rho_0 \right) + (c^2)' \boldsymbol{u}_0 \cdot \nabla \rho_0$$

- **En résumé (nos équations de référence)**

$$
\left\{
\begin{array}{l}
\dfrac{\partial \rho'}{\partial t} + \boldsymbol{u}_0 \cdot \nabla \rho' + \boldsymbol{u}' \cdot \nabla \rho_0 + \rho_0 \nabla \cdot \boldsymbol{u}' = 0 \\[2ex]
\rho_0 \left(\dfrac{\partial \boldsymbol{u}'}{\partial t} + \boldsymbol{u}_0 \cdot \nabla \boldsymbol{u}' + \boldsymbol{u}' \cdot \nabla \boldsymbol{u}_0 \right) + \nabla p' - \dfrac{\rho'}{\rho_0} \nabla p_0 = 0 \qquad \left(\boldsymbol{g} = \boldsymbol{u}_0 \cdot \nabla \boldsymbol{u}_0 + \dfrac{1}{\rho_0} \nabla p_0 \right) \\[2ex]
\dfrac{\partial s'}{\partial t} + \boldsymbol{u}_0 \cdot \nabla s' + \boldsymbol{u}' \cdot \nabla s_0 = 0 \\[2ex]
\left\{
\begin{array}{l}
p' = c_0^2 \rho' + \dfrac{p_0}{c_v} s' \qquad \text{(perfect gas)} \\[2ex]
\dfrac{\partial p'}{\partial t} + \boldsymbol{u}_0 \cdot \nabla p' + \boldsymbol{u}' \cdot \nabla p_0 = c_0^2 \left(\dfrac{\partial \rho'}{\partial t} + \boldsymbol{u}_0 \cdot \nabla \rho' + \boldsymbol{u}' \cdot \nabla \rho_0 \right) + (c^2)' \boldsymbol{u}_0 \cdot \nabla \rho_0
\end{array}
\right.
\end{array}
\right.
$$

- **Cas de référence (et le plus simple !)**

$\rho_0 = \text{cte}, \boldsymbol{u}_0 = 0, p_0 = \text{cte}$

(et donc $s_0 = \text{cte}$)

$$
\begin{cases}
\dfrac{\partial \rho'}{\partial t} + \rho_0 \nabla \cdot \boldsymbol{u}' = 0 & (1) \\[2ex]
\rho_0 \dfrac{\partial \boldsymbol{u}'}{\partial t} + \nabla p' = 0 & (2) \\[2ex]
\dfrac{\partial s'}{\partial t} = 0 & (3) \\[2ex]
p' = c_0^2 \rho' + \dfrac{p_0}{c_v} s' & (4) \\
\text{(gaz parfait)} &
\end{cases}
$$

- On a $s' \equiv 0$ et $p' = c_0^2 \rho'$
- On forme alors l'équation des ondes en éliminant la vitesse fluctuante :

$$\partial / \partial t (1) - \nabla \cdot (2) = 0$$

$$\frac{\partial^2 p'}{\partial t^2} - c_0^2 \nabla^2 p' = 0$$

En prenant le rotationnel de l'équation (2)

$$\frac{\partial}{\partial t} \nabla \times \boldsymbol{u}' = 0 \qquad \Longrightarrow \qquad \boldsymbol{u}' = \nabla \phi$$

ϕ potentiel acoustique

en milieu homogène et au repos, le champ acoustique est irrotationnel

Équation d'Helmholtz

Pour une perturbation harmonique $p'(\boldsymbol{x}, t) = \tilde{p}(\boldsymbol{x})e^{-i\omega t}$

$$(-i\omega)^2\tilde{p} - c_0^2\nabla^2\tilde{p} = 0$$

$$\nabla^2\tilde{p} + k_0^2\tilde{p} = 0 \qquad k_0 \equiv \frac{\omega}{c_0} \quad \text{nombre d'onde}$$

Solution élémentaire : onde plane (progressive)

$$p'(\boldsymbol{x}, t) = Ae^{i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)} \qquad \boldsymbol{u}'(\boldsymbol{x}, t) = \frac{A}{\rho_0 c_0}\, e^{i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)}\boldsymbol{\nu} = \frac{p'}{\rho_0 c_0}\boldsymbol{\nu}$$

$\boldsymbol{\nu} \equiv \boldsymbol{k}/k$ vecteur unitaire associé au nombre d'onde $\boldsymbol{k}$
(dans la direction de propagation ici, car sans écoulement moyen)

Propagation dans l'océan ou dans l'atmosphère

$c_0 = c_0(\boldsymbol{x})$, $\rho_0 = \rho_0(\boldsymbol{x})$, $s_0 = s_0(\boldsymbol{x})$, $p_0 = p_0(\boldsymbol{x})$ et $\boldsymbol{u}_0 \equiv 0$

$$\begin{cases} \dfrac{\partial \rho'}{\partial t} + \boldsymbol{u}' \cdot \nabla \rho_0 + \rho_0 \nabla \cdot \boldsymbol{u}' = 0 & (1) \\ \rho_0 \dfrac{\partial \boldsymbol{u}'}{\partial t} + \nabla p' - \dfrac{\rho'}{\rho_0} \nabla p_0 = 0 & (2) \\ \dfrac{\partial p'}{\partial t} + \boldsymbol{u}' \cdot \nabla p_0 = c_0^2 \left(\dfrac{\partial \rho'}{\partial t} + \boldsymbol{u}' \cdot \nabla \rho_0 \right) & (3) \end{cases}$$

L'éq. (3) remplace
$p' = c_0^2 \rho'$
en milieu homogène
(air ou eau)

Équation des ondes ?

... impossible d'écrire une équation portant uniquement sur p' dans le cas général

Ordre de grandeur des termes en $\nabla p_0 = \rho_0 \boldsymbol{g}$

Peut-on négliger les effets de la gravité sur la propagation acoustique ?

- Perturbation caractéristique : $p' \simeq \tilde{p} e^{i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)}$ $\quad \boldsymbol{u}' \simeq \dfrac{p'}{\rho_0 c_0}\boldsymbol{\nu}$ $\quad \rho' \simeq p'/c_0^2$

Équation (2) :

$$\rho_0 \frac{\partial \boldsymbol{u}'}{\partial t} \quad + \quad \nabla p' \quad - \frac{\rho'}{\rho_0}\nabla p_0 = 0$$

$$\sim \rho_0 \omega \frac{\tilde{p}}{\rho_0 c_0} \qquad \sim \frac{\omega}{c_0}\tilde{p} \qquad \sim \frac{\tilde{p}}{c_0^2} g \qquad\qquad (\nabla p_0 = \rho_0 \boldsymbol{g})$$

- Le terme en ∇p_0 peut être négligé lorsque $\omega \gg g/c_0$

$$f \gg \frac{1}{2\pi}\frac{g}{c_0} \approx 10^{-3}\ \text{Hz} \quad \text{(eau)} \qquad\qquad f \gg \frac{1}{2\pi}\frac{g}{c_0} \approx 5\times 10^{-3}\ \text{Hz} \quad \text{(air)}$$

Dans toute la suite, on négligera la contribution du terme en ∇p_0 dans les équations (2) & (3) pour la propagation en milieu inhomogène (atmosphère ou océan).

Ordre de grandeur des termes en $\nabla p_0 = \rho_0 \boldsymbol{g}$

- En complément, quid du terme de pression pour l'Éq. (3)

Équation (3) :
$$\frac{\partial p'}{\partial t} + \boldsymbol{u}' \cdot \nabla p_0 = c_0^2 \left(\frac{\partial \rho'}{\partial t} + \boldsymbol{u}' \cdot \nabla \rho_0 \right)$$
$$= c_0^2 \rho_0 \nabla \cdot \boldsymbol{u}' \quad \text{avec l'Éq. (1)}$$
$$\sim \omega \tilde{p} \qquad \sim \frac{g}{c_0} \tilde{p} \qquad \sim \omega \tilde{p}$$

On retrouve bien ainsi la condition précédente sur la fréquence,
$$f \gg \frac{1}{2\pi} \frac{g}{c_0}$$

Il existe des discussions plus sophistiquées dans la littérature, Bergmann (*J. Acoust. Soc. Am.*, 1946)

Équation des ondes

- On forme $\partial/\partial t(1) - \nabla\cdot(2) = 0$

$$\frac{\partial^2\rho'}{\partial t^2} + \underbrace{\frac{\partial}{\partial t}\nabla\cdot(\rho_0\boldsymbol{u}') - \nabla\cdot\left(\rho_0\frac{\partial\boldsymbol{u}'}{\partial t}\right)}_{=0} - \nabla^2 p' = 0$$

- Avec (3) & (2), on a : $$\frac{\partial^2\rho'}{\partial t^2} = \frac{1}{c_0^2}\frac{\partial^2 p'}{\partial t^2} - \frac{\partial\boldsymbol{u}'}{\partial t}\cdot\nabla\rho_0 = \frac{1}{c_0^2}\frac{\partial^2 p'}{\partial t^2} + \frac{1}{\rho_0}\nabla p'\cdot\nabla\rho_0$$

- On note aussi que : $$\frac{1}{\rho_0}\nabla\rho_0\cdot\nabla p' - \nabla^2 p' = -\rho_0\nabla\cdot\left(\frac{1}{\rho_0}\nabla p'\right)$$

L'équation des ondes s'écrit alors :

$$\frac{1}{c_0^2(\boldsymbol{x})}\frac{\partial^2 p'}{\partial t^2} - \rho_0(\boldsymbol{x})\nabla\cdot\left(\frac{1}{\rho_0(\boldsymbol{x})}\nabla p'\right) = 0$$

Équation d'Helmholtz à indice variable

- Pour une perturbation harmonique $p'(\boldsymbol{x},t) = \tilde{p}(\boldsymbol{x})\, e^{-i\omega t}$

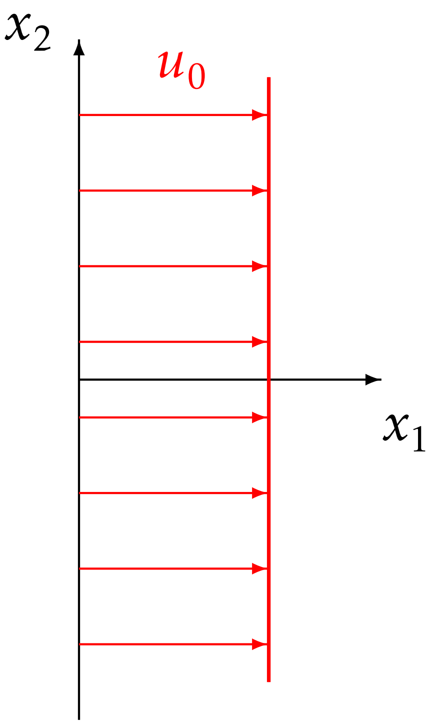
$$\rho_0(\boldsymbol{x})\nabla\cdot\left(\frac{1}{\rho_0(\boldsymbol{x})}\nabla\tilde{p}\right) + \frac{\omega^2}{c_0^2(\boldsymbol{x})}\,\tilde{p} = 0$$

- En introduisant l'indice de réfraction $n = c_\infty/c_0(\boldsymbol{x})$ avec c_∞ valeur moyenne (ou de référence) et $k_\infty = \omega/c_\infty = \text{cst}$

$$\rho_0(\boldsymbol{x})\nabla\cdot\left(\frac{1}{\rho_0(\boldsymbol{x})}\nabla\tilde{p}\right) + k_\infty^2 n^2(\boldsymbol{x})\,\tilde{p} = 0$$

$\rightsquigarrow$ Cette équation est le point de départ pour développer les *approximations paraboliques*, permettant le calcul de la propagation acoustique en milieu inhomogène sur de grandes distances (devant λ_a).

- **Milieu uniforme avec $\boldsymbol{u}_0 = u_0\, \boldsymbol{x}_1$ avec $u_0 = $ cte**



$$
\begin{cases}
\dfrac{D_0\rho'}{Dt} + \rho_0 \nabla\cdot\boldsymbol{u}' = 0 & (1)\\[2mm]
\rho_0 \dfrac{D_0\boldsymbol{u}'}{Dt} + \nabla p' = 0 & (2)\\[2mm]
\dfrac{D_0 s'}{Dt} = 0 & (3)\\[2mm]
p' = c_0^2\rho' + \dfrac{p_0}{c_v} s' & (4)
\end{cases}
$$

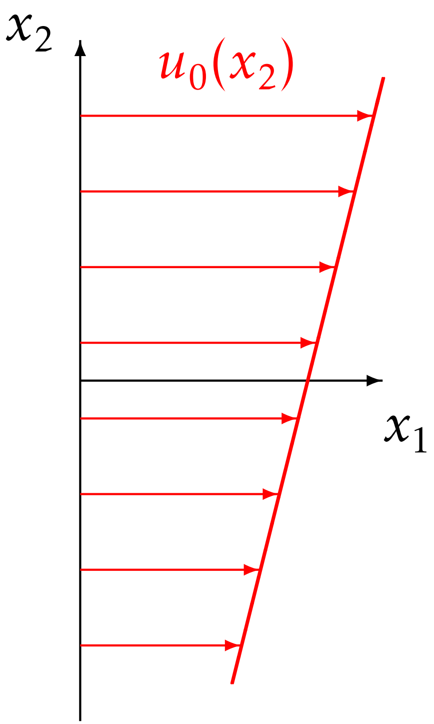
équation des ondes convectées

$$\frac{D_0^2 p'}{Dt^2} - c_0^2\nabla^2 p' = 0$$

D_0/Dt opérateur de dérivation en suivant l'écoulement moyen

$$\frac{D_0}{Dt} \equiv \frac{\partial}{\partial t} + \boldsymbol{u}_0\cdot\nabla = \frac{\partial}{\partial t} + u_0 \frac{\partial}{\partial x_1}$$

- **Écoulement moyen uniformément cisaillé** $\boldsymbol{u}_0 = u_0(x_2, x_3)\boldsymbol{x}_1,\ \rho_0 = \rho_0(x_2, x_3)$



Pridmore-Brown (1958)
Lilley (1972)

Sir Geoffrey Lilley (1919-2015)

$$\frac{D_0}{Dt}\left[\frac{1}{c_0^2}\frac{D_0^2 p'}{Dt^2} - \rho_0 \nabla \cdot \left(\frac{1}{\rho_0}\nabla p'\right)\right] + 2\frac{\partial u_0}{\partial x_i}\frac{\partial^2 p'}{\partial x_1 \partial x_i} = 0$$

$(i = 2, 3)$

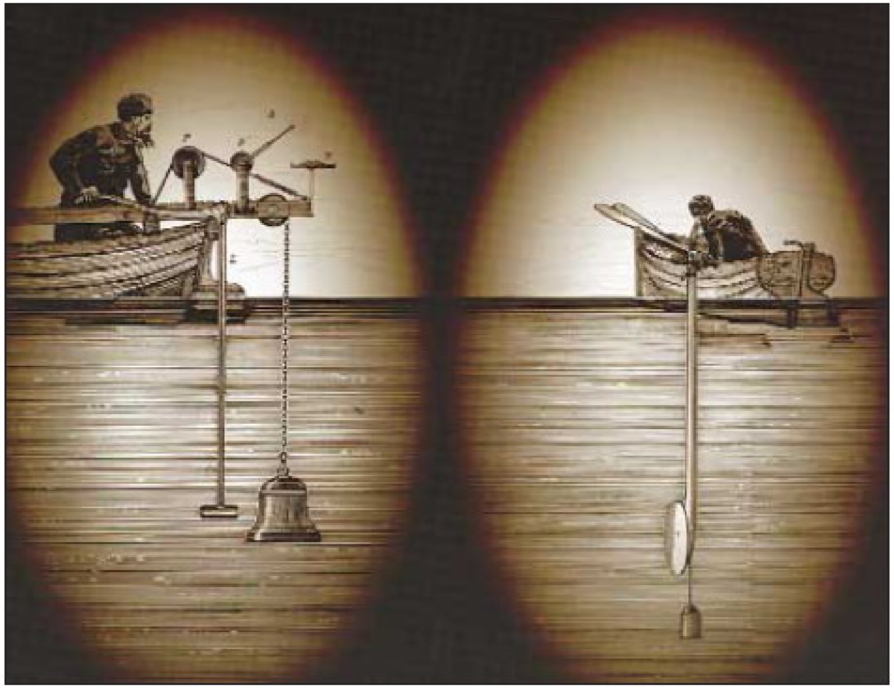
Dans le cas général où $\boldsymbol{u}_0 = \boldsymbol{u}_0(x_1, x_2, x_3)$, il n'est pas possible d'éliminer la fluctuation de vitesse à partir des équations d'Euler linéarisées
(cf. cours master 3A)

- Except in a few simple cases, it is difficult or impossible to write a wave equation on p', and to solve this equation analytically.

- In many applications, we are concerned with long range sound propagation and / or with propagation of high frequency signals with respect to the characteristic length scale of the medium.
  - Underwater sound propagation
    detection, communication, tomography, ...
  - Atmospheric propagation
    noise environment, meteorology - Doppler SODAR (*sonic detection and ranging*), infrasound monitoring, ...

SODAR – $1\ \text{kHz} \lesssim f \lesssim 4\ \text{kHz}$
detection up to 2 km
REMTECH

Measurement of the speed of sound in water

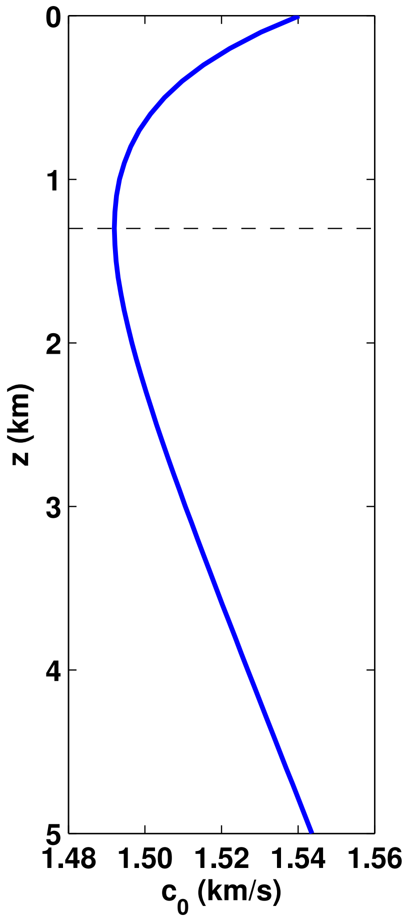


In 1826, Charles Sturm (left, French mathematicien) and Jean-Daniel Colladon (right, Swiss physicist) made the first accurate measurement of the speed of sound in the waters of Lake Geneva. Their result, $c_0 = 1435$ m.s^{-1} in water at a temperature of 8° C was only 3 m.s^{-1} off from what is known today.

Note – William Derham (1657-1735) produced the earliest, reasonably accurate estimate of the speed of sound in air.

Profile of the speed of sound in ocean

(z is usually defined downward for underwater acoustics)



Munk, *J. Acoust. Soc. Am.* (1974)

- Variable thin layer at the ocean's surface (day/night, season, ...), which is rather difficult to model, $z \lesssim 100$ m

- Mean thermocline, characterized by a negative gradient of $c_0(z)$

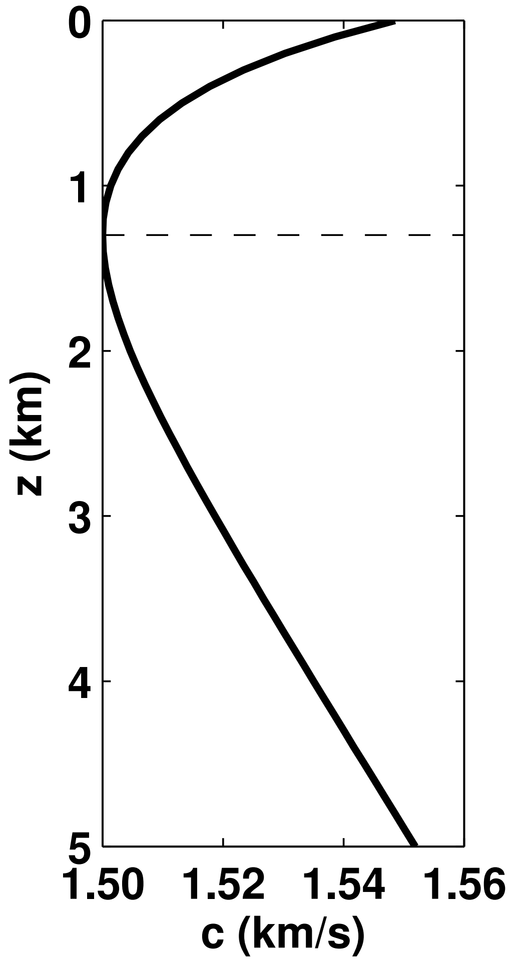
- Minimum at a depth around $z \simeq 1300$ m : sound channel axis, discovered by Ewing & Worzel (1943) and by Brekhovskikh (1943).

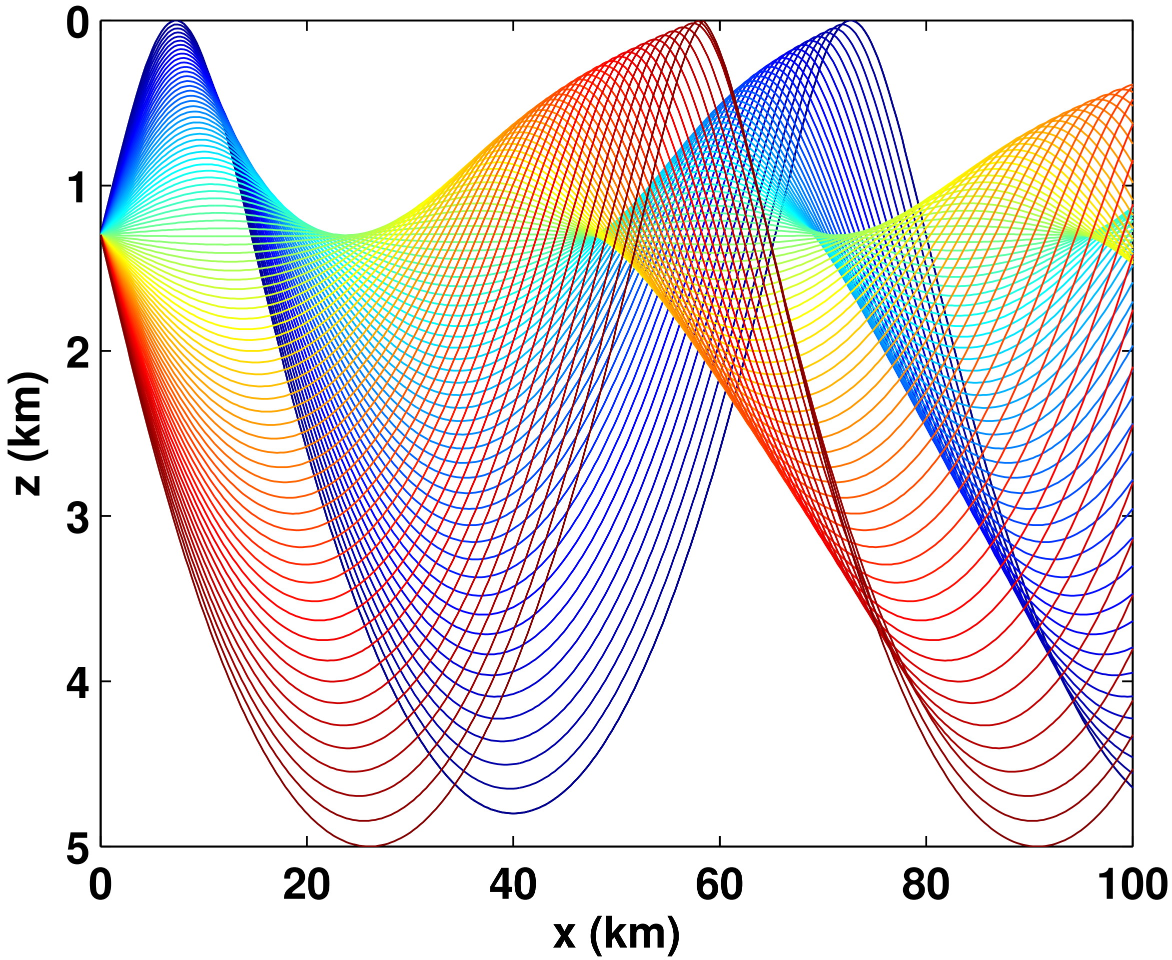
  SOund Fixing And Ranging (SOFAR)
  Underwater Sound Channel (USC)

- For $z \geq 500$ m, thermal equilibrium with $T_0 \simeq 4^{\circ}$ C

- For $z \geq 1300$ m, increase of c_0 by 1.7 m.s^{-1} every 100 m only due to pressure

- SOFAR (SOund Fixing And Ranging)





Quelques échelles caractéristiques

- Longueur caractéristique sur laquelle varie la vitesse du son : $L \simeq 100$ m
- $\Delta z = 100$ m $\rightsquigarrow \Delta c_0 = 1.7$ m.s^{-1}, $\Delta T_0 = 1^\circ$ C $\rightsquigarrow \Delta c_0 = 3$ m.s^{-1},
  $\Delta S = 1‰ \rightsquigarrow \Delta c_0 = 1.3$ m.s^{-1}
- SONAR de pêche : $f \simeq 100$ kHz, $\lambda_a \simeq 0.015$ m,
  distance de propagation $\simeq 1$ km

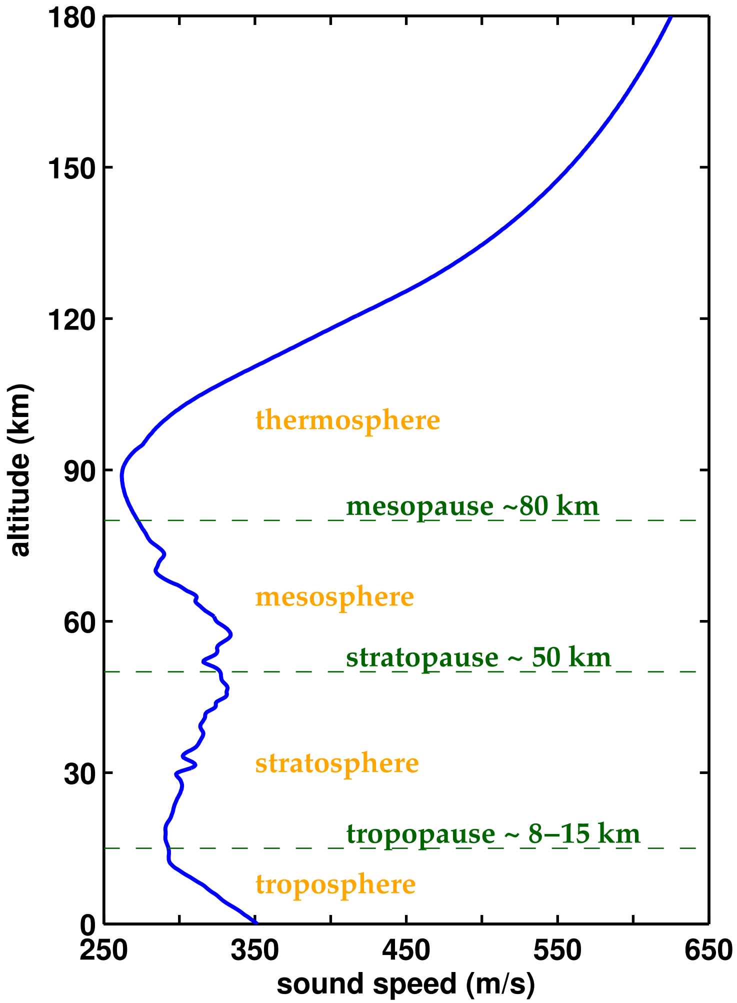
  Longueur d'onde plus grande que la taille des organismes planctoniques, et du même ordre de grandeur que les vessies natatoires (poches d'air) de nombreuses espèces de poisson
- SOFAR : $f \simeq 100$ Hz, $\lambda_a \simeq 15$ m, distance de propagation $\simeq 1000$ km

  Il faut que la fréquence soit inférieure au spectre de bruit de fond de la turbulence et de plus que l'atténuation reste faible, pour une propagation sur de grandes distances (plusieurs centaines de km).

- **Ghost octopus 'Casper'**

Octopus observed at a depth of 4290 meters by the remotely operated vehicle *Deep Discoverer* (Hawaiian island of Necker ; NOAA, 2016)

Profil de vitesse du son dans l'atmosphère



- 2 guides d'ondes naturels (chenals sonores), pour $10 \leq z \leq 30$ km et pour $z \simeq 100$ km.

Misty picture experiment (Los Alamos National Laboratory - CEA)

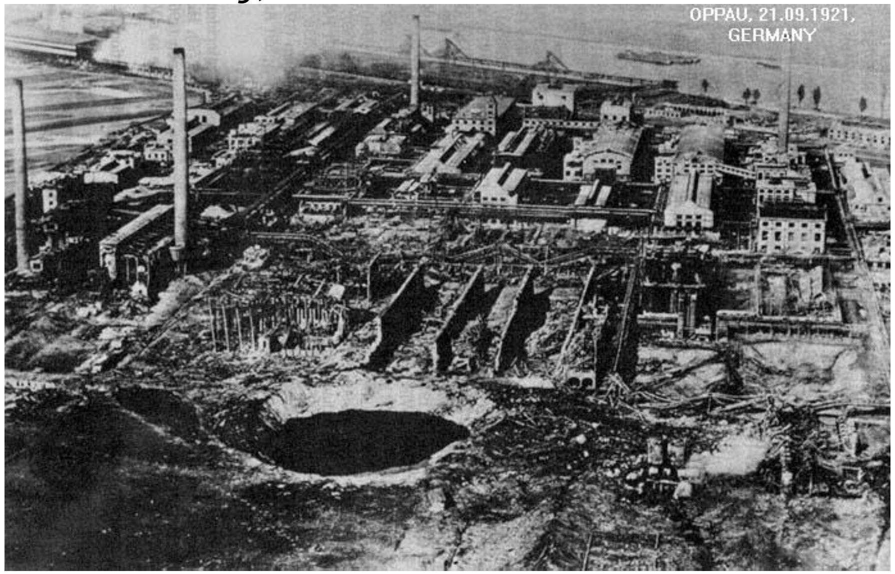
Gainville, O. *et al.*, 2010, chapter 18 in *Infrasound monitoring for atmospheric studies*, A. Le Pichon, E. Blanc, A. Hauchecorne Eds., *Springer Dordrecht*

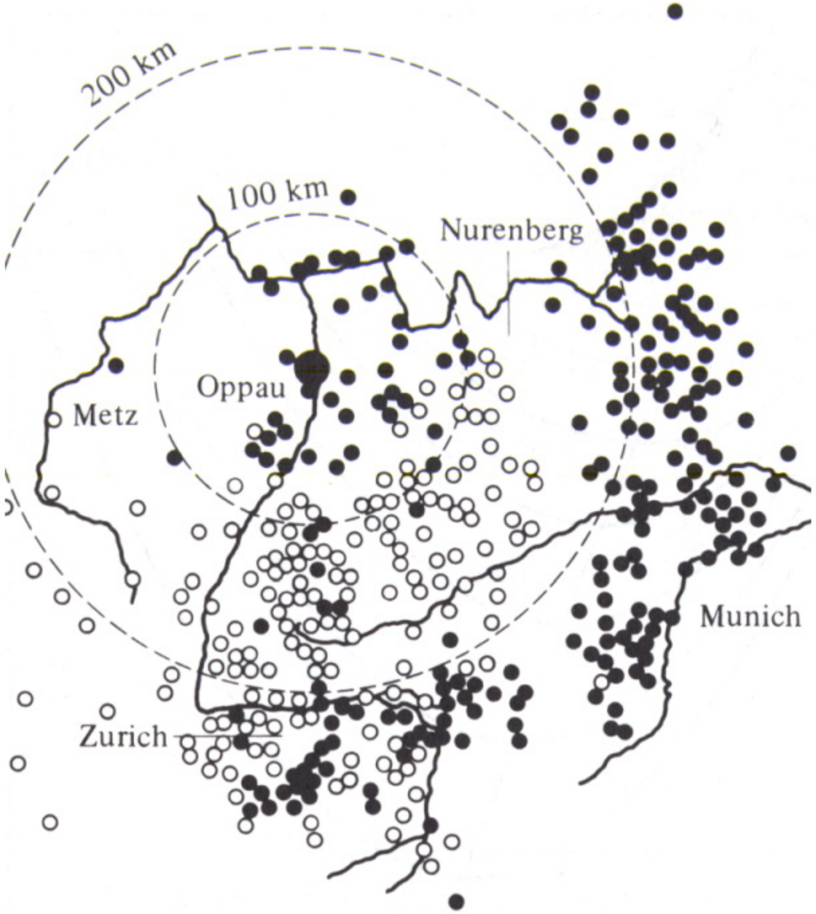
Mean flow effects on sound propagation

Explosion at Oppau, Germany, on sept. 21 1921 (561 deaths)

Locations where sound was heard ●
and not heard ○

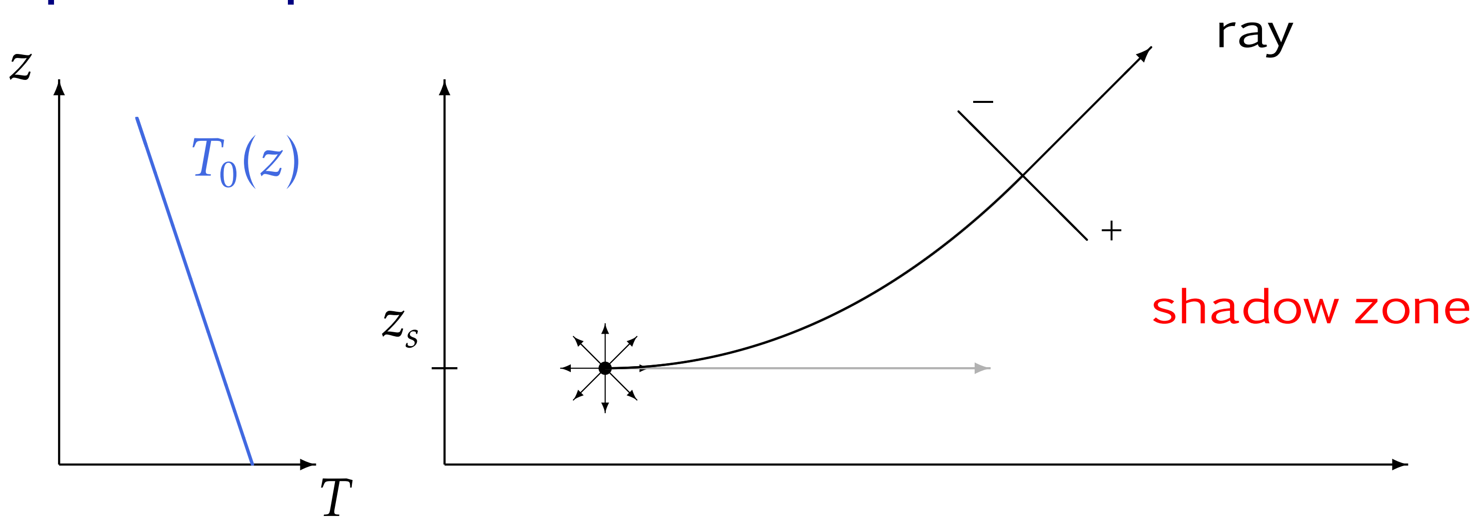
BASF factory, ammonium nitrate



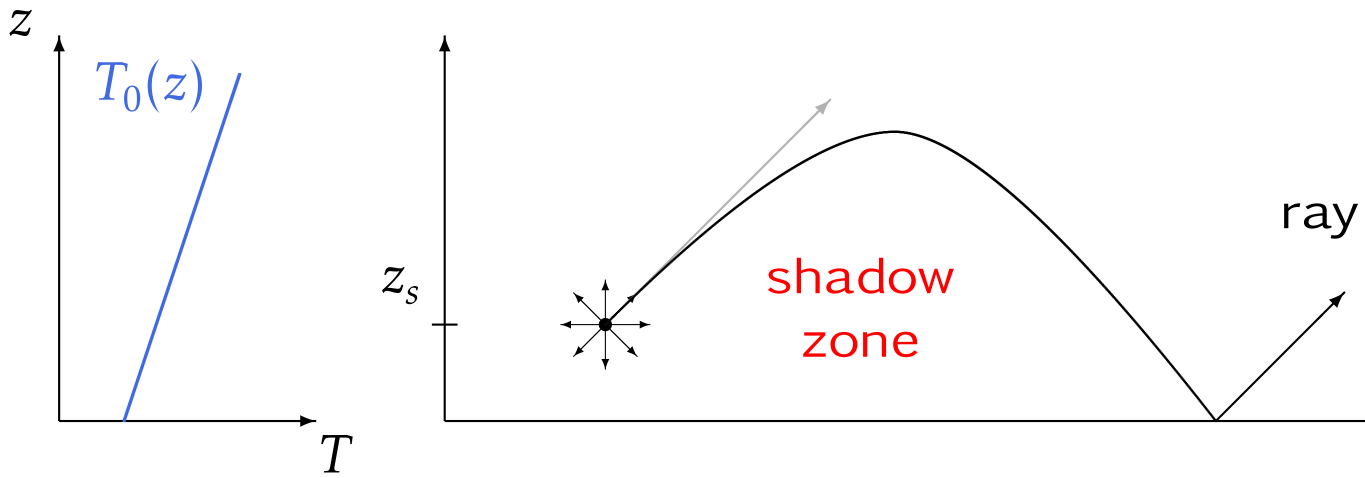


Cook, R.K., 1962, Strange sounds in the Atmosphere, *Sound*, **1**(2)

- **Standard temperature profile**



- **Temperature profile inversion (pollution)**



Geometrical (or kinematic) wave theory

Hayes (*Proc. Roy. Soc. Lond.* 1970), Candel (*J. Fluid Mech.*, 1977)

- On cherche une solution pour $\lambda_a/L \to 0$, en espérant obtenir des équations simples à résoudre numériquement (temps restitution court) tout en pouvant traiter des configurations relativement complexes : inhomogénéïtés du milieu, gradients de vitesse et réflexions (spéculaires) sur des surfaces.

- Propagation sur de très grandes distances, ~ 100 à 1000 km, en milieu stratifié comme dans l'océan ou bien dans l'atmosphère : difficile actuellement d'utiliser une résolution directe des équations d'Euler linéarisées par exemple.

► **high-performance computing**

High-frequency solution of linearized Euler's equations

Non homogeneous medium, $c_0 = c_0(\boldsymbol{x})$ or/and $\boldsymbol{u}_0 = \boldsymbol{u}_0(\boldsymbol{x})$

This leads to partial differential equations with non-constant coefficients, and thus, it is no longer possible to apply a simple Fourier transform.

Extention of Fourier's solutions for a medium with slowly varying properties with respect to the wavelength : geometrical or high frequency approximation

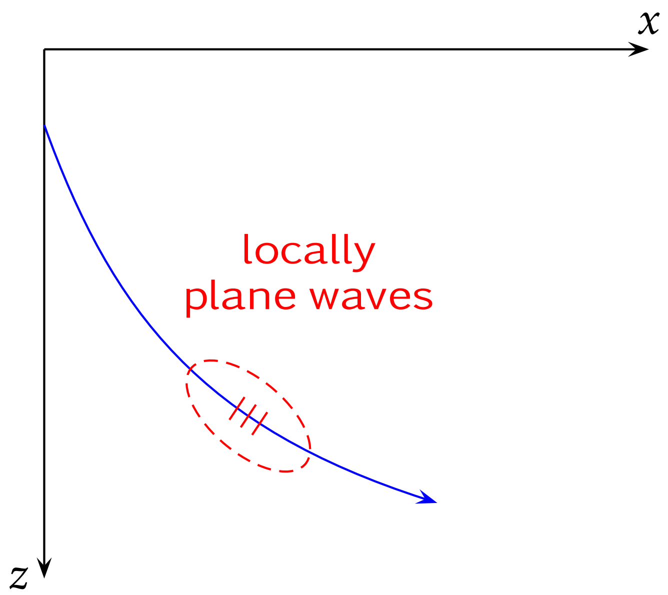
The solution is now sought as a local plane wave,

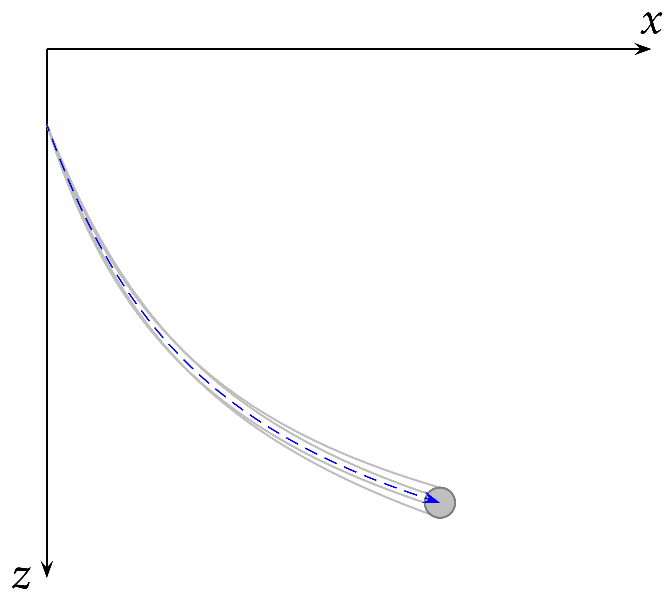
$$p'(\boldsymbol{x}, t) = \tilde{p}(\boldsymbol{x}) e^{i\Theta} \qquad \Theta = \Theta(\boldsymbol{x}, t) \quad \text{phase}$$

wavenumber vector $\boldsymbol{k} = \nabla\Theta$
angular frequency $\omega = -\partial_t \Theta$

The amplitude $\tilde{p}(\boldsymbol{x})$ and the wave number $\boldsymbol{k}(\boldsymbol{x}) = \nabla\Theta$
vary slowly with position $\boldsymbol{x}$ on scale $\lambda_a = 2\pi/k$, or equivalently $\lambda_a/L \ll 1$,
$\rightsquigarrow$ locally plane waves

- **Sketch of the propagation of a local plane wave**





Slowly varying medium $\lambda_a \ll L$

Ray tube (to follow the propagation of acoustic energy)

High-frequency solution of linearized Euler's equations

- Dérivée particulaire : $\rho' = \tilde{\rho}(\boldsymbol{x})e^{i\Theta(\boldsymbol{x},t)}$

$$\frac{D_0\rho'}{Dt} = \frac{\partial \rho'}{\partial t} + \boldsymbol{u}_0 \cdot \nabla\rho' = \Big(\underbrace{-i\omega\tilde{\rho} + i\boldsymbol{k}\cdot\boldsymbol{u}_0\tilde{\rho}}_{\text{(a)} \sim \frac{1}{\lambda_a} \sim \omega} + \underbrace{\boldsymbol{u}_0 \cdot \nabla\tilde{\rho}}_{\text{(b)} \sim \frac{1}{L}} \Big) e^{i\Theta}$$

$\rightsquigarrow$ milieu lentement variable, $\lambda_a/L \ll 1$, ou approximation HF : le terme (a) est au moins d'un ordre de grandeur plus grand que le terme (b) (amplitude de l'onde lentement variable)

- On va donc rechercher une solution d'onde plane localement pour les équations d'Euler linéarisées, en isolant les termes dominants, de type (a), par rapport aux autres, de type (b).

Conservation de la masse

$$\begin{cases} \dfrac{\partial \rho'}{\partial t} + \boldsymbol{u}_0 \cdot \nabla \rho' + \boldsymbol{u}' \cdot \nabla \rho_0 + \rho_0 \nabla \cdot \boldsymbol{u}' + \rho' \nabla \cdot \boldsymbol{u}_0 = 0 \\[2ex] (-i\omega + i\boldsymbol{k} \cdot \boldsymbol{u}_0)\tilde{\rho} + \boldsymbol{u}_0 \cdot \nabla \tilde{\rho} + \tilde{\boldsymbol{u}} \cdot \nabla \rho_0 + \rho_0 i\boldsymbol{k} \cdot \tilde{\boldsymbol{u}} + \rho_0 \nabla \cdot \tilde{\boldsymbol{u}} + \tilde{\rho} \nabla \cdot \boldsymbol{u}_0 = 0 \end{cases}$$

$\rightsquigarrow$ termes prépondérants en ω ($\omega \to \infty$)

$$(-i\omega + i\boldsymbol{k} \cdot \boldsymbol{u}_0)\tilde{\rho} + \rho_0 i\boldsymbol{k} \cdot \tilde{\boldsymbol{u}} = A$$

- **Équation d'Euler**
  (en négligeant le terme en ∇p_0, cf. discussion en amont)

$$\begin{cases} \rho_0\left(\dfrac{\partial \boldsymbol{u}'}{\partial t} + \boldsymbol{u}_0 \cdot \nabla \boldsymbol{u}' + \boldsymbol{u}' \cdot \nabla \boldsymbol{u}_0\right) + \nabla p' = 0 \\ \\ \rho_0(-i\omega + i\boldsymbol{k} \cdot \boldsymbol{u}_0)\tilde{\boldsymbol{u}} + \rho_0 \boldsymbol{u}_0 \cdot \nabla \tilde{\boldsymbol{u}} + \rho_0 \tilde{\boldsymbol{u}} \cdot \nabla \boldsymbol{u}_0 + i\boldsymbol{k}\tilde{p} + \nabla \tilde{p} = 0 \end{cases}$$

$\rightsquigarrow$ termes prépondérants en ω ($\omega \rightarrow \infty$)

$$\rho_0(-i\omega + i\boldsymbol{k} \cdot \boldsymbol{u}_0)\tilde{\boldsymbol{u}} + i\boldsymbol{k}\tilde{p} = \boldsymbol{B}$$

Équation de conservation de l'entropie

$$\left\{\begin{aligned} &\frac{\partial s'}{\partial t} + \boldsymbol{u}_0 \cdot \nabla s' + \boldsymbol{u}' \cdot \nabla s_0 = 0 \\ &(-i\omega + i\boldsymbol{k} \cdot \boldsymbol{u}_0)\tilde{s} + \boldsymbol{u}_0 \cdot \nabla \tilde{s} + \tilde{\boldsymbol{u}} \cdot \nabla s_0 = 0 \end{aligned}\right.$$

$\rightsquigarrow$ termes prépondérants en ω ($\omega \to \infty$)

$$(-i\omega + i\boldsymbol{k} \cdot \boldsymbol{u}_0)\tilde{s} = C$$

- **Au final :**

$$\begin{aligned}(-i\omega + i\boldsymbol{k}\cdot\boldsymbol{u}_0)\tilde{\rho} + \rho_0 i\boldsymbol{k}\cdot\tilde{\boldsymbol{u}} &= A\\ \rho_0(-i\omega + i\boldsymbol{k}\cdot\boldsymbol{u}_0)\tilde{\boldsymbol{u}} + i\boldsymbol{k}\tilde{p} &= \boldsymbol{B}\\ (-i\omega + i\boldsymbol{k}\cdot\boldsymbol{u}_0)\tilde{s} &= C\end{aligned}$$

On cherche généralement à éliminer la masse volumique $\tilde{\rho}$ au profit de la pression $\tilde{p}$ et de l'entropie $\tilde{s}$ par l'intermédiaire de l'équation d'état :

$$\tilde{p} = c_0^2\tilde{\rho} + \alpha\tilde{s} \qquad \text{en notant} \qquad \alpha \equiv \left.\frac{\partial p}{\partial s}\right|_{\rho,0}$$

$$(-i\omega + i\boldsymbol{k}\cdot\boldsymbol{u}_0)\tilde{p} - (-i\omega + i\boldsymbol{k}\cdot\boldsymbol{u}_0)\alpha\tilde{s} + \rho_0 c_0^2 i\boldsymbol{k}\cdot\tilde{\boldsymbol{u}} = c_0^2 A$$

$$A \leftarrow c_0^2 A$$

- The system of equations can be recast in matrix form as,

$$\underbrace{\begin{pmatrix} id & \rho_0 c_0^2 i\boldsymbol{k}\cdot & -i\alpha d \\ i\boldsymbol{k} & \rho_0 id & 0 \\ 0 & 0 & id \end{pmatrix}}_{\boldsymbol{H}} \begin{pmatrix} \tilde{p} \\ \tilde{\boldsymbol{u}} \\ \tilde{s} \end{pmatrix} = \underbrace{\begin{pmatrix} A \\ \boldsymbol{B} \\ C \end{pmatrix}}_{\boldsymbol{\Lambda}} \qquad d \equiv \boldsymbol{k}\cdot\boldsymbol{u}_0 - \omega$$

General eikonal method. The amplitude of each variable $(\tilde{p}, \tilde{\boldsymbol{u}}, \tilde{s})$ is then expanded in an asymptotic series in power of the small parameter $\epsilon \sim \lambda_a/L \sim 1/\omega$

$$\tilde{p}(\boldsymbol{x}) = \sum_{n=0}^{\infty} \epsilon^n \, \tilde{p}_n(\boldsymbol{x}) = \tilde{p}_0 + \epsilon\tilde{p}_1 + \dots \qquad \text{caution, } \tilde{p}_0(\boldsymbol{x}) \neq p_0(\boldsymbol{x})$$

By noting that $\boldsymbol{\Lambda}/\boldsymbol{H} = \mathcal{O}(\epsilon)$, we get the following condition at zeroth order,

$$\boldsymbol{H}\left[\tilde{p}_0, \tilde{\boldsymbol{u}}_0, \tilde{s}_0\right]^t = 0 \quad \Longrightarrow \quad \det(\boldsymbol{H}) = 0 \quad \Longrightarrow \quad D(\boldsymbol{k}, \omega, \boldsymbol{x}) = 0$$

- The condition $\det(\boldsymbol{H}) = 0$ for having non-trivial solutions, leads to the **dispersion relation** $D(\boldsymbol{k},\omega,\boldsymbol{x}) = 0$. In 2-D with $\boldsymbol{k} = (k_x, k_y)$

$$\boldsymbol{H} = \begin{pmatrix} id & \rho_0 c_0^2 ik_x & \rho_0 c_0^2 ik_y & -i\alpha d \\ ik_x & \rho_0 id & 0 & 0 \\ ik_y & 0 & \rho_0 id & 0 \\ 0 & 0 & 0 & id \end{pmatrix} \qquad d = \boldsymbol{k}\cdot\boldsymbol{u}_0 - \omega$$

$$\begin{aligned} \text{dét}(\boldsymbol{H}) &= id\Big[id\,(\rho_0 id)^2 - ik_x\rho_0 c_0^2 ik_x \rho_0 id + ik_y(-\rho_0 id \rho_0 c_0^2 ik_y)\Big] \\ &= id\Big[\rho_0^2 id(-d^2 + c_0^2k_x^2 + c_0^2k_y^2)\Big] \end{aligned}$$

Dispersion relation $\quad D(\boldsymbol{k},\omega,\boldsymbol{x}) = (\boldsymbol{k}\cdot\boldsymbol{u}_0 - \omega)^2\Big[c_0^2k^2 - (\boldsymbol{k}\cdot\boldsymbol{u}_0 - \omega)^2\Big] = 0$

$$\boldsymbol{k}\cdot\boldsymbol{u}_0 - \omega = 0 \qquad \text{or} \qquad c_0^2k^2 - (\boldsymbol{k}\cdot\boldsymbol{u}_0 - \omega)^2 = 0$$

- Relation de dispersion : $\boldsymbol{k}\cdot\boldsymbol{u}_0 - \omega = 0$

  - vitesse de groupe : $\boldsymbol{v}_g = \left.\dfrac{\partial\omega}{\partial\boldsymbol{k}}\right|_{\boldsymbol{x}} = \boldsymbol{u}_0$
  - en reportant la relation de dispersion dans le système à l'ordre zéro :

$$\begin{pmatrix} 0 & \rho_0 c_0^2 i\boldsymbol{k}\cdot & 0 \\ i\boldsymbol{k} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \tilde{p}_0 \\ \tilde{\boldsymbol{u}}_0 \\ \tilde{s}_0 \end{pmatrix} = 0$$

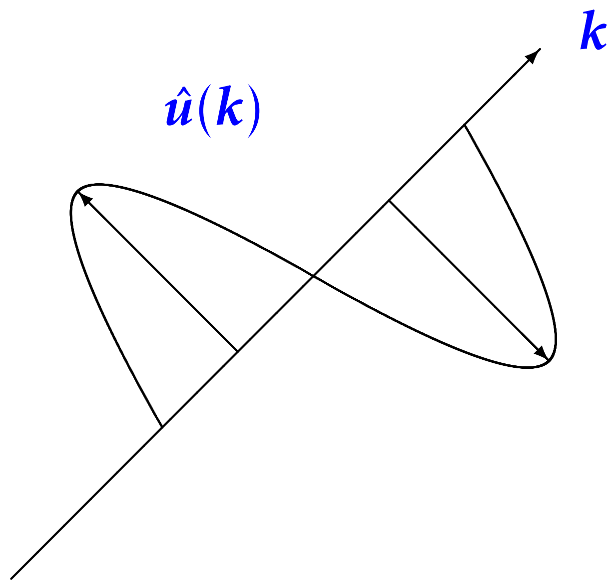
- soit $\tilde{\boldsymbol{u}}_0 \neq 0$ avec $\boldsymbol{k}\cdot\tilde{\boldsymbol{u}}_0 = 0$: fluctuations de vitesse (particulaire) perpendiculaires à la direction de propagation $\boldsymbol{\nu} = \boldsymbol{k}/k$, $\tilde{p}_0 \equiv 0$, $\tilde{s}_0 = 0$ (et $\tilde{\rho}_0 = 0$)
$\rightsquigarrow$ mode tourbillonnaire (ondes transverses, cf. interprétation à la suite)

- soit $\tilde{s}_0 \neq 0$: fluctuations d'entropie avec $\tilde{\rho}_0 = -(\alpha/c_0^2)\,\tilde{s}_0$,
avec $\tilde{\boldsymbol{u}}_0 = 0$ et $\tilde{p}_0 \equiv 0$
$\rightsquigarrow$ mode entropique (convection de fluctuations de température)

Interprétation du mode tourbillonnaire

Des fluctuations de vitesse incompressibles, $\nabla \cdot \boldsymbol{u}' = 0$, sont des perturbations rotationnelles : $\boldsymbol{u}' = \nabla \times \psi$

$$\boldsymbol{u}' = \int \hat{\boldsymbol{u}}(\boldsymbol{k}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}} d\boldsymbol{k} \qquad \Longrightarrow \qquad \nabla \cdot \boldsymbol{u}' = \int i\boldsymbol{k} \cdot \hat{\boldsymbol{u}}(\boldsymbol{k}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}} d\boldsymbol{k} = 0$$



- Relation de dispersion : $c_0^2 k^2 - (\boldsymbol{k}\cdot\boldsymbol{u}_0 - \omega)^2 = 0$ soit $\omega = \boldsymbol{k}\cdot\boldsymbol{u}_0 \pm c_0 k$

  - vitesse de groupe : $\boldsymbol{v}_g = \left.\dfrac{\partial\omega}{\partial\boldsymbol{k}}\right|_{\boldsymbol{x}} = \boldsymbol{u}_0 \pm c_0\boldsymbol{\nu} \qquad \boldsymbol{\nu} \equiv \dfrac{\boldsymbol{k}}{k}$

  - en reportant la relation de dispersion dans le système à l'ordre zéro :

$$\begin{pmatrix} \mp i c_0 k & \rho_0 c_0^2 i\boldsymbol{k}\cdot & \pm i\alpha c_0 k \\ i\boldsymbol{k} & \mp i\rho_0 c_0 k & 0 \\ 0 & 0 & \mp i c_0 k \end{pmatrix}\begin{pmatrix} \tilde{p}_0 \\ \tilde{\boldsymbol{u}}_0 \\ \tilde{s}_0 \end{pmatrix} = 0$$

  et par conséquent $\tilde{s}_0 \equiv 0$ avec,

$$\tilde{\boldsymbol{u}}_0 = \pm\frac{\tilde{p}_0}{\rho_0 c_0}\boldsymbol{\nu} \qquad \tilde{p}_0 = c_0^2\tilde{\rho}_0$$

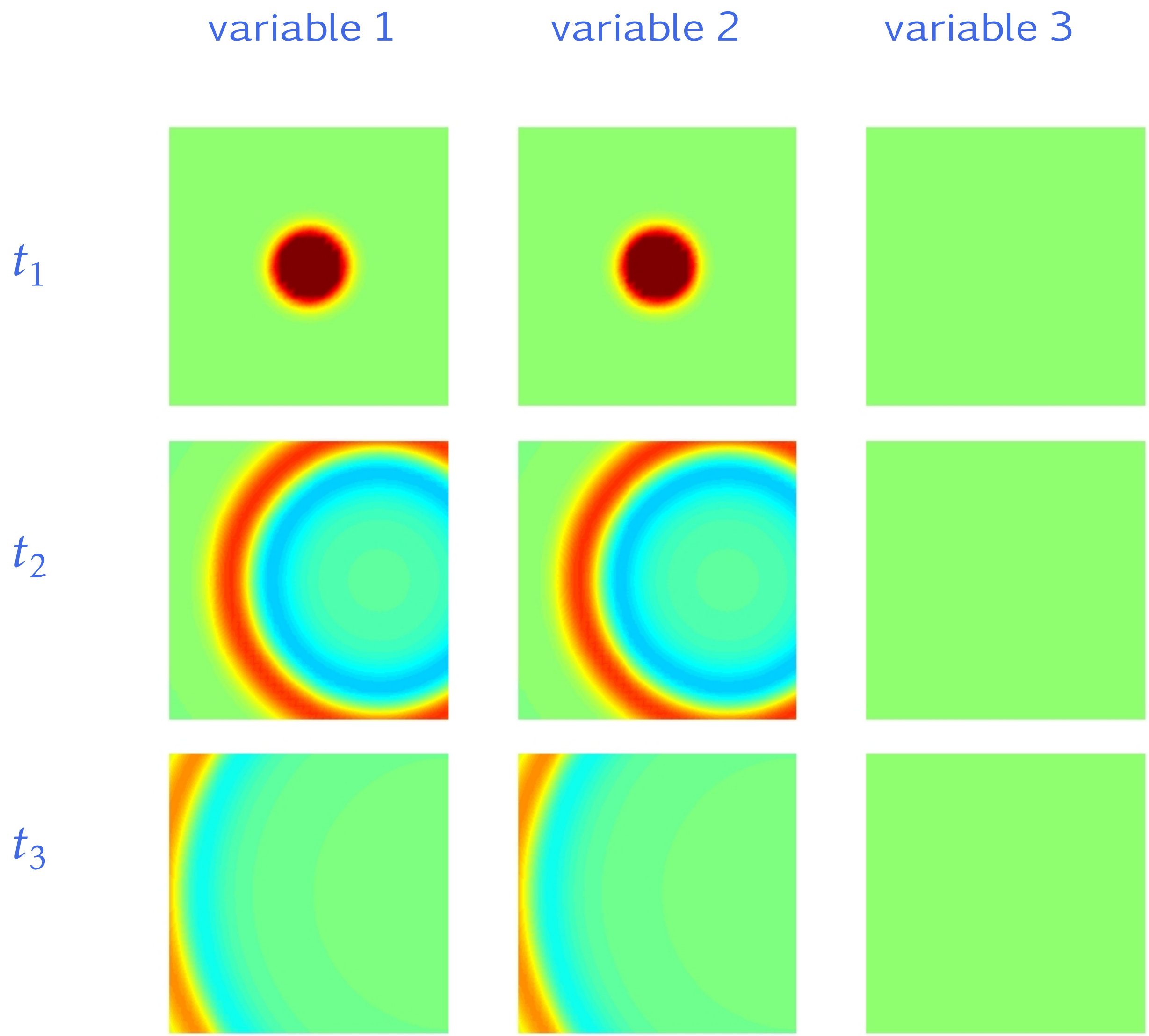
$\rightsquigarrow$ mode acoustique, ondes localement longitudinales, la perturbation de vitesse $\tilde{u}_0$ est colinéaire à $\boldsymbol{k}$ ($\neq$ direction de propagation de l'onde)

- Ces trois modes, *i.e.* le mode acoustique, le mode tourbillonnaire et le mode entropique, sont généralement découplés uniquement dans l'approximation haute fréquence (c'est-à-dire l'acoustique géométrique) ou lorsque l'écoulement de base est homogène $(\lambda_a/L \rightarrow 0)$.

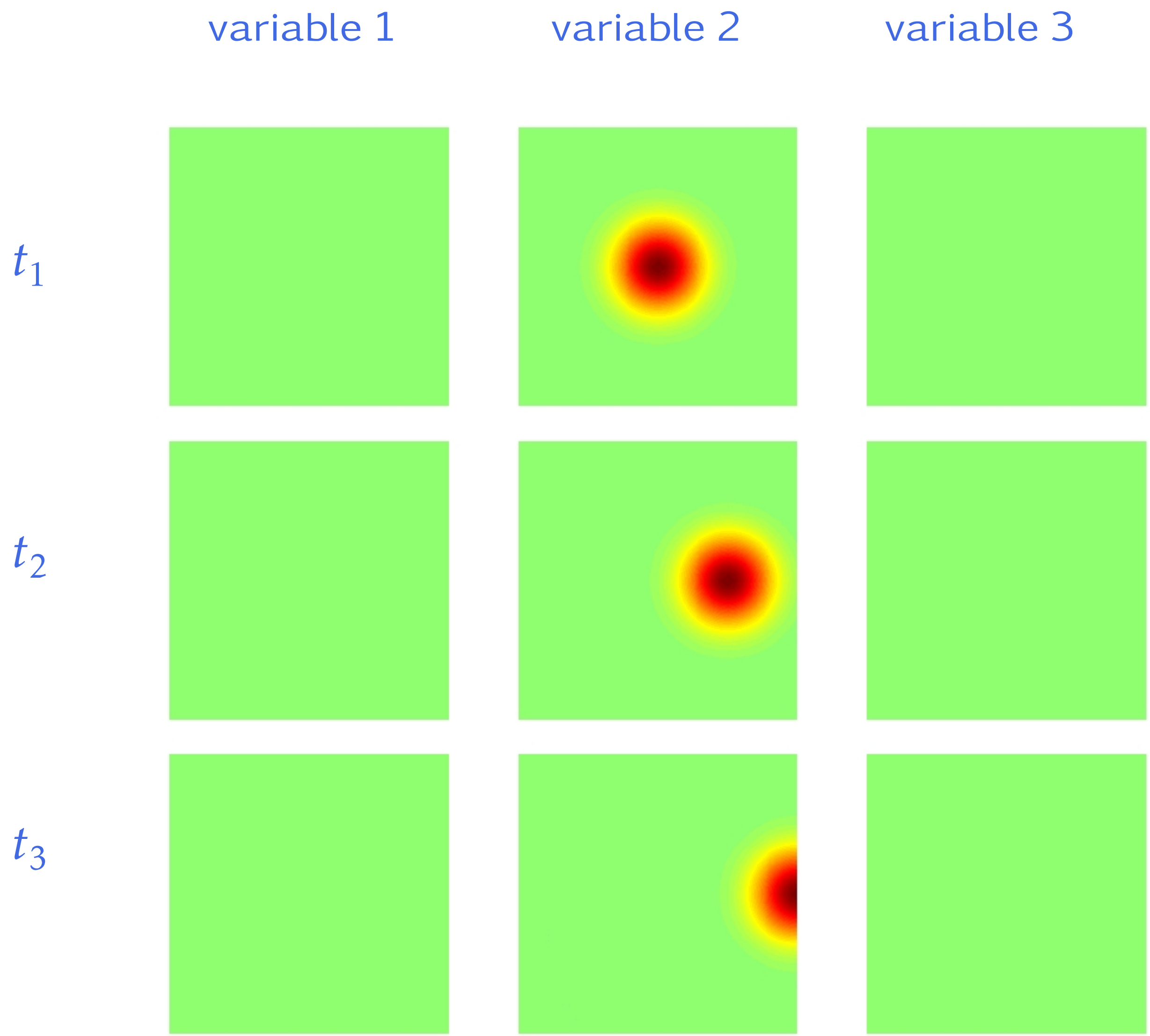
  On a alors un résultat remarquable :

$$\text{solution générale} \quad = \quad \left\{ \begin{array}{c} \text{perturbation acoustique} \\ + \\ \text{perturbation tourbillonnaire} \\ + \\ \text{perturbation entropique} \end{array} \right.$$

- **Test 1**

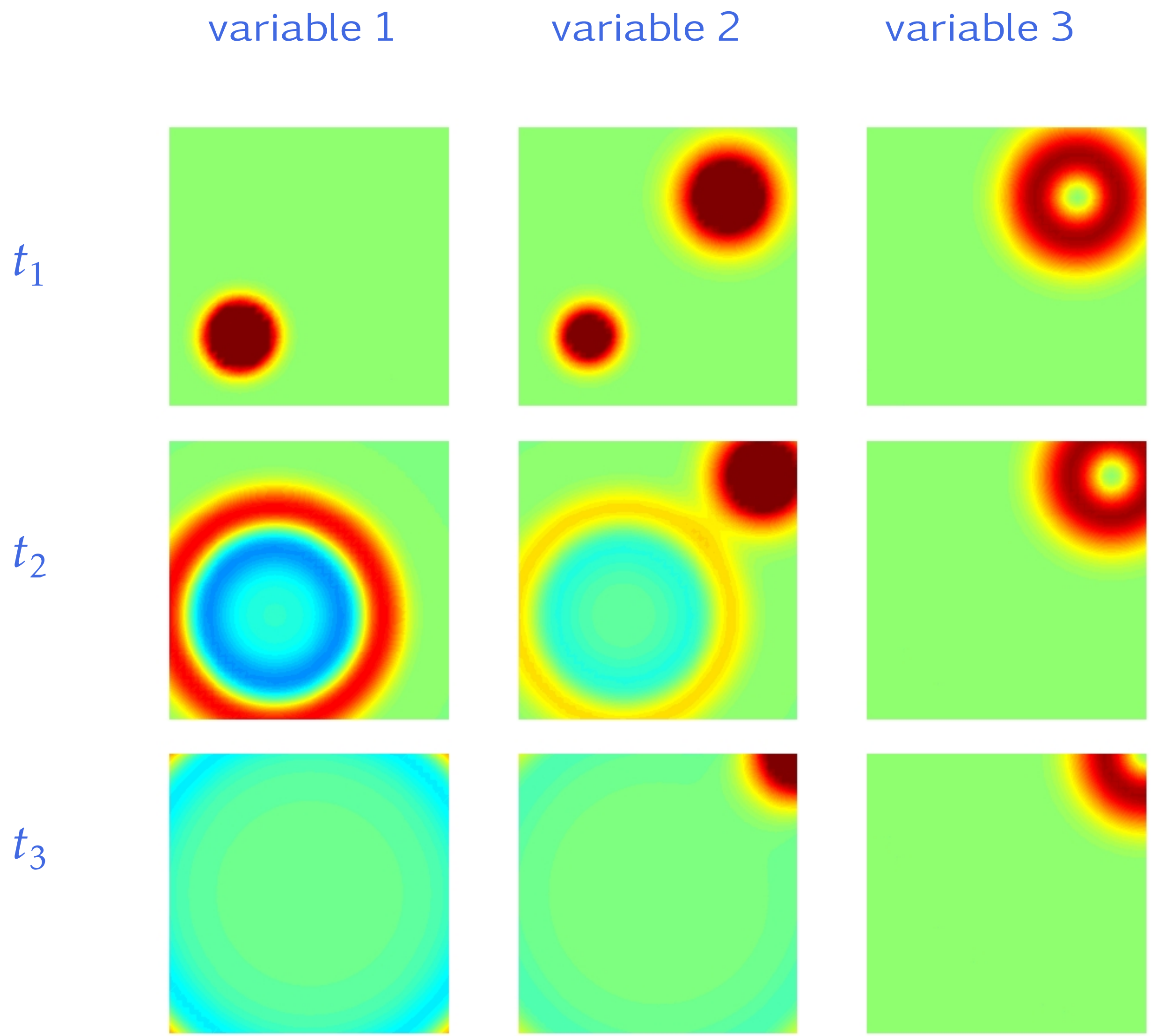


Test 2



- **Test 3**

- **Test 4**

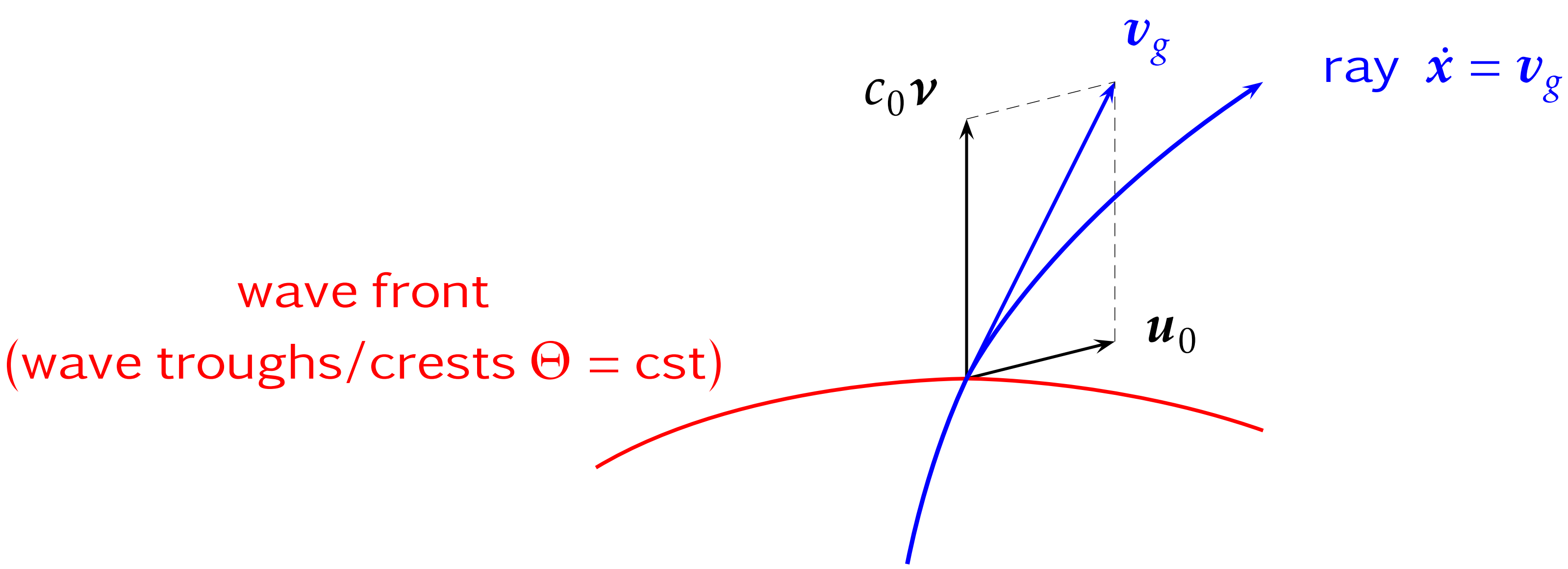


Ray tracing

The dispersion relation $\omega = \boldsymbol{k}\cdot\boldsymbol{u}_0 + c_0 k$ obtained by the eikonal method can be solved by the method of characteristics, commonly called rays and defined as the solution of

$$\dot{\boldsymbol{x}} = \boldsymbol{v}_g \qquad \boldsymbol{v}_g = c_0\boldsymbol{\nu} + \boldsymbol{u}_0 = c_0\left(\boldsymbol{\nu} + \boldsymbol{M}_0\right)$$

$\rightsquigarrow$ rays : curves $\boldsymbol{x}$ tangent to the group velocity at each point, and thus not perpendicular to wavefronts in a moving medium $(\boldsymbol{u}_0 \neq 0)$.



Wave kinematics (Whitham, 1960)

The orientation of the normal vector $\boldsymbol{\nu} = \boldsymbol{k}/k$ to the wavefront must be determined along the ray path, through the evolution of $\boldsymbol{k}(\boldsymbol{x}, t)$ along this ray, that is $\partial_t \boldsymbol{k} + \boldsymbol{v}_g \cdot \nabla \boldsymbol{k} = \ ?$

Dispersion relation $\omega = \Omega(\boldsymbol{k}, \boldsymbol{x})$

$$\begin{cases} \dfrac{\partial \omega}{\partial t} = \left.\dfrac{\partial \Omega}{\partial t}\right|_{\boldsymbol{k},\boldsymbol{x}} + \nabla_{\boldsymbol{k}}\Omega \cdot \dfrac{\partial \boldsymbol{k}}{\partial t} = \nabla_{\boldsymbol{k}}\Omega \cdot \dfrac{\partial \boldsymbol{k}}{\partial t} \\ \boldsymbol{v}_g \equiv \nabla_{\boldsymbol{k}}\Omega \qquad \dfrac{\partial \boldsymbol{k}}{\partial t} = \dfrac{\partial}{\partial t}\nabla\Theta = \nabla\dfrac{\partial \Theta}{\partial t} = -\nabla\omega \qquad \Longrightarrow \qquad \dfrac{\partial \omega}{\partial t} + \boldsymbol{v}_g \cdot \nabla\omega = 0 \end{cases}$$

The angular frequency is simply convected along rays $\dot{\boldsymbol{x}} = \boldsymbol{v}_g$ if the medium is independent of time (time-independent mean flow)

Wave kinematics

In a similar way, one has for the wavevector $\boldsymbol{k}$

$$\frac{\partial k_i}{\partial t} = \frac{\partial}{\partial t}\frac{\partial \Theta}{\partial x_i} = \frac{\partial}{\partial x_i}\frac{\partial \Theta}{\partial t} = -\frac{\partial \omega}{\partial x_i} = -\left.\frac{\partial \Omega}{\partial x_i}\right|_{\boldsymbol{k}} - \frac{\partial \Omega}{\partial \boldsymbol{k}}\frac{\partial \boldsymbol{k}}{\partial x_i} = -\left.\frac{\partial \Omega}{\partial x_i}\right|_{\boldsymbol{k}} - \underline{v_{gj}\frac{\partial k_j}{\partial x_i}}$$

In order to form the material derivative with the last term, it should be noted that $\nabla \times \boldsymbol{k} = 0$ by construction since $\boldsymbol{k} = \nabla \Theta$. It yields

$$\frac{\partial k_j}{\partial x_i} - \frac{\partial k_i}{\partial x_j} = 0 \quad \Longrightarrow \quad \underline{v_{gj}\frac{\partial k_j}{\partial x_i}} = v_{gj}\frac{\partial k_i}{\partial x_j} = \boldsymbol{v}_g \cdot \nabla k_i$$

and the transport equation can be rewritten

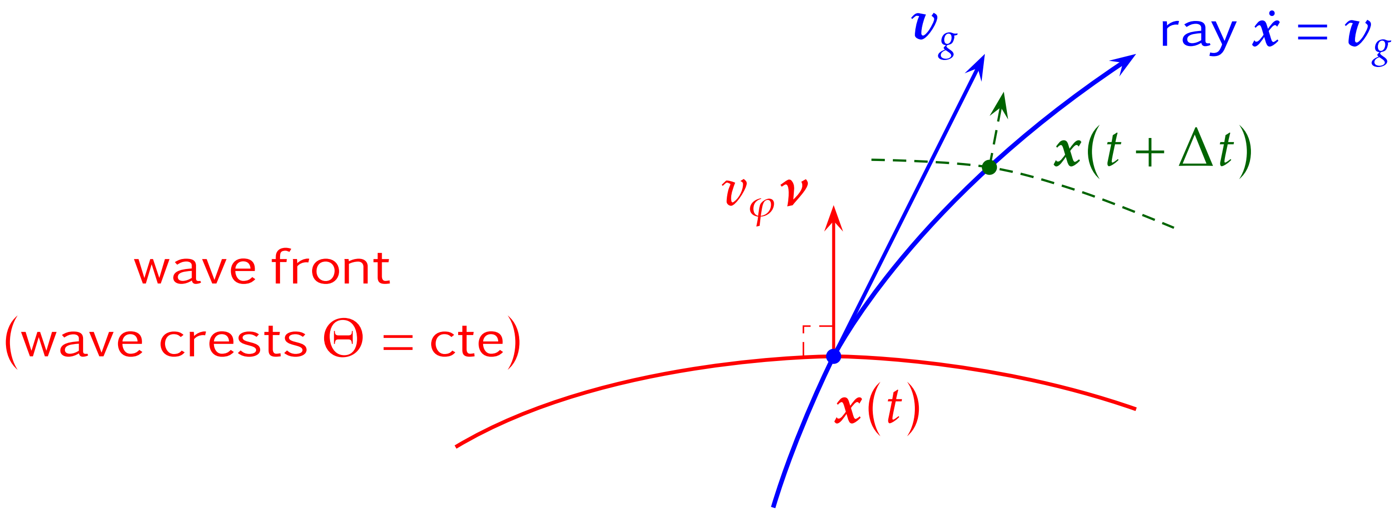
$$\frac{\partial \boldsymbol{k}}{\partial t} + \boldsymbol{v}_g \cdot \nabla \boldsymbol{k} = -\nabla \Omega|_{\boldsymbol{k}}$$

where the term $\nabla \Omega|_{\boldsymbol{k}}$ is linked to the explicit dependence on space of the medium and thus to refraction effects.

- **Ray tracing equations** (for a medium independent of time)
  System of differential equations to solve

$$\begin{cases} \dfrac{d\boldsymbol{x}}{dt} = \boldsymbol{v}_g & (1) \\[2ex] \dfrac{d\boldsymbol{k}}{dt} = -\nabla\Omega|_{\boldsymbol{k}} & (2) \end{cases}$$

Eq. (1) provides rays whereas Eq. (2) provides the orientation of wave fronts along the rays. Refraction effects are included in the term $-\nabla\Omega$ (frequency remains constant along these rays : steady medium)



Ray tracing equations

$$\left\{\begin{aligned} \frac{dx_i}{dt} &= c_0\frac{k_i}{k} + u_{0i} \\ \frac{dk_i}{dt} &= -k\frac{\partial c_0}{\partial x_i} - k_j\frac{\partial u_{0j}}{\partial x_i} \end{aligned}\right.$$

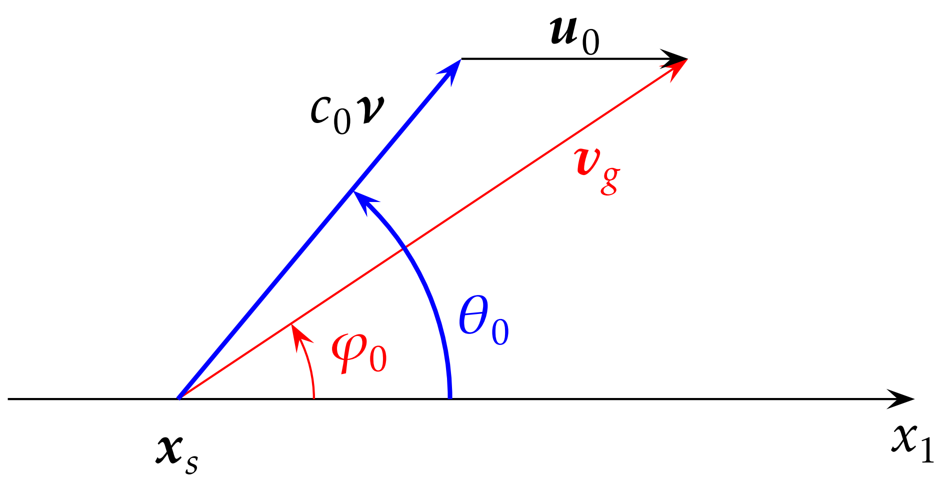
Initial boundary conditions (2-D)

$\boldsymbol{x} = \boldsymbol{x}_s$ (source position)
$\boldsymbol{k} = k_0(\cos\theta_0, \sin\theta_0)$

(note the ray-tracing is independent of ω ...)

Orientation of the wavefront,
with shooting angle θ_0,
$M_0 = u_0(\boldsymbol{x}_s)/c_0$,

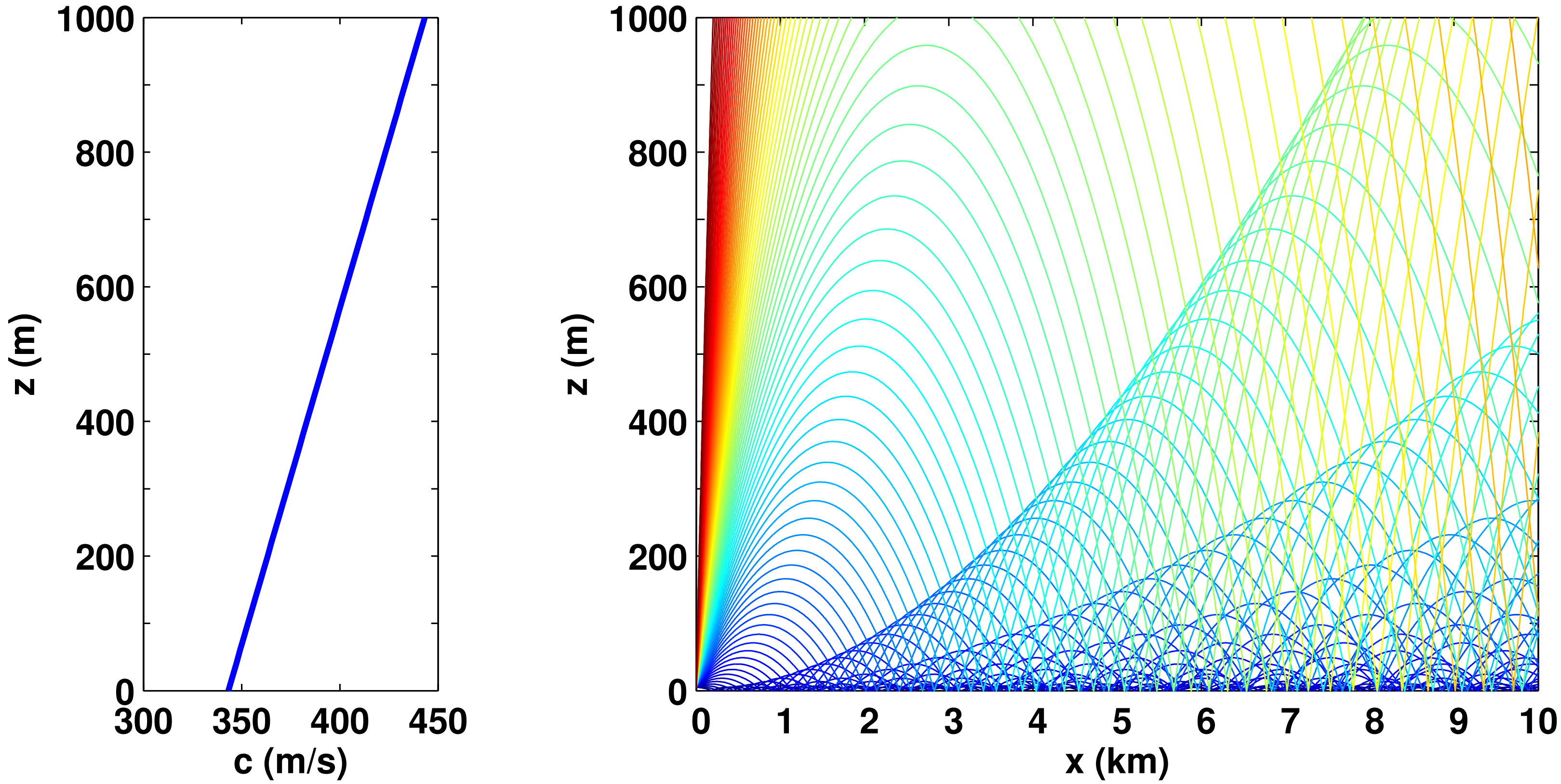
$$\cos\varphi_0 = \frac{M_0 + \cos\theta_0}{\sqrt{(M_0 + \cos\theta_0)^2 + \sin^2\theta_0}}$$



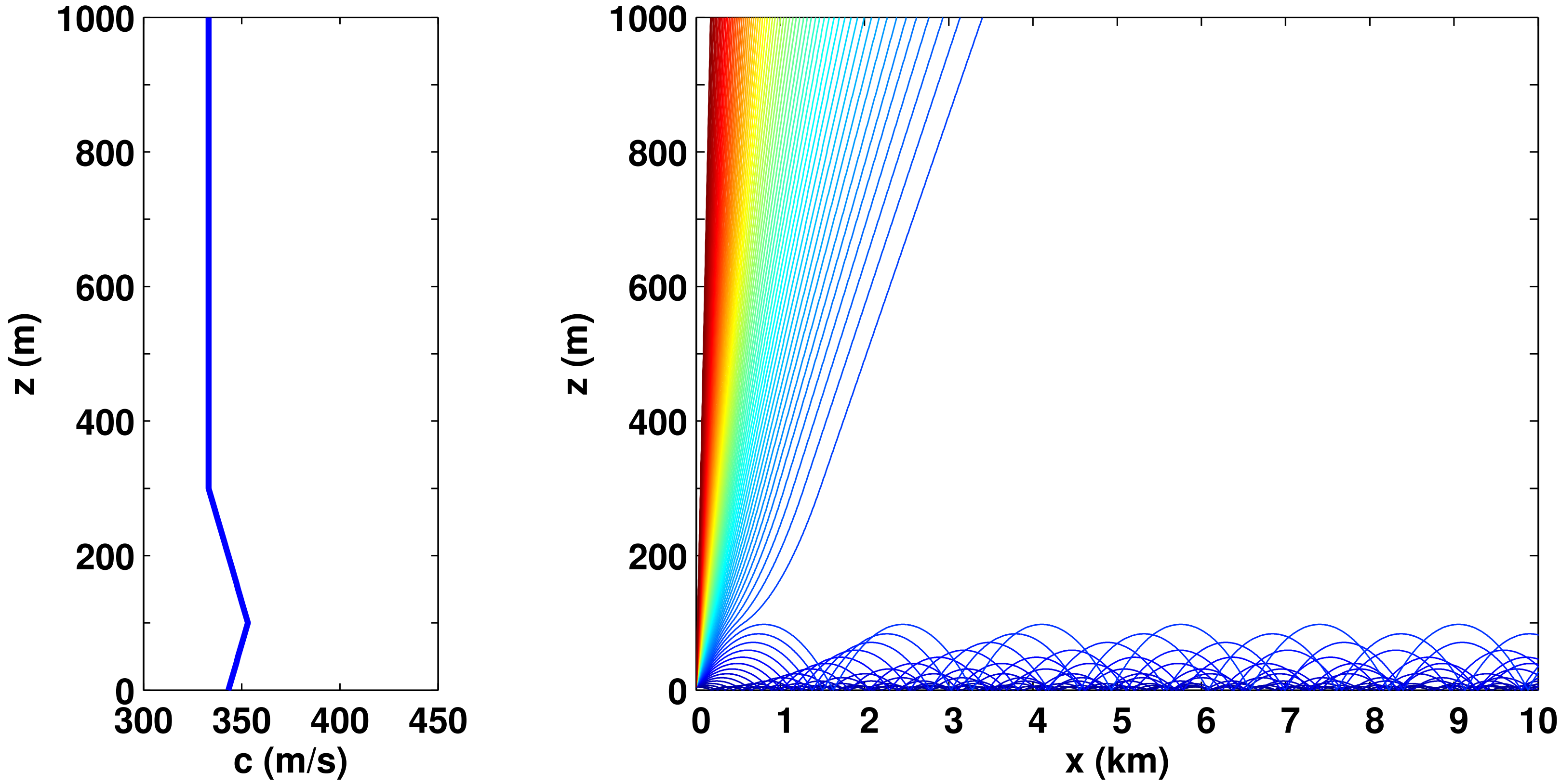
(velocity along the x_1 axis)

Outdoor sound propagation

$\rightsquigarrow$ strong positive sound speed gradient of $0.1\ \mathrm{s}^{-1}$

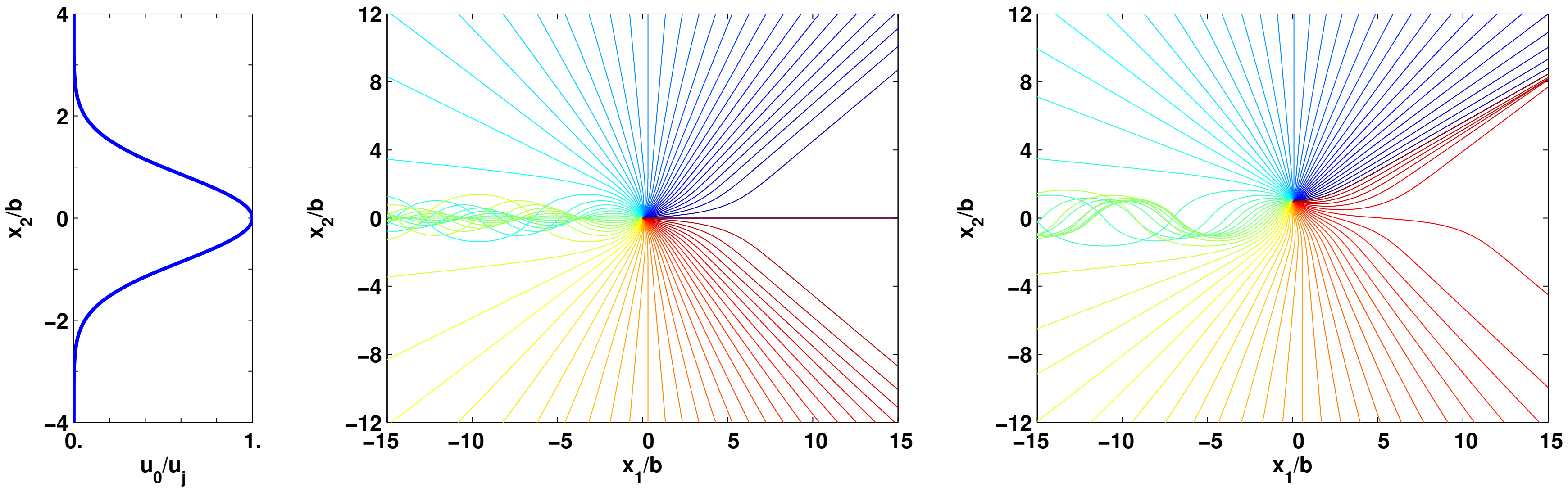


- **Outdoor sound propagation**
  $\rightsquigarrow$ more typical sound speed profile
  test case #4 from Attenborough *et al.* (1995)



Gaussian subsonic jet

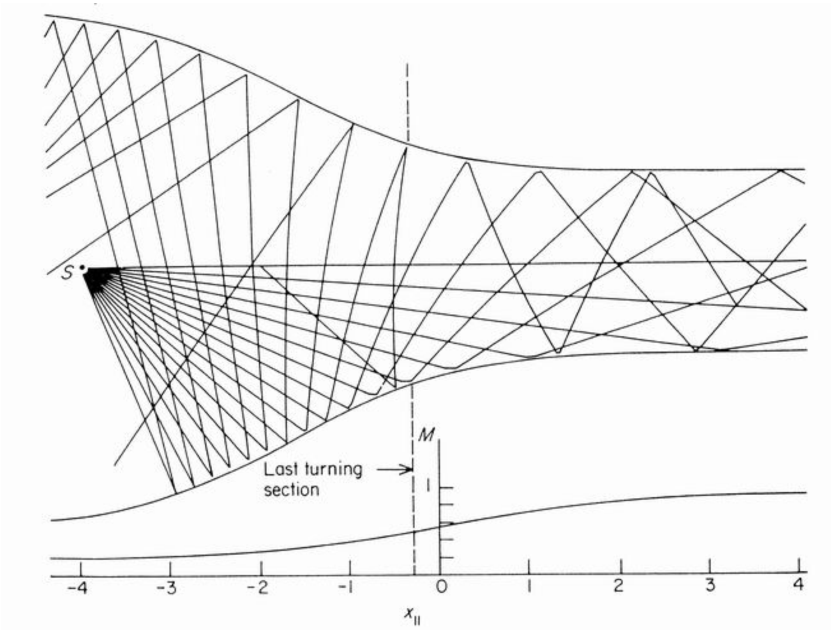
$$u_{01}/u_j = \exp\left(-\ln 2\,(x_2/b)^2\right) \qquad M_j = u_j/c_\infty = 0.5$$

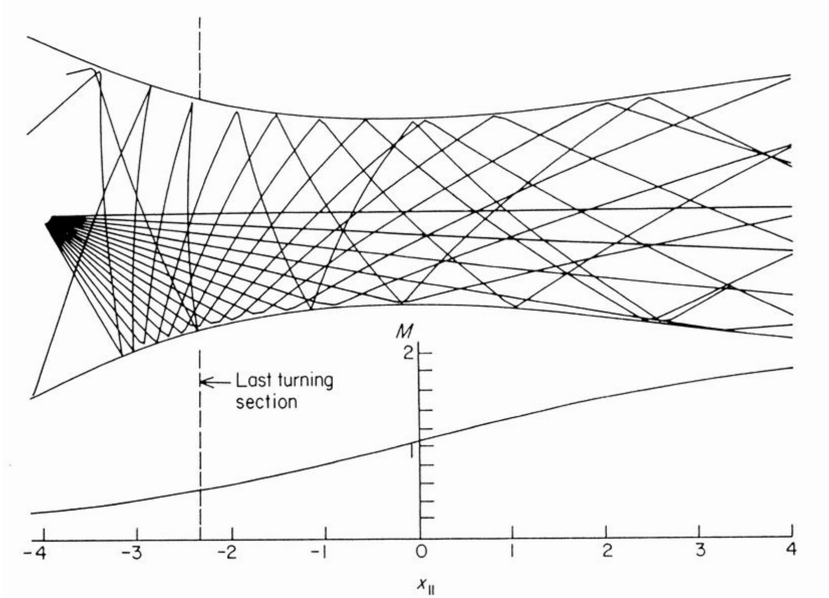


$\boldsymbol{x}_s = (0, 0)$ $\qquad$ $\boldsymbol{x}_s = (0, b)$

Sound propagation in nozzles

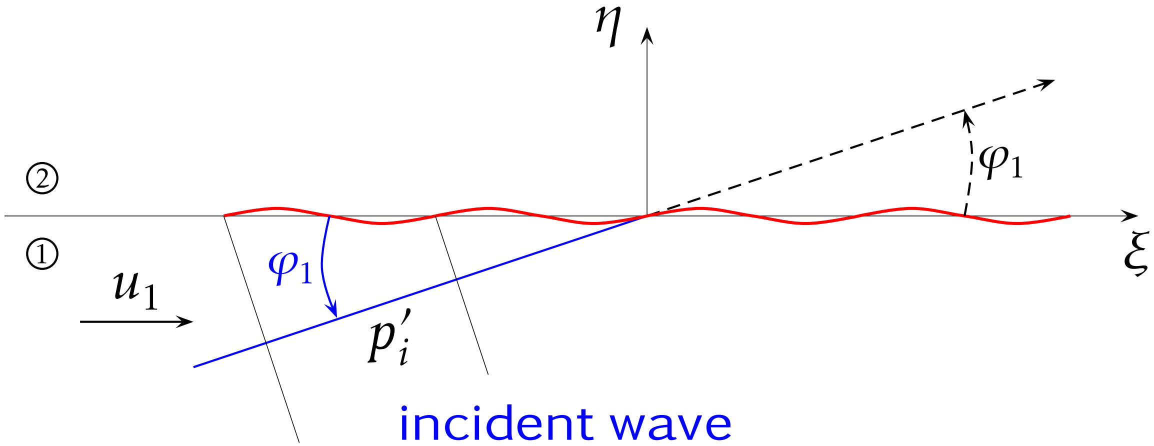
Candel (*J. Sound Vib.*, 1975)





tuyère subsonique tuyère supercritique

Transmission (and reflexion) of sound between two media



$$\frac{D^2 p_i'}{Dt^2} - c_1^2 \nabla^2 p_i' = 0$$

$$D/Dt \equiv \partial/\partial t + u_1 \partial/\partial x_1$$

$$p_i' = \tilde{p}_i e^{i(\boldsymbol{k}\cdot\boldsymbol{x} - \omega t)} \qquad \boldsymbol{x} \equiv (\xi, \eta)$$

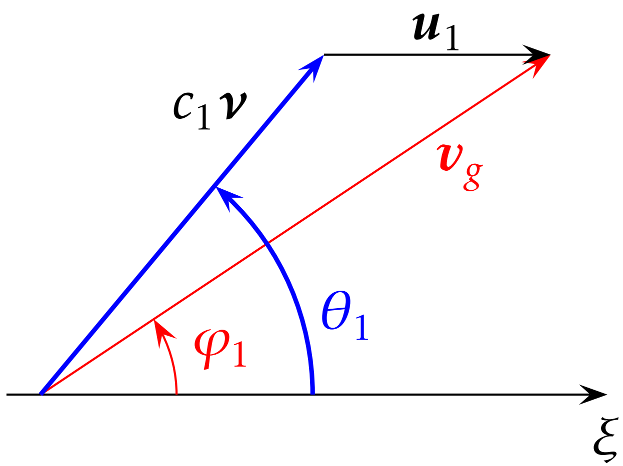
$\omega = \boldsymbol{k}\cdot\boldsymbol{u}_1 + c_1 k$ (and direction of $\boldsymbol{k}$ is unchanged)

$\omega = k(\boldsymbol{\nu}\cdot\boldsymbol{u}_1 + c_1) = kc_1(1 + M_1\cos\varphi_1)$

$D_1 = 1 + M_1 \cos\varphi_1$ Doppler factor

$k_1 = \omega/c_1$ wavenumber when no flow, $k_1 = kD_1$

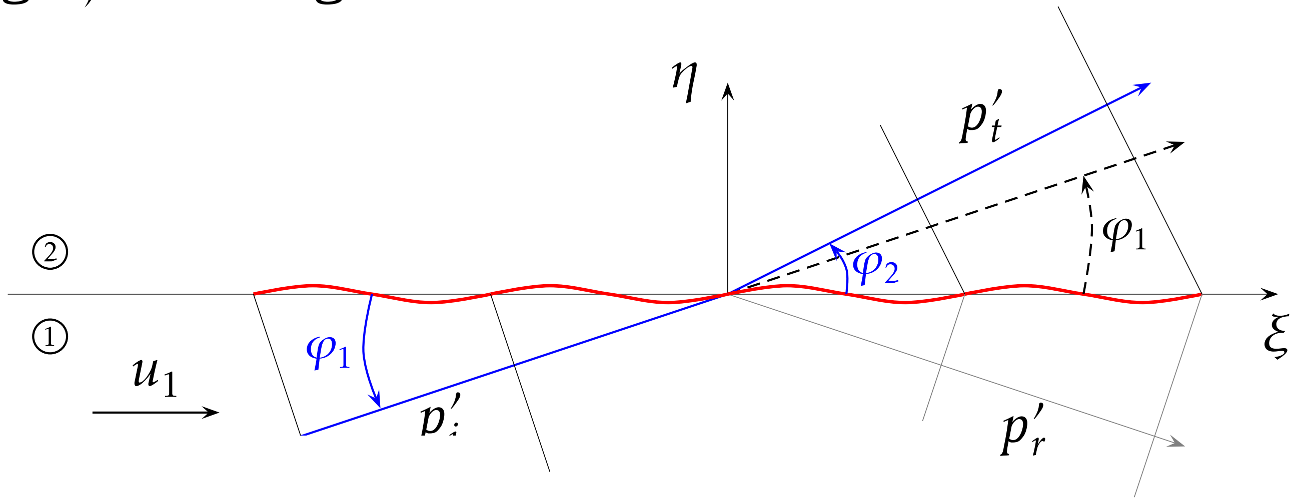
$$p_i' = \tilde{p}_i e^{i\left(\frac{k_1\cos\varphi_1}{D_1}\xi + \frac{k_1\sin\varphi_1}{D_1}\eta - \omega t\right)}$$

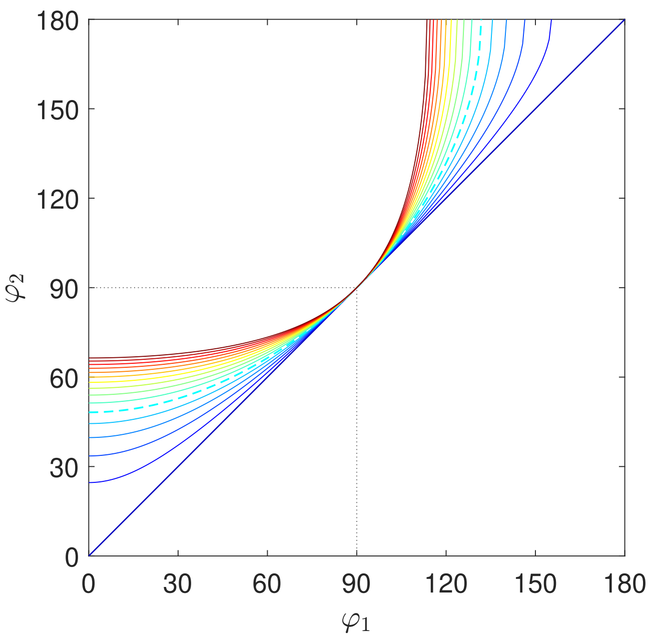


Transmission (and reflexion) of sound between two media (cont.)

Phase velocity (wavelength) matching at the interface

$$c_1\left(\frac{1}{\cos\varphi_1}+M_1\right)=\frac{c_2}{\cos\varphi_2}$$





$$\cos\varphi_2=\frac{\cos\varphi_1}{1+M_1\cos\varphi_1}\qquad(0\leq\varphi_1\leq\pi, c_2=c_1)$$

$$\varphi_{2\mathrm{min}}=\frac{1}{1+M_1}$$ for $\varphi_1=0^\circ$ (grazing incidence)

$\varphi_2=f(\varphi_1)$ for $M_1=0:0.1:1.5$

Eikonal method

- The zeroth order approximation provides the **dispersion relation** (integrated by the method of characteristics – ray tracing)

$$\boldsymbol{H}\begin{pmatrix}\tilde{p}_0\\ \tilde{\boldsymbol{u}}_0\\ \tilde{s}_0\end{pmatrix} = 0 \qquad \Longrightarrow \qquad D(\boldsymbol{k},\omega,\boldsymbol{x}) = 0$$

- Determination of the **amplitudes**

$$\boldsymbol{H}\begin{pmatrix}\tilde{p}_1\\ \tilde{\boldsymbol{u}}_1\\ \tilde{s}_1\end{pmatrix} = \boldsymbol{\Lambda}(\tilde{p}_0,\tilde{\boldsymbol{u}}_0,\tilde{s}_0) \qquad \text{with} \qquad \det(\boldsymbol{H}) = 0$$

The first order provides an **energy transport equation** (written in a conservative form for the *wave action* density)

Amplitude of the acoustic mode

- The first order approximation yields,

$$\boldsymbol{H}\begin{pmatrix} \tilde{p}_1 \\ \tilde{\boldsymbol{u}}_1 \end{pmatrix} = \boldsymbol{\Lambda}_0 \qquad \text{with} \qquad \boldsymbol{H} = \begin{pmatrix} -ic_0k & \rho_0 c_0^2 i\boldsymbol{k}\cdot \\ i\boldsymbol{k} & -\rho_0 c_0 ik \end{pmatrix}$$

where $\boldsymbol{\Lambda}_0 = \boldsymbol{\Lambda}(\tilde{p}_0, \tilde{\boldsymbol{u}}_0)$, and under the condition $\det(\boldsymbol{H}) = 0$ associated with the dispersion relation $\omega = \boldsymbol{k} \cdot \boldsymbol{u}_0 + c_0 k$

- The system is singular since the rank of the scalar matrix is 2 : a non-trivial solution is then obtained by cancelling the bordering determinant, obtained by replacing the first column of the matrix $\boldsymbol{H}$ by the right-hand member $\boldsymbol{\Lambda}_0$. One finds

$$\nabla \cdot (E\boldsymbol{v}_g) = 0 \qquad E = E_0(1 + \boldsymbol{M}_0 \cdot \boldsymbol{\nu}) \qquad E_0 = \frac{\tilde{p}_0^2}{\rho_0 c_0^2}$$

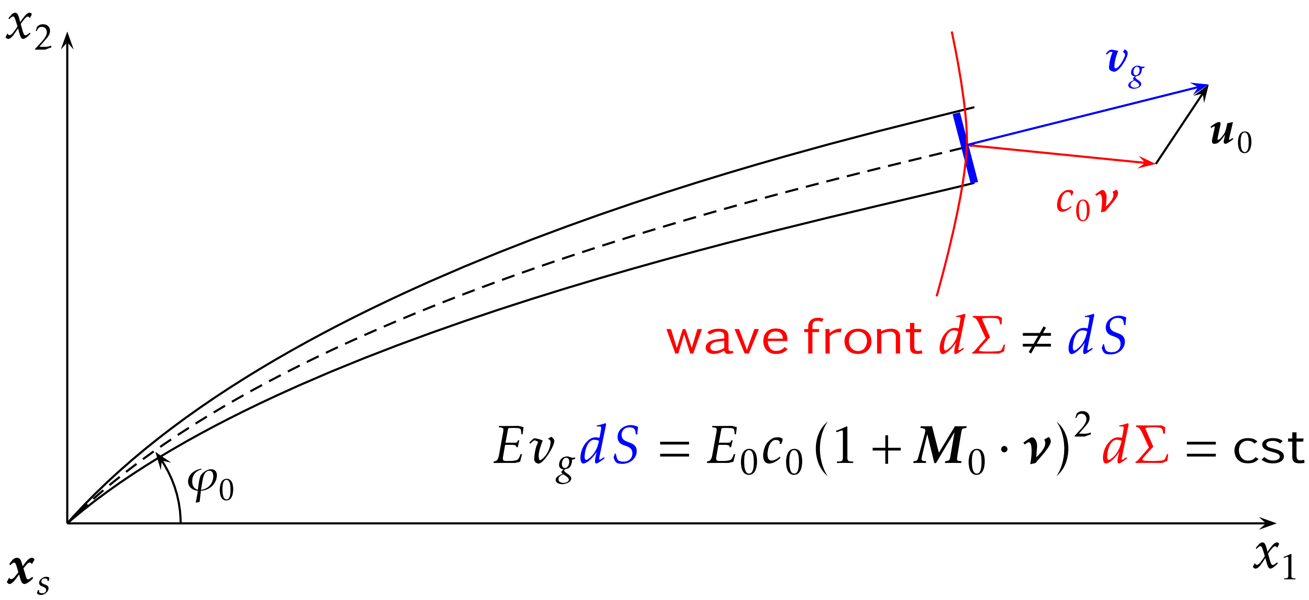
$$\boldsymbol{I} = E\boldsymbol{v}_g = E_0 c_0 (1 + \boldsymbol{M}_0 \cdot \boldsymbol{\nu})(\boldsymbol{\nu} + \boldsymbol{M}_0) \quad \equiv \quad \text{acoustic flux}$$

Amplitude of the acoustic mode

Conservation of the acoustic flux in a ray tube

$$\int_S E\,\boldsymbol{v}_g \cdot \boldsymbol{n}\,dS = 0 \qquad \boldsymbol{n} = \frac{\boldsymbol{v}_g}{v_g} \qquad E c_0\,|\boldsymbol{M}_0 + \boldsymbol{\nu}|\,dS = \text{cst}$$

The calculation of the surface element dS allows to determine the local amplitude of the acoustic field, not always easy to implement in practice.



Advantages of ray tracing over other methods

- Easy to implement to get a full picture of the sound field pattern
- Describe sound propagation in a general medium
  (mean flow with gradient, 3-D, ...)

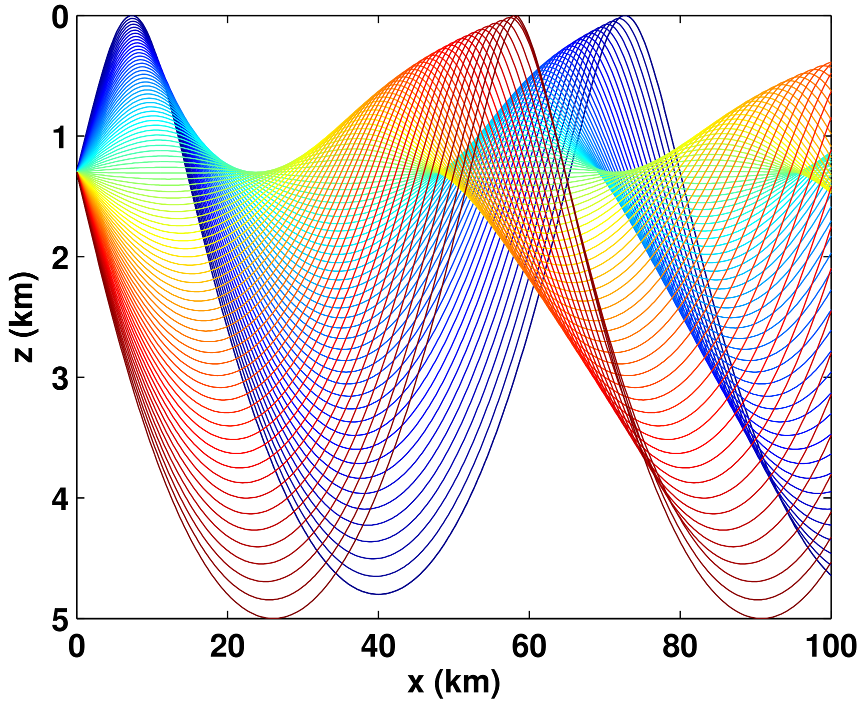
Difficulties

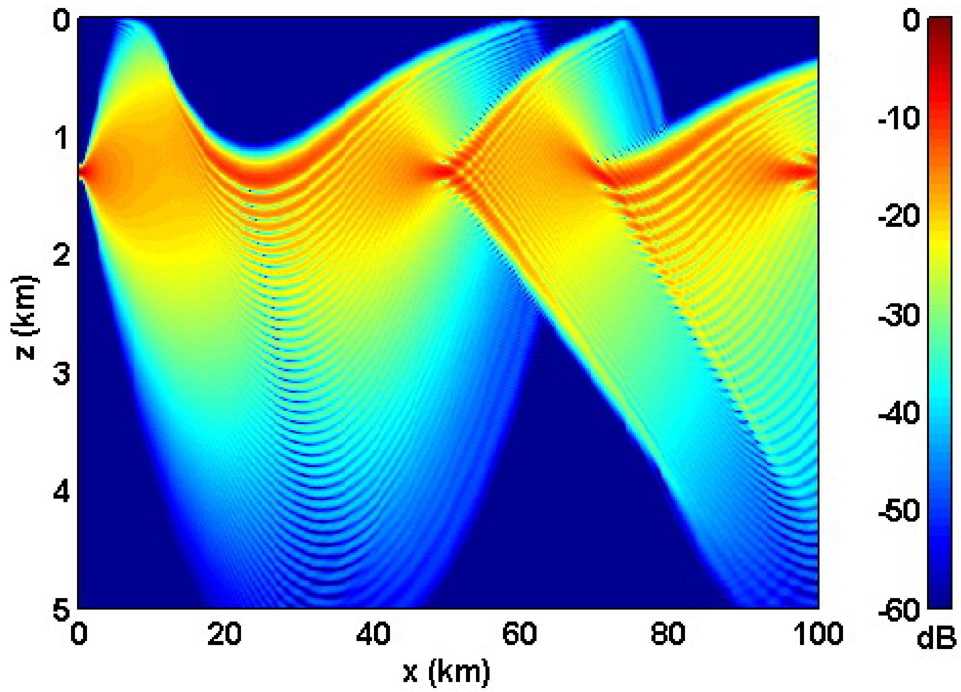
- Computation of the acoustic intensity is often cumbersome
- Singularities with the treatment of cautics
- By hypothesis, specular approach for reflexion at solid boundaries : no diffraction.

  The **Geometrical Theory of Diffraction** (GTD) provides a possible extension of the usual ray-tracing (geometrical acoustics), *e.g.* to include scattering from a wedge.

Ray-tracing versus parabolic approximation in underwater acoustics

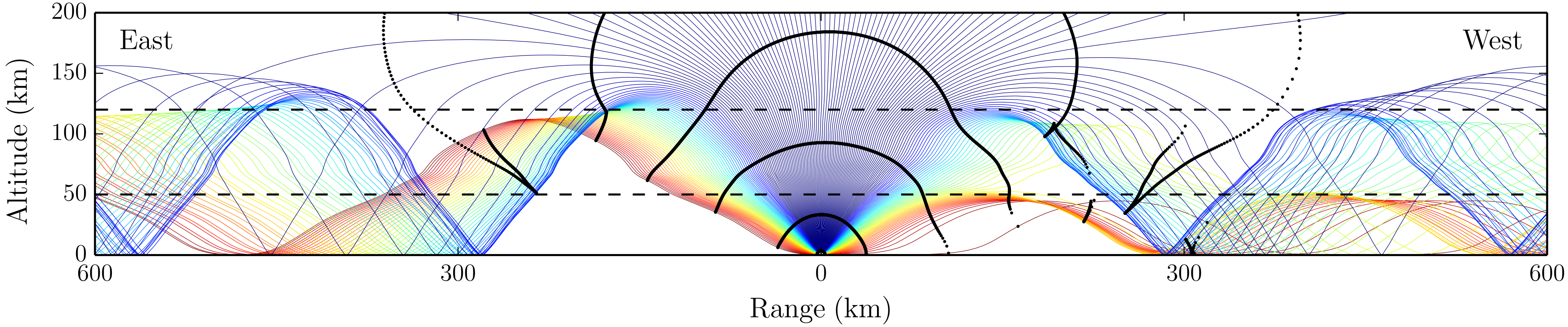
(Munk's profile for the speed of sound)

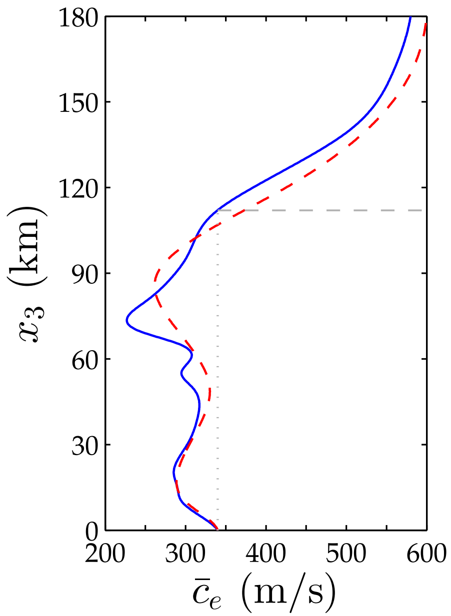




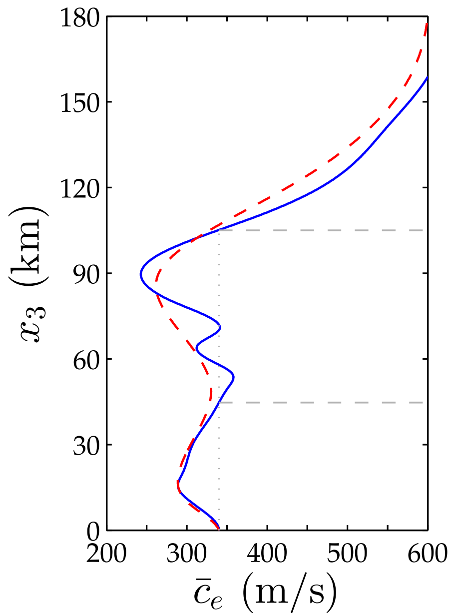
Infrasound propagation in the Earth's atmosphere

Illustration with a ray tracing (refer to slide 15)



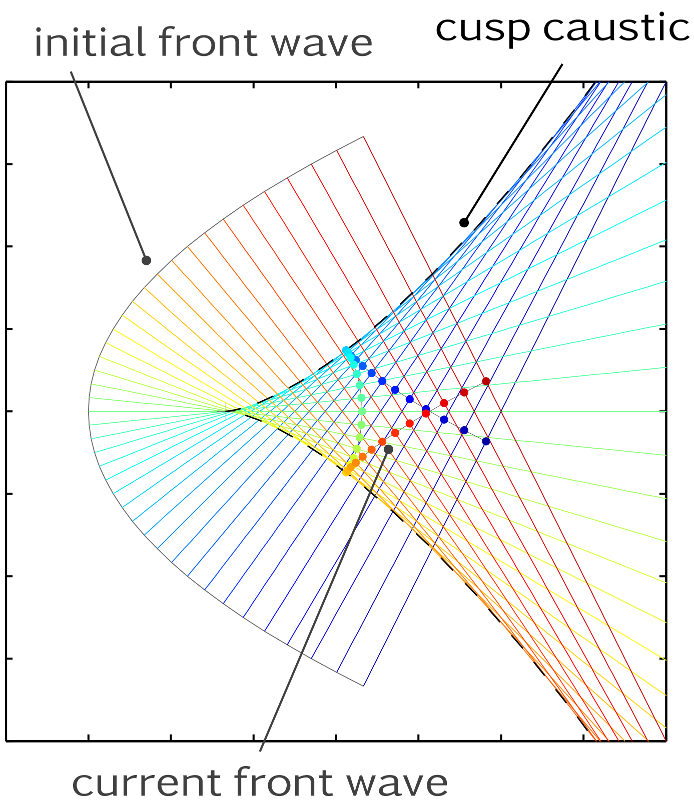


Propagation in a windy atmosphere, inducing two wave guides in the West direction, and only one in the East direction



Caustics

comes from the Greek word *kaustikos*, which burns!



cardioïde

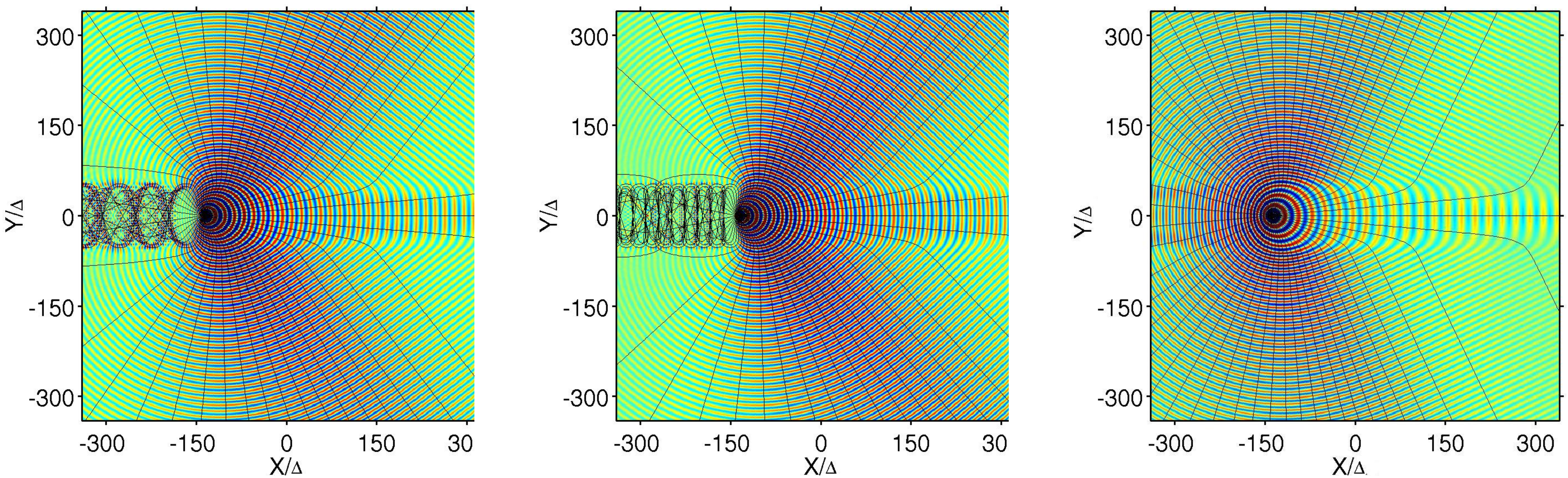
http://images.math.cnrs.fr/Cafe-ou-lait.html

Ehrenfried Walther von Tschirnhaus (1651-1708)

Final quiz !

Ray tracing superimposed to the linearized Euler solution for three subsonic jets and a point source located on the jet axis (same velocity, jet diameter, ...)

Can you identify the three cases $T_j = T_\infty$, $T_j \gg T_\infty$ and $T_j \ll T_\infty$ below ?



- Enjeux et Motivations
- Propagation en écoulement
- **Introduction to aeroacoustics**
  - Elementary sources of sound
  - Moving source
  - Thermoacoustic noise source
  - Cavity noise
  - Lighthill's theory of aerodynamic noise
  - Jet noise
- Advances in aeroacoustics research

Équations d'Euler linéarisées

$(\rho_0 = \text{cte}, p_0 = \text{cte}, \boldsymbol{u}_0 \equiv 0)$

$$\frac{\partial \rho'}{\partial t} + \rho_0 \nabla \cdot \boldsymbol{u}' = q_m$$

q_m source (apport) de masse

$$\rho_0 \frac{\partial \boldsymbol{u}'}{\partial t} = -\nabla p' + \boldsymbol{f}_v$$

$\boldsymbol{f}_v$ forces volumiques

$$\rho_0 T_0 \frac{\partial s'}{\partial t} = q_e$$

q_e source (apport) de chaleur

et pour un gaz parfait $p' = c_0^2 \rho' + (p_0/c_v) s'$.

On forme alors l'équation des ondes :

$$\frac{1}{c_0^2}\frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = \underbrace{\frac{\partial q_m}{\partial t} - \nabla \cdot \boldsymbol{f}_v + \frac{\gamma - 1}{c_0^2}\frac{\partial q_e}{\partial t}}_{\text{sources acoustiques, } \equiv S}$$

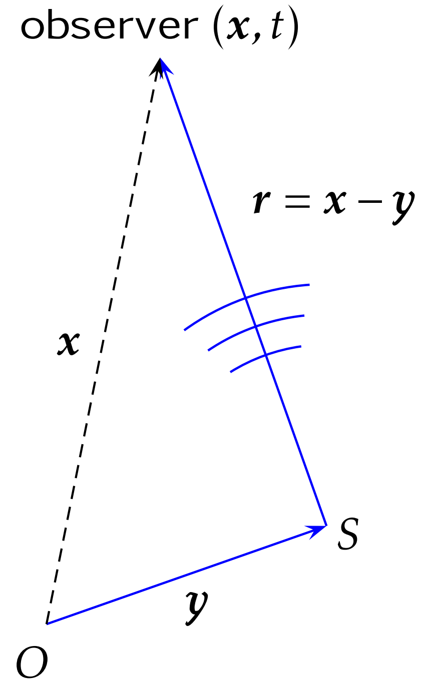
Fonction de Green G_∞ : réponse impulsionnelle d'une source ponctuelle

$$\frac{1}{c_0^2}\frac{\partial^2 G_\infty}{\partial t^2} - \nabla^2 G_\infty = \delta(\boldsymbol{x}-\boldsymbol{y})\,\delta(t-\tau)$$

$G_\infty = 0$ pour $t < \tau$
(causalité)

En espace libre 3-D,

$$G_\infty(\boldsymbol{x}, t|\boldsymbol{y}, \tau) = \frac{1}{4\pi r}\,\delta\left(t - \tau - \frac{r}{c_0}\right) \qquad \boldsymbol{r} = \boldsymbol{x} - \boldsymbol{y} \quad r = \|\boldsymbol{r}\|$$



Cette solution représente un pulse sphérique arrivant à l'observateur au temps $t = \tau + r/c_0$, et décroissant en $1/r$.

Fonction de Green G_∞

- Solution of a non-homogeneous linear equation $\mathcal{L}(p') = S$

$$\mathcal{L} \equiv \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2 \qquad\qquad \mathcal{L}(G_\infty) = \delta$$

$$p' = p' * \delta = p' * \mathcal{L}(G_\infty) = \mathcal{L}(p') * G_\infty = S * G_\infty \tag{3}$$

- La solution générale de l'équation des ondes s'écrit alors :

$$p'(\boldsymbol{x}, t) = S * G_\infty = \frac{1}{4\pi}\int_{\mathcal{V}(\boldsymbol{y})}\int_\tau S(\boldsymbol{y}, \tau)\,\delta\left(t - \tau - \frac{r}{c_0}\right)\frac{d\boldsymbol{y}}{r}d\tau$$

$$= \frac{1}{4\pi}\int_{\mathcal{V}} S\left(\boldsymbol{y}, t - \frac{r}{c_0}\right)\frac{d\boldsymbol{y}}{r} \tag{4}$$

Harmonic point source

- For a source of mass (in the continuity equation)
  $Q_m \sim$ kg.s^{-1} is the monopole source strength

$$S(\boldsymbol{y},t) = \frac{\partial q_m}{\partial t} = \frac{\partial}{\partial t}\left(Q_m e^{-i\omega t}\delta(\boldsymbol{y}) \right)$$

- The radiated acoustic field is then given by Eq. (4)
  (we recall that $\boldsymbol{r} \equiv \boldsymbol{x} - \boldsymbol{y}$)

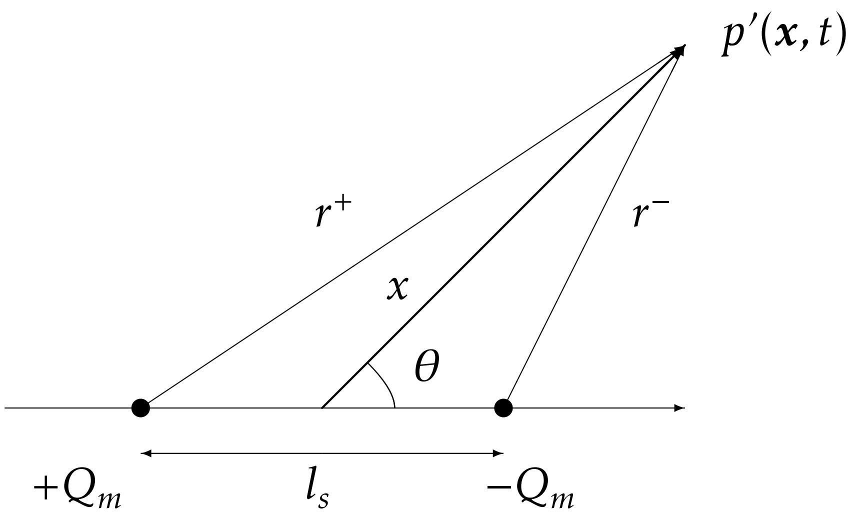
$$p'(\boldsymbol{x},t) = \frac{1}{4\pi}\int_{\mathcal{V}} S\left(\boldsymbol{y}, t - \frac{r}{c_0}\right)\frac{d\boldsymbol{y}}{r} = \frac{-i\omega Q_m}{4\pi}\int_{\mathcal{V}} e^{-i\omega(t-r/c_0)}\delta(\boldsymbol{y})\frac{d\boldsymbol{y}}{r}$$

$$p'(\boldsymbol{x},t) = \frac{-i\omega Q_m}{4\pi}\, e^{-i\omega t}\,\frac{e^{ik_0 x}}{x} \qquad k_0 \equiv \frac{\omega}{c_0}$$

Radiated acoustic power W (in Watts)

$$W = \frac{1}{2}p'p'^{\star} \times \frac{4\pi x^2}{\rho_0 c_0} = \frac{\omega^2 Q_m^2}{8\pi\rho_0 c_0} \qquad (5)$$

Dipole source



$x \gg d$

$$p'(\boldsymbol{x},t) = -\frac{i\omega Q_m}{4\pi}\, e^{-i\omega t}\left(\frac{e^{ik_0 r^+}}{r^+} - \frac{e^{ik_0 r^-}}{r^-}\right)$$

$$r^+ \simeq x + \frac{l_s}{2}\cos\theta \qquad r^- \simeq x - \frac{l_s}{2}\cos\theta$$

$$p'(\boldsymbol{x},t) \simeq -\frac{i\omega Q_m}{4\pi}\, e^{-i\omega t}\, l_s \cos\theta\, \frac{\partial}{\partial x}\left(\frac{e^{ik_0 x}}{x}\right)$$

as $x \rightarrow \infty$ (far field, wrt λ_a and l_s)

$$p'(\boldsymbol{x},t) \simeq \frac{c_0 Q_m l_s}{4\pi x}\, k_0^2 \cos\theta\; e^{i(k_0 x - \omega t)} \qquad \overline{p'^2} \sim \cos^2\theta \quad \forall\; k_0 l_s$$

Radiated acoustic power W

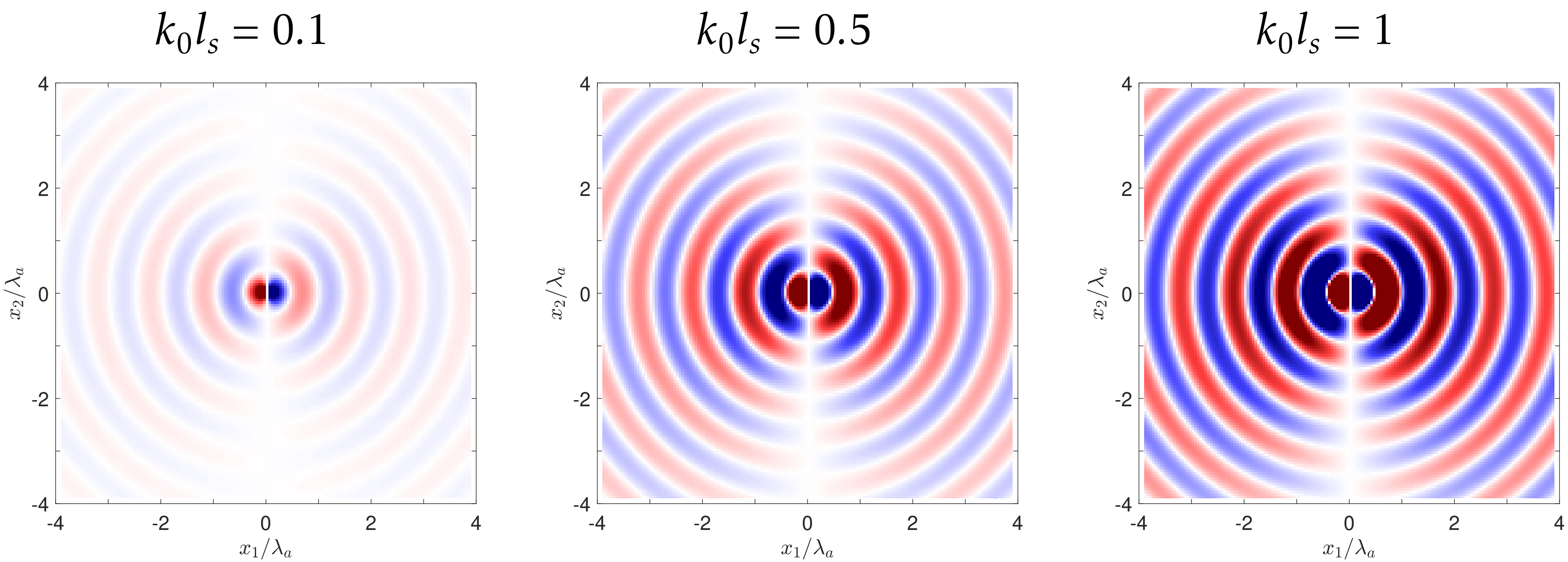
$$W = \int_0^{\pi} \frac{1}{2}\frac{p'p'^{\star}}{\rho_0 c_0} 2\pi x^2 \sin\theta\, d\theta = \frac{\omega^2 Q_m^2}{24\pi\rho_0 c_0}\,(k_0 l_s)^2 \qquad (6)$$

Dipole source : efficiency

(in the geometric, $x \gg l_s$, and acoustic far field, $x \gg \lambda_a$)

$$\frac{W_{\text{dipole}}}{W_{\text{monopole}}} \sim (k_0 l_s)^2$$

$\rightarrow 0$ as $k_0 l_s \rightarrow 0$ compact dipole ($l_s/\lambda_a \ll 1$)



Solution en champ acoustique lointain

See Eq. (4),

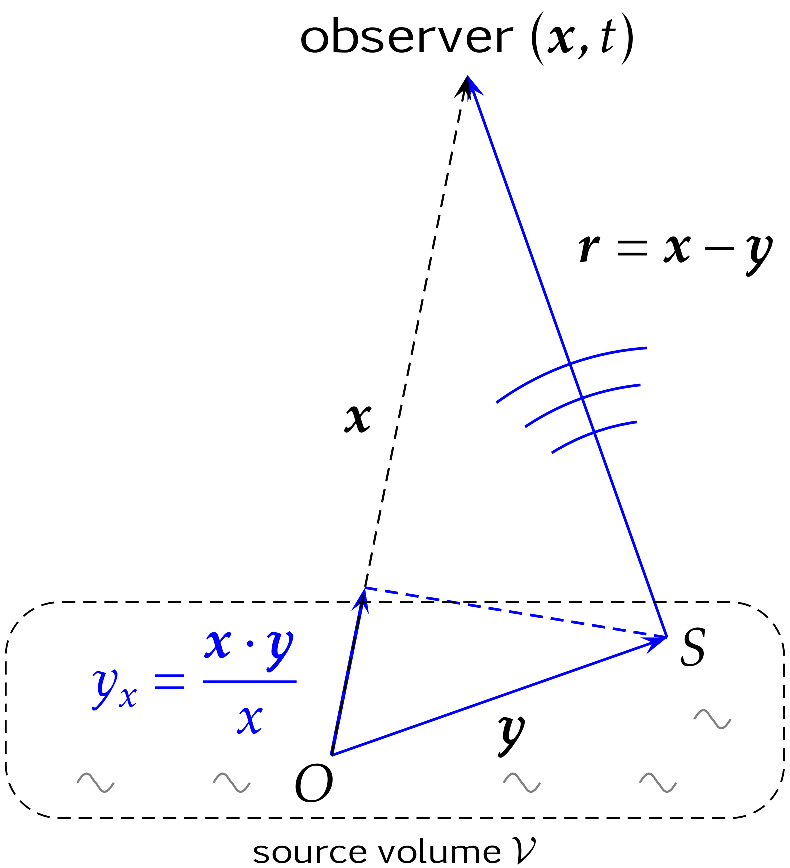
$$p'(\boldsymbol{x},t) = \frac{1}{4\pi}\int_{\mathcal{V}} S\left(\boldsymbol{y}, t - \frac{r}{c_0}\right)\frac{d\boldsymbol{y}}{r}$$

- temps retardé $\tau_r = t - r/c_0$
- champ acoustique lointain

$$r = \left|\boldsymbol{x}-\boldsymbol{y}\right| \simeq x - \frac{\boldsymbol{x}\cdot\boldsymbol{y}}{x} + O\left(\frac{y^2}{x}\right) \qquad x \gg y$$

$$p'(\boldsymbol{x},t) \simeq \frac{1}{4\pi x}\int_{\mathcal{V}} S\left(\boldsymbol{y}, t - \frac{x}{c_0} + \frac{\boldsymbol{x}\cdot\boldsymbol{y}}{x c_0}\right) d\boldsymbol{y}$$

(approximation de Fraunhofer)



Source acoustique spatialement compacte

$$\tau_r = t - \frac{r}{c_0} \simeq t - \frac{x}{c_0} + \frac{\boldsymbol{x}\cdot\boldsymbol{y}}{xc_0} \simeq t - \frac{x}{c_0} \quad ? \qquad \Delta\tau_r = \frac{\boldsymbol{x}\cdot\boldsymbol{y}}{xc_0} \sim \frac{l_s}{c_0}$$

La variation du temps de retard $\Delta\tau_r$ sur le volume source $\mathcal{V} \sim l_s^3$ doit être petite devant le temps caractéristique d'évolution de la source $\tau_s \sim l_s/u$

$$\frac{\Delta\tau_r}{\tau_s} \sim \frac{u}{c_0} \sim M_t \ll 1 \qquad \text{avec} \qquad M_t = \frac{u}{c_0}$$

et alors

$$p'(\boldsymbol{x},t) \simeq \frac{1}{4\pi x}\int_{\mathcal{V}} S\left(\boldsymbol{y}, t - \frac{x}{c_0} + \frac{\boldsymbol{x}\cdot\boldsymbol{y}}{xc_0}\right) d\boldsymbol{y} \simeq \frac{1}{4\pi x}\int_{\mathcal{V}} S\left(\boldsymbol{y}, t - \frac{x}{c_0}\right) d\boldsymbol{y}$$

source acoustique spatialement compacte

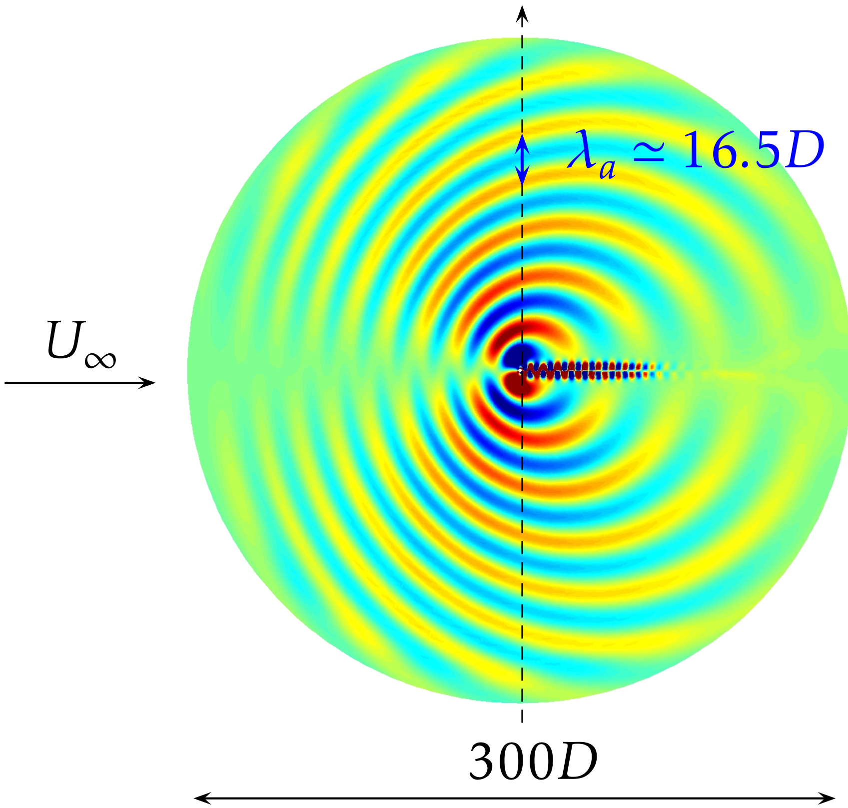
La longueur d'onde λ_a du rayonnement acoustique est alors grande devant la taille caractéristique l_s de la source :

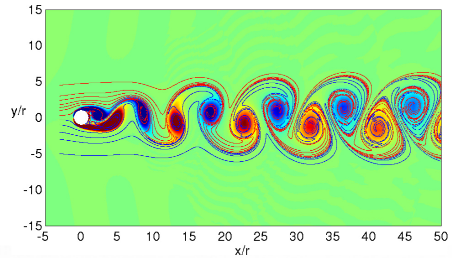
$$\frac{\lambda_a}{l_s} = \frac{c_0}{U_\infty} \times \frac{U_\infty}{f l_s} = \frac{1}{M_a} \times \frac{1}{St} \qquad (M_a \propto M_t)$$

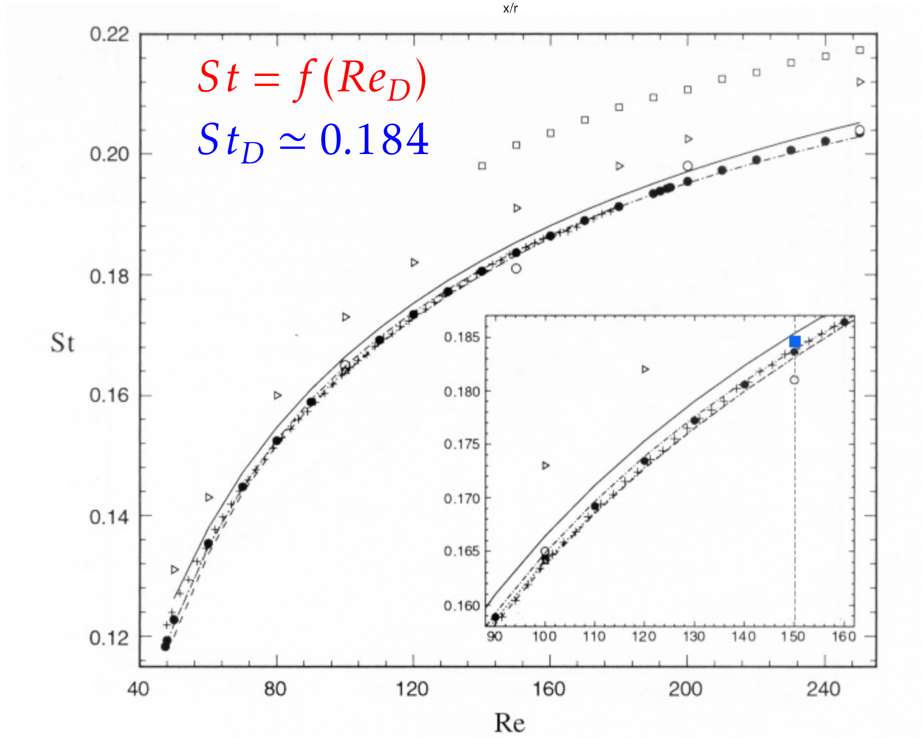
Low-Reynolds number flow around a cylinder

Instantaneous fluctuating pressure field

$Re_D = 150$ and $M_a \simeq 0.33$







$$\frac{\lambda_a}{D} = \frac{1}{M_a \times St} \simeq 16.5$$

$$C_p = \frac{p_w - p_\infty}{\rho_\infty U_\infty^2/2} \simeq 1.32$$

Marsden *et al.*, *J. Comput. Acoust.*, 13(4), 2005

Source acoustique spatialement non compacte, $M \gg 1$

On revient à l'expression générale (pour $x \gg y$), $\boldsymbol{y} = (\boldsymbol{y}_x, \boldsymbol{y}_s)$ $\quad y_x = \dfrac{\boldsymbol{x} \cdot \boldsymbol{y}}{x}$

$$p'(\boldsymbol{x},t) \simeq \frac{1}{4\pi x} \int_{\mathcal{V}} \int_{\tau} S\,(y_x, \boldsymbol{y}_s, \tau)\, \delta\left(t - \tau - \frac{x}{c_0} + \frac{y_x}{c_0}\right) dy_x\, d\boldsymbol{y}_s\, d\tau$$

$$\simeq \frac{c_0}{4\pi x} \int_{\mathcal{V}} \int_{\tau} S\,(x - c_0 t + c_0 \tau, \boldsymbol{y}_s, \tau)\, d\boldsymbol{y}_s\, d\tau$$

On compare alors la variation spatiale du terme source S sur la distance $c_0\tau$ dans la direction de l'observateur y_x :

$$\frac{c_0 \tau}{l_s} \sim \frac{c_0\, l_s/u}{l_s} \sim \frac{1}{M} \ll 1$$

$$p'(\boldsymbol{x},t) \simeq \frac{c_0}{4\pi x} \int_{\mathcal{V}} \int_{\tau} S\,(x - c_0 t, \boldsymbol{y}_s, \tau)\, d\boldsymbol{y}_s\, d\tau$$

source acoustique spatialement non compacte

Cas d'une source de masse

$$S = \frac{\partial q_m}{\partial t} \sim \frac{\rho_0 u/l_s}{l_s/u} \sim \rho_0 \frac{u^2}{l_s^2} \qquad (q_m \sim \text{kg.m}^{-3}.\text{s}^{-1})$$

- Source spatialement compacte $\quad M \ll 1$

$$p'(\boldsymbol{x},t) \simeq \frac{1}{4\pi x}\int_{\mathcal{V}} S\left(\boldsymbol{y}, t - \frac{x}{c_0}\right) d\boldsymbol{y} \sim \frac{1}{x}\rho_0 \frac{u^2}{l_s^2} l_s^3 \sim \rho_0 c_0^2 \frac{l_s}{x} M^2$$

rayonnement d'une source monopolaire compacte, $W \sim U^4$

- Source spatialement non compacte $\quad M \gg 1$

$$p'(\boldsymbol{x},t) \simeq \frac{c_0}{4\pi x}\int_{\mathcal{V}}\int_{\tau} S\left(x - c_0 t, \boldsymbol{y}_s, \tau\right) d\boldsymbol{y}_s d\tau \sim \frac{c_0}{x}\rho_0 \frac{u^2}{l_s^2} l_s^2 \frac{l_s}{u} \sim \rho_0 c_0^2 \frac{l_s}{x} M$$

rayonnement d'une source monopolaire non compacte, $W \sim U^2$

Cas d'une source de masse (suite)

Lorsque la source est compacte, ce sont les variations temporelles de la source $S = \partial q_m / \partial t$ qui imposent le rayonnement acoustique :

$$p'(\boldsymbol{x}, t) \simeq \frac{1}{4\pi x} \frac{\partial}{\partial t} \int_{\mathcal{V}} q_m\left(\boldsymbol{y}, t - \frac{x}{c_0}\right) d\boldsymbol{y}$$

alors que lorsque la source est non compacte, ce sont les variations spatiales de la source dans la direction de l'observateur qui imposent les caractéristiques du rayonnement sonore :

$$p'(\boldsymbol{x}, t) \simeq -\frac{1}{4\pi x} \int_{\mathcal{V}} \int_{\tau} \frac{\partial q_m}{\partial y_r}(x - c_0 t, \boldsymbol{y}_s, \tau)\, d\boldsymbol{y}_s d\tau$$

Petit exemple instructif (Crighton, 1974)

$$S(\boldsymbol{y},\tau) = \rho_0 \frac{u^2}{l_s^2}\, e^{-\frac{y^2}{l_s^2} - \frac{u^2\tau^2}{l_s^2}}$$

$$p'(\boldsymbol{x},t) = \rho_0 c_0^2 \frac{\sqrt{\pi}}{4}\, \frac{l_s}{x}\, \frac{M^2}{(1+M^2)^{1/2}}\, e^{-\frac{u^2}{l_s^2}\left(t-\frac{x}{c_0}\right)^2 \frac{1}{1+M^2}} \qquad x \gg y$$

$M \rightarrow 0$

$$p'(\boldsymbol{x},t) = \rho_0 c_0^2 \frac{\sqrt{\pi}}{4}\, \frac{l_s}{x}\, M^2\, e^{-\frac{u^2}{l_s^2}\left(t-\frac{x}{c_0}\right)^2}$$

$$W \sim M^4$$

fréquence source $\sim u/l_s$

fréquence rayonnement $\sim u/l_s$

$M \rightarrow \infty$

$$p'(\boldsymbol{x},t) = \rho_0 c_0^2 \frac{\sqrt{\pi}}{4}\, \frac{l_s}{x}\, M\, e^{-\frac{c_0^2}{l_s^2}\left(t-\frac{x}{c_0}\right)^2}$$

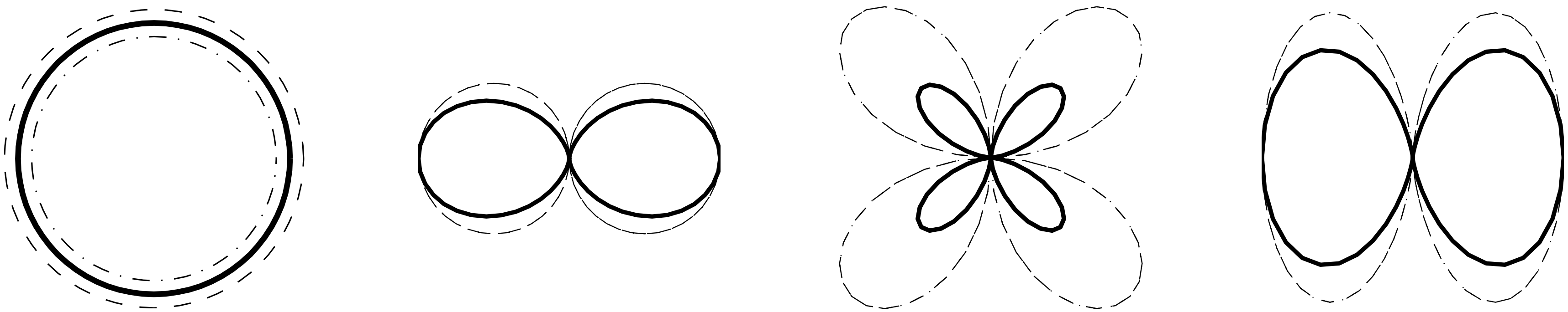
$$W \sim M^2$$

fréquence source $\sim u/l_s$

fréquence rayonnement $\sim c_0/l_s$

- **Monopoles, dipoles et quadripoles compacts**

monopôle	dipôle	quadripôle latéral	quadripôle longitudinal



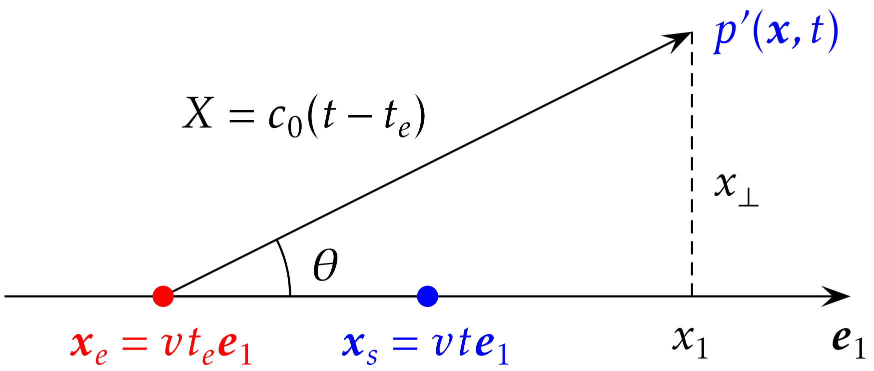
$I \sim 1$	$I \sim \cos^2\theta$	$I \sim \cos^2\theta \sin^2\theta$	$I \sim \cos^4\theta$
$W \sim U^4$	$W \sim U^6$	$W \sim U^8$	$W \sim U^8$

Rayonnement acoustique d'une source mobile ponctuelle

Source de masse mobile $M = v/c_0 < 1$
dans un milieu homogène et au repos

$$q_m(t,\boldsymbol{x}) = Ae^{-i\omega t}\delta(x_1 - vt)\,\delta(x_2)\delta(x_3)$$

$$\frac{1}{c_0^2}\frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = \frac{\partial q_m}{\partial t}$$



Il s'avère plus simple de travailler avec le potentiel acoustique, $p' \equiv \partial\psi/\partial t$

$$\frac{1}{c_0^2}\frac{\partial^2 \psi}{\partial t^2} - \nabla^2 \psi = q_m$$

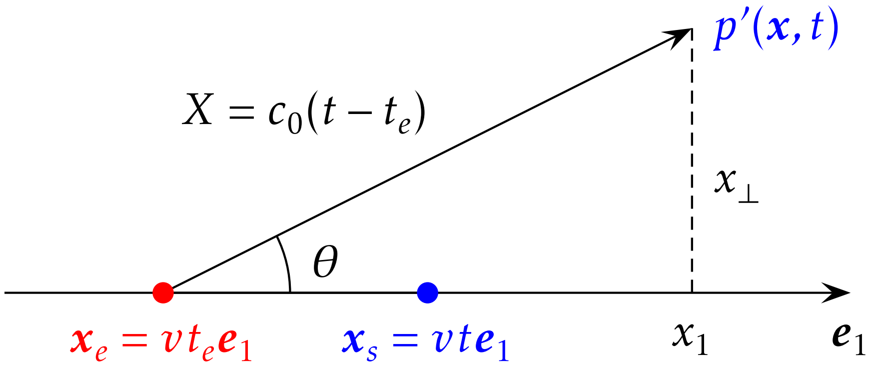
Transformation de Lorentz avec $\beta = 1/\sqrt{1 - M^2}$

$$\begin{cases} \tilde{x}_1 = \beta(x_1 - vt) \\ \tilde{x}_2 = x_2 \\ \tilde{x}_3 = x_3 \\ \tilde{t} = \beta(t - Mx_1/c_0) \end{cases} \quad \Longrightarrow \quad \frac{1}{c_0^2}\frac{\partial^2 \psi}{\partial \tilde{t}^2} - \tilde{\nabla}^2 \psi = A\beta e^{-i\omega\beta\tilde{t}}\delta(\tilde{\boldsymbol{x}})$$

Rayonnement acoustique d'une source mobile (suite)

En utilisant la solution générale, voir l'Éq. (4) et l'application qui suit,

$$\psi(\tilde{\boldsymbol{x}},\tilde{t}) = \frac{A\beta}{4\pi\tilde{x}}\, e^{-i\omega\beta(\tilde{t}-\tilde{x}_1/c_0)}$$



En ne conservant que le terme dominant en champ lointain,

$$p'(\boldsymbol{x},t) \simeq \frac{-i\omega A}{4\pi X(1-M\cos\theta)^2}\, e^{-i\omega(t-X/c_0)}$$

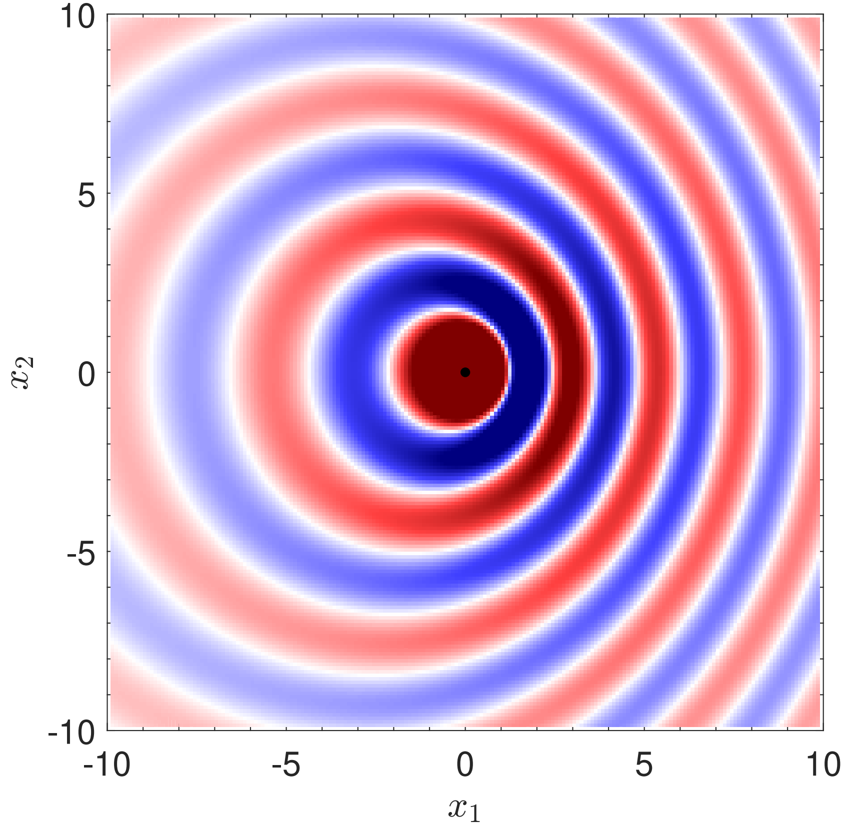
$$X = c_0(t-t_e) = \|\boldsymbol{x}-\boldsymbol{x}_e\|$$

$$X = \frac{M(x_1 - vt) + X_1}{1-M^2} \quad (M<1)$$

$$X_1 = \sqrt{(x_1-vt)^2 + (1-M^2)\,x_\perp^2} \quad x_\perp^2 = x_2^2 + x_3^2$$

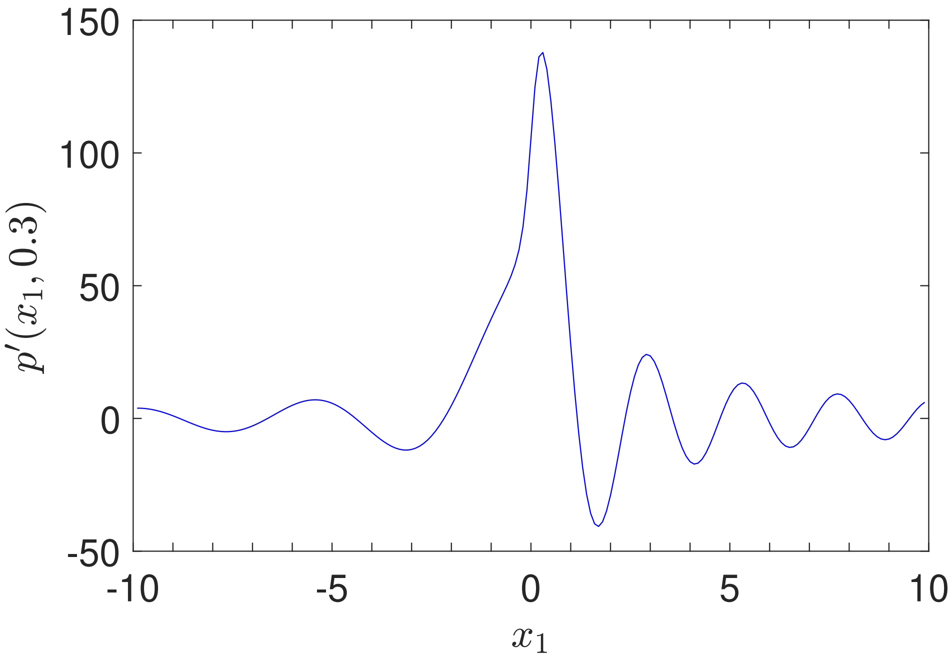
$$X_1 = X(1-M\cos\theta)$$

Rayonnement acoustique d'une source mobile (suite)



Décalage en fréquence, effet Doppler
$\omega = \omega_e/(1 - M\cos\theta)$

et pour la longueur d'onde
$\lambda_a = (1 - M\cos\theta)\lambda_{a_{M=0}}$



$M = 0.3$

$f = 100$ Hz, (x_1, x_2) en m

Rayonnement acoustique d'une source mobile (suite)

Puissance acoustique rayonnée par la source,

$$W = \int_0^{\pi} I(X,\theta)\, 2\pi X^2 \sin\theta d\theta \qquad I = \frac{1}{2} p \boldsymbol{u}^{\star} \cdot \boldsymbol{e}_X$$

$$= \frac{\omega^2 A^2}{8\pi\rho_0 c_0} \frac{1}{(1-M^2)^2}$$

Le premier terme correspond à la puissance acoustique rayonnée par un monopole fixe, voir l'Éq. (5), et le second terme est un facteur d'amplification lié à la convection :

$$\frac{1}{(1-M^2)^2} > 1$$

La même source acoustique n'émet pas la même puissance acoustique suivant qu'elle est dans un milieu au repos ou dans un écoulement ...

Sound field of a moving point source (general approach)

$$\frac{1}{c_0^2}\frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = S(\boldsymbol{x},t) \qquad S(\boldsymbol{x},t) = q(t)\delta(\boldsymbol{x}-\boldsymbol{x}_s) \qquad \boldsymbol{x}_s = \boldsymbol{x}_s(t)$$

Eq. (4) $\Longrightarrow$

$$p'(\boldsymbol{x},t) = \frac{1}{4\pi}\int_{\mathcal{V}}\int_{-\infty}^{+\infty} S(\boldsymbol{y},\tau)\delta\left(t-\tau-\frac{r}{c_0}\right)\frac{d\boldsymbol{y}}{r}d\tau$$

$$= \frac{1}{4\pi}\int_{-\infty}^{+\infty} q(\tau)\delta\left(g(\tau)\right)\frac{d\tau}{r} \qquad g(\tau) = t-\tau-\frac{\|\boldsymbol{x}-\boldsymbol{x}_s(\tau)\|}{c_0}$$

$$= \frac{1}{4\pi r}\frac{q(\tau)}{|dg(\tau)/d\tau|}\bigg|_{\tau=\tau_e} \qquad \tau_e \text{ root of } g(\tau_e)=0$$

$$= \frac{1}{4\pi r}\frac{q(\tau)}{|1-M_r|}\bigg|_{\tau=\tau_e}$$

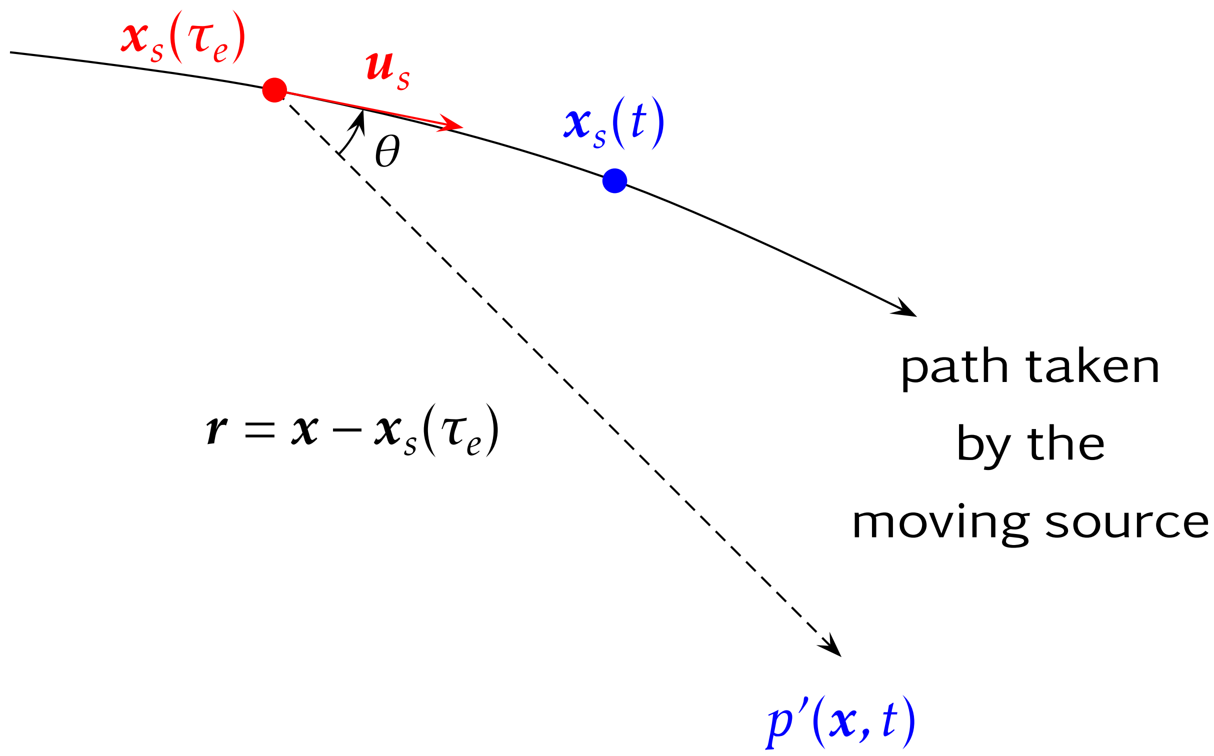
- reference time t is observer time at $(\boldsymbol{x},t)$
- implicit equation for emission time τ_e : $c_0(t-\tau_e) = \|\boldsymbol{x}-\boldsymbol{x}_s(\tau_e)\|$ (only one real root for subsonic motion)

Sound field of a moving point source (cont.)

$$M = \frac{u_s}{c_0} \qquad \boldsymbol{u}_s = \frac{d\boldsymbol{x}_s}{dt}$$

$$M_r = \frac{\boldsymbol{u}_s}{c_0}\cdot\frac{\boldsymbol{r}}{r} = M\cos\theta$$

$1 - M_r \equiv$ Doppler factor



Source of mass

$$S(\boldsymbol{x},t) = \frac{\partial}{\partial t}\left[q_m(t)\delta(\boldsymbol{x}_s(t)-\boldsymbol{x})\right]$$

Using a convolution property, $p' = \partial_t S * G = \partial_t (S * G)$,
the solution is given by :

$$p'(\boldsymbol{x},t) = \frac{\partial}{\partial t}\left\{\frac{1}{4\pi r}\frac{q_m(\tau)}{|1-M_r|}\bigg|_{\tau=\tau_e}\right\} = \frac{\partial \tau_e}{\partial t}\frac{\partial}{\partial \tau_e}\frac{1}{4\pi r}\frac{q_m(\tau)}{|1-M_r|}\bigg|_{\tau=\tau_e}$$

Noting that : $c_0(t-\tau_e) = |\boldsymbol{x}-\boldsymbol{x}_s(\tau_e)| \quad \rightsquigarrow \quad \dfrac{\partial \tau_e}{\partial t} = \dfrac{1}{1-M_r}$

$$p'(\boldsymbol{x},t) = \frac{1}{4\pi r}\frac{\partial q_m/\partial \tau}{(1-M_r)^2}\bigg|_{\tau=\tau_e} \qquad \text{(usually dominant term)}$$

$$+ \frac{1}{4\pi r}\frac{q_m}{|1-M_r|^3}M_r'\bigg|_{\tau=\tau_e} \qquad \text{(a)}$$

$$+ \frac{1}{4\pi r^2}\frac{q_m}{|1-M_r|^3}c_0(M_r-M^2)\bigg|_{\tau=\tau_e} \qquad \text{(b)}$$

Source of mass (cont.)

(a) non uniform motion of the source (*e.g.* uniform rotation), term in $1/r$, present for a steady source

(b) near-field term in $1/r^2$

Dipole point source

$$S(\boldsymbol{x},t) = \frac{\partial f_i}{\partial x_i}\,\delta(\boldsymbol{x}_s - \boldsymbol{x}) \qquad \frac{\partial}{\partial x_i} \rightsquigarrow -\frac{1}{c_\infty}\frac{x_i}{x}\frac{\partial}{\partial t}$$

The solution for the dominant term is

$$p'(\boldsymbol{x},t) \simeq -\frac{1}{4\pi r}\frac{1}{c_\infty}\frac{x_i}{x}\frac{f_i'}{|1-M_r|^2}\bigg|_{\tau=\tau_e} \qquad x \gg x_s$$

Quadrupole point source

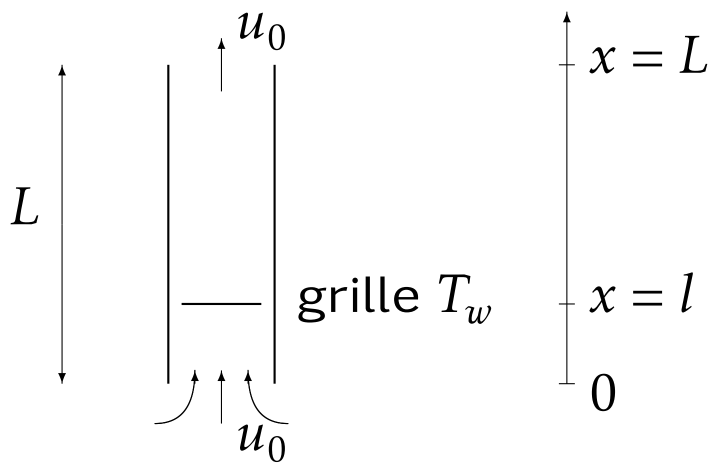
$$S(\boldsymbol{x},t) = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}\,\delta(\boldsymbol{x}_s - \boldsymbol{x}) \qquad \frac{\partial^2}{\partial x_i \partial x_j} \rightsquigarrow \frac{1}{c_\infty^2}\frac{x_i x_j}{x^2}\frac{\partial^2}{\partial t^2}$$

The solution for the dominant term is

$$p'(\boldsymbol{x},t) \simeq \frac{1}{4\pi r}\frac{1}{c_\infty^2}\frac{x_i x_j}{x^2}\frac{T_{ij}''}{|1-M_r|^3}\bigg|_{\tau=\tau_e} \qquad x \gg x_s$$

Présence d'une source de chaleur : tube de Rijke (1859)

$$\frac{\partial s'}{\partial t} = \frac{1}{\rho_0 T_0}\, q_e'$$



Équation des ondes

$$\frac{1}{c_0^2}\frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = \frac{\gamma - 1}{c_0^2}\frac{\partial q_e'}{\partial t}$$

Bilan d'énergie dans le tube $\quad \mathcal{E} = S\int_0^L E_a dx \qquad E_a = \dfrac{p'^2}{2\rho_0 c_0^2} + \rho_0 \dfrac{\boldsymbol{u}'^2}{2}$

$$\frac{\partial \mathcal{E}}{\partial t} + \left[S p' \boldsymbol{u}' \cdot \boldsymbol{n}\right]_0^L = \frac{\gamma - 1}{\rho_0 c_0^2}\int_{\mathcal{V}} p' q_e' d\boldsymbol{x}$$

$\downarrow$ énergie perdue par rayonnement aux extrémités du tube

$\downarrow$ terme source : couplage avec la source de chaleur q_e'

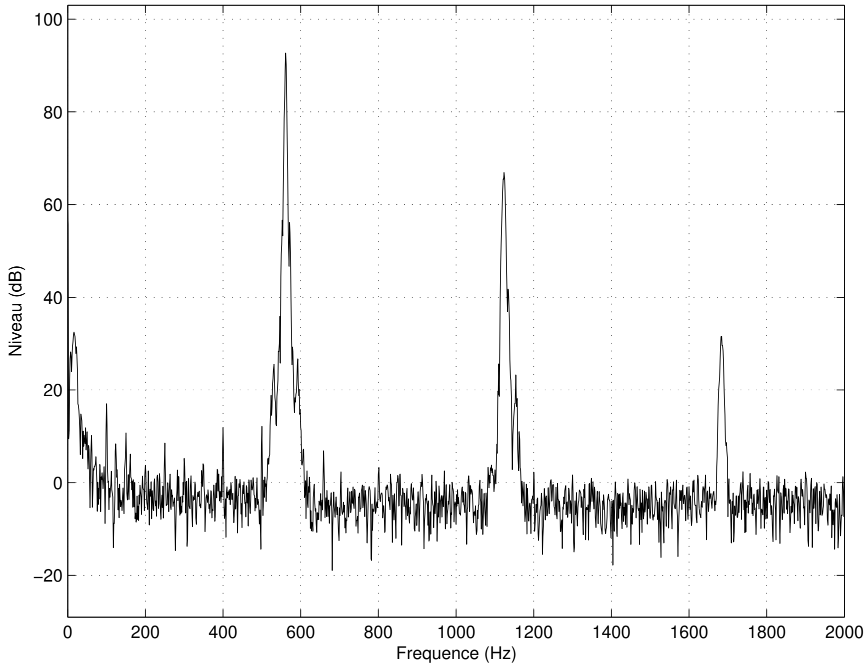
- Pour avoir augmentation de l'énergie acoustique dans le tube, et donc une configuration instable, il est nécessairement d'avoir,

$$\Delta \bar{E} = \frac{\gamma - 1}{\rho_0 c_0^2} \int_{\mathcal{V}} \overline{p' q'_e} d\boldsymbol{x} \geq 0 \qquad (\rho_0 c_0^2 = \gamma p_0)$$

critère de Rayleigh (1894)

Lord Rayleigh
(1842 - 1919)

prix Nobel de physique (1904) pour la découverte de l'argon en 1895



Spectre acoustique mesuré

Pour $n = 1$, avec $L = 30$ cm et $c_0 = 343.2$ m.s^{-1}, on a $f \simeq 572$ Hz. Avec la correction d'extrémité, $L_{eq} \simeq 0.30 + 2 \times 0.31 \times D$, et on obtient $f \simeq 540$ Hz.

$L_{eq} = L + \delta L$, D diameter

unflanged pipe (Levine & Schwinger, 1948)
$\delta L \simeq 0.31 D$

flanged pipe (Norris & Sheng, 1989)
$\delta L \simeq 0.41 D$

Tube de Rijke

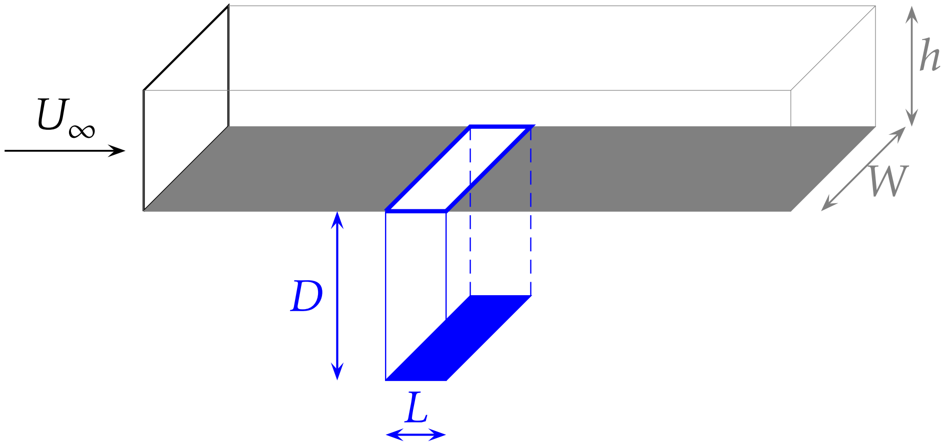
Perturbation acoustique sur le mode fondamental
(ondes planes, justifié par le spectre mesuré)

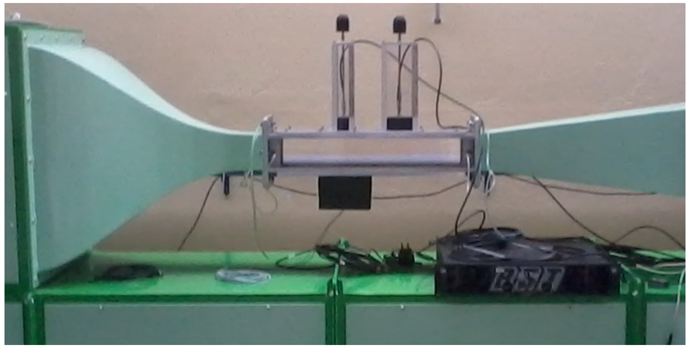
$$\left| \begin{array}{ll} p'(x,t) = \hat{p}\cos(kx)\cos(\omega t) & k = \dfrac{\omega}{c_0} = \dfrac{\pi}{L} \\ q'_e(t) = (\beta_w/S)u'(l,t-\tau)\,\delta(x-l) & \tau = \tau(u_0) \qquad u' = p'/(\rho_0 c_0) \end{array} \right.$$

$$\Delta\bar{E} = \frac{\gamma-1}{\rho_0 c_0^2}\,\frac{\beta_w \hat{p}^2}{4\rho_0 c_0}\sin(2kl)\sin(\omega\tau)$$

Instabilité (couplage) thermo-acoustique lorsque la grille est dans la partie inférieure du tube, $0 < l < L/2$ (avec optimum en $x \simeq L/4$), et lorsqu'il y a un temps de retard $\omega\tau \neq 0$ (et $\omega\tau < \pi$).

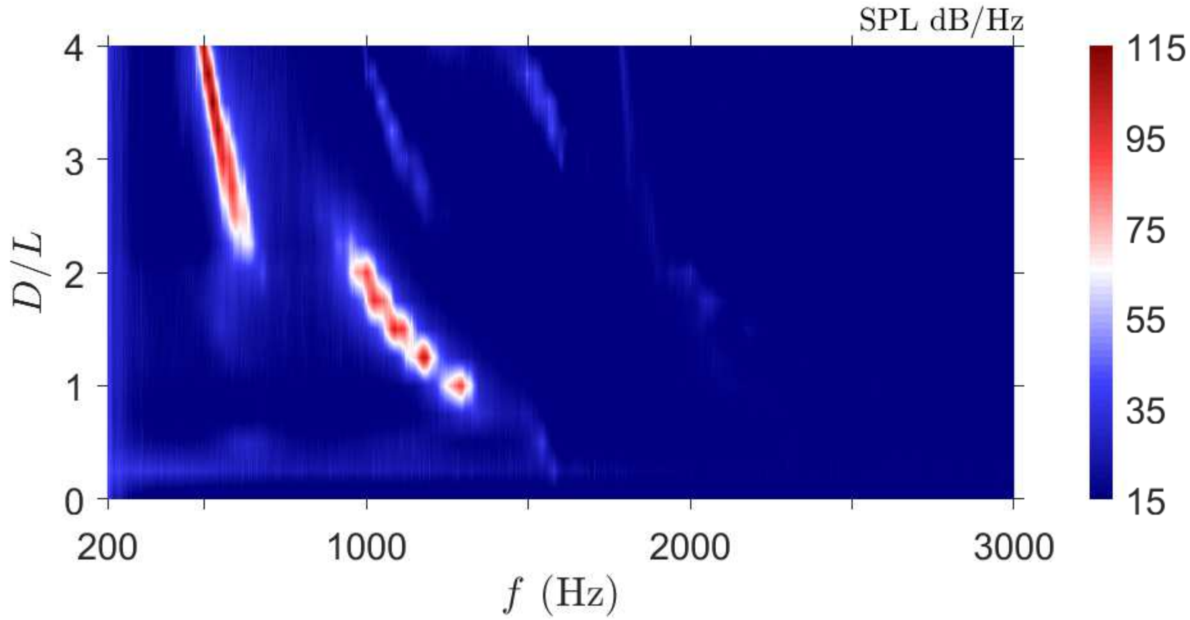
Flow-induced cavity oscillations





$$\begin{cases} L = 0.04 \text{ m} \\ h = 0.1 \text{ m} \\ U_\infty = 52.2 \text{ m.s}^{-1} \\ 0 \leq D \leq 4L \end{cases}$$

SPL measured inside the cavity



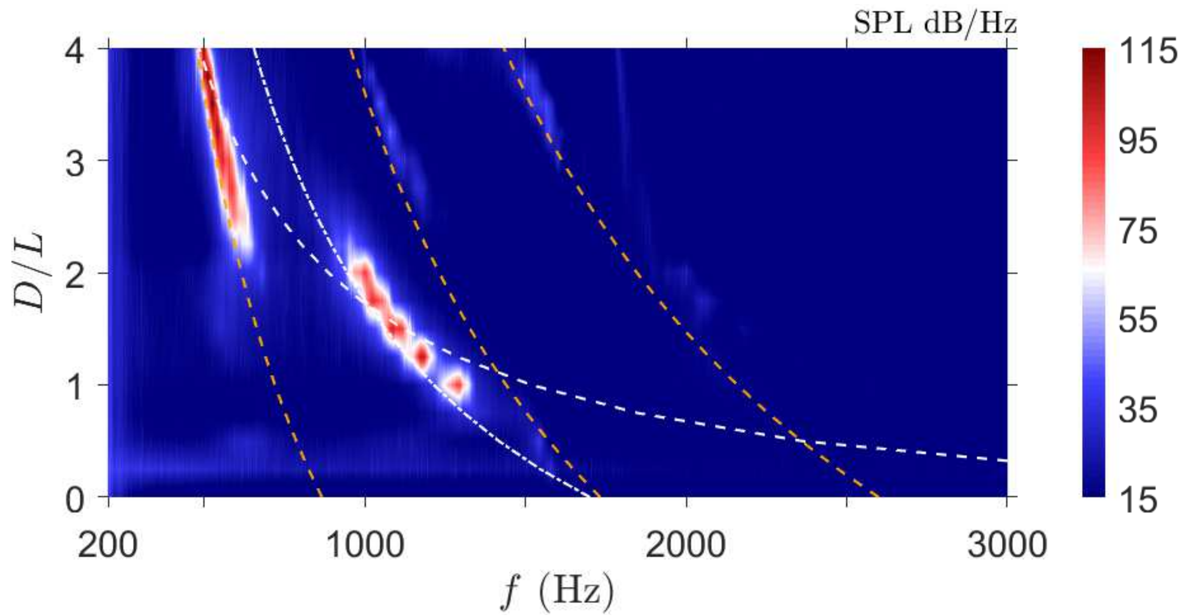
Flow-induced cavity oscillations (cont.)

Cavity depth modes $---\ n=0$
(cavity+end-correction)

$$f=(2n+1)\frac{c_\infty}{4(D+0.41L)}$$

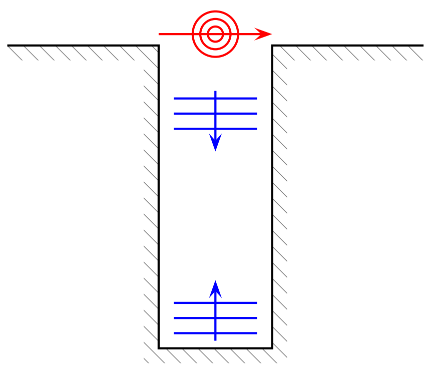
Acoustic modes $-\cdot-\cdot-\ n=1$
(cavity+channel height)

$$f=n\frac{c_\infty}{2(h+D)}$$



Aeroacoustic feedback loop
$---\ n=1,2,3$

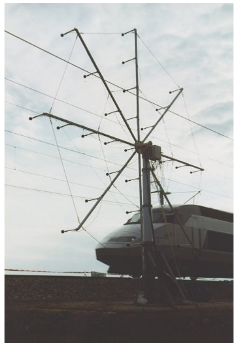
$$f=\frac{n}{L/U_c+2D/c_\infty}$$

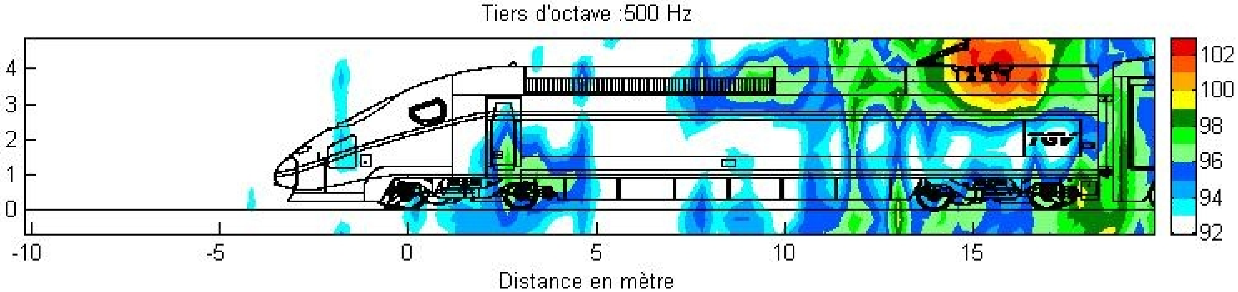


Convection of turbulent structures within the shear layer, which impact at the downstream corner, generation of acoustic waves (depth modes) and excitation of the shear layer upstream.

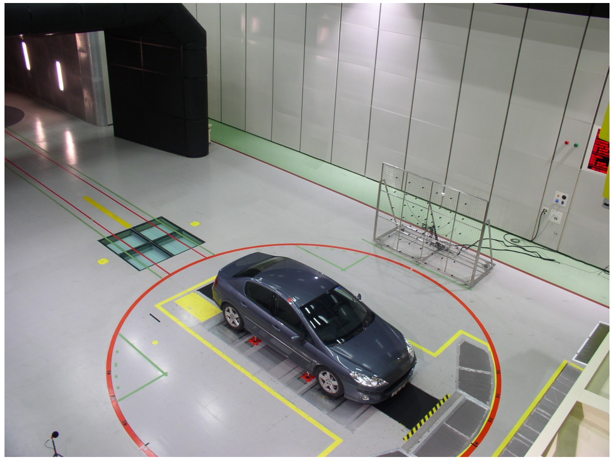
Modelling : frequency selection + gain $\rightsquigarrow$ whistle

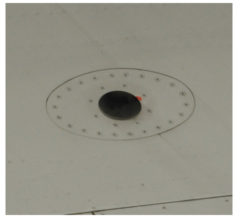
- **Cavity noise : a long-lasting problem in transport**





TGV - high speed train (rear engine) - courtesy of SNCF



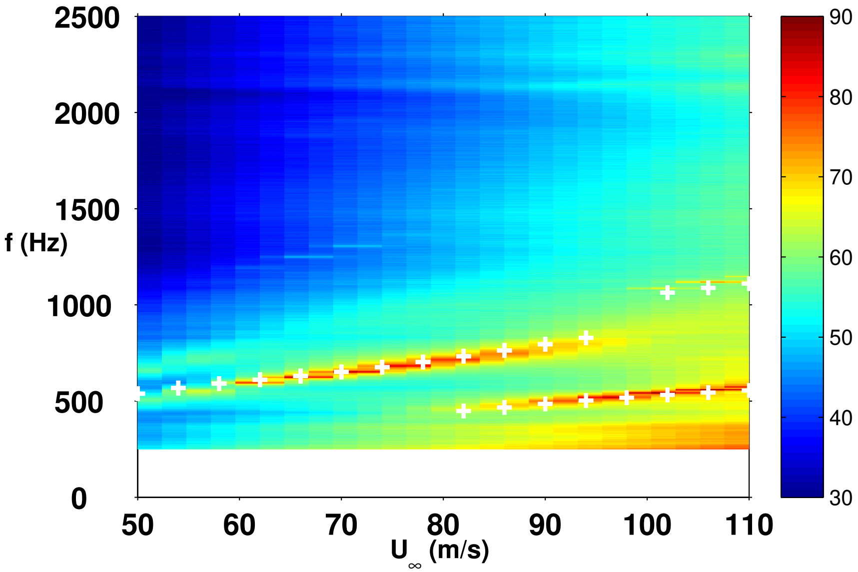


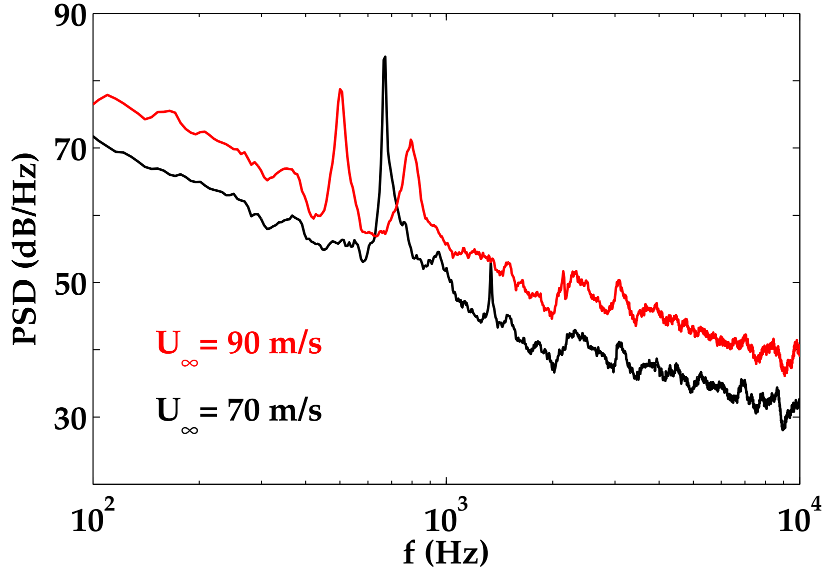
fuel pressure relief vent on a A319

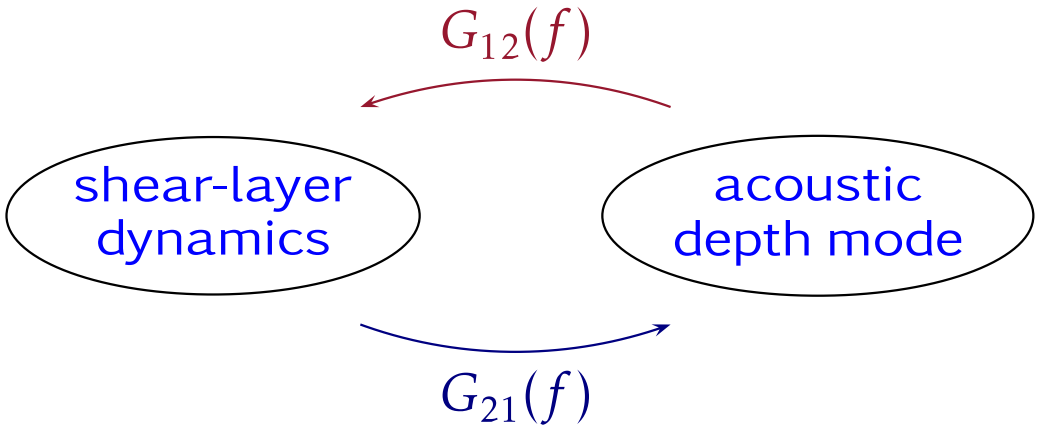
Courtesy of Jan Delf (DLR), refer to *AIAA Paper 2002-2470*

Measured acoustic spectra at 1 m above a cylindrical cavity

Diameter $D = 10$ cm, depth $H = D$, flow speed $50 \leq U_\infty \leq 110$ m/s TR-PIV 







$$\mathfrak{Im}\,(G_{12} \times G_{21}) = 0$$

$\Longrightarrow$ resonance frequencies +

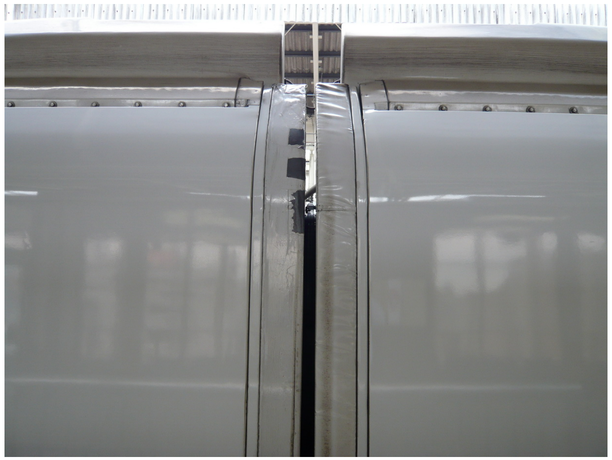
Marsden *et al.*, 2012, *J. Sound Vib.*, **331**

Elder, 1978, *J. Acoust. Soc. Am.*, **63**(3)

Shinkansen (Tokyo – Sendai, july '08)

E2 series – 275 km/h

E4 series – 240 km/h



aerodynamic noise generated by intercoach spacing

- New Hayabusa Shinkensan train ('the bullet train')

THE DAILY YOMIURI

SUNDAY, MARCH 6, 2011

THE DAILY YOMIURI (2011)

PRICE ¥120 (¥2,650 per calendar month)—tax included

The Yomiuri Shimbun

Fast trainspotting

Train fans snap pictures as the new Hayabusa bullet train on the Tohoku Shinkansen line leaves Tokyo Station on its maiden trip to Shin-Aomori Station on Saturday morning. Th[illegible] train, the first entirely new mod[illegible] introduced by East Japan Railway Co. since 1997, achieved speeds of up to 300 kph, 25 kph faster and quieter than the conventional Hayate superexpress. It connects Tokyo and Shin-Aomori in three hours [illegible] 10 minutes. (Related story on Page 2)

China tar[illegible]

7% annual growth eyed in new 5-yr plan

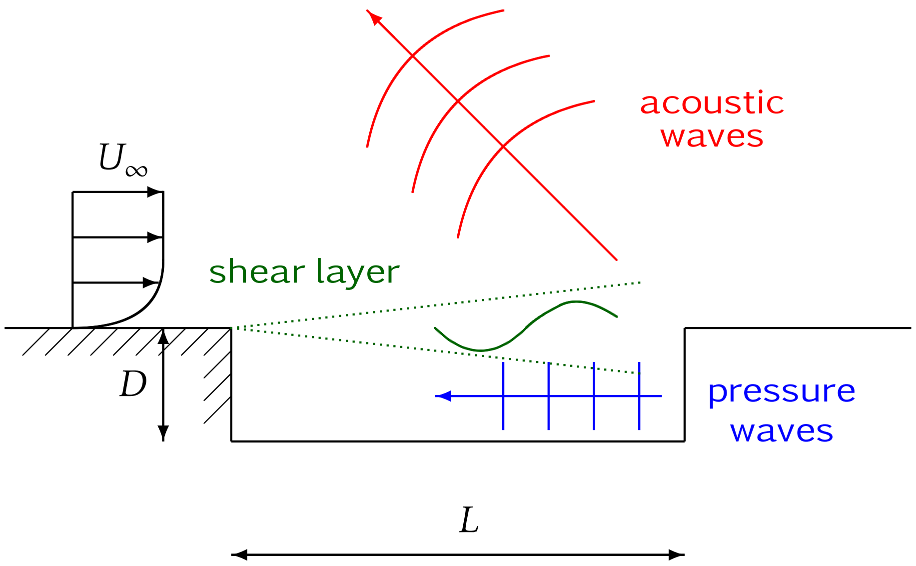
BEIJING (Reuters)—China is on course for another five years of robust growth although inflation threatens social stability and must be tamed, Premier Wen Jiabao said Saturday.

In China's version of a "State of the Union" address to the annual parliament session, Wen said the top priority this year was to curb price rises that were making [illegible] for ordinary people.

faster
and
QUIETER !

Flow-induced cavity oscillations : aeroacoustic feedback loop for shallow cavities (at rather high Mach numbers)

$L/U_c + L/c = n/f$



Rossiter formula (1964)

$$\mathrm{St} = \frac{fL}{U_\infty} = \frac{n-\alpha}{M + \dfrac{1}{\kappa}}$$

f frequency
L length
U_∞ free stream velocity
n number of vortices
α phase lag
$\kappa = U_c/U_\infty$, U_c convection velocity
$M = U_\infty/c$

- No information about amplitude or mode selection (L/δ_θ)
- Acoustic resonance can superimpose (longitudinal or depth mode)

Lighthill's theory of aerodynamic noise (1952)

What is the simplest equation that can be derived from the basic equations of fluid mechanics?

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_i)}{\partial x_i} = 0 \qquad (7)$$

$$\frac{\partial (\rho u_i)}{\partial t} + \frac{\partial (\rho u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} \qquad (8)$$

Sir James Lighthill (1924-1998)

By taking $\partial(7)/\partial t - \partial(8)/\partial x_i$, and observing that

$$c_\infty^2 \nabla^2 \rho = c_\infty^2 \frac{\partial^2}{\partial x_i \partial x_j}(\rho \delta_{ij})$$

$$\frac{\partial^2 \rho}{\partial t^2} - c_\infty^2 \nabla^2 \rho = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} \qquad \text{with} \qquad T_{ij} = \rho u_i u_j + (p - c_\infty^2 \rho)\delta_{ij} - \tau_{ij}$$

Lighthill's tensor

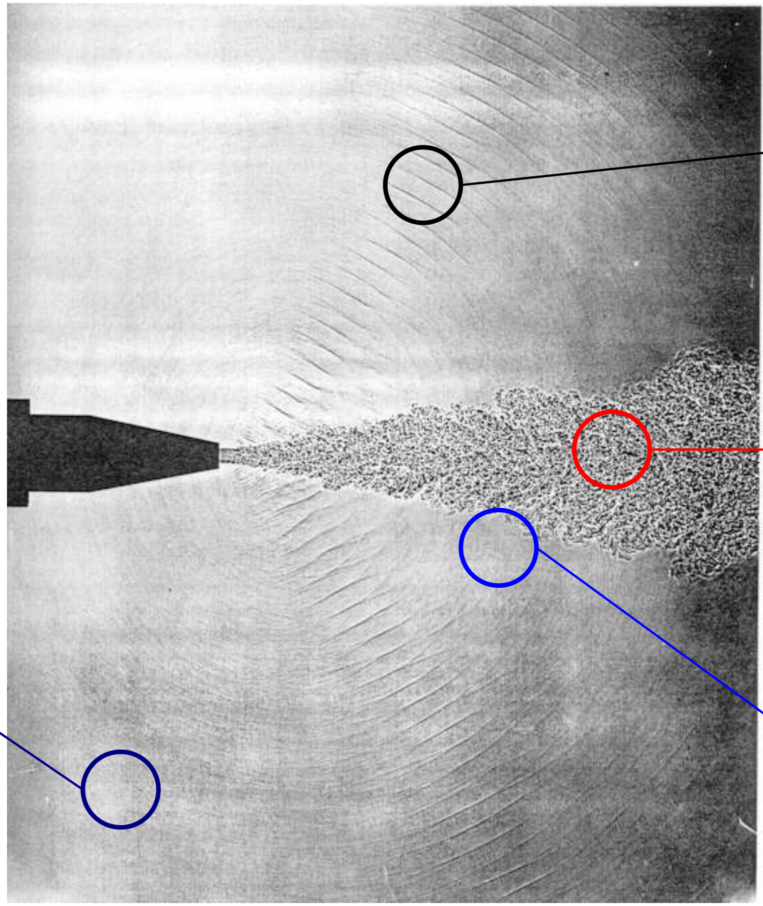
- **Interpretation of Lighthill's equation $\square \rho = \Lambda$**

  $\square \equiv \partial_{tt} - c_\infty^2 \nabla^2 \qquad \Lambda = \nabla \cdot \nabla \cdot \overline{\overline{\boldsymbol{T}}}$

In a uniform medium at rest

$\rho_\infty, p_\infty, c_\infty$

$\square \rho = 0$



$\square \rho \simeq \Lambda_{\text{nonlinear effects}}$

$\square \rho \simeq \Lambda_{\text{turbulence}}$

$\square \rho \simeq \Lambda_{\text{mean flow effects}}$

Quadrupole feature of the source

Solution obtained by a convolution product $*$ with the Green function in free space G_∞, refer to Eq. (3)

$$c_\infty^2 \rho(\boldsymbol{x}, t) = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} * G_\infty = T_{ij} * \frac{\partial^2 G_\infty}{\partial x_i \partial x_j}$$

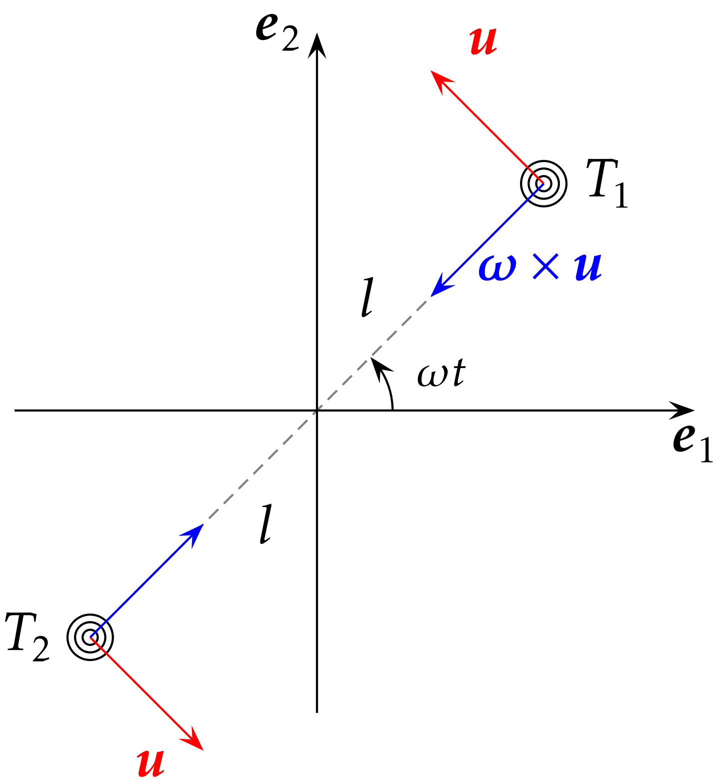
$$\frac{\partial^2 G_\infty}{\partial x_i \partial x_j} = \frac{1}{x}\frac{\delta''[\,]}{4\pi c_\infty^2}\frac{x_i x_j}{x^2} + \frac{1}{x^2}\frac{\delta'[\,]}{4\pi c_\infty}\left(3\frac{x_i x_j}{x^2} - \delta_{ij}\right) + \frac{1}{x^3}\frac{\delta[\,]}{4\pi}\left(3\frac{x_i x_j}{x^2} - \delta_{ij}\right)$$

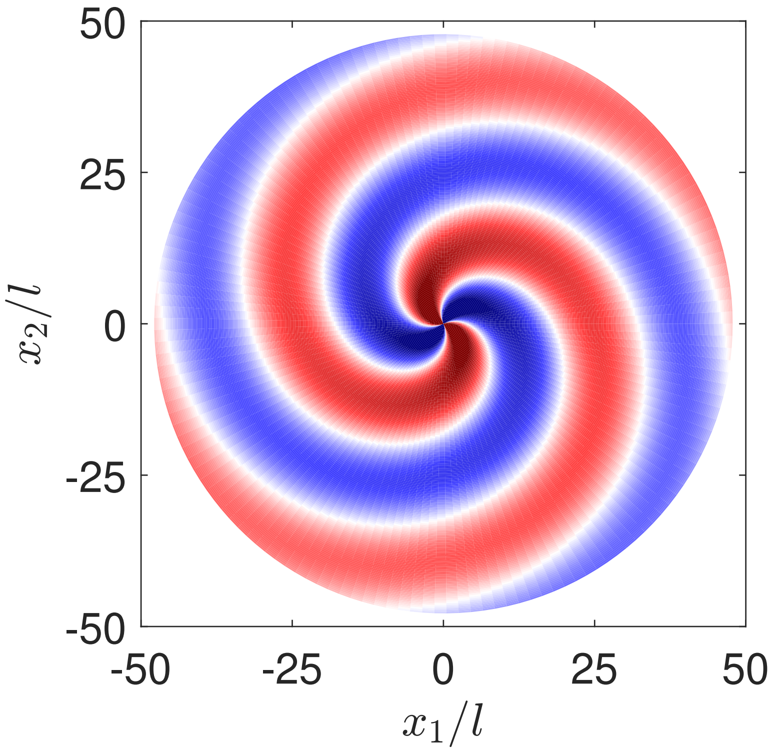
by noting $[\,] \equiv t - x/c_\infty$

As an illustration, compact longitudinal quadripole T_{33} avec $x_3 = r\cos\theta$ $(x_1 = r\sin\theta\cos\varphi, x_2 = r\sin\theta\sin\varphi)$

$$\frac{\partial^2 G_\infty}{\partial x_3 \partial x_3} \sim \underbrace{\frac{1}{r}\frac{\delta''[\,]}{4\pi c_\infty^2}\cos^2\theta}_{I \sim \cos^4\theta} + \frac{1}{r^2}\frac{\delta'[\,]}{4\pi c_\infty}\left(3\cos^2\theta - 1\right) + \frac{1}{r^3}\frac{\delta[\,]}{4\pi}\left(3\cos^2\theta - 1\right)$$

Noise radiated by spinning vortices – Powell (1964) & Howe (1998)





Direct noise computation
(Navier-Stokes)

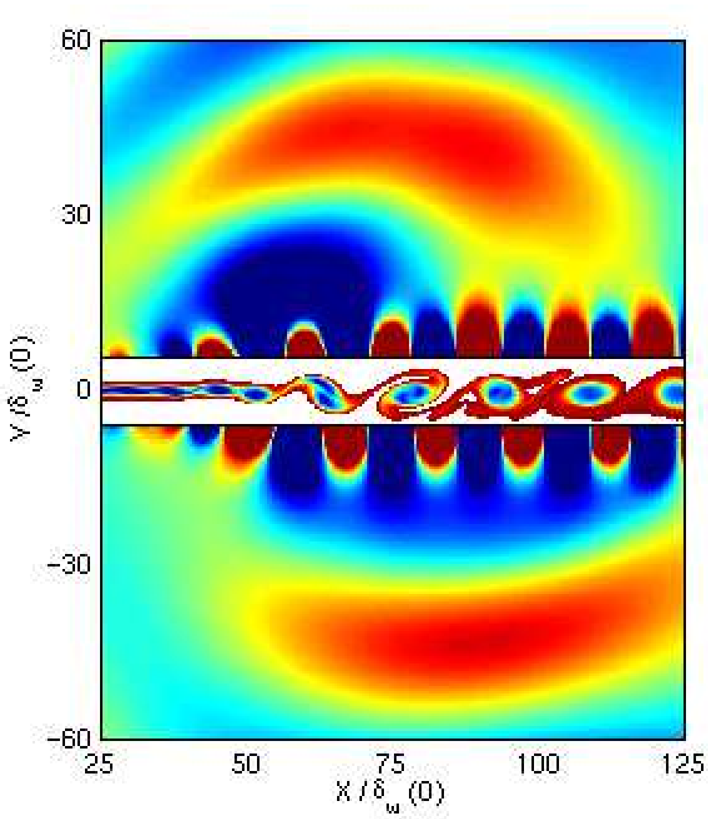
$$\rho'(r,\theta,t) \simeq -\rho_\infty \, 4\sqrt{\pi} \, \sqrt{\frac{l}{r}} \, M^{7/2} \, \cos\left[2\theta - 2\omega\left(t - \frac{r}{c_\infty}\right) + \frac{\pi}{4}\right]$$

$\mathrm{M} = \mathcal{U}/c_\infty, \mathcal{U} \sim \Gamma/(4\pi l)$

$1/\sqrt{r}$ decay of the pressure

$W_a \sim M^7$ per unit length in z (2-D flow)

Noise generated by vortex pairings in a mixing layer



Bogey *et al.*, 2000, AIAA J., **38**(12)

Forcing at f_0 (fundamental frequency of the mixing layer) and $f_0/2$, vortex pairing locations fixed around $75\delta_{\omega(0)}$

Wave fronts coming from pairing location with a wavelength $\lambda_{f_p} \simeq 51\delta_\omega(0)$ corresponding to frequency $f_p = f_0/2$

Convection effects (particularly in the upper rapid flow, $U_1 = 160$ m.s^{-1}, $U_2 = 40$ m.s^{-1})

Noise generation by pairing process : double spiral pattern, similar to a quadrupole structure already described in the work by Powell (1964)

Retarded-time solution of Lighthill's equation

Using the general expression (4)

$\mathcal{L} = (1/c_\infty^2)\partial_{tt}^2 - \nabla^2 \qquad S = (1/c_\infty^2)\partial_{x_i x_j}^2 T_{ij} \qquad G_\infty(\boldsymbol{x},t) = 1/(4\pi x)\,\delta(t - x/c_\infty)$

$\rho = \rho * \delta = \rho * \mathcal{L}(G_\infty) = \mathcal{L}(\rho) * G_\infty = S * G_\infty$ (in free space)

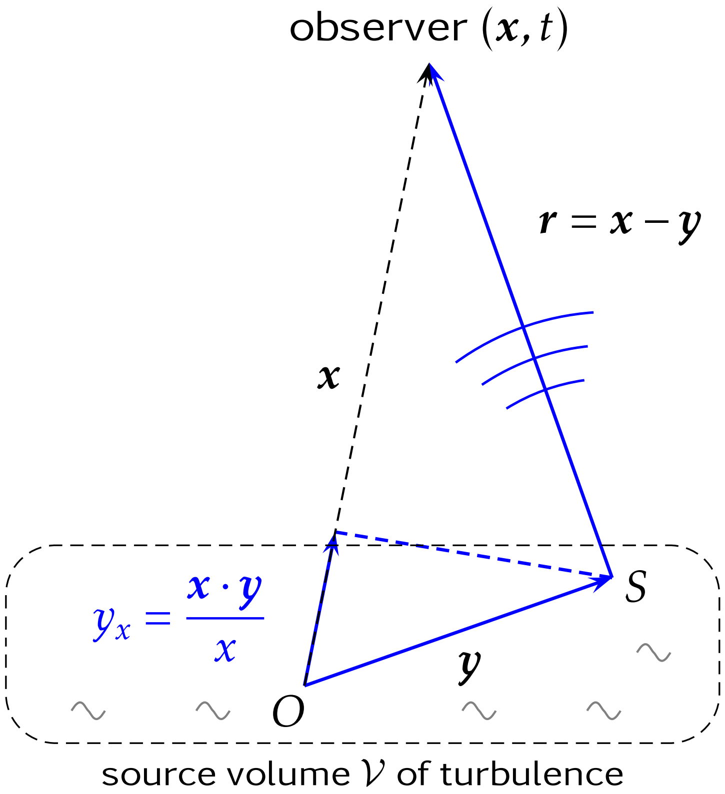
$$\rho(\boldsymbol{x},t) = S * G_\infty = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} * \frac{1}{4\pi c_\infty^2 x}\,\delta\left(t - \frac{x}{c_\infty}\right)$$

$$\simeq \frac{1}{4\pi c_\infty^2 x}\frac{\partial T_{ij}}{\partial x_j} * \frac{\partial}{\partial x_i}\delta\left(t - \frac{x}{c_\infty}\right) \simeq \frac{1}{4\pi c_\infty^2 x}\frac{\partial T_{ij}}{\partial x_j} * \left(-\frac{1}{c_\infty}\frac{x_i}{x}\right)\frac{\partial}{\partial t}\delta\left(t - \frac{x}{c_\infty}\right)$$

as $x \to \infty$

Retarded-time solution of Lighthill's equation

$$\rho(\boldsymbol{x},t) = \frac{1}{4\pi c_\infty^4}\int_{\mathcal{V}} \frac{\partial^2 T_{ij}}{\partial y_i \partial y_j}\left(\boldsymbol{y}, t - \frac{r}{c_\infty}\right)\frac{d\boldsymbol{y}}{r}$$



By using the rule (see the previous slide) in the far field approximation

$$\frac{\partial}{\partial y_i} \rightsquigarrow -\frac{1}{c_\infty}\frac{x_i}{x}\frac{\partial}{\partial t} \qquad x \gg y$$

$$\rho'(\boldsymbol{x},t) \simeq \frac{1}{4\pi c_\infty^4 x}\frac{x_i x_j}{x^2}\int_{\mathcal{V}} \frac{\partial^2 T_{ij}}{\partial t^2}\left(\boldsymbol{y}, t - \frac{r}{c_\infty}\right) d\boldsymbol{y} \tag{9}$$

Some remarks about these subtle integral formulations

Refer to Crighton (1975) & Ffowcs Williams (1992)

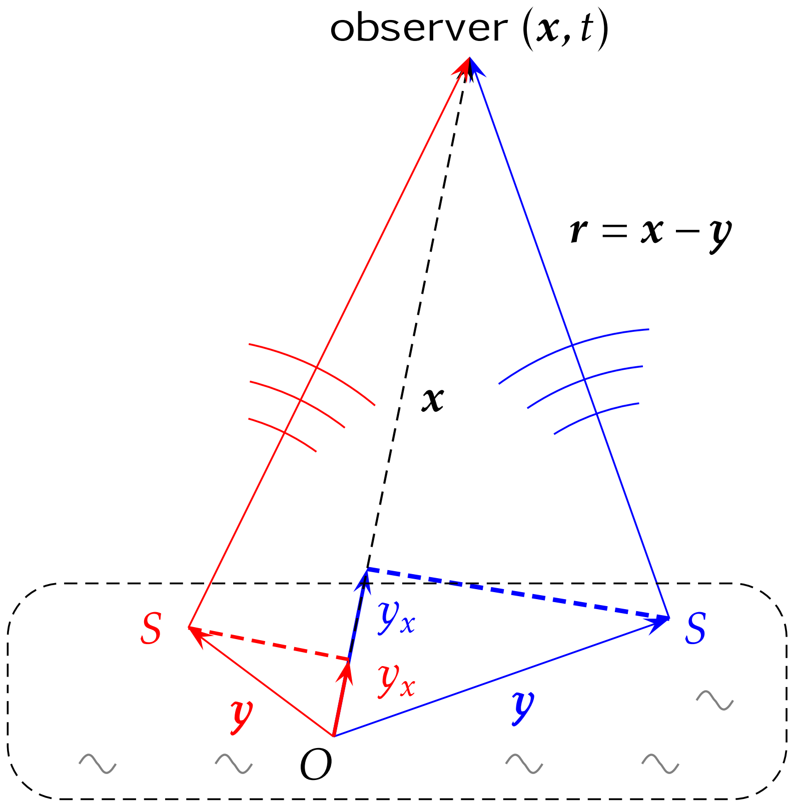
May we neglect the retarded time differences in the integral solution, induced by the term $y_x = \boldsymbol{y} \cdot \boldsymbol{x}/x$?

$$t - \frac{r}{c_\infty} = t - \frac{|\boldsymbol{x} - \boldsymbol{y}|}{c_\infty} \simeq t - \frac{x}{c_\infty} + \frac{\boldsymbol{x} \cdot \boldsymbol{y}}{x c_\infty} + \cdots$$

difference in time emission $\quad \dfrac{\boldsymbol{x} \cdot \boldsymbol{y}}{x c_\infty} \sim \dfrac{l_s}{c_\infty}$

$$\frac{\text{difference in time emission}}{\text{source turbulent time}} \sim \frac{l_s/c_\infty}{l_s/u'} \sim M_t$$

$\rightsquigarrow$ Yes if $M_t \ll 1$, that is for compact sources ($M_t = u'/c_\infty$ turbulent Mach number)



source volume $\mathcal{V} \sim l_s^3$ of turbulence

Some remarks (cont)

Viscous effects are very weak noise sources. For relatively low Mach numbers and flows nearly isentropic, $p' = c_\infty^2 \rho' + (p_\infty / c_v) s'$ for an ideal gas, Lighthill's tensor is often simplified as follows

$$T_{ij} \simeq \bar{\rho} u_i u_j \simeq \rho_\infty u_i u_j$$

...but acoustic - mean flow interactions are definitively lost
(see first part – propagation effects in a non-uniform mean flow : there is an acoustic part in T_{ij} with ρ', implicit formulation)

Jet noise scaling (dimensional law)

In the far field and for $M_t \leq 1$ (compact sources)

$$\rho'(\boldsymbol{x},t) \simeq \frac{1}{4\pi c_\infty^4 x}\frac{x_i x_j}{x^2}\int_{\mathcal{V}} \frac{\partial^2 T_{ij}}{\partial t^2}\left(\boldsymbol{y}, t - \frac{x}{c_\infty}\right) d\boldsymbol{y}$$

$$\sim \frac{1}{c_\infty^4 x}\frac{\rho_j U_j^2}{(D/U_j)^2} D^3 \qquad \begin{cases} \text{jet nozzle diameter } D \\ \text{jet exit velocity } U_j \end{cases}$$

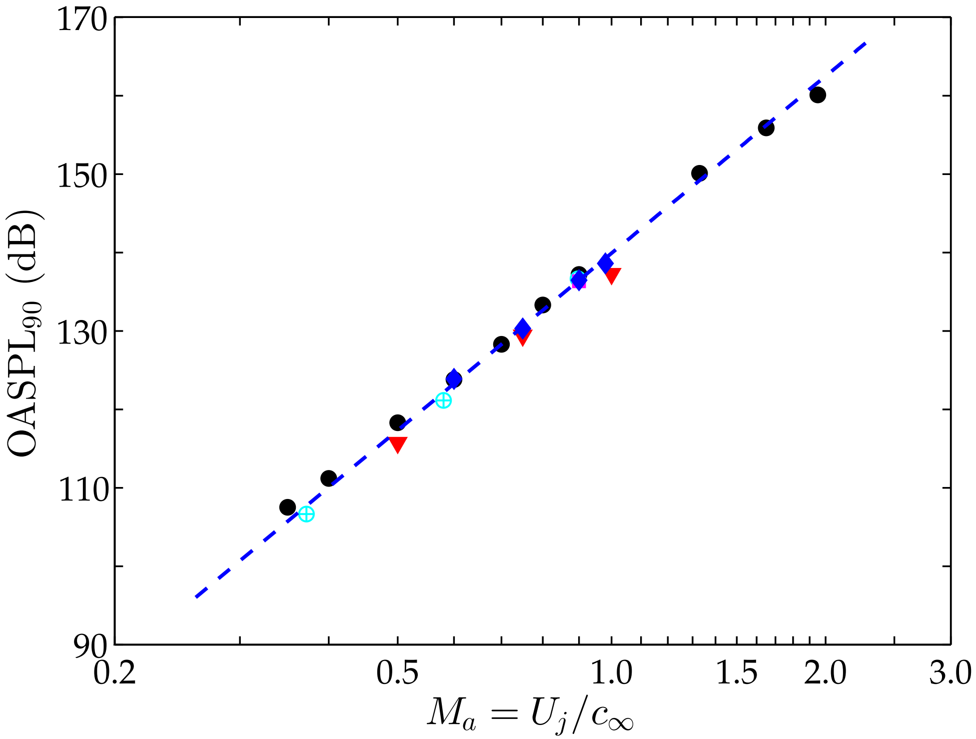
Hence,

$$W \sim \frac{\rho_j}{\rho_\infty}\frac{U_j^5}{c_\infty^5} A \rho_j U_j^3 \qquad (A = \pi D^2/4)$$

Lighthill's eigth power law (1952)

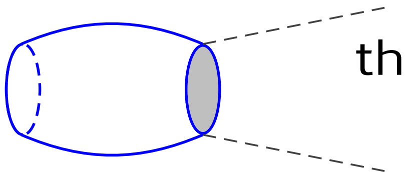
$$\overline{p'^2}\Big|_{\theta=90^\circ} \simeq K \rho_\infty^2 c_\infty^4 \frac{A}{r^2}\left(\frac{\rho_j}{\rho_\infty}\right)^2 M^{7.5} \qquad K \simeq 1.9 \times 10^{-6}$$

Jet noise scaling – acoustic efficiency η



$$\eta = \frac{W_{\text{acoustic}}}{W_{\text{mechanical}}} \simeq 1.2\times10^{-4}\,(\rho_j/\rho_\infty)\,M^5$$

$$W_{\text{acoustic}} \sim A U_j^8$$
$$W_{\text{mechanical}} = A\rho_j U_j^3/2$$



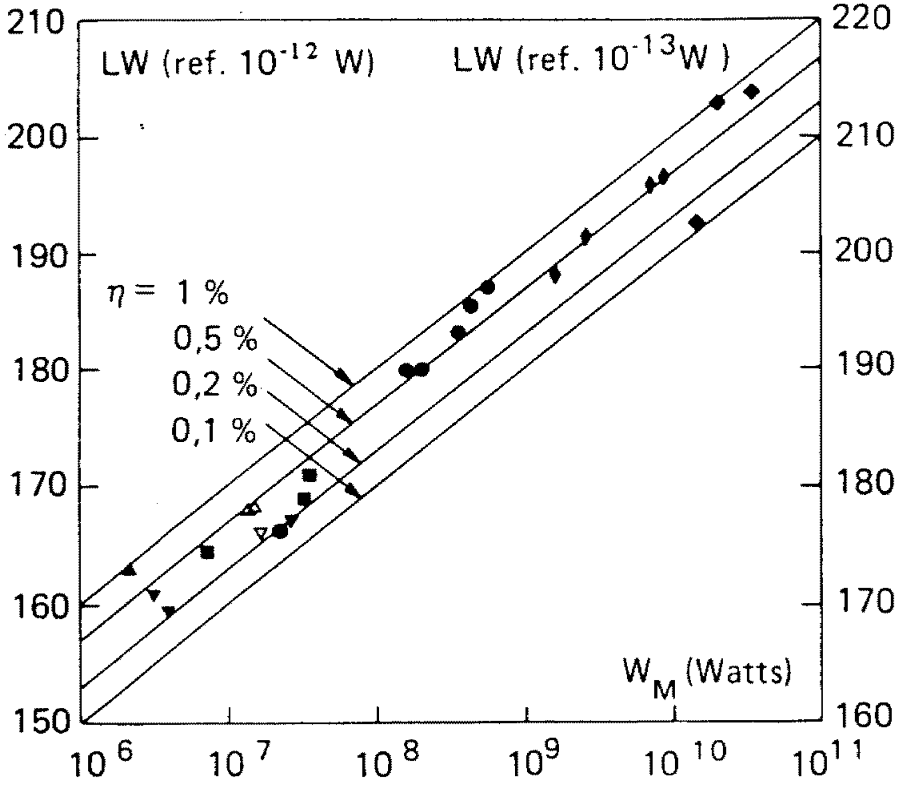
$$\text{thrust} = \dot{m}_j U_j$$
$$= A\rho_j U_j^2$$

♦ Bogey *et al.* (2007), • Tanna (1977), ⊕ Lush (1971),
▼ QinetiQ 1983 NTF data, ■ MolloChristensen (1964)

Free-field loss-less data scaled to a nozzle exit area A of 1 m^2, $T_j/T_\infty = 1$

- **Acoustic efficiency η for supersonic jet noise**

$$\eta \sim (\rho_j/\rho_\infty) \qquad \text{(Ffowcs Williams, 1963)}$$

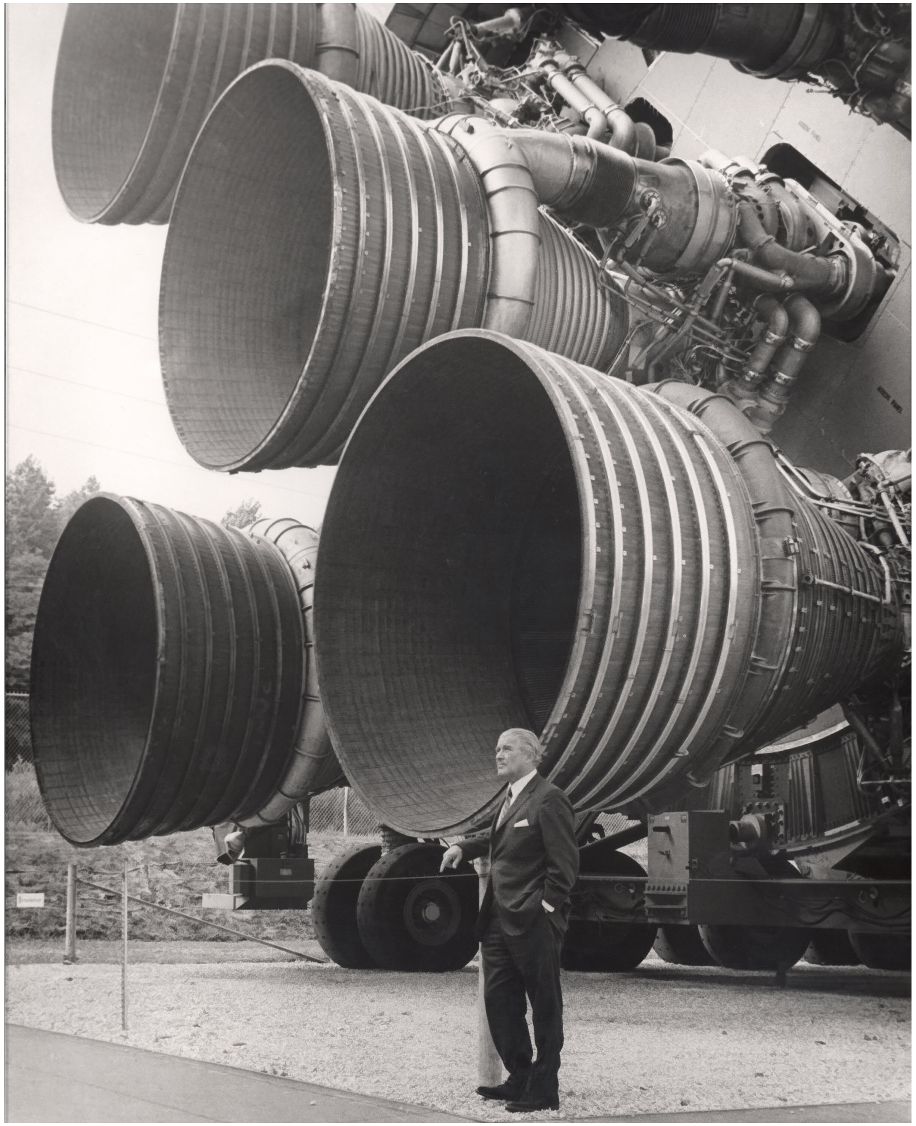


Ref. Eldred, NASA SP-8072 (1971) & Candel (1983)

rocket engines
(solid and liquid propergol)
$1.56 \leq T \leq 31100$ *kN*

– combustion noise
– Reynolds number effects

Von Braun (1912 - 1977) / Saturn V

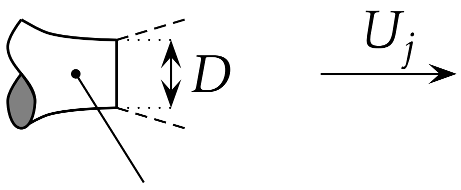


- Acoustic Mach number M_a

$$M_a = \frac{U_j}{c_\infty} \qquad \text{noise} \propto M_a^n$$

- Reynolds number Re_D

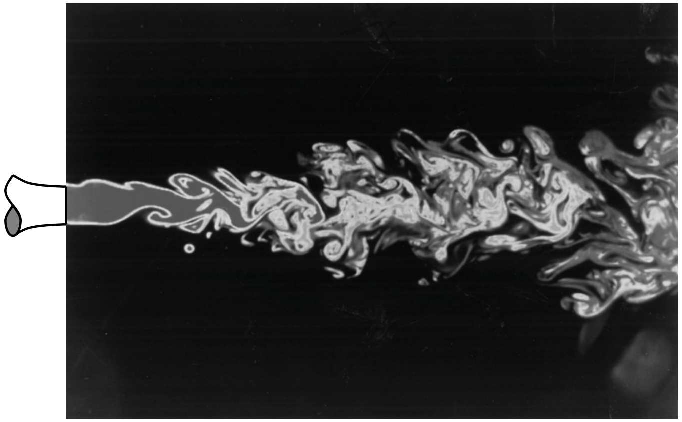
$$Re_D = \frac{U_j D}{\nu} = \frac{D^2/\nu}{D/U_j} \sim \frac{\text{viscous time}}{\text{convective time}}$$



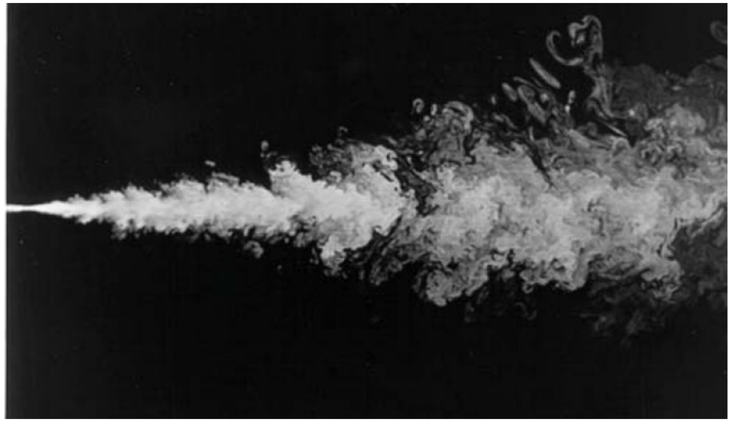
- Strouhal number St

$$St = \frac{fD}{U_j} = \frac{f}{U_j/D} \qquad \text{non-dimension frequency}$$

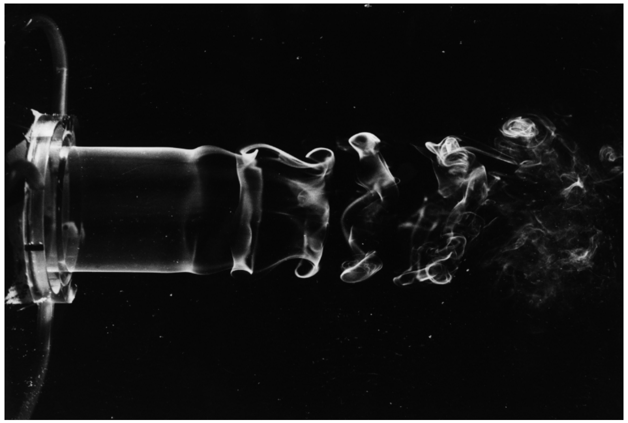
- **Subsonic turbulent jets, Reynolds number $\mathrm{Re}_D = U_j D/\nu$**



Prasad & Sreenivasan (1989)
$\mathrm{Re}_D \simeq 4000$

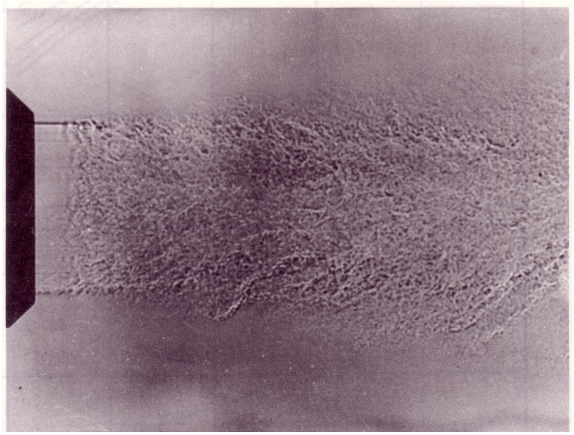


Dimotakis *et al.* (1983)
$\mathrm{Re}_D \simeq 10^4$



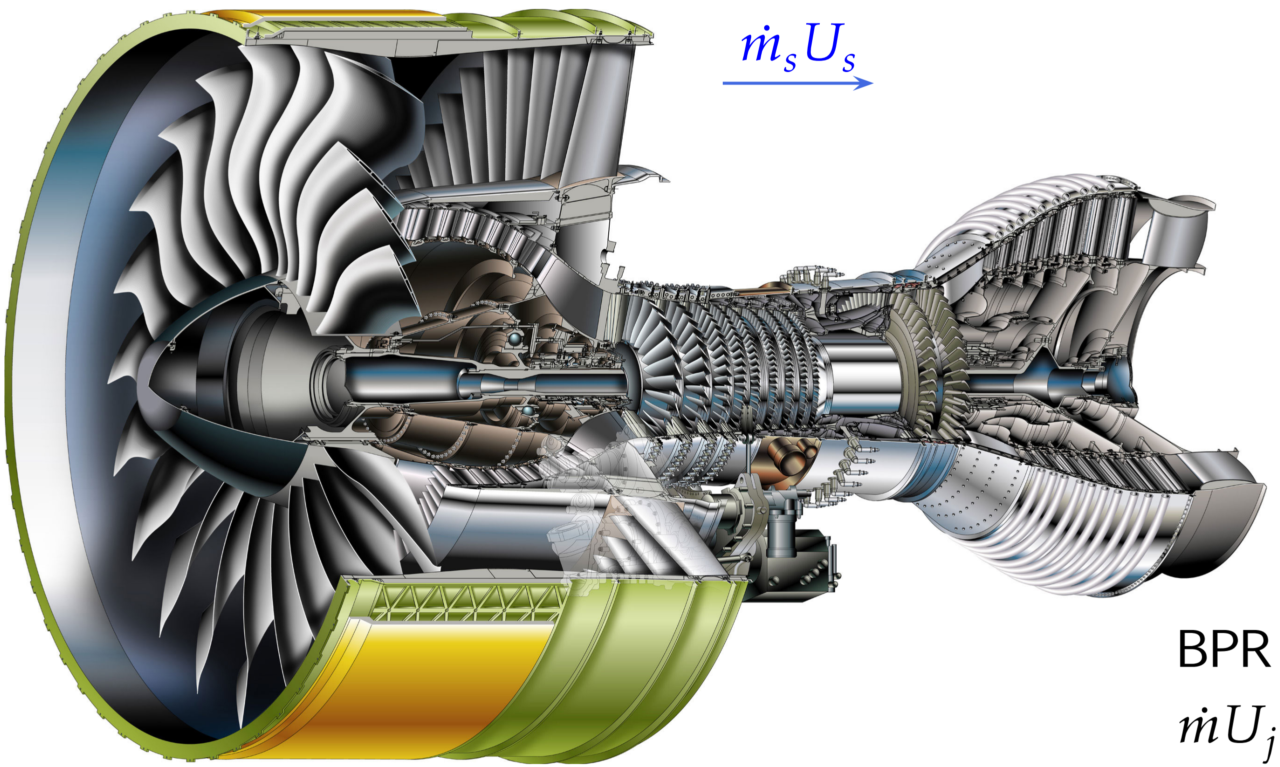
Kurima, Kasagi & Hirata (1983)
$\mathrm{Re}_D \simeq 5.6 \times 10^3$

Ayrault, Balint & Schon (1981)
$\mathrm{Re}_D \simeq 1.1 \times 10^4$



Mollo-Christensen (1963)
$\mathrm{Re}_D = 4.6 \times 10^5$

- **Engine Alliance GP7200** (A380, BPR = 8.7, $D_{\text{fan}} = 2.95$ m)



$$\text{BPR} \equiv \dot m_s / \dot m_p$$
$$\dot m U_j = \dot m_p U_p + \dot m_s U_s$$
$$\delta U = U_j - U_f$$

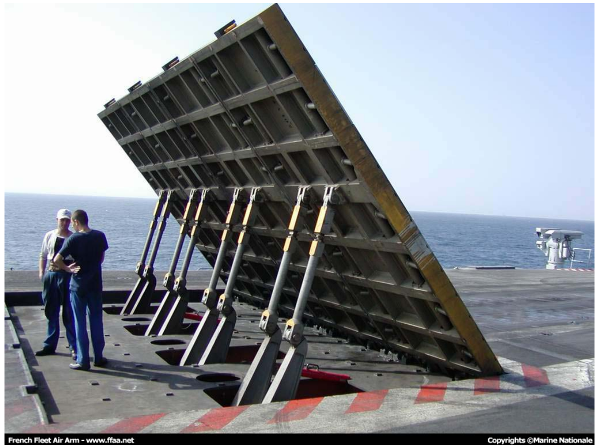
Trust $T = \dot m \delta U$ Propulsive efficiency $\eta_p = \dfrac{1}{1 + \delta U / (2U_f)}$

$\eta_p \nearrow \quad \Longrightarrow \quad \delta U \searrow \quad \Longrightarrow \quad \dot m, \text{BPR} \nearrow$

Motivation

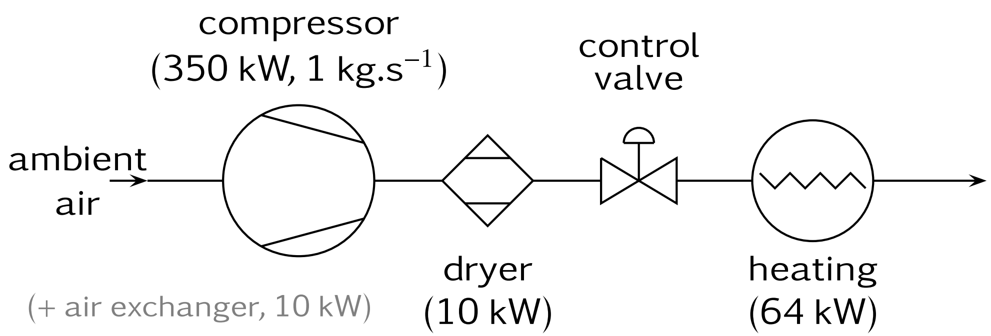
Tanegashima Space Center (JAXA, Japan)

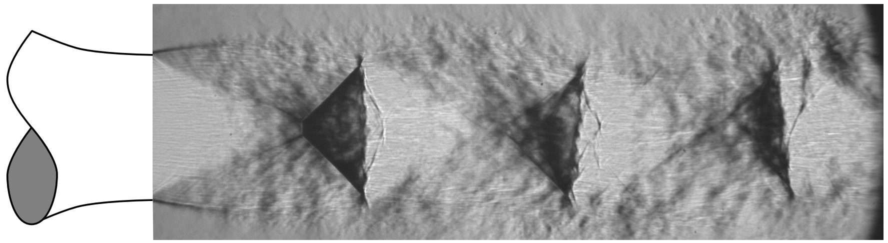
- acoustic environment of space launcher at lift-off and protection of payload
- military aircraft (*e.g.* hearing protection of naval crew on aircraft carrier deck)
- coaxial propulsive jet of turbofan engine in cruise condition (cabin noise)



Take off from the CDG aircraft carrier (noise levels exceed 140 dB)

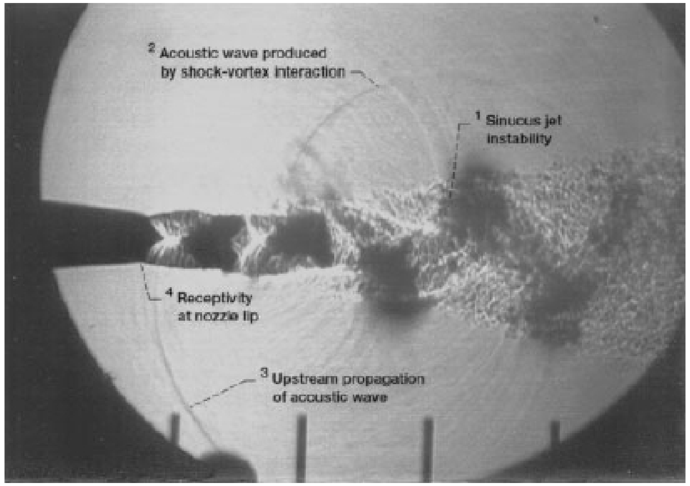
Supersonic jet flow in a laboratory



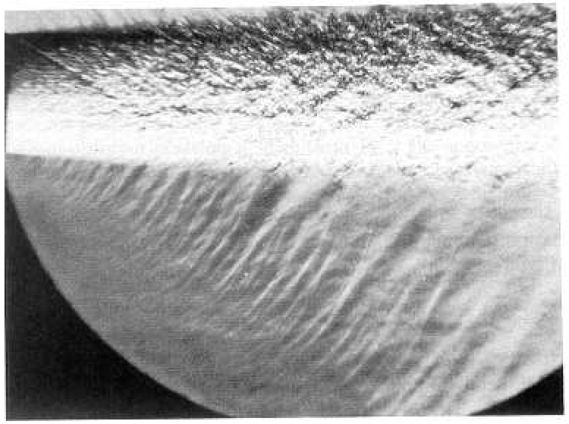


 $M_j = 1.50$

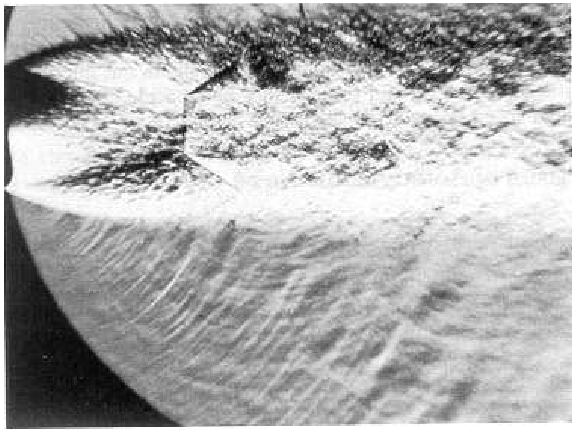
A shock wave is a thin region of space (a few micrometers thick) where the flow velocity, pressure, density and temperature change extremely quickly



Raman, *J. Fluid Mech.* (1997)

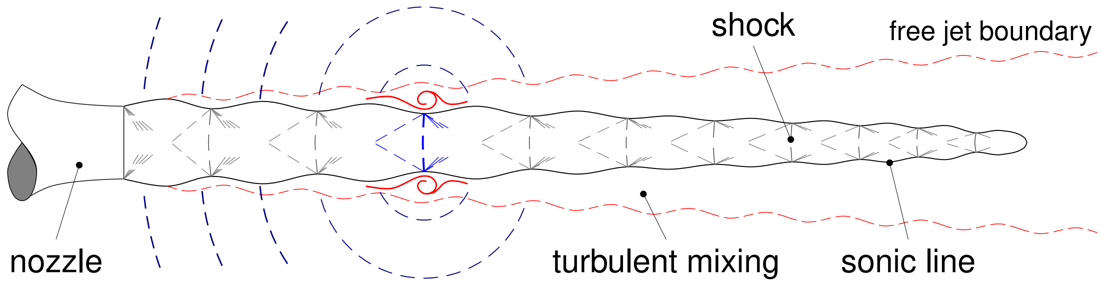


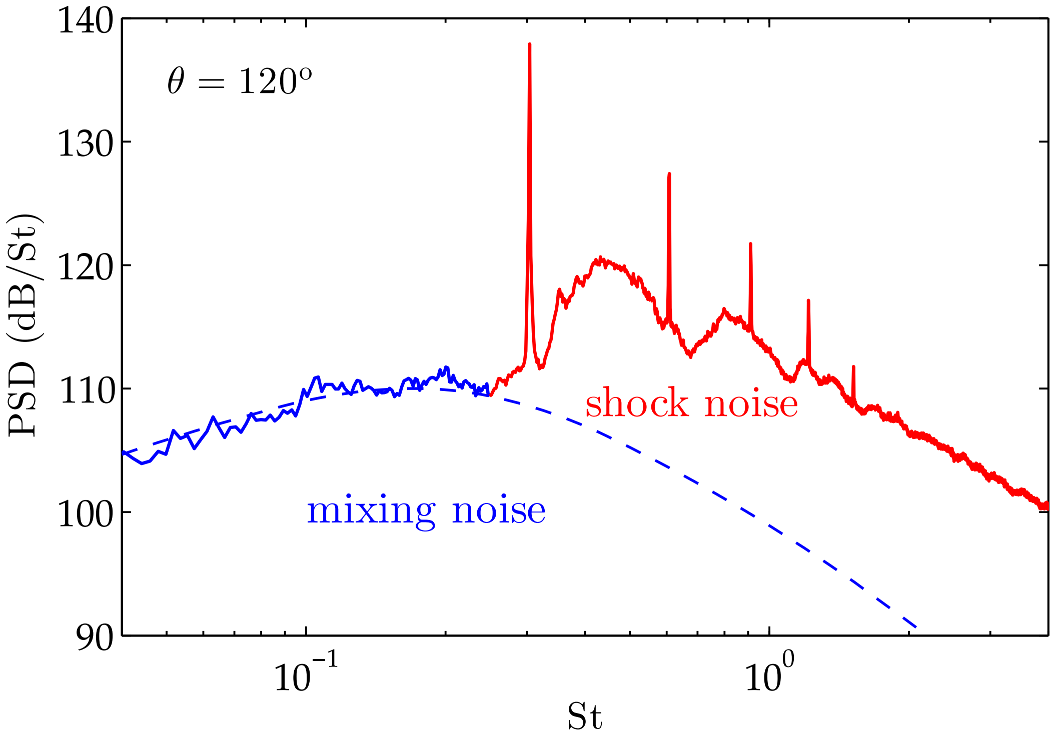
Seiner, AIAA Paper 84-2275
M= $2,\ p_j/p_\infty = 1$

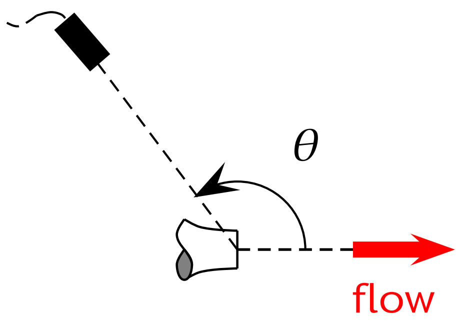


Seiner, AIAA Paper 84-2275
M= $2,\ p_j/p_\infty = 3.05$

- Mixing noise including Mach waves and shock noise : broadband shock-associated noise (BBSAN) + screech tone (strong acoustic feedback)







$M_j = 1.35$ $D = 0.38$ mm

Acoustic spectrum at 2 m

$M_f = 0.39$

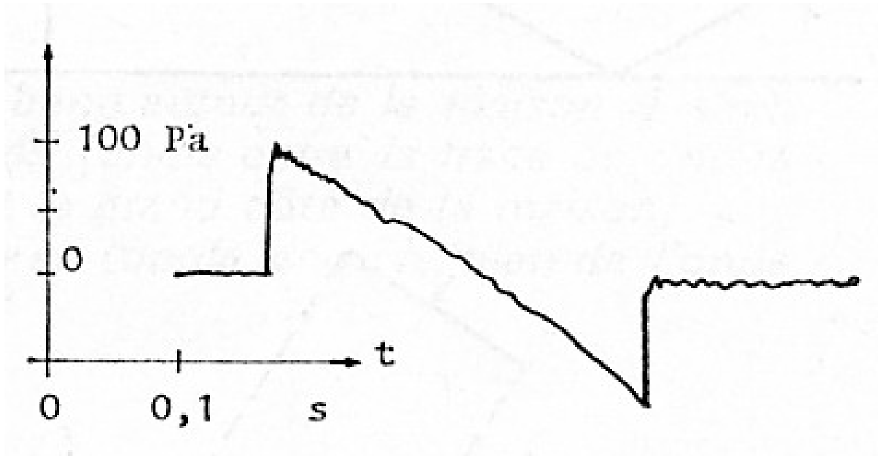
André *et al.*, *Phys. Fluids*, **23**, 126102 (2011)

Sonic boom : a special case of airframe noise

F/A-18 Hornet
passing through the sound barrier
(Navy Ensign John Gay, July 7, 1999)

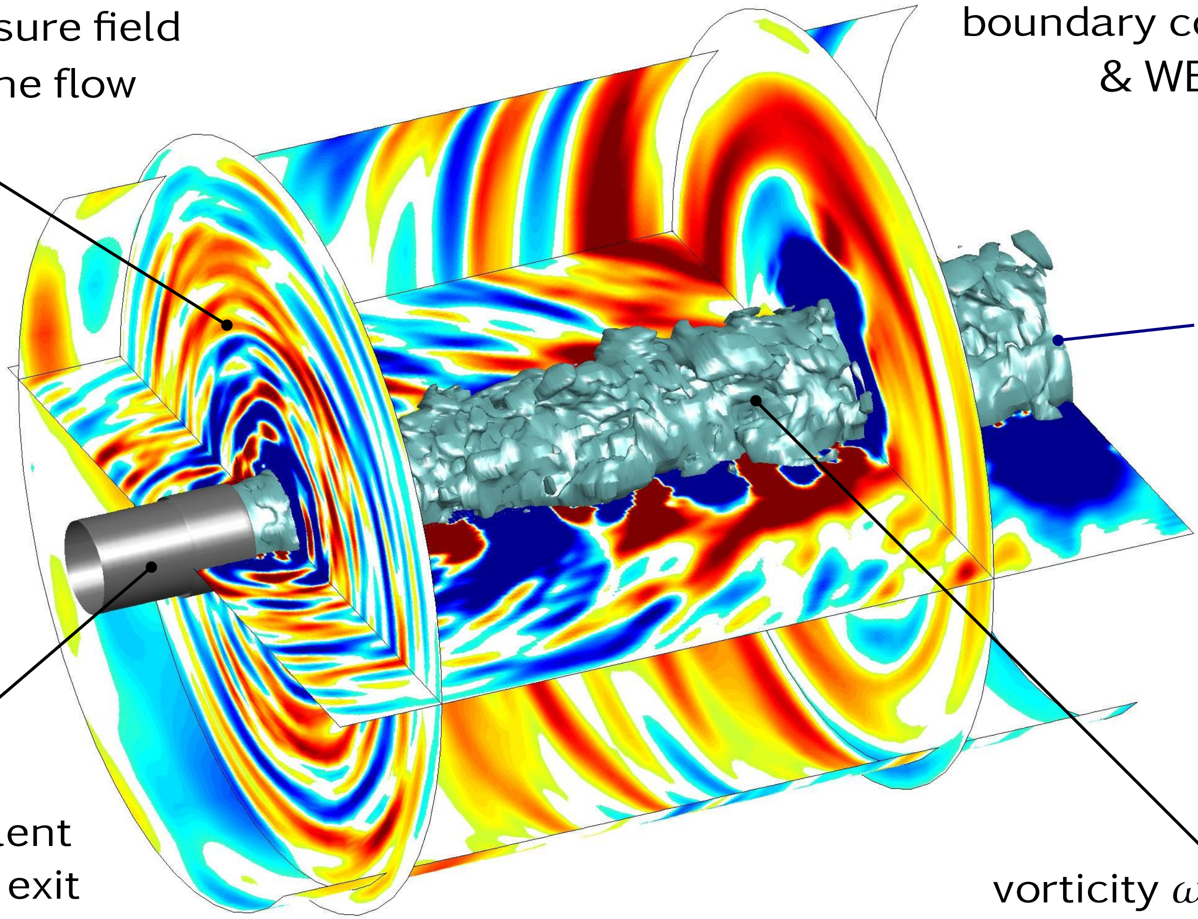
Concorde - Shock waves at Mach 2.2 in wind tunnel (ONERA)

N-wave pattern measured close to the ground from Concorde



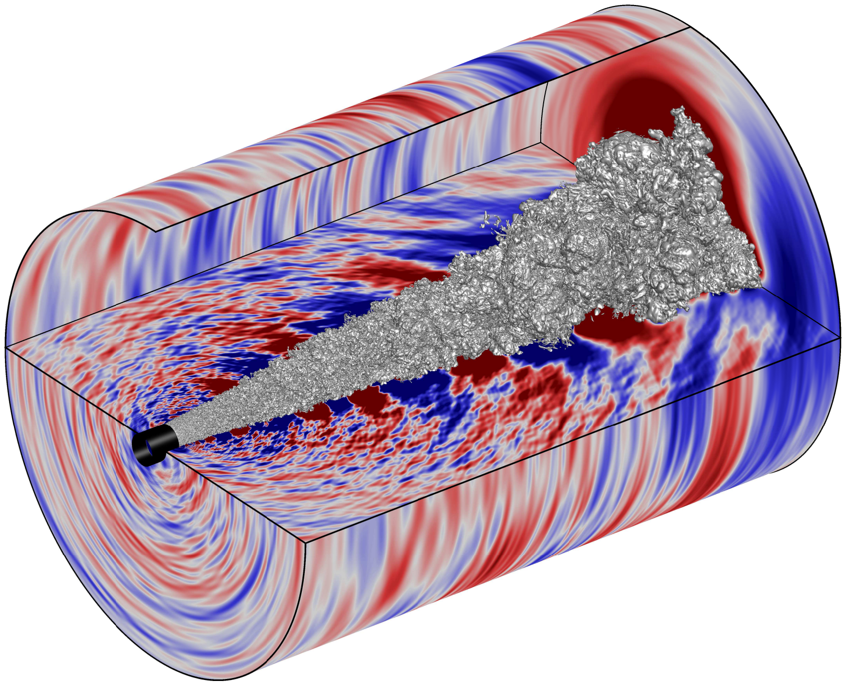
- Enjeux et Motivations
- Propagation en écoulement
- Introduction to aeroacoustics
- Advances in aeroacoustics research

- **High fidelity flow/noise simulation in a physically and numerically controlled environment**

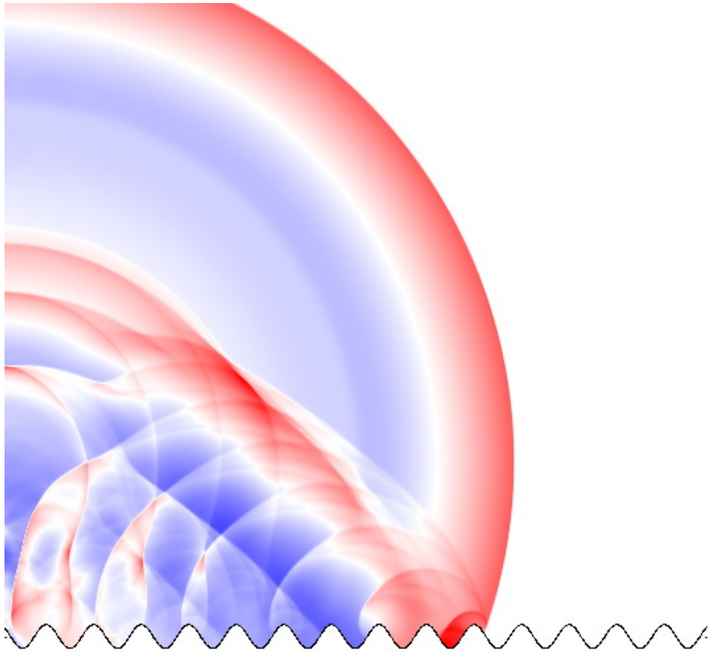


Barré *et al., Int. J. AeroAcous.* (2006)
Bogey & Bailly, *J. Fluid Mech.* (2007)

Quelques autres sujets



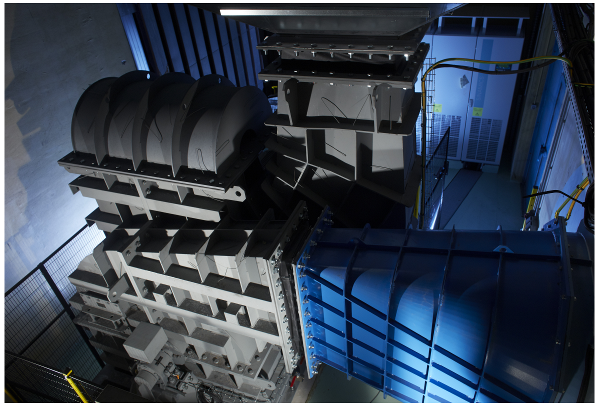
Simulation numérique d'un jet subsonique et de son acoustique

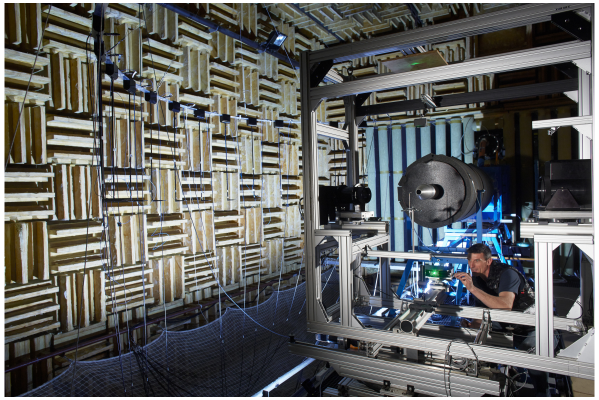


Propagation non linéaire d'une onde de choc sur une surface rugueuse

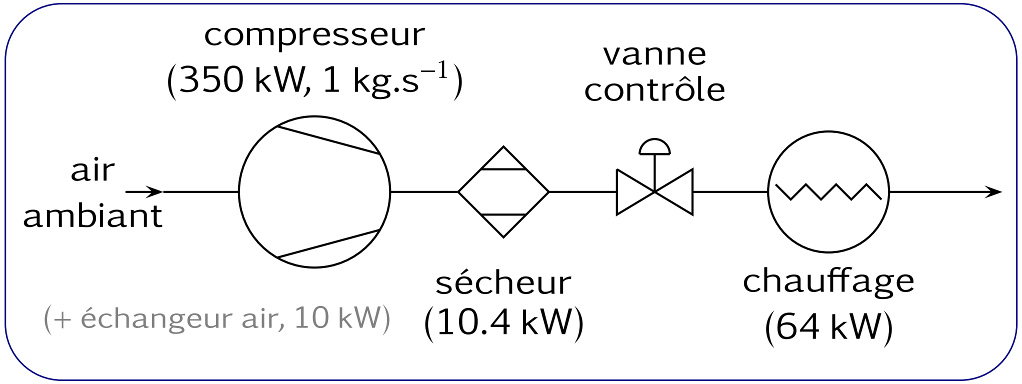
... et beaucoup d'autres sujets non abordés dans ce cours faute de temps, en particulier le bruit des marchines tournantes.

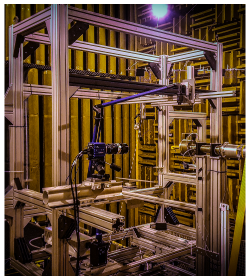
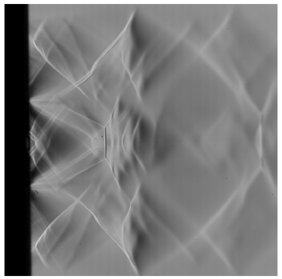
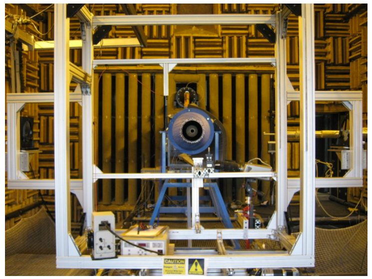
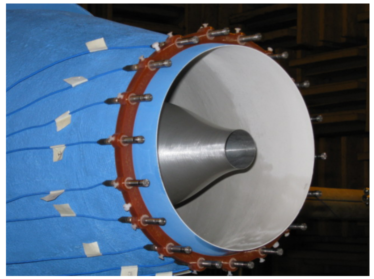
Soufflerie anéchoïque subsonique et supersonique





Soufflerie supersonique · $M \leq 1.59$ (NPR $\simeq 4.19$)



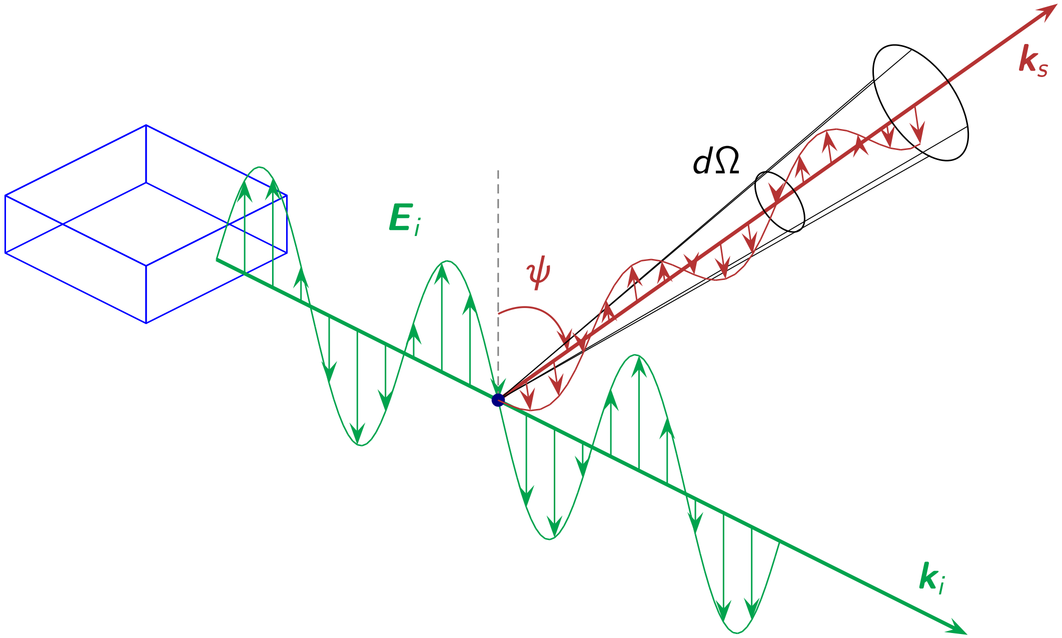





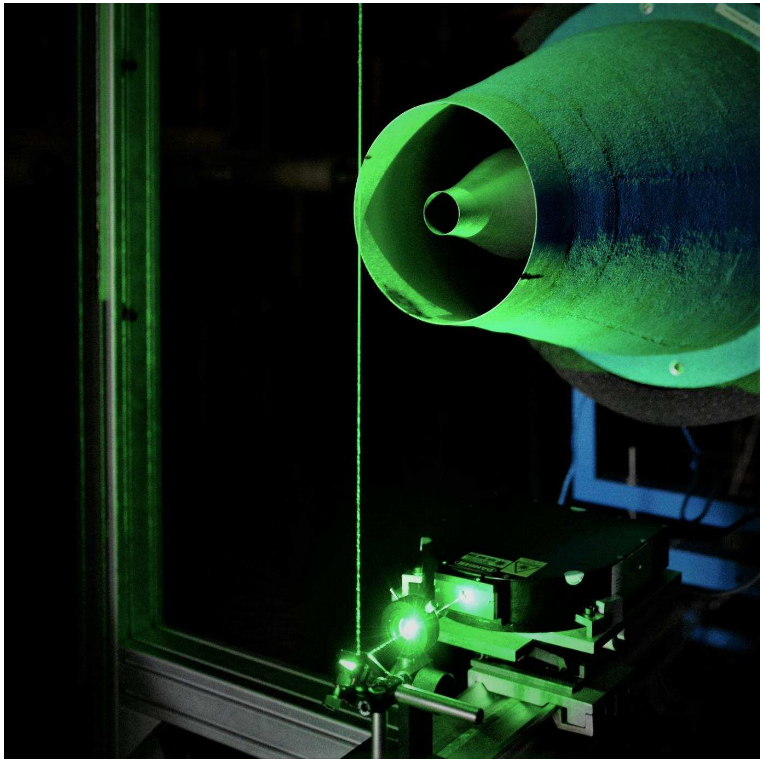
Soufflerie subsonique · ventilateur centrifuge double étage 800 kW · 0.5% de taux de turbulence en entrée de chambre sourde · $M \simeq 0.5$, $\dot{m} \simeq 19.4$ kg.s^{-1} dans une section de 30 cm × 40 cm pour étudier le bruit de profil & $M \simeq 0.8$, $\dot{m} \simeq 10.4$ kg.s^{-1} dans un écoulement secondaire de diamètre 20 cm pour reproduire l'effet de vol sur un jet subsonique ou supersonique

Grand chambre anéchoïque $(10 \times 8 \times 8$ m$^3)$ et silencieux baffles

- **Rayleigh scattering** to measure time-resolved density fluctuations induced by the turbulent jet flow, correlated to its acoustic

Based on light scattered by the molecules constituting the flow - non intrusive method ($ka \ll 1$ with $a \simeq 1$ nm and $\lambda = 532$ nm)



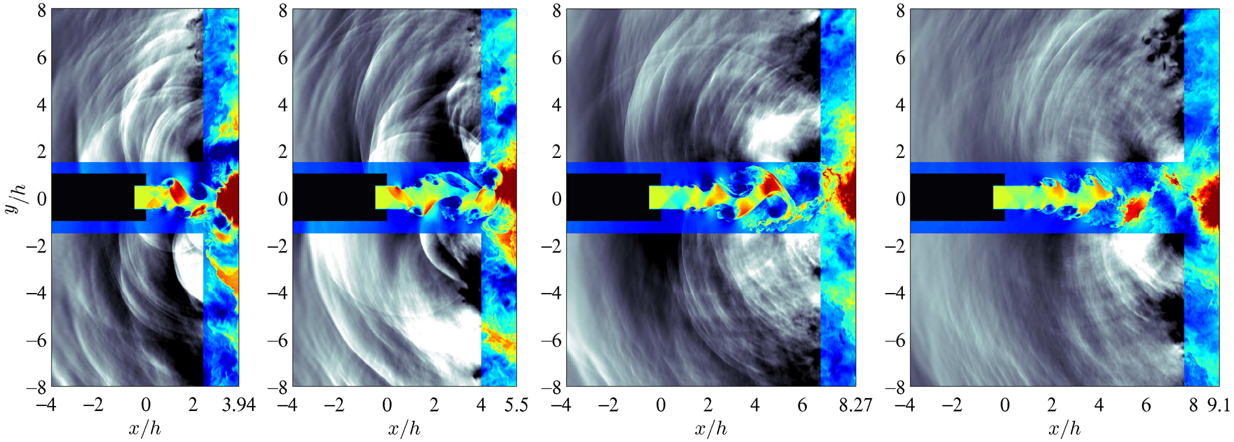


$\rightsquigarrow$ photon flux proportional to density ...

Highly qualified candidates are encouraged to apply at any time!

http://acoustique.ec-lyon.fr

Turbulence and Aeroacoustics – Investigation of tone generation in ideally expanded supersonic planar impinging jets



Density and pressure fields, $L/h = 3.94, 5.5, 8.27, 9.1$

$M_j = 1.28$, $\mathrm{Re}_h = 5 \times 10^4$

(Gojon, Bogey & Marsden, *J. Fluid Mech.*, 2016)