Tutorials - Exercises

Guidelines

- You must reuse the results mentioned in the course slides without demonstration (quoting only the relevant slide number, for example)
- An essential part of your work is to provide valuable / insightfull comments on your results.

Added mass

1. A steel pipeline 1 m in diameter and 0.05 m thick runs from the sea floor to an offshore platform. Calculate the added mass per unit length of the empty pipe in air and in water. The density of steel if 7800 kg.m^{-3}, the density of air is 1.2 kg.m^{-3} and the density of sea water is 1020 kg.m^{-3} What is the ratio of added mass to structural mass ?

2. The added mass of a circular cylinder in a viscous fluid is approximately

$$\rho_f \pi a^2 b \left[1 + 2(\pi^2 a f/\nu)^{-1/2}\right]$$

where f is the frequency of oscillation. Does viscosity increase or decrease added mass ? Justify your answer. What contribution does viscosity make to added mass in the previous question if $f = 1$ Hz, $\nu_f = 1.2 \times 10^{-6}$ m^2.s^{-1} in water and $\nu_f = 2 \times 10^{-5}$ m^2.s^{-1} in air

References

[1] Naudascher E. & Rockwell D., 1994, *Flow-induced vibrations, an engineering guide*, Springer, New York.

Acceleration of a sphere in a fluid medium at rest

To be complete regarding the determination of the added mass of a moving sphere $\xi_1(t)$, the force exerted by the fluid on the sphere should take the form $F_a = -m_a \ddot{\xi}_1$ (using the notations of the lectures). For the demonstration, we need to involve the generalized Bernoulli equation for an irrotational unsteady flow

$$\rho_f \frac{\partial \phi}{\partial t} + \frac{1}{2}\rho_f \boldsymbol{u}^2 + p = h(t) \qquad (1)$$

where h is a function of time only to be determine.

1. Recall the expression for the velocity potential ϕ and the velocity field in spherical coordinates

$$u_r = \frac{\partial \phi}{\partial r} \qquad u_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} \qquad (u_\varphi = 0)$$

2. Show that $h(t) = p_\infty$ in our case.

3. Show that the force exerted by the flow to the sphere can be recast as

$$F_a = -2\pi a^2 \int_0^\pi p|_{r=a} \cos\theta \sin\theta d\theta \qquad (2)$$

4. Before to susbtitute the expression of pressure p using Bernoulli Eq. (1) in the previous integral, we need to determine $\partial\phi/\partial t$. For a current point (x_1, x_2, x_3) in the fluid domain in Cartesian coordinates, one has $r\cos\theta = x_1 - \xi_1$ and $r^2 = (x_1 - \xi_1)^2 + x_2^2 + x_3^2$ with $\xi_1 = \xi_1(t)$. Using the chain rule for differentiation, show that

$$r\frac{\partial r}{\partial t} = -(x_1 - \xi_1)\dot{\xi}_1$$

and

$$\frac{\partial\phi}{\partial t} = -\frac{a^3}{2r^3}\ddot{\xi}_1(x_1 - \xi_1) + \frac{a^3}{2r^3}\dot{\xi}_1^2 + \frac{3}{2}\frac{a^3}{r^4}\dot{\xi}_1(x_1 - \xi_1)\frac{\partial r}{\partial t}$$

5. Determine the pressure on the sphere surface

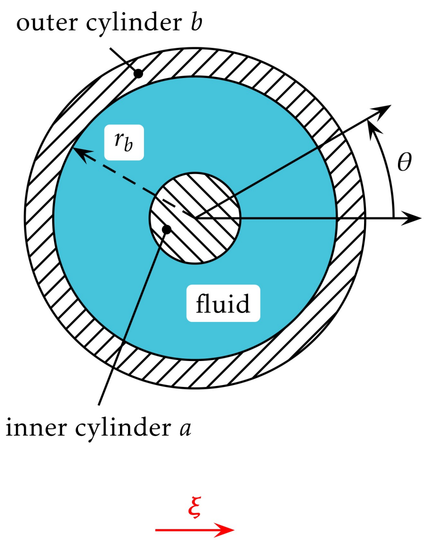
$$p|_{r=a} = p_\infty + \frac{1}{2}\rho_f\dot{\xi}_1^2\left(1 - \frac{9}{4}\sin^2\theta\right) + \frac{1}{2}\rho_f a\cos\theta\,\ddot{\xi}_1$$

6. Finally calculate the force exerted on the sphere using Eq. (2) and show that

$$F_a = -m_a\ddot{\xi}_1$$

Provide an interpretation of this result. Compute the pressure coefficient $C_p = (p - p_\infty)/(\rho_f\dot{\xi}_1^2/2)$ for a uniform motion ($\dot{\xi}_1 = \text{cst}$)

Fluid coupling between two concentric cylinders



Two long concentric cylinders are coupled through a fluid-filled gap. Following Fritz,[1] the following assumptions are made to derive the model. The flow is two-dimensional, incompressible and inviscid. The flow is also assumed to be initially at rest with the aim to determine impulsive fluid loads on naval components during a typical military shock.[2]

Let r_a be the radius and ξ_a the lateral displacement of the inner cylinder from the concentric starting point, r_b the radius and ξ_b the lateral displacement of the outer cylinder from the concentric starting point.

The Laplacian operator in cylindrical coordinates reads

$$\nabla^2\phi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} \tag{3}$$

1. Write the equation and the boundary conditions verified by the velocity potential ϕ.

2. Find a solution of the form $\phi = f(r)\cos\theta$ and show that

$$\phi = a_1 r\cos\theta + a_{-1}\frac{1}{r}\cos\theta \qquad a_1 = \frac{r_b^2\dot{\xi}_b - r_a^2\dot{\xi}_a}{r_b^2 - r_a^2} \qquad a_{-1} = \frac{(\dot{\xi}_b - \dot{\xi}_a)r_a^2 r_b^2}{r_b^2 - r_a^2} \tag{4}$$

3. Provide the expression of the velocity field $\boldsymbol{u} = \nabla\phi = (\partial\phi/\partial r, (1/r)\partial\phi/\partial\theta)$ and of $\boldsymbol{u}^2$

4. Using the generalized Bernoulli Eq. (1), show that there is no contribution of the term $\boldsymbol{u}^2$ to the force F_a acting on the inner cylinder.

5. Using the chain rule to compute $\partial\phi/\partial t$