Laboratoire de Mécanique des Fluides et d'Acoustique
LMFA UMR 5509

CENTRALE LYON

Fluid-structure interaction (part II)

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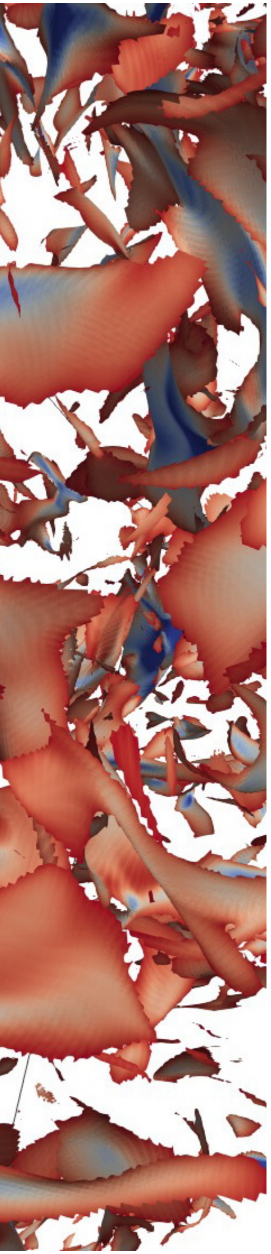
(work in progress!)

http://acoustique.ec-lyon.fr

Outline



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Notation

c_b	phase speed of bending waves (m.s^{-1})
c_0	phase speed of sound in the fluid (m.s^{-1})
c_P	phase speed of longitudinal waves (m.s^{-1})
c_S	phase speed of transverse waves (m.s^{-1})
Ca	Cauchy number, see slide 48
E	Young's modulus (Pa)
$(\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3)$	unit vectors of the Cartesian frame
$\overline{\overline{\boldsymbol{\epsilon}}}$	strain tensor, see slide 14
f_{vs}	vortex shedding frequency (s^{-1})
$\boldsymbol{g}$	gravity vector (m.s^{-2})
k_0	acoustic wavenumber (m^{-1}), $k_0 = \omega / c_0$
$\overline{\overline{\boldsymbol{I}}}$	identity tensor
λ_s	Lamé constant (Pa)
m_s	solid mass (kg)
m_a	added mass (kg)
μ_f	dynamic fluid viscosity (Pa.s^{-1})
μ_s	Lamé constant (Pa), shear modulus
ν_f	kinematic viscosity (m^2.s^{-1}), $\mu_f = \rho_f \nu_f$

Notation

ν_s	Poisson's ratio
ω_0	natural angular frequency ($\omega_0 = 2\pi f_0$)
ρ_f, ρ_s	fluid and solid density (kg.m^{-3})
r_ρ	density ratio (reduced mass) $r_\rho = \rho_f/\rho_s$
U_∞	free stream velocity
U_R	reduced velocity, see slide 48
$\boldsymbol{u}(\boldsymbol{x}, t)$	fluid velocity (m.s^{-1})
$\overline{\overline{\boldsymbol{\sigma}}}_f, \overline{\overline{\boldsymbol{\sigma}}}_s$	fluid and solid stress tensor, see slides 13 & 14
$\boldsymbol{\xi}$	displacement vector in the solid domain, $\boldsymbol{x} = \boldsymbol{x}_0 + \boldsymbol{\xi}$, $\boldsymbol{x}_0$ original position
$.^T$	transpose

Additional relations

$$\lambda_s = \frac{E\nu_s}{(1+\nu_s)(1-2\nu_s)} \quad \mu_s = \frac{E}{2(1+\nu_s)} \qquad \frac{\lambda_s + 2\mu_s}{\rho_s} = \frac{E}{\rho_s}\frac{1-\nu_s}{(1+\nu_s)(1-2\nu_s)}$$

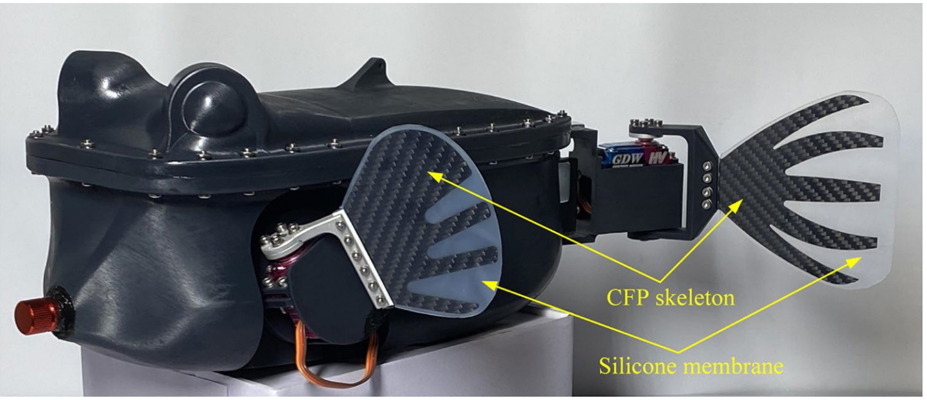
$$E = \mu_s \frac{3\lambda_s + 2\mu_s}{\lambda_s + \mu_s}$$

$$\epsilon_{ij} = \frac{1+\nu_s}{E}\sigma_{s,ij} - \frac{\nu_s}{E}\sigma_{s,kk}\delta_{ij} \quad \text{(Hooke's law)}$$

1 - Introduction to fluid-structure interaction
(*i.e.* between movable or deformable solids and flow)

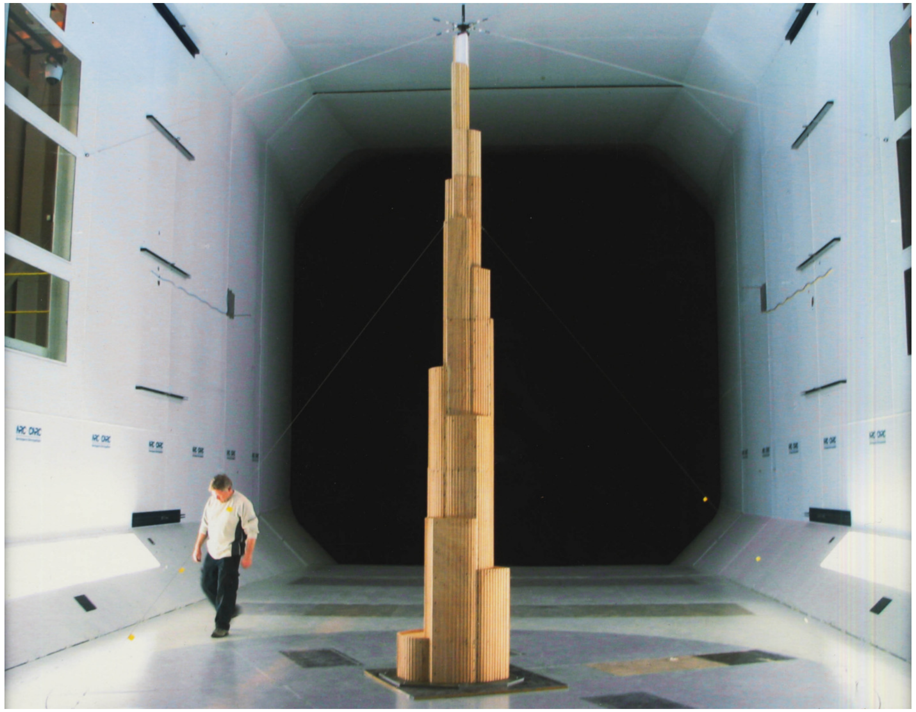
...

Motivation



Qiu *et al.*, 2023, *J. Bionic Eng.*

Robotic boxfish locomotion (CFP = Carbon Fiber Plate) : fluid-structure interaction (in place of flow active control)



Burj Khalifa tower model in wind tunnel (RWDI Consulting) to study wind loading and influence of vortex shedding

Inauguration in 2010, height 829.8 m.
At its tallest point, the tower sways a total of 1.5 m

Motivation

Aerial cableway in Pralognan-la-Vanoise (Mont Bochor)
$\rightsquigarrow$ wind-induced oscillations

Aircraft flutter

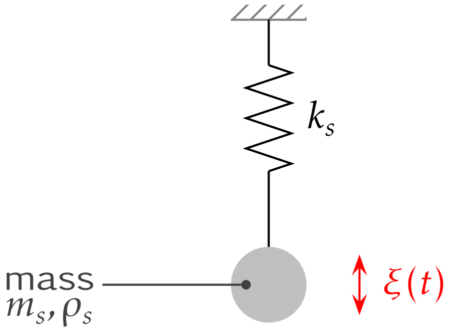
Icing on power network cables
$\rightsquigarrow$ galloping (wind-induced instability)

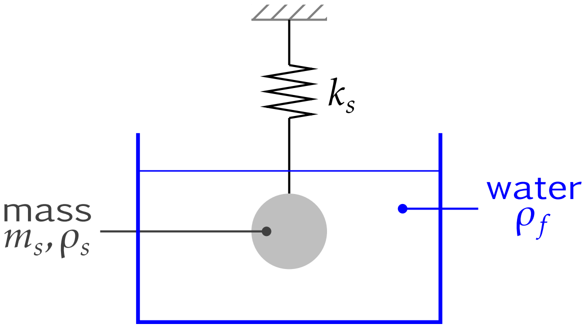
Motivation

Floating offshore wind turbine

Focus on key dates

- Concept of added mass introduced by Bessel (1828) and Stokes (1851)





Can we predict the evolution of the natural frequency of the system ?

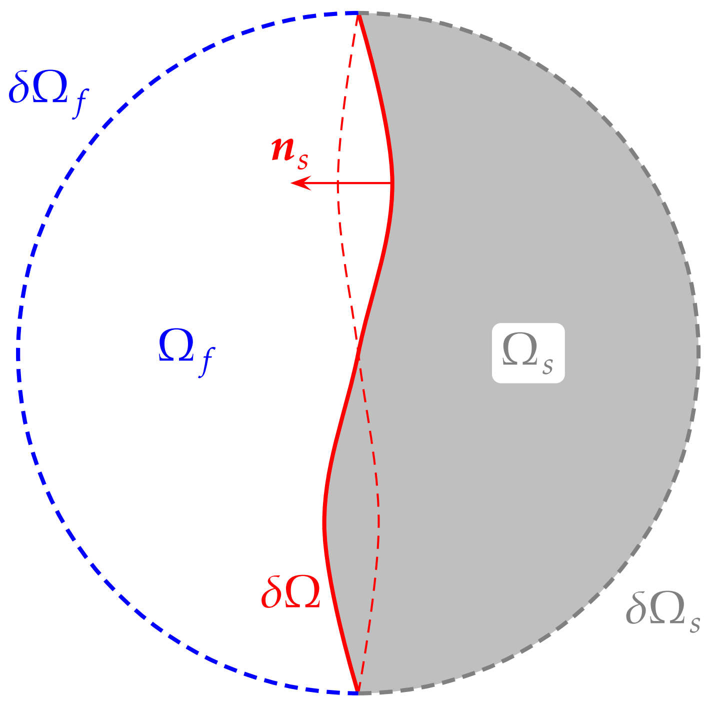
- Major milestone : Tacoma Narrows bridge (1940)

British Pathé

(collapse cannot be explained by vortex shedding frequency close to the natural structural frequency, see slide 56)

Formulation

Generic sketch of the fluid and solid domains



Ω_s solid domain, and $\delta\Omega_s$ boundary of the solid domain

Ω_f fluid domain, and $\delta\Omega_f$ boundary of the fluid domain

$\delta\Omega(t)$ interface between the fluid and the solid

Outward normal to the solid (fluid) domain $\boldsymbol{n}_s$ ($\boldsymbol{n}_f = -\boldsymbol{n}_s$)

Modeling the fluid domain

For a Newtonian incompressible flow, the equation conservation of mass and momentum for a fluid particle (Eulerian description, use of the material derivative $D \cdot / Dt$) reads

$$\begin{cases} \nabla \cdot \boldsymbol{u} = 0 \\ \rho_f \dfrac{D\boldsymbol{u}}{Dt} = \nabla \cdot \overline{\overline{\boldsymbol{\sigma}}}_f + \rho_f \boldsymbol{g} \end{cases}$$

where the fluid stress tensor is

$$\overline{\overline{\boldsymbol{\sigma}}}_f = -p\overline{\overline{\boldsymbol{I}}} + \overline{\overline{\boldsymbol{\tau}}} \qquad \overline{\overline{\boldsymbol{\tau}}} = 2\mu_f \overline{\overline{\boldsymbol{D}}} \qquad \overline{\overline{\boldsymbol{D}}} = \frac{1}{2}(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T)$$

Navier-Stokes equation (ρ_f = cst and μ_f = cst here)

$$\rho_f \left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} \right) = -\nabla p + \mu_f \nabla^2 \boldsymbol{u} + \rho_f \boldsymbol{g} \tag{1}$$

Compressible effects for adiabatic small perturbations will be introduce later to take into account acoustic waves

Modeling the solid domain

For a linear elastic and isotropic material, equilibrium of forces reads

$$\rho_s \frac{\partial^2 \boldsymbol{\xi}}{\partial t^2} = \nabla \cdot \overline{\overline{\boldsymbol{\sigma}}}_s + \rho_s \boldsymbol{g}$$

where the stress tensor is given by Hooke's law

$$\overline{\overline{\boldsymbol{\sigma}}}_s = \lambda_s \nabla \cdot \boldsymbol{\xi}\, \overline{\overline{\boldsymbol{I}}} + 2\mu_s \overline{\overline{\boldsymbol{\epsilon}}} \qquad \overline{\overline{\boldsymbol{\epsilon}}} = \frac{1}{2}(\nabla \boldsymbol{\xi} + \nabla \boldsymbol{\xi}^T) \qquad \Theta_s = \nabla \cdot \boldsymbol{\xi} = \mathrm{tr}\, \overline{\overline{\boldsymbol{\epsilon}}} \ \text{(dilatation)}$$

Navier equation ($\lambda_s = \mathrm{cst}$, $\mu_s = \mathrm{cst}$)

$$\rho_s \frac{\partial^2 \boldsymbol{\xi}}{\partial t^2} = (\lambda_s + \mu_s)\nabla(\nabla \cdot \boldsymbol{\xi}) + \mu_s \nabla^2 \boldsymbol{\xi} \qquad (2)$$

$$\left.\begin{array}{l} \text{vibrating wire } \boldsymbol{\xi} = (0, \xi_2(x_1, t), 0) \\ \text{membrane } \boldsymbol{\xi} = (0, 0, \xi_3(x_1, x_2, t)) \end{array}\right\} \qquad \nabla \cdot \boldsymbol{\xi} = 0 \qquad \rho_s \frac{\partial^2 \boldsymbol{\xi}}{\partial t^2} = \mu_s \nabla^2 \boldsymbol{\xi}$$

Interface

Boundary conditions at the current fluid-structure interface $\delta\Omega(t)$,

- kinematic condition : continuity of the velocity at the moving interface
  $\boldsymbol{u}_s = \boldsymbol{u}_f$, that is $\dot{\boldsymbol{\xi}} = \boldsymbol{u}_f$
- dynamic condition : Newton's third law $(\boldsymbol{F}_{s\to f} = -\boldsymbol{F}_{f\to s})$
  $\overline{\overline{\boldsymbol{\sigma}}}_s \cdot \boldsymbol{n}_s + \overline{\overline{\boldsymbol{\sigma}}}_f \cdot \boldsymbol{n}_f = 0$

These boundary conditions are not so easy to apply in practice because of the mixed Lagrangian-Eulerian description used for solid and fluid domain respectively

A classical approach in numerical simulation is to consider the Arbitrary Lagrangian-Eulerian (ALE) formulation. For small displacements of the structure, the fluid domain mesh is deformed to follow the interface, without remeshing. Interface terms can then be computed with high accuracy (see textbooks for a review, not addressed in this course)

To summarize

The complete set of equations are generally complicated to solve, whether by coupling of numerical codes, or by using advanced ALE formulations. In addition, simplifications are often relevant,

- Weakly coupled fluid-structure system : small-amplitude vibrations of the structure without affecting the flow field
- Strongly coupled fluid-structure system : modifications of the flow field can not be neglected, induced by high-amplitude vibrations or large displacements

To well understand physics in fluid-structure interaction, introduce relevant scales and key concepts, a nice step is to first consider simple structures, with a small number of degrees of freedom. It can be also seen as a preliminary step before considering modal analysis of a full structure, where the behaviour of each mode can be seen as a body oscillator (mass-spring-damper system)

2 - Body oscillator as a paradigm*

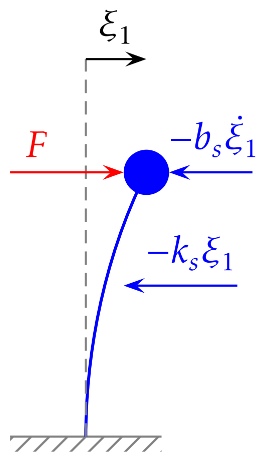
* The paradigm is what is shown as an example, what is called, what exemplifies a ruler and can serve as a model. A paradigm is the *easy object* on which we practice before processing an object resembling the first, but *more complex*

Mass-spring-damper system

Basic one-dimensional mass-spring-damper system in a vacuum

$$m_s\ddot{\xi}_1 + b_s\dot{\xi}_1 + k_s\xi_1 = F \qquad (3)$$

- $m_s\ddot{\xi}_1$ acceleration
- $k_s\xi_1$ restering force (spring / body stiffness)
- $b_s\dot{\xi}_1$ damping force (internal friction)
- F unsteady excitation force (wind for instance)



flow-induced oscillations

The natural frequency ω_0 of the system is found by searching perturbations of the form $\xi_1 \sim e^{i\omega_0 t}$ for the undamped free system (no driving force)

$$-m_s\omega_0^2 + k_s = 0 \qquad \Longrightarrow \qquad \omega_0^2 = \frac{k_s}{m_s}$$

Mass-spring-damper system

which leads to the following canonical form for the ode on ξ_1

$$\ddot{\xi}_1 + 2\beta\omega_0\dot{\xi}_1 + \omega_0^2\xi_1 = \frac{F}{m_s} \qquad \beta \equiv \frac{b_s}{2\sqrt{k_s m_s}}$$

Analytical solution for an harmonic forcing $F = F_0 \cos(\omega t)$

$\xi_1(t) = (F_0/k_s)\, S(\omega)\, \cos(\omega t - \varphi)$ ($\beta < 1$ and excluding transient)

$$\varphi = \mathrm{atan}\left[\frac{2\beta\omega/\omega_0}{(\omega/\omega_0)^2 - 1}\right] \qquad S(\omega) = \frac{1}{\left[(1-(\omega/\omega_0)^2)^2 + 4\beta^2(\omega/\omega_0)^2\right]^{1/2}}$$

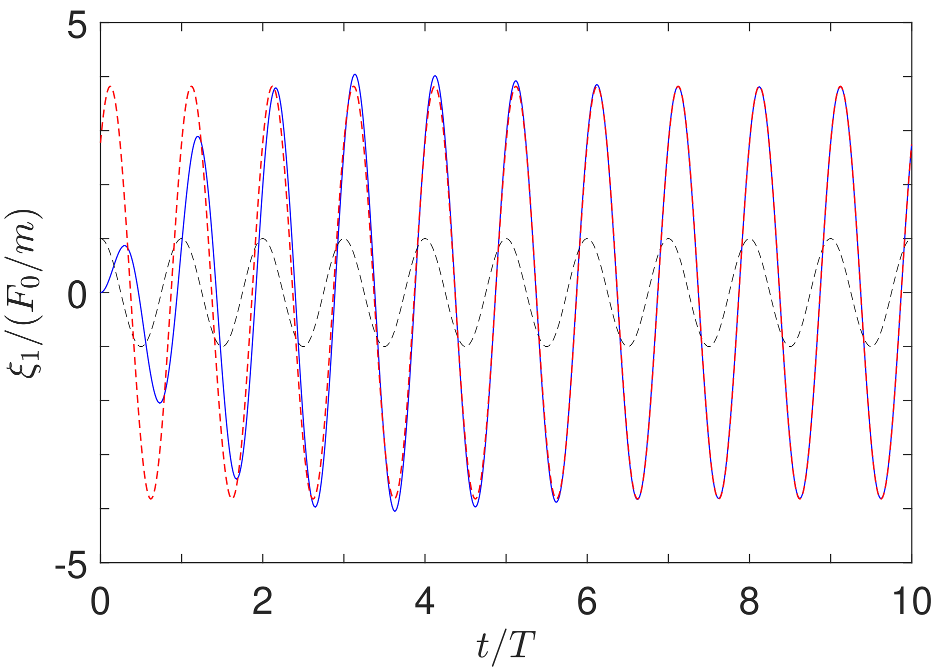
Phase shift φ linked to damping (response lags the excitation force)

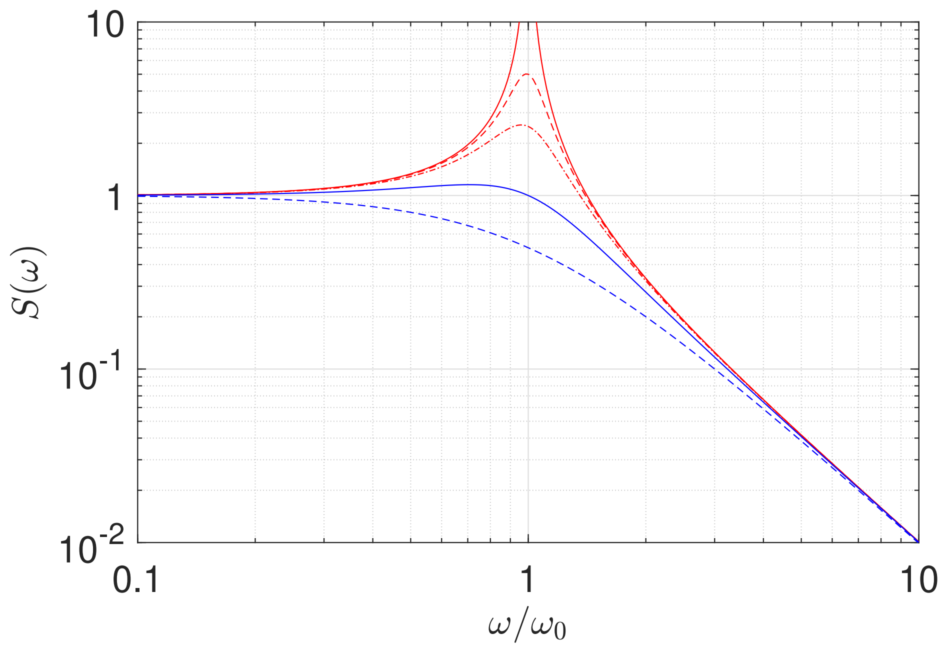
$\varphi = \pi/2$ at the resonance $\omega = \omega_0$

Oscillations (for $\beta < 1$) with the same frequency as the driving force (linear system), strong amplification near the natural frequency ω_0 of the system

Mass-spring-damper system

Solution for an harmonic excitation $F = F_0 \cos(\omega t)$





$\omega/\omega_0 = 0.9, \beta = 0.1, \xi_1|_{t=0} = 0, \dot{\xi}_1|_{t=0} = 0$ (numerical solution)

analytical solution

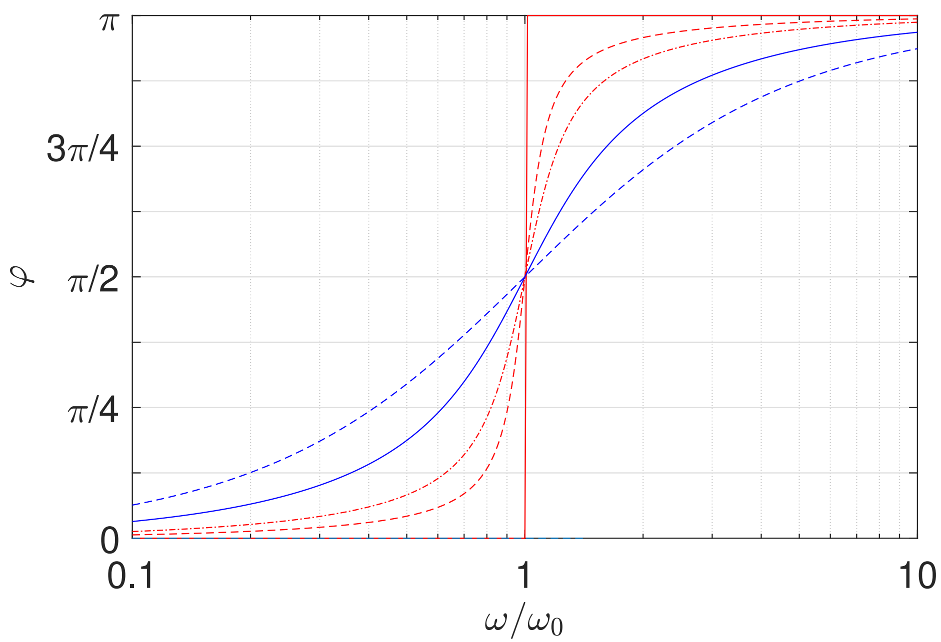
excitation $F/(F_0/m) = \cos(\omega t)$

$\omega/\omega_0 = 0.9$

$\beta = 0.0$, $\beta = 0.1$, $\beta = 0.2$,
$\beta = 0.5$, $\beta = 1.0$

Mass-spring-damper system

Solution for an harmonic excitation $F = F_0 \cos(\omega t)$



$\omega/\omega_0 = 0.9$

$\beta = 0.0$, $\beta = 0.1$, $\beta = 0.2$,
$\beta = 0.5$, $\beta = 1.0$

Mass-spring-damper system

General solution to $\mathcal{L}(\xi_1) = F$,
where $\mathcal{L}$ is a linear operator, see Eq. (3) for instance
with the initial conditions $\xi_1|_{t=0} = 0$ and $\dot{\xi}_1|_{t=0} = 0$

We first need to determine the impulse response g, solution to $\mathcal{L}(g) = \delta$, and then, we consider the following convolution product

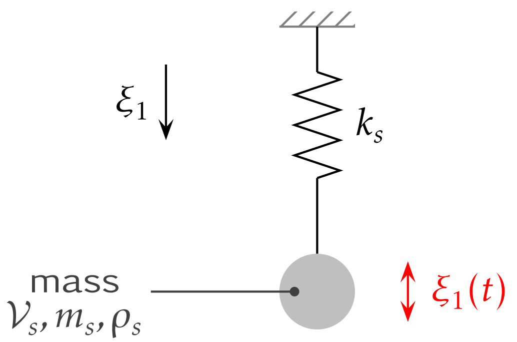
$$\xi_1 = \xi_1 * \delta = \xi_1 * \mathcal{L}(g) = \mathcal{L}(\xi_1) * g = F * g$$

The general solution reads as the convolution product of the source term F by the impulse response g, as known as the Green's function of the problem,

$$\xi_1(t) = \int_{-\infty}^{t} F(\tau) g(t-\tau) d\tau \tag{4}$$

Influence of the fluid flow

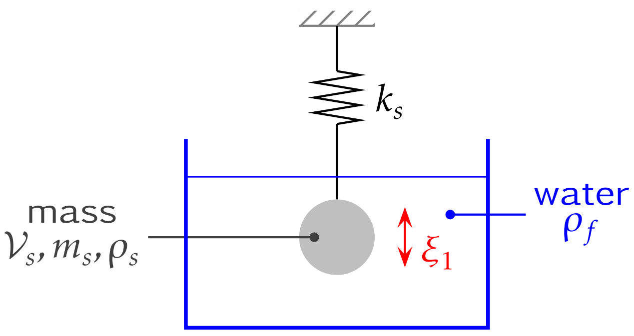
Natural frequency of the system (without damping)



At equilibrium $k_s\xi_1^\star = m_s g$ where $\xi_1^\star$ is the spring extension

Oscillations ξ_1 around this equilibrium position $\xi_1^\star$,
$m_s g - k_s(\xi_1^\star + \xi_1) = m_s\ddot{\xi}_1$

$$m_s\ddot{\xi}_1 + k_s\xi_1 = 0 \qquad \omega_0^2 = k_s/m_s$$



The buoyancy force (Archimede) changes the equilibrium position $\xi_1^\star$, $k_s\xi_1^\star = \rho_s\mathcal{V}_s g - \rho_f\mathcal{V}_s g = m_s g(1 - r_\rho)$

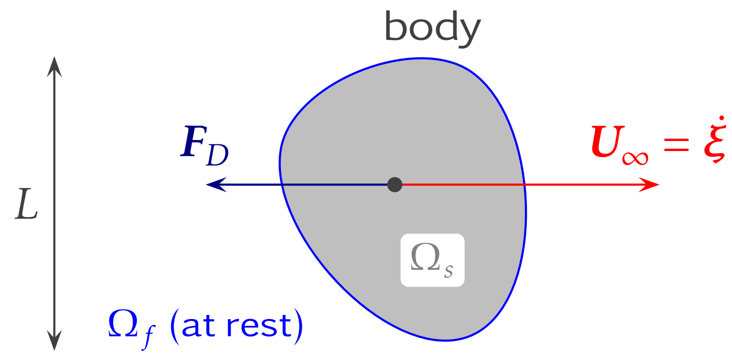
$r_\rho = \rho_f/\rho_s$ density ratio

Forces acting on a moving/accelerating body?
need to introduce the drag force and added mass

Influence of the fluid flow : drag force

For a constant motion U_∞ of the body in a fluid domain at rest, the drag force F_D exerted by the flow on the body can be expressed in the form

$$F_D = C_D \frac{1}{2} \rho_f U_\infty^2 S$$



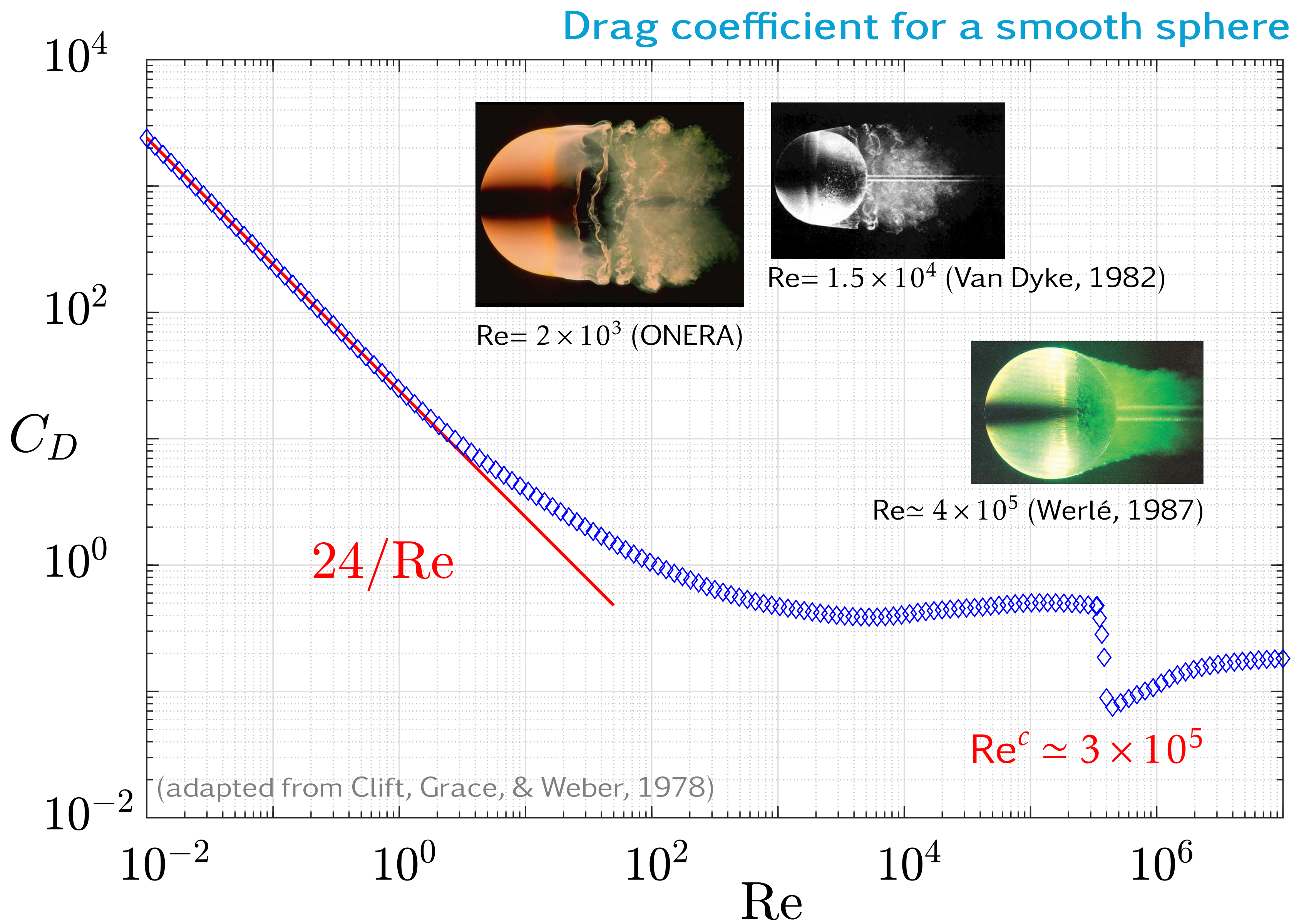
where $S \propto L^2$ is usually the cross-sectional area, and $C_D = C_D(\text{Re})$ is the drag coefficient, a function of the Reynolds number (that is the flow regime)

$$\text{Re} = \frac{\rho_f U_\infty L}{\mu_f}$$

For low Reynolds number values, $C_D \sim 1/\text{Re}$ and $F_D \sim U_\infty = \dot{\xi}_1$ with $\boldsymbol{U}_\infty = U_\infty \boldsymbol{e}_1$, instead of $F_D \sim U_\infty^2 = |\dot{\xi}_1|\dot{\xi}_1$ for high values of Re, for which $C_D \simeq \text{cst}$

The drag force is opposite to the velocity, and results in the dissipation of kinetic energy by the viscous flow

Influence of the fluid flow : drag force

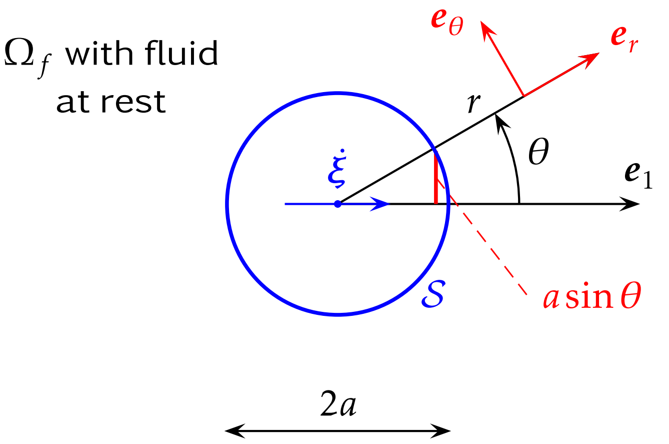


Influence of the fluid flow : added mass

The concept of virtual mass added to the body is associated with the necessary work to be done to change the kinetic energy of the fluid when the body accelerates or decelerates.

Incompressible flow $\nabla \cdot \boldsymbol{u} = 0$, velocity potential ϕ (with $\boldsymbol{u} = \nabla\phi$, inviscid irrotational flow) is solution of $\nabla^2\phi = 0$. For a sphere of velocity $\dot{\xi}(t)$

$$\phi = -\frac{a^3}{2r^2}\dot{\xi}_1 \cos\theta$$



Kinetic energy of the fluid

$$E_c = \frac{1}{2}\rho_f \int_{\Omega_f} (\nabla\phi)^2 \, dv = \frac{1}{2}\rho_f \int_{\mathcal{S}} (\phi\nabla\phi)\cdot\boldsymbol{n} \, ds$$

by using the vectorial identity $\nabla\phi\cdot\nabla\phi = \nabla\cdot(\phi\nabla\phi) - \nabla^2\phi$ and by noting that $\phi \to 0$ when $r \to \infty$

Influence of the fluid flow : added mass

Kinetic energy of the fluid (cont.)

$$E_c = -\frac{1}{2}\rho_f \int_{\mathcal{S}} \phi \frac{\partial \phi}{\partial r}\, ds = \frac{1}{2}\rho_f \frac{2}{3}\pi a^3 \dot{\xi}_1^2 \qquad \left.\frac{\partial \phi}{\partial r}\right|_{r=a} = \dot{\xi}_1 \cos\theta \qquad ds = 2\pi a \sin\theta \, a d\theta$$

$E_c = (1/2) m_a \dot{\xi}_1^2$ where $m_a = \rho_f (2/3)\pi a^3$ is the added mass, one half of the displaced fluid mass $\rho_f \mathcal{V}_s$ for a sphere

$E_c =$ cst for a constant velocity $\dot{\xi}_1$ (steady configuration). However, when the sphere is submitted to acceleration or deceleration, the kinetic energy is no longer constant. Its variation $dE_c = \boldsymbol{F}_a \cdot d\boldsymbol{\xi} = -F_a d\xi_1 = -F_a \dot{\xi}_1 dt$ is associated with the work (per unit time) of an additional drag $\boldsymbol{F}_a$. The minus sign indicates that the force $\boldsymbol{F}_a$ will act on the opposite direction to sphere's motion

$$F_a = -\frac{1}{\dot{\xi}_1}\frac{dE_c}{dt} = -\frac{1}{\dot{\xi}_1} m_a \dot{\xi}_1 \ddot{\xi}_1 = -m_a \ddot{\xi}_1$$

Influence of the fluid flow : added mass

Application to the previous mass-spring system

Oscillations of the sphere are then governed by $m_s\ddot{\xi}_1 = -k_s\xi_1 - m_a\ddot{\xi}_1$, that is $(m_s + m_a)\ddot{\xi}_1 = -k_s\xi_1$, $m_s + m_a$ is the virtual mass of the sphere and the natural frequency ω_0 becomes

$$\omega_0^2 = \frac{k_s}{m_s + m_a} \qquad m_s + m_a = m_s\left(1 + \frac{1}{2}\frac{\rho_f}{\rho_s}\right) = m_s(1 + r_\rho/2)$$

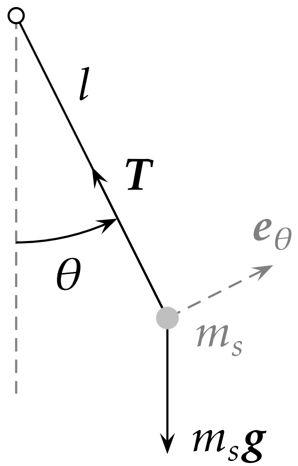
The physical problem is actually more complex in fluid dynamics (but fall out of the scope of this course) ; another example with Stokes problems is developed in the next chapter

Influence of the fluid flow : added mass

body geometry	added mass m_a
circular cylinder of radius a	$\rho_f \pi a^2 b$
square section of side $2a$	$\rho_f 1.51 \pi a^2 b$
elliptical section with major axis a	$\rho_f \pi a^2 b$
flat thin plate of height $2a$	$\rho_f \pi a^2 b$
sphere of radius a	$\rho_f (2/3) \pi a^3$
cube of side a	$\rho_f 0.7 a^3$

b is the span for 2-D geometry (from Blevins, Table 2-2, page 25)

Pendulum



Fundamental principle of dynamics projected along the direction $\boldsymbol{e}_\theta$ provides $m_s l\ddot{\theta} = -m_s g \sin\theta$. Hence

$$\ddot{\theta} + \omega_0^2 \sin\theta = 0 \qquad \omega_0 = \sqrt{g/l}$$

... apparently simple but non linear !
(ideal pendulum in a vacuum)

- For small displacements, $\sin\theta \simeq \theta$, and the solution reads $\theta = \theta_0 \cos(\omega_0 t + \varphi_0)$, where θ_0 and φ_0 are determined by initial conditions.

  Isochronic oscillations, same time period $T_0 = 2\pi/\omega_0$ whatever the amplitude, as for the free mass-spring-damper system with Eq. (3)

Pendulum

Physical pendulum oscillations : influence of the ambiant fluid

Fundamental principle of dynamics projected along the direction $\boldsymbol{e}_\theta$ including buoyancy force, added mass and drag

$$m_s l\ddot{\theta} = -m_s g \sin\theta + m_f g \sin\theta - m_a l\ddot{\theta} - \frac{1}{2}\rho_f S C_D |l\dot{\theta}| l\dot{\theta}$$

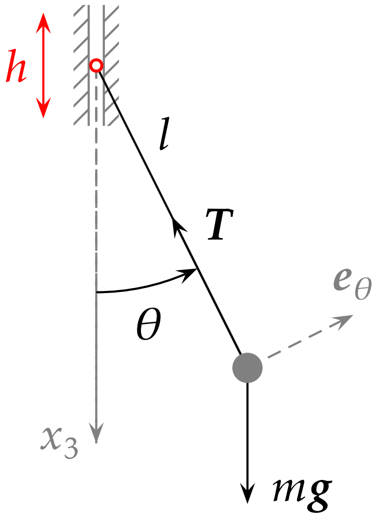
and the previous equation be recast as

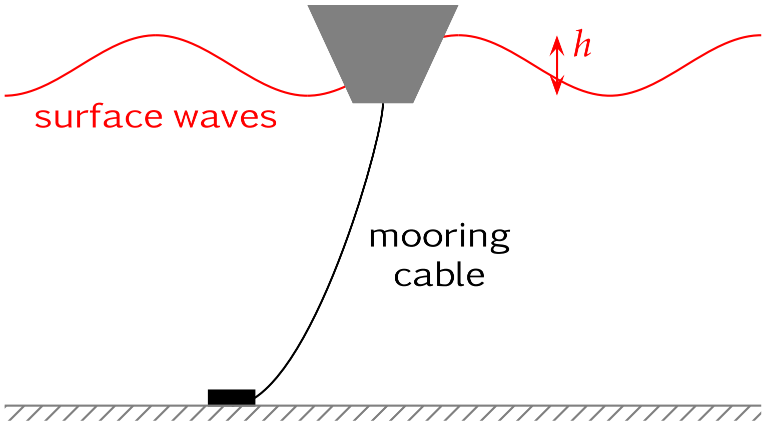
$$(1 - r_\rho/2)\ddot{\theta} + (1 - r_\rho)\omega_0^2 \sin\theta + \frac{\rho_f S C_D l}{2m_s} |\dot{\theta}|\dot{\theta} = 0$$

$$r_\rho = \rho_f/\rho_s \quad \omega_0^2 = g/l \quad S = \pi a^2$$

Parametric oscillator

A parametric oscillator is a system in which one of the coefficient depends on time. It can be represented mechanically by considering a simple pendulum whose support is subjected to a vertical (periodic) motion



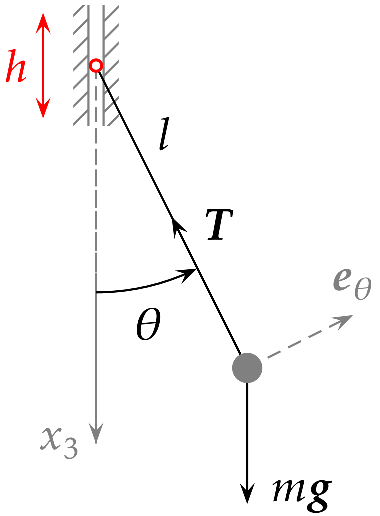


The Botafumeiro is a famous thurible used at the Santiago de Compostela Cathedral (Spain)

Mooring cable linked to a buoyant body : floating wind turbine, offshore structure, ...

Parametric oscillator

This system is equivalent to a simple pendulum subjected to a time-dependent gravity field $g_e(t)$



$h = h_0 \cos(2\omega t)$ along x_3

Fundamental principle of dynamics projected along $\boldsymbol{e}_\theta$

$ml\ddot{\theta} - m\ddot{h}\sin\theta = -mg\sin\theta$

$$\ddot{\theta} + \omega_0^2(1 - \ddot{h}/g)\sin\theta = 0 \qquad \omega_0 = \sqrt{g/l}$$

$$\ddot{\theta} + \omega_0^2\left[1 + \tilde{h}_0\cos(2\omega t)\right]\sin\theta = 0 \qquad \tilde{h}_0 = 4\omega^2 h_0/g$$

Mathieu Eq. (1868)

By comparaison with the free oscillator, an effective gravity can be introduced $g_e(t) = g + 4\omega^2 h_0 \cos(2\omega t)$

Other interpretation : pendulum in θ, forced by an independant oscillator (clock) with $\zeta = \tilde{h}_0\cos(2\omega t)$

$$\begin{cases} \ddot{\theta} + \omega_0^2(1 + \zeta^2)\sin\theta = 0 \\ \ddot{\zeta} + 4\omega^2\zeta = 0 \end{cases}$$

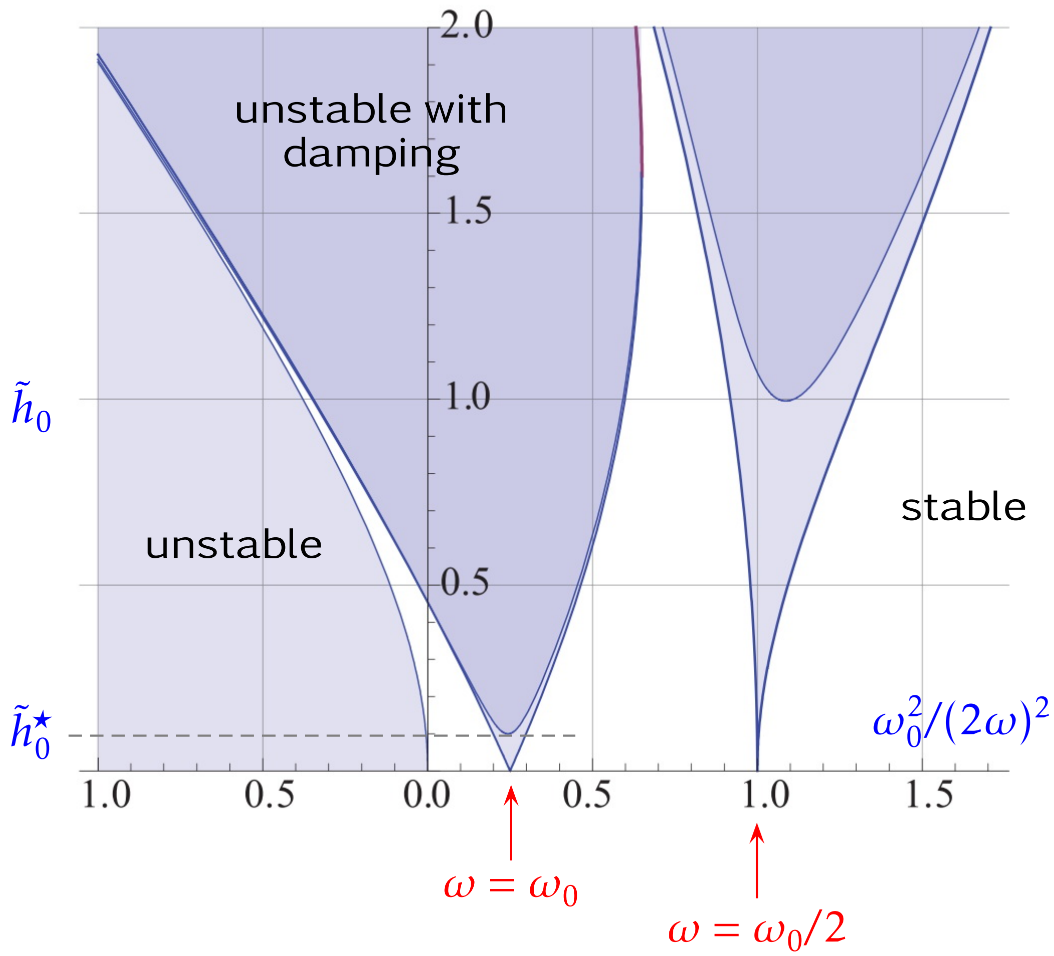
Parametric oscillator

Stability analysis for small $\tilde{h}_0$

For $\omega = \omega_0$, an infinitesimal perturbation is sufficient to start instability (without damping, otherwise there is a threshold $\tilde{h}_0^\star$)

Other parametric resonances for $\omega/\omega_0 = 1/n$, $n \in \mathbb{N}$, with narrower tongues and higher instability threshold in the presence of damping (the resonance for $n = 1$ is the most efficient)

Instability or resonance for $\omega = \omega_0$ is a subharmonic resonance since the suspension motion $h = h_0 \cos(2\omega t)$ oscillates twice as fast as the θ motion



Must be read carefully! graph in plane $(\omega_0^2/(2\omega)^2, \tilde{h}_0)$

(Butikov, 2018, *Am. J. Phys.*)

To resume

- Body oscillator, a one-degree-of-freedom system governs by

$$\ddot{\xi}_1 + \omega_0^2 \xi_1 + g(\xi_1, \dot{\xi}_1, \ddot{\xi}_1) = F(t)$$

  where the two first terms ensure that the structure is able to oscillate

  General solution (4) for a linear problem through a convolution product

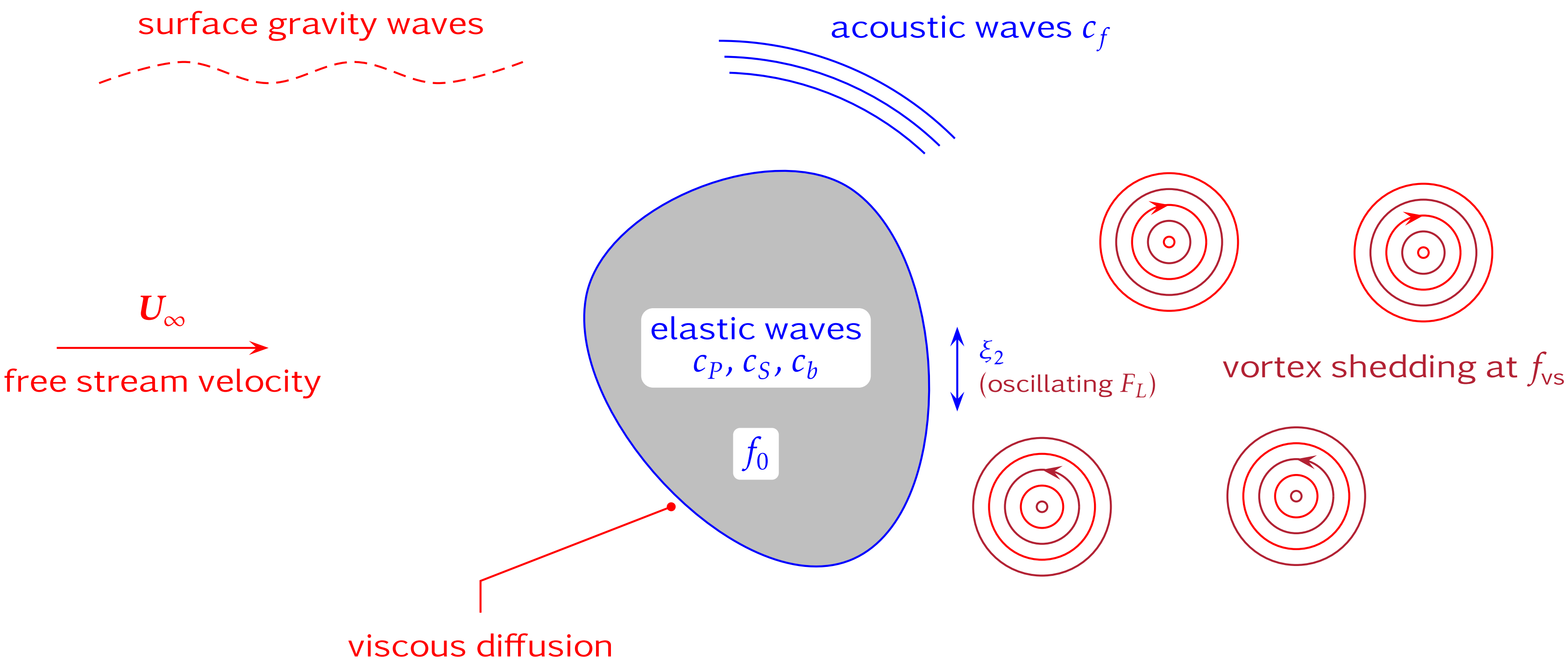
- Influence of the flow : classical drag force (steady configuration), buoyancy effect (Archimede) and added mass : see exercises

Wind-induced motion in vegetation
De Langre (2000), Sellier & Suzuki (2021)

3 - Flow-induced vibration

Introduction

Introduction to physical mechanisms
and associated scales



(Adapted from E. de Langre, 2000)

Fluid viscosity effects

Introduction to fluid viscosity effects

In the previous lecture, we have introduced the added mass to account for the change in kinetic energy induced by the body acceleration or deceleration (vibrations). The added mass has been estimated with the assumption of a potential flow, that is an inviscid irrotational flow.

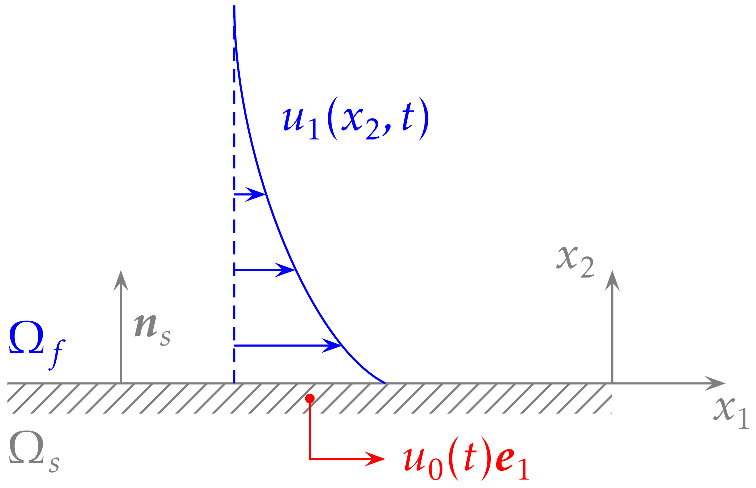
The two Stokes problems are quickly presented in the following to introduce viscous effects

Fluid viscosity effects

1 – Solid wall suddently set into motion along the direction x_1

- viscous fluid at rest for $x_2 \geq 0$, constant pressure in Ω_f, velocity field sought in the form $u_1(x_2, t)$
- boundary condition at $x_2 = 0$

$$u_0 = \begin{cases} 0 & t \leq 0 \\ u_e = \text{cst} & t > 0 \end{cases}$$



(first Stokes problem, 1856)

In the regime of small Reynolds number value flow, Re ≤ 1 (creeping or Stokes flow), Navier-Stokes equation reduces to

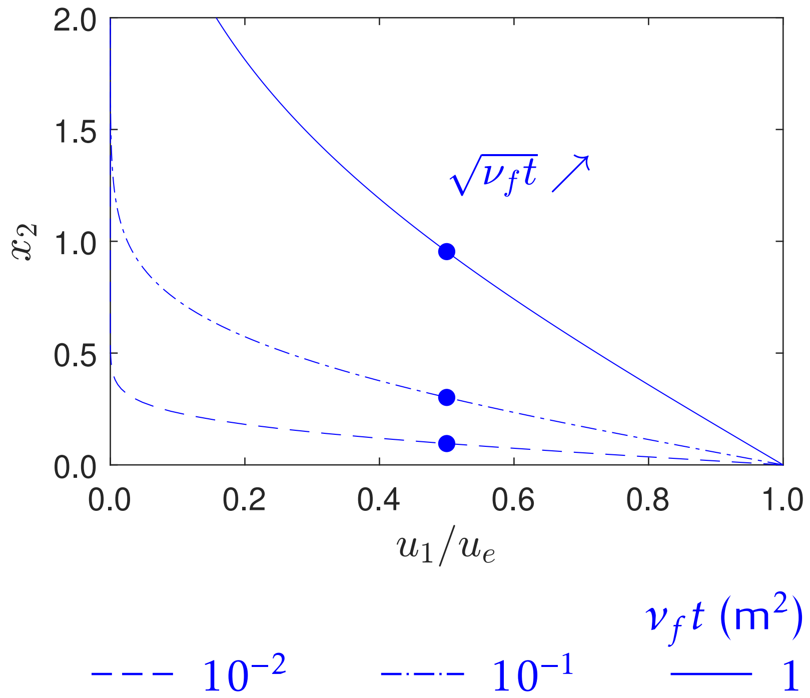
$$\frac{\partial u_1}{\partial t} = \nu_f \frac{\partial^2 u_1}{\partial x_2^2} \qquad (5)$$

Fluid viscosity effects

The self-similar solution reads as (the check is left as an exercise)

$$u_1/u_e = 1 - \mathrm{erf}(\eta) \qquad (6)$$

$$\eta = \frac{x_2}{2\sqrt{\nu_f t}} \qquad \mathrm{erf}(\eta) = \frac{1}{\sqrt{\pi}} \int_0^{\eta} e^{-\tilde{\eta}^2} d\tilde{\eta}$$



Viscous diffusion normal to the wall $\propto \sqrt{\nu_f t}$

The force $d\boldsymbol{F} = \overline{\overline{\boldsymbol{\tau}}} \cdot \boldsymbol{n}_s|_{x_2=0}\ d\mathcal{S}$ with $\boldsymbol{n}_s = \boldsymbol{e}_2$ applied to the plate by the viscous flow is calculated from the wall shear stress value $\tau_w = \tau_{12}(x_2 = 0)$

$$\tau_w = \mu_f \left.\frac{\partial u_1}{\partial x_2}\right|_{x_2=0} = -\rho_f u_e \sqrt{\frac{\nu_f}{\pi t}}$$

and the force acting on the plate is $d\boldsymbol{F} = \tau_w \boldsymbol{e}_1\ d\mathcal{S}$

Fluid viscosity effects

Since the problem is linear, it can be shown (tutorials) that the general solution $\tilde{u}_1(x_2, t)$ for an arbitrary motion $f(t)u_e$ of the wall can be determined from the previous particular solution (6) for u_1

$$\tilde{u}_1(x_2, t) = \int_0^t \dot{f}(\tau)\, u_1(x_2, t-\tau)\, d\tau$$

and in the same way for the force applied to the plate per unit surface

$$\tilde{\tau}_w(t) = \int_0^t \dot{f}(\tau)\, \tau_w(t-\tau)\, d\tau$$

The viscous force $\tilde{\tau}_w \boldsymbol{e}_1$ (per unit surface here) cannot be reduced to a basic added mass because of the time delay $t-\tau$ in the integrand : the force depends of the state of the flow at earlier times through the convolution product

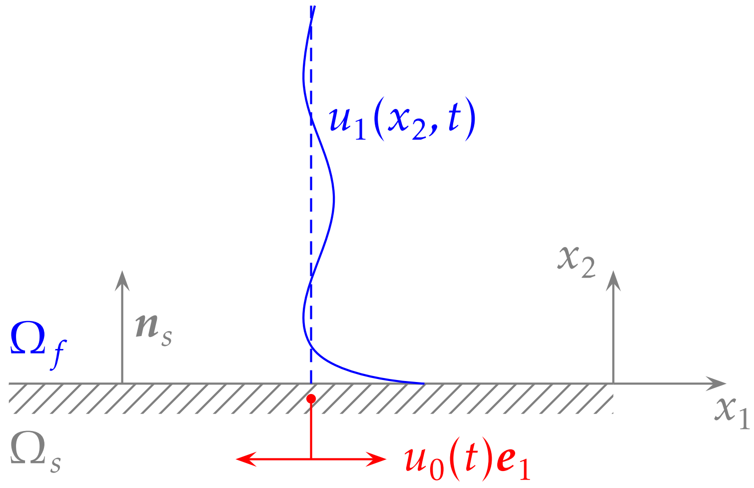
Fluid viscosity effects

2 – Oscillating wall along the direction x_1, with
$u_0(t) = u_e \cos(\omega t)$

It can be shown that the solution to Eq. (5) reads

$$u_1(x_2,t) = u_e e^{-x_2/\delta} \cos(\omega t - x_2/\delta)$$

where $\delta = \sqrt{2\nu_f/\omega}$ is the viscous layer thickness



(Second Stokes problem, 1856)

The force (per unit surface) acting on the plate $\tau_w \boldsymbol{e}_1$ can be recast as follows

$$\tau_w = -\frac{\mu}{\delta^2\omega}\,\dot{u}_0(t) \; - \frac{\mu}{\delta^2}\,u_0(t) = \underbrace{-\frac{\rho_f}{2}\,\dot{u}_0(t)}_{\text{(a)}} \; \underbrace{-\frac{\rho_f\omega}{2}\,u_0(t)}_{\text{(b)}}$$

(a) added mass acting in the opposite direction to acceleration

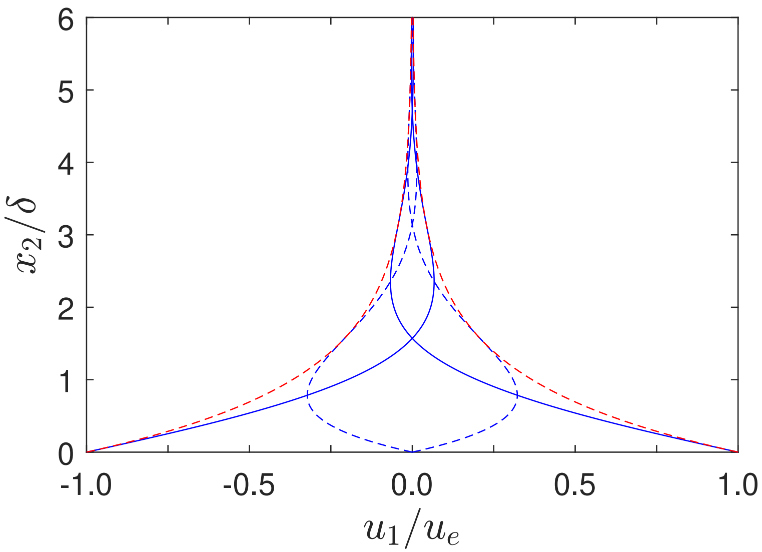
(b) viscous damping acting in the opposite direction to velocity

Fluid viscosity effects

δ is the depth of penetration of the viscous wave (normal to the oscillating wall)

phase velocity $v_\varphi = \sqrt{2\nu_f \omega}$
in observing that $\omega t - x_2/\delta = (v_\varphi t - x_2)/\delta$

$\omega \nearrow \quad \delta \searrow \quad v_\varphi \nearrow$



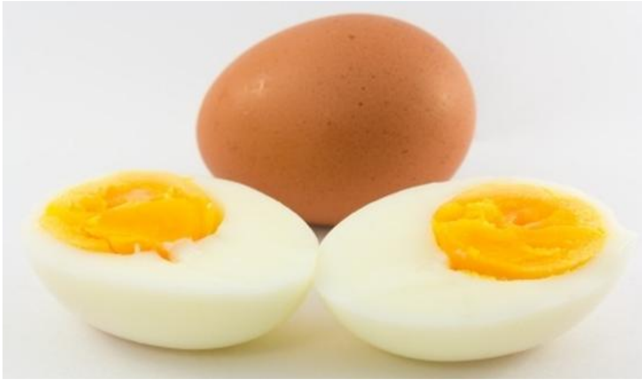
$e^{-x_2/\delta}$

$\omega t = \pm\pi$

$\omega t = \pi/2 \pm \pi$

Fluid viscosity effects

How can you tell a raw egg from a hard-boiled egg ?



This can be quickly distinguished between a raw egg and a hardboiled one by spinning each one on a table. Also, if it will be stopped a spinning raw egg with anyone's finger and released it very quickly. It will resume quickly.

When the raw egg is stopped in the midst of spinning, the liquid inside continue to move, causing the egg to resume spinning when released.

Waves in elastic solid

Let's take a look at the waves that can propagate in an 3-D unbounded elastic body. Helmholtz decomposition with $\Theta_s = \nabla \cdot \xi_s$ and $\nabla \times \xi_s$ leads to (see slide 6)

$$\frac{\partial^2 \Theta_s}{\partial t^2} - \frac{\lambda_s + 2\mu_s}{\rho_s} \nabla^2 \Theta_s = 0 \qquad \left[\frac{\partial^2}{\partial t^2} - \frac{\mu_s}{\rho_s} \nabla^2 \right] (\nabla \times \xi_s) = 0$$

Longitudinal waves (P-wave), velocity $c_P = \sqrt{(\lambda_s + 2\mu_s)/\rho_s}$

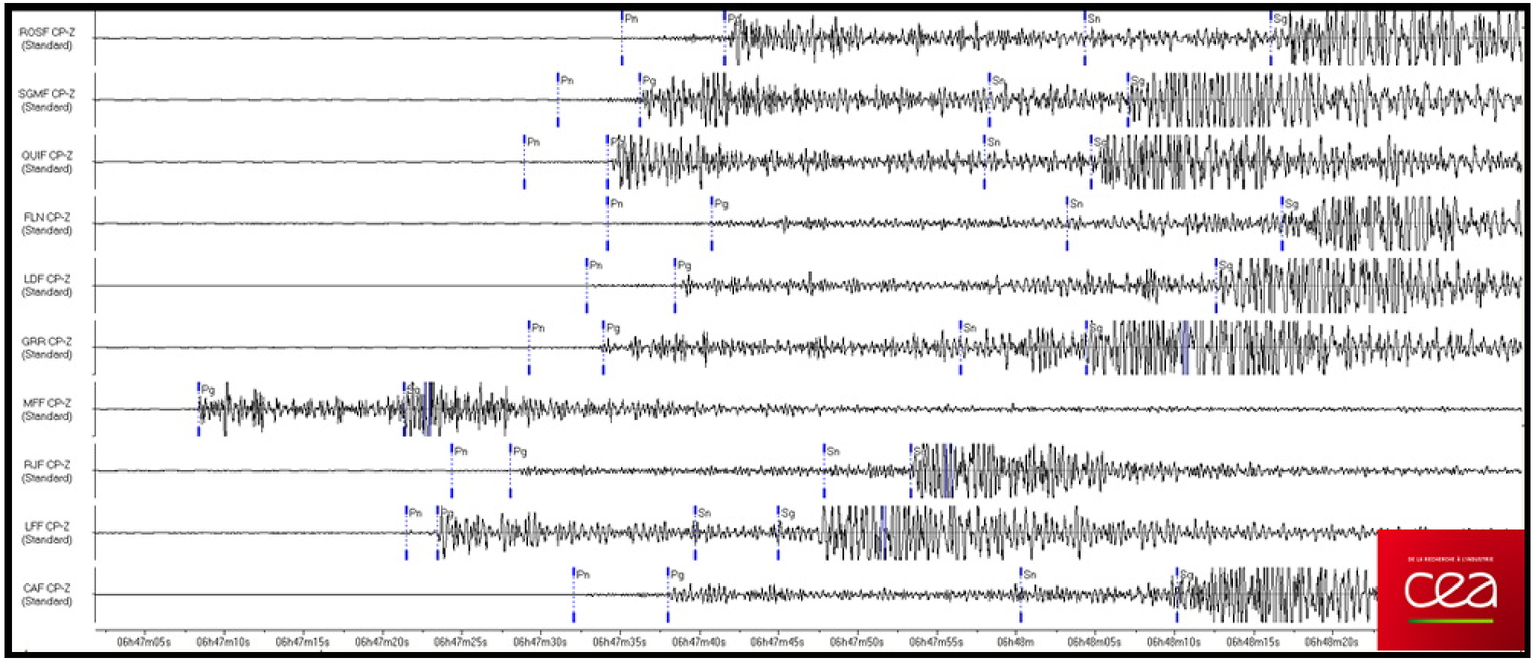
Transverse shear waves (S-wave), velocity $c_S = \sqrt{\mu_s/\rho_s} < c_P$

where P stands for primary, and S for secondary

Non-dispersive waves (phase velocity is independent of frequency)

Waves in elastic solid

Seismic waves (île d'Oléron, France, 28-Apr-2016 at 06h46 TU)



Phases : Pn P-wave from Earth mantle, Pg P-wave from Earth crust, Sn S-wave from mantle, Sg S-wave from crust

https://www-dase.cea.fr/

Waves in elastic solid

Waves in finite elastic bodies, that is with geometric confinement (c_S is however never modified)

- bars or beams, quasi-longitudinal propagation : $\nu_s = 0$, $c'_P = \sqrt{E/\rho_s}$
- thin plates :

$$c''_P = \sqrt{\frac{E}{\rho_s(1-\nu_s^2)}}$$

material	E	ρ_s	ν_s	c_P	c_S
steel	2.0×10^{11}	7.8×10^{3}	0.28	5900	3160
rubber (soft)	5.0×10^{6}	9.5×10^{2}	0.50	∞	40

$c_P \to \infty$ as $\nu_s \to 0.5$, incompressible (rubber)

Dimensionless variables

The various configurations encountered in practice can be classified from a dimensional analysis and the introduction of key dimensionless parameters

For the fluid domain, see slide 13

$$x_i^\star = \frac{x_i}{L} \quad t^\star = \frac{t}{\tau_c} \quad u_i^\star = \frac{u_i}{U_\infty} \quad \sigma_{ij}^\star = \frac{\sigma_{ij}}{\rho_f U_\infty^2} \qquad \tau_c = L/U_\infty$$

$$\frac{\partial u_i^\star}{\partial t^\star} + u_j^\star \frac{\partial u_i^\star}{\partial x_j^\star} = -\frac{\partial p^\star}{\partial x_i^\star} + \frac{1}{\mathrm{Re}} \frac{\partial^2 u_i^\star}{\partial x_j^\star \partial x_j^\star} - \frac{1}{\mathrm{Fr}^2}\delta_{3i} \qquad \frac{\partial u_i^\star}{\partial x_i^\star} = 0$$

where Re is the Reynolds number, Fr is the Froude number,

$$\mathrm{Re} = \frac{\rho_f U_\infty L}{\mu_f} \qquad \mathrm{Fr} = \frac{U_\infty}{\sqrt{Lg}}$$

Dimensionless variables

For the solid domain, see slide 14

$$\xi_i^\star = \frac{\xi_i}{\xi_0} \qquad t^\star = \frac{t}{L/\tau_s} \qquad \overline{\overline{\sigma}}_s^\star = \frac{\overline{\overline{\sigma}}_s}{E} \qquad \tau_s = L/\sqrt{E/\rho_s}$$

$$\mathcal{D}\frac{\partial^2 \xi_i^\star}{\partial t^{\star 2}} = \frac{\partial \sigma_{s,ij}^\star}{\partial x_j^\star} - \frac{\mathrm{U}_R^2}{\mathrm{Fr}^2}\delta_{3i}$$

where $\mathcal{D}$ is the vibration amplitude and U_R is the reduced velocity

$$\mathcal{D} = \frac{\xi_0}{L} \qquad \mathrm{U}_R = \frac{U_\infty}{\sqrt{E/\rho_s}} = \frac{\tau_s}{\tau_c}$$

Dimensionless variables

Boundary conditions at the interface, see slide 15

$$\mathcal{D}\,\partial\boldsymbol{\xi}^\star/\partial t^\star|_{X^\star} = \mathrm{U}_R\,\boldsymbol{u}^\star|_{x^\star} \qquad (\boldsymbol{x}^\star = \boldsymbol{X}^\star + \mathcal{D}\boldsymbol{\xi}^\star)$$

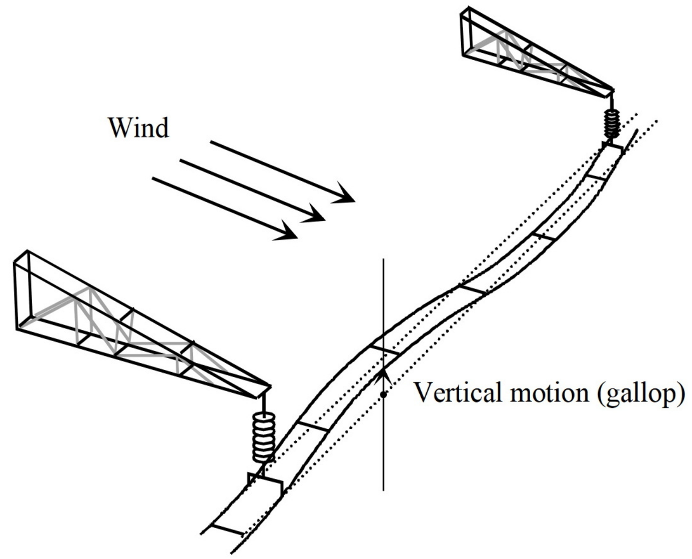
$$\overline{\overline{\boldsymbol{\sigma}}}_s^\star \cdot \boldsymbol{n}_s + \mathrm{Ca}\,\overline{\overline{\boldsymbol{\sigma}}}_f^\star \cdot \boldsymbol{n}_f = 0 \qquad \mathrm{Ca} = \frac{\rho_f U_\infty^2}{E}$$

Ca is the Cauchy number, defined as the ratio between aerodynamic forces (fluid) and elastic forces (solid)

When Ca $\rightarrow 0$, that is for a rigid body or a low density fluid, $\overline{\overline{\boldsymbol{\sigma}}}_s^\star \cdot \boldsymbol{n}_s \simeq 0$, corresponding to a free-surface boundary condition for the solid

- fluid-structure interaction (FSI) : FSI generally involves two-way interaction
- flow-induced vibration (FIV) : interaction normally is one-way

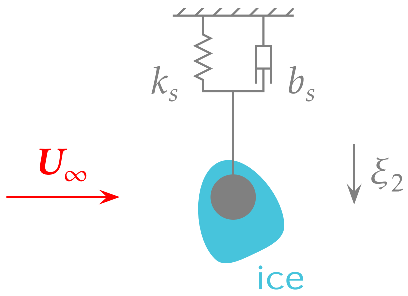
Galloping

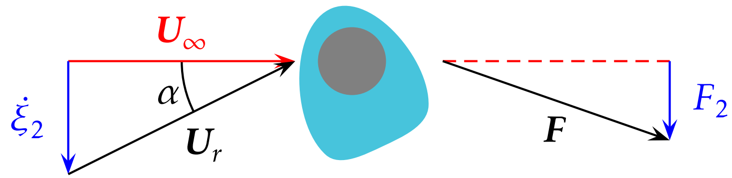


Krysinski & Malburet (2011, fig. 5.17)

Flexible cable, cross-sectionally bluff, submitted to a steady flow : large oscillations in the transverse direction ξ_2 at low frequency (~ 1 Hz)

The vertical cable velocity $\dot{\xi}_2$ and the flow force $\boldsymbol{F}$ acting on the cable along $\boldsymbol{e}_2$ can be in the same direction, providing energy to the structure and self-sustained oscillations (a circular cylinder is immune to galloping)

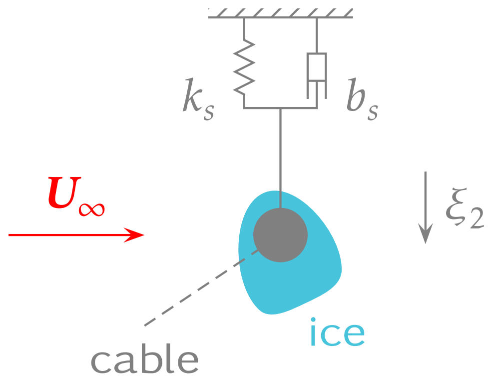




Galloping

The vertical motion ξ_2 is modeled by a single-degree-of-freedom system,

$$m_s\ddot{\xi}_2 + b_s\dot{\xi}_2 + k_s\xi_2 = F_2 \tag{7}$$



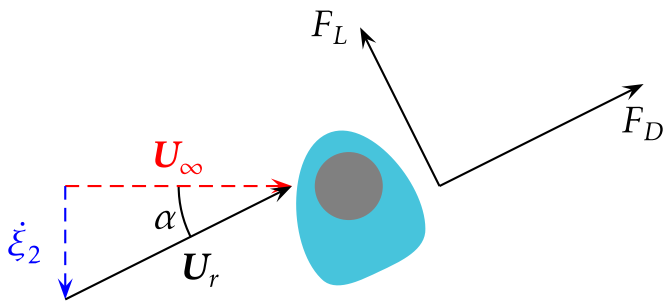
where ξ_2 and F_2 are taken positive downwards. The mass per unit length m_s includes the added mass, b_s is the structural damping and k_s the stiffness (force per unit displacement)

The parameters m_s, b_s and k_s are identified by the behaviour of the cable with no wind (eigenfrequency and damping)

As a reminder, $\omega_0 = \sqrt{k_s/m_s}$ is the natural frequency ; there is a canonical expression with $b_s = 2\beta_s m_s \omega_0$

Galloping

The force component F_2 is defined from the lift $F_L = F_L(\alpha)$ and the drag $F_D = F_D(\alpha)$ with respect to the relative wind velocity $\boldsymbol{U}_r$, by noting cable's motion implies a change of angle of attack α



$$F_L(0) = F_L^0 = C_L \frac{1}{2}\rho_f U_\infty^2 S$$

$$F_D(0) = F_D^0 = C_D \frac{1}{2}\rho_f U_\infty^2 S$$

For a small incident angle α, a Taylor series at the first order in α provides

$$-F_2 = F_L \cos\alpha + F_D \sin\alpha \simeq F_L^0 + \frac{\partial F_L^0}{\partial \alpha}\alpha + F_D^0 \alpha \qquad (8)$$

and furthermore,

$$\frac{\dot{\xi}_2}{U_\infty} = \tan\alpha \simeq \alpha \quad \Longrightarrow \quad -F_2 \simeq F_L^0 + \left(\partial_\alpha F_L^0 + F_D^0\right)\frac{\dot{\xi}_2}{U_\infty}$$

Galloping

Eq. (7) of motion for the cable is

$$m_s\ddot{\xi}_2 + \left[b_s + \frac{1}{U_\infty}\left(\partial_\alpha F_L^0 + F_D^0\right)\right]\dot{\xi}_2 + k_s\xi_2 = -F_L^0$$

Stability is ensured when the damping term remains positive, and self-sustained oscillations appear when this term becomes negative. The cancellation of damping leads to the following condition for the critical wind velocity U_∞^c

$$U_\infty^c = \frac{-b_s}{(\partial_\alpha F_L^0 + F_D^0)\rho_f S/2}$$

In neglecting the structural damping b_s, an aerodynamic instability may be observed for negative values of the aerodynamic damping (Den Hartog, 1932)

$$\partial_\alpha F_L^0 + F_D^0 < 0 \qquad (9)$$

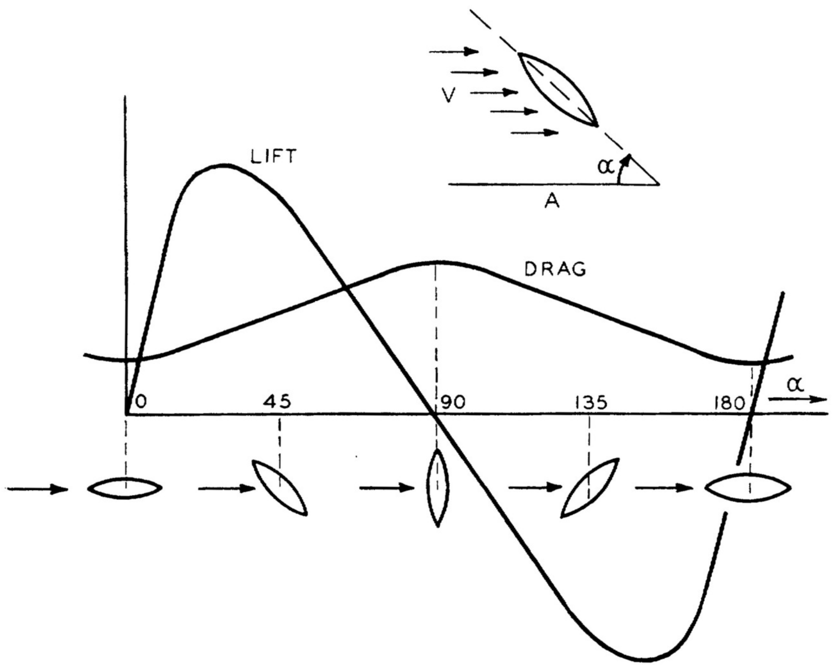
This model has been developed in the framework of a quasi-steady approximation, involving drag and lift forces defined for a steady flow

Galloping

A simple interpretation of this result can be given. If we consider an increase of $\Delta\dot{\xi}_2$, and thus of the incident angle $\Delta\alpha$, oscillations will grow for an increase in the force F_2, that is $\partial F_2/\partial\alpha > 0$. From Eq. (8), a Taylor series about α leads to

$$-\frac{\partial F_2}{\partial \alpha} \simeq \partial_\alpha F_L^0 + F_D^0$$

and the previous condition $\partial_\alpha F_L^0 + F_D^0 < 0$ is retrieved.



Lift C_L and drag C_D forces as a function of the angle of incidence α for an elongated symmetrical cross section (elliptic cable). Instabilities are susceptible to develop when the slope of the lift curve is negative, for roughly $30° < \alpha < 150°$

From Den Hartog, *Transactions AIEE*, 1932

Vortex shedding behind a bluff body

Galloping is a low-frequency aerodynamic instability of non-axisymmetric structures caused by a self-excitation when the aerodynamic damping becomes negative, see Eq. (9), with an onset velocity U_∞^c

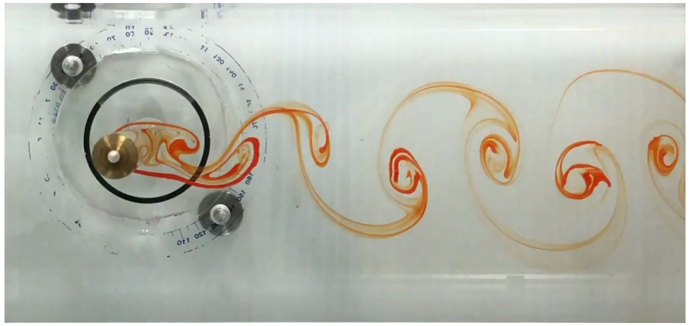
Vortex-induced vibration is produced by vortex shedding around the structure which its frequency coincides with the natural frequency of the structure : resonance mechanim or lock-in, with a frequency selection

Largest vibration amplitudes are observed when the vortex shedding frequency f_{vs} (D diameter cylinder, U_∞ free stream velocity)

$$\mathrm{St} = \frac{f_{\mathrm{vs}}\, D}{U_\infty} \qquad (10)$$

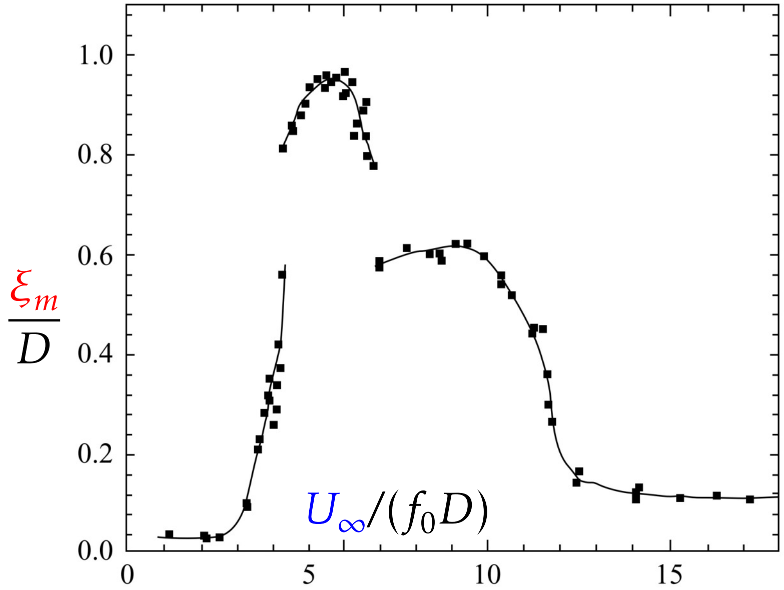
matches the natural frequency f_0 of the structure (see slide 23)

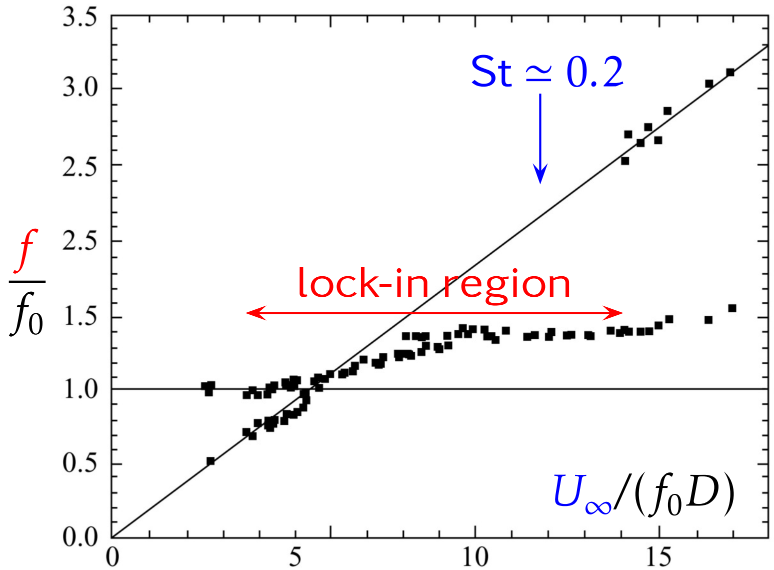
$$\omega_0 = \sqrt{k_s/(m_s + m_a)}$$



$\mathrm{Re} = 171$ (Albright, 2017)

Vortex shedding behind a bluff body





Frequency f and amplitude ξ_m of cylinder vibration as a function of the free stream velocity U_∞, normalized by the cylinder diameter D and its natural frequency f_0 ;

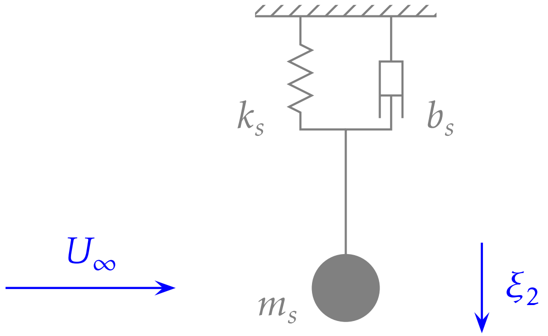
Cylinder mounted in a water channel flow, $\rho_s/\rho_f = 2.4$
Data from Khalak & Williamson (1996, 1997)
Paidoussis, Price & De Langre (2010)

Some devices for suppression of vortex-induced vibration
(Wang, *J. Hydrodynamics*, 2020)

Vortex shedding behind a bluff body

Vortex shedding is basically an harmonic phenomenon, see Eq. (10). Using again a single-degree-of-freedom system to model the motion of a cylinder of diameter D attached with spring and damper and submitted to vortex shedding,

$$m_s\ddot{\xi}_2 + b_s\dot{\xi}_2 + k_s\xi_2 = \frac{1}{2}\rho_f U_\infty^2 D C_L \sin(\omega_{\mathrm{vs}} t) \tag{11}$$



where $\omega = 2\pi f_{\mathrm{vs}}$, f_{vs} being the vortex shedding frequency. Force per unit length of the cylinder,

$$F_L = \frac{1}{2}\rho_f U_\infty^2 D\, C_L \sin(\omega_{\mathrm{vs}} t)$$

The solution has been given in the first lecture, including the resonance case when $\omega = \omega_0$, the natural frequency of the structure ($\omega_0^2 = k_s/m_s$)

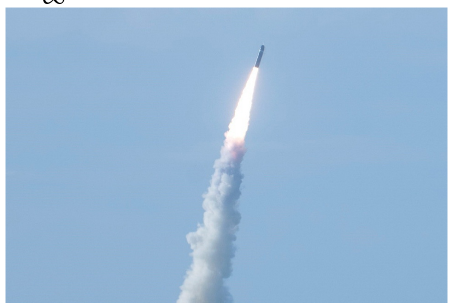
4 - Flow-induced vibration and sound (vibroacoustics)

$M_\infty \simeq 10^{-2}$

$M_\infty \simeq 10^{-1}$

$M_\infty \simeq 0.8$

$M_\infty \simeq 15$



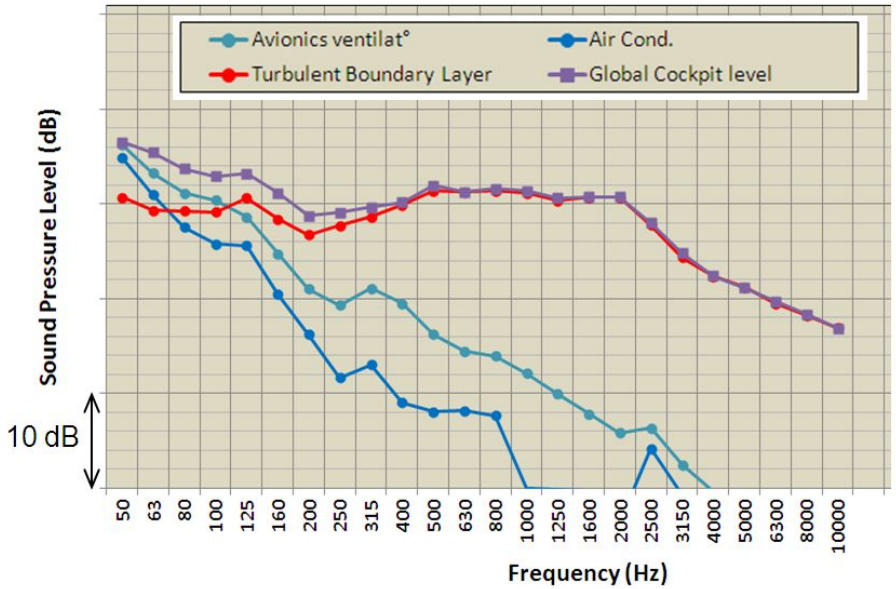
Cabin noise issues in various configurations

Motivation

Turbulent boundary layer excitation is the main contribution to interior noise for commercial aircraft at cruise conditions

Noise source breakdown in an A380 cockpit

(from Airbus)



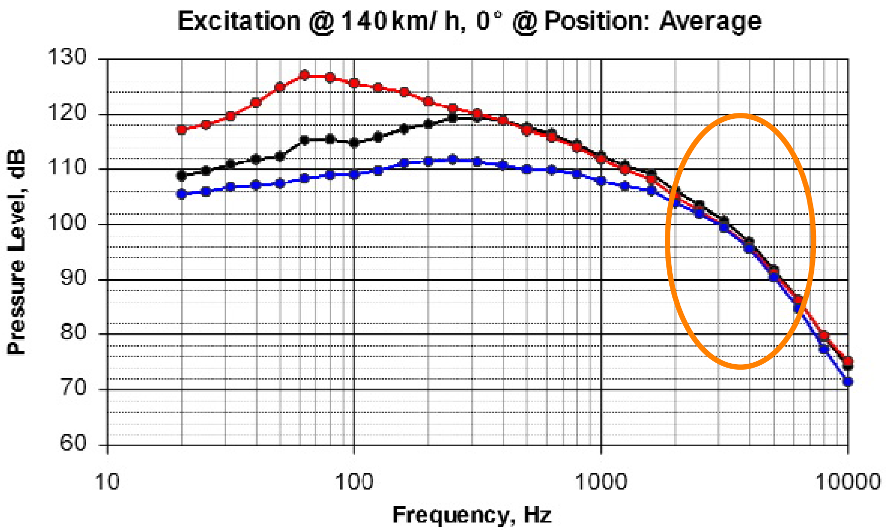
Flight test campaign on Airbus A350-1000 with flush-mounted microphones (characterization of the acoustic loading) on the fuselage

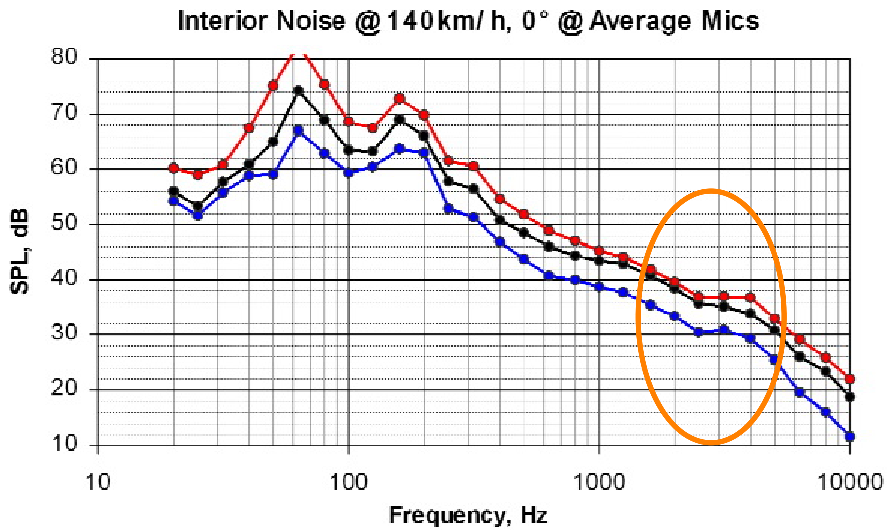
(from Helffer, ICSV25, 2018)

Motivation

Characterization of the turbulent boundary layer excitation (indirect contribution through panel vibration) and of the direct acoustic contribution

External pressure fluctuations and interior SPLs for different car mirror setups :
● baseline mirror, ● enlarged mirror, ● no mirror
$U_\infty = 140$ km/h

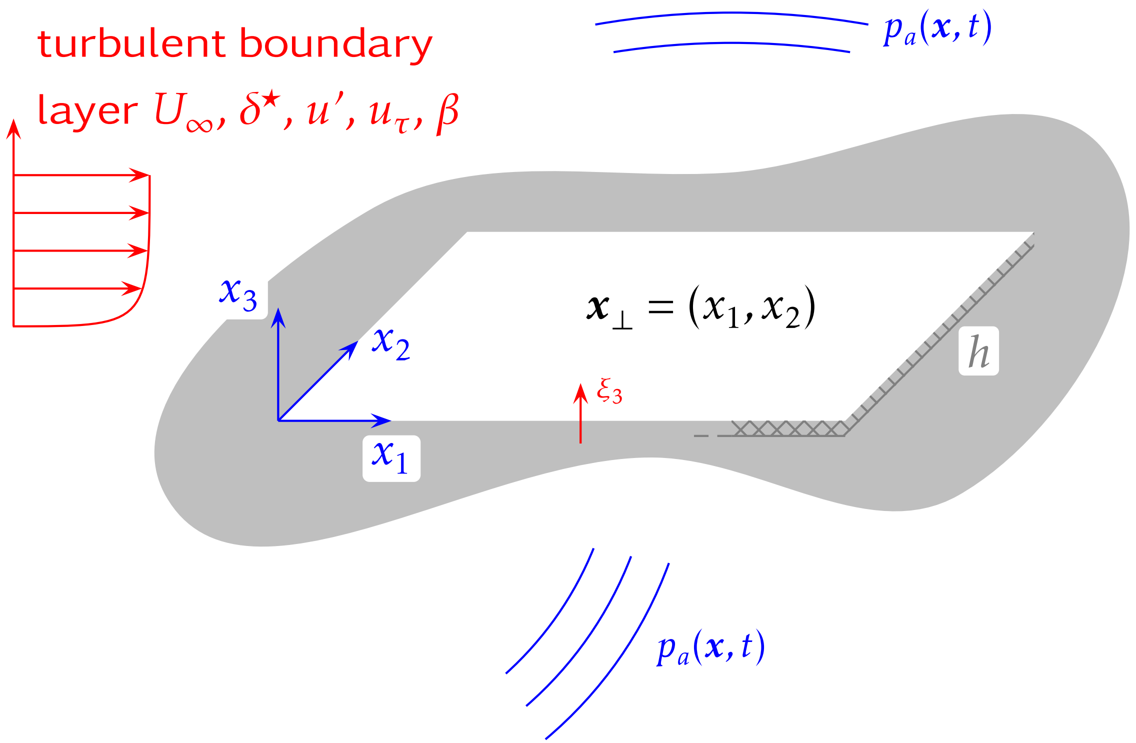




Hartmann *et al.*, AIAA Paper 2012-2205 ; Bremmer, cfa 2014 (No. 418, 2139-2146)

Turbulence-induced plate vibrations

Most sound radiation is caused by bending waves, that deform the structure transversely as they propagate.



Thin finite plate with thickness h (infinite or of length a and width b)

Turbulent boundary layer over the upper side of the plate $(x_3 > 0)$, but the wall pressure field $p_t(\boldsymbol{x}_\perp, t)$ is not influenced by plate vibrations (p_t is also called the blocked pressure)

Pressure acoustic field $p_a(\boldsymbol{x}_\perp, x_3, t)$ on either side of the plate induced by the plate vibrations

Turbulence-induced plate vibrations

Equation governing the transverse displacement $\xi_3 = \xi_3(\boldsymbol{x}_\perp, t)$ of a thin plate in the x_3 direction

$$D\nabla^4\xi_3 + m_s\beta_s\frac{\partial\xi_3}{\partial t} + m_s\frac{\partial^2\xi_3}{\partial t^2} = -[\![p]\!] \qquad D = \frac{Eh^3}{12(1-\nu_s^2)} \tag{12}$$

$m_s = \rho_s h$ mass per unit area
β_s damping per unit area
D flexural rigidity (bending stiffness), $K = D/m_s$

fluid loading $[\![p]\!] = p(\boldsymbol{x}_\perp, 0^+, t) - p(\boldsymbol{x}_\perp, 0^-, t) = p_t + p_a|_{0^+} - p_a|_{0^-}$
p_t turbulent boundary layer pressure
p_a acoustic pressure field

Turbulence-induced plate vibrations

The acoustic field p_a is solution of the (convected, if $x_3 > 0$) wave equation, with c_0 the speed of sound in the fluid and k_a the acoustic wavenumber

$$\frac{\partial^2 p_a}{\partial t^2} - c_0^2 \nabla^2 p_a = 0 \qquad k_a = \frac{\omega}{c_0}$$

The continuity of the velocity at $x_3 = 0$ requires that

$$\underbrace{-\frac{1}{\rho_f}\frac{\partial p_a}{\partial x_3}\bigg|_{x_3=0^\pm} = \frac{\partial u_a}{\partial t}\bigg|_{x_3=0^\pm}}_{\text{linearized Euler Eq.}} = \frac{\partial^2 \xi_3}{\partial t^2}$$

Linearized Euler equations for the acoustic field $(\rho_a, \boldsymbol{u}_a, p_a)$

$$\frac{\partial \rho_a}{\partial t} + \rho_f \nabla \cdot \boldsymbol{u}_a = 0 \qquad \frac{\partial \boldsymbol{u}_a}{\partial t} = -\frac{1}{\rho_f}\nabla p_a \qquad p_a = c_0^2 \rho_a \tag{13}$$

Turbulence-induced plate vibrations

Bending (flexural) waves

Plane wave solution with $\xi_3 = \xi_e e^{i(k_b x_1 - \omega t)} = \xi_e e^{ik_b(x_1 - c_b t)}$
in Eq. (12) without damping to determine the dispersion relation

$$D k_b^4 - m_s \omega^2 = 0 \qquad k_b = K^{-1/4}\omega^{1/2} \qquad c_b = \omega/k_b = K^{1/4}\omega^{1/2}$$

Bending waves are dispersive :
the phase speed c_b is a function of the frequency ω

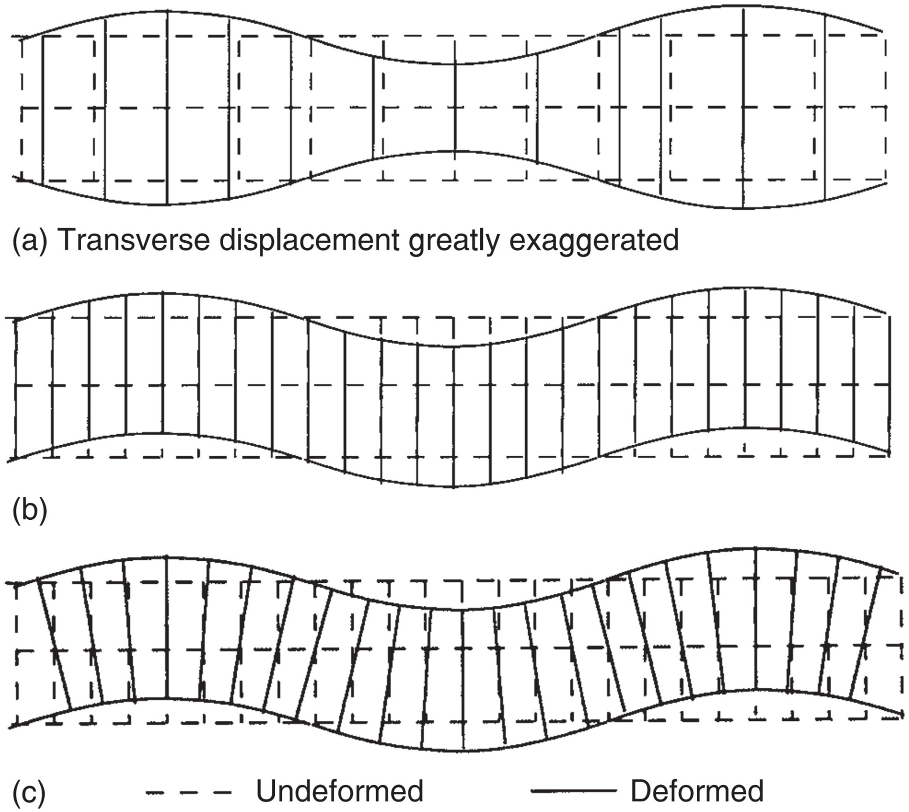
e.g. aluminum plate with $h = 5$ mm,
$E = 7.2 \times 10^{10}$ Pa, $\rho_s = 2.7 \times 10^3$ kg.m^{-3} and $\nu_s = 0.34$

$$\left| \begin{array}{lll} f = 1000 \text{ Hz} & c_b \simeq 220 \text{ m.s}^{-1} < c_0 & \text{subsonic bending wave} \\ f = 4000 \text{ Hz} & c_b \simeq 440 \text{ m.s}^{-1} > c_0 & \text{supersonic bending wave} \end{array} \right.$$

(see also Table slide 47 by comparison)

Turbulence-induced plate vibrations

Waves propagating in an elastic solid (see also slide 46)



Quasi-longitudinal waves (geometric confinement), phase velocity c'_P

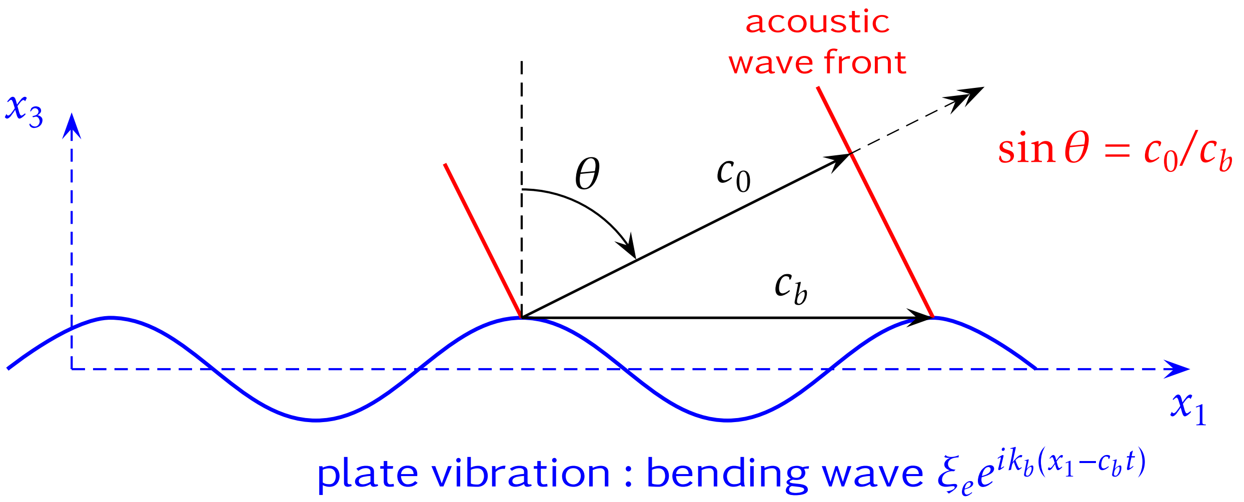
Transverse (shear) waves, phase velocity c_S

Bending waves, phase velocity c_b (dispersive waves). Unlike a transverse wave, the plate cross sections rotate about the neutral axis for bending waves

(Fahy & Gardonio, fig. 1.18)

Turbulence-induced plate vibrations

Supersonic bending wave, $c_b > c_0$ or equivalently $k_b < k_0$



Radiated acoustic pressure field p_a

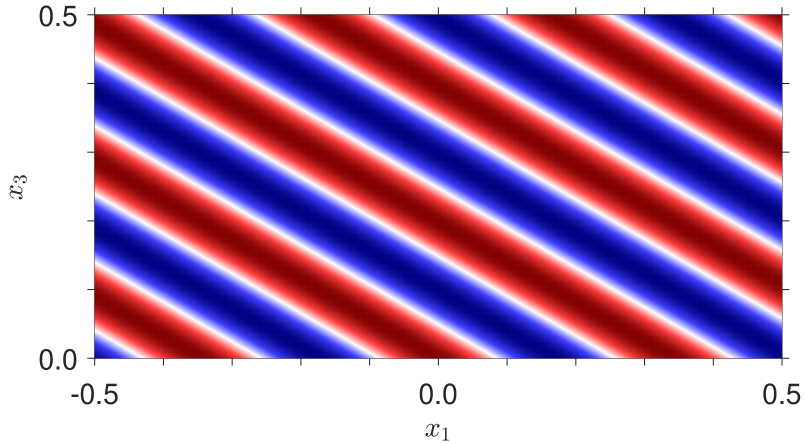
obtained by imposing continuity of the normal velocity at $x_3 = 0$

$$\boldsymbol{k} = \begin{cases} k_1 = k_b \\ k_3 = (k_0^2 - k_b^2)^{1/2} \end{cases} \qquad p_a(x_1, x_3) = \frac{\rho_f v_e \omega}{k_3}\, e^{i(\boldsymbol{k}\cdot\boldsymbol{x} - \omega t)} \qquad k_0^2 = (k_1^2 + k_3^2)^{1/2} \tag{14}$$

$(v_e = -i\omega\xi_e)$

Turbulence-induced plate vibrations

Supersonic bending wave, $c_b > c_\infty$ or equivalently $k_b < k_0$



$c_b = 2c_0$, $f = 2$ kHz

Plane wave propagating with an angle θ from the solid interface

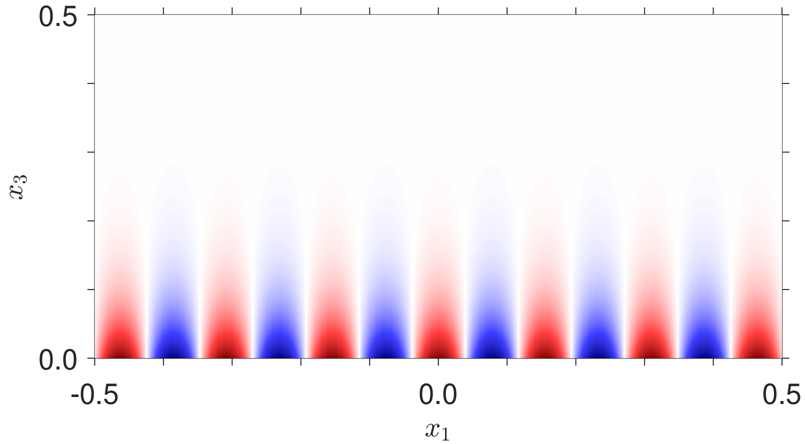
$$\sin\theta = c_0/c_b$$
$$\cos\theta = (1 - c_0^2/c_b^2)^{1/2}$$

$$\boldsymbol{k} = k_0 \left| \begin{array}{l} \sin\theta \\ \cos\theta \end{array} \right.$$

Turbulence-induced plate vibrations

Subsonic bending wave, $c_b < c_0$ or equivalently $k_b > k_0$

$$\boldsymbol{k} = \begin{cases} k_1 = k_b \\ k_3 = (k_b^2 - k_0^2)^{1/2} \end{cases} \qquad p_a(x_1, x_3) = i\frac{\rho_f v_e \omega}{k_3}\, e^{i(k_1 x_1 - \omega t)} e^{-k_3 x_3} \tag{15}$$



$c_b = 0.9c_0$, $f = 2$ kHz

Evanescent plane wave in the x_3 direction, propagating in the x_1 direction at c_b

Turbulence-induced plate vibrations

Acoustic power W_a radiated by the plate

$$W_a = \int_{\mathcal{S}} \boldsymbol{I} \cdot \boldsymbol{n} \, ds \qquad \boldsymbol{I} = p_a \boldsymbol{u}_a \qquad \bar{I} = \frac{1}{2} \mathfrak{Re}\left\{ p_a u_{3a}^{\star} \right\}_{x_3=0}$$

where $\bar{I}$ is the time-average power per unit surface, which can be evaluated at the center of the plate in our case (homogeneous configuration). The velocity $u_{3a} = \boldsymbol{u}_a \cdot \boldsymbol{n}$ is also the normal velocity of the vibrating plate

For supersonic bending waves using Eq. (14), one obtains

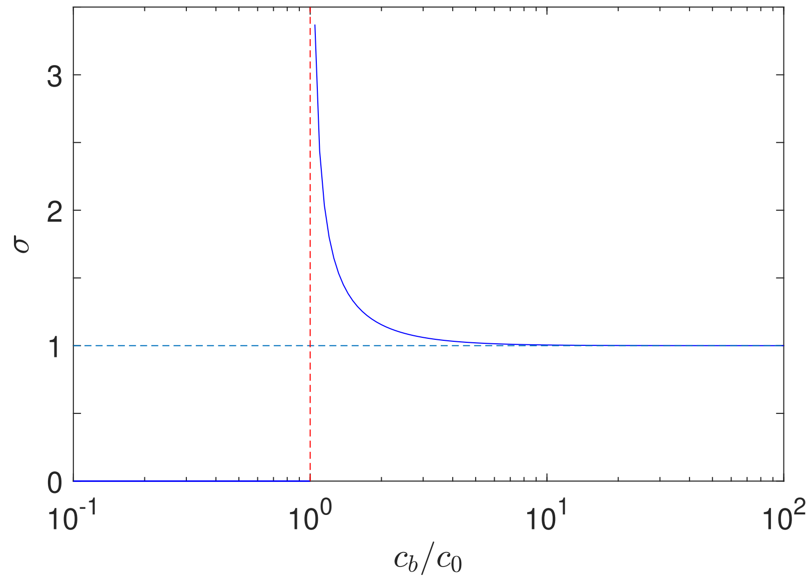
$$\bar{I} = \frac{1}{2} v_e^2 \, \rho_f c_0 \, \frac{1}{\cos\theta}$$

where $v_e^2/2$ is the plate vibration energy, that is the averaged mean square normal velocity, and $\rho_f c_0$ is the acoustic impedance of the fluid, and the last factor is the radiation efficiency $\sigma = 1/\cos\theta$

For subsonic bending waves, $\sigma = 0$, refer to Eq. (15)

Turbulence-induced plate vibrations

Critical frequency defined as $c_b = c_0$
or equivalently $\lambda_b = \lambda_0$: note that $\lambda_b \sim \sqrt{f}$ whereas $\lambda_0 \sim 1/f$



infinite plate
asymptotic behaviour, plane waves perpendicular to the plate (as a piston)

critical frequency f_c for $c_b = c_0$

$$\omega_c = 2\pi f_c = \frac{c_0^2}{K^{1/2}} = \frac{c_0^2}{h}\sqrt{\frac{12(1-\nu_s^2)\rho_s}{E}}$$

The radiation efficiency of a structure is usually very high near this critical frequency

Turbulence-induced plate vibrations

When a structure is excited acoustically, the frequency at which the speed of the forced bending wave in the structure and the speed of the free bending wave are equal is called the coincidence frequency. Sound transmission is highest near this coincidence frequency

Turbulence-induced plate vibrations

Modes of a finite rectangular thin plate

Plate of length a and width b, simply supported edge ($\xi_3 = 0$ on the edges). By taking the Fourier transform in time of Eq. (12), the normalized eigenfunction or modes can be determined

$$\phi_{mn}(\boldsymbol{x}_\perp) = \frac{2}{\sqrt{ab}} \sin(k_m x_1) \sin(k_n x_2) \qquad k_m = \frac{m\pi}{a} \quad k_n = \frac{n\pi}{b}$$

with $< \phi_{mn}, \phi_{pq} >= \delta_{mn,pq}$ since modes are orthogonal with respect to the following inner product,

$$< \phi_{mn}, \phi_{pq} >= \int_0^a \int_0^b \phi_{nm}(\boldsymbol{x}_\perp) \phi_{pq}(\boldsymbol{x}_\perp) d x_1 d x_2$$

Modal development for the displacement, where a_{mn} is the amplitude of the (m, n)-th mode

$$\xi_3(\boldsymbol{x}_\perp) = \sum_{m,n} a_{mn}\, \phi_{mn}(\boldsymbol{x}_\perp)$$

Turbulence-induced plate vibrations

Modes of a finite rectangular thin plate

The corresponding resonance frequencies (dispersion relation) are given by

$$\omega_{mn} = K^{1/2}(k_m^2 + k_n^2) \qquad k_b = (k_m^2 + k_n^2)^{1/2} \sim \omega_{mn}^{1/2}$$

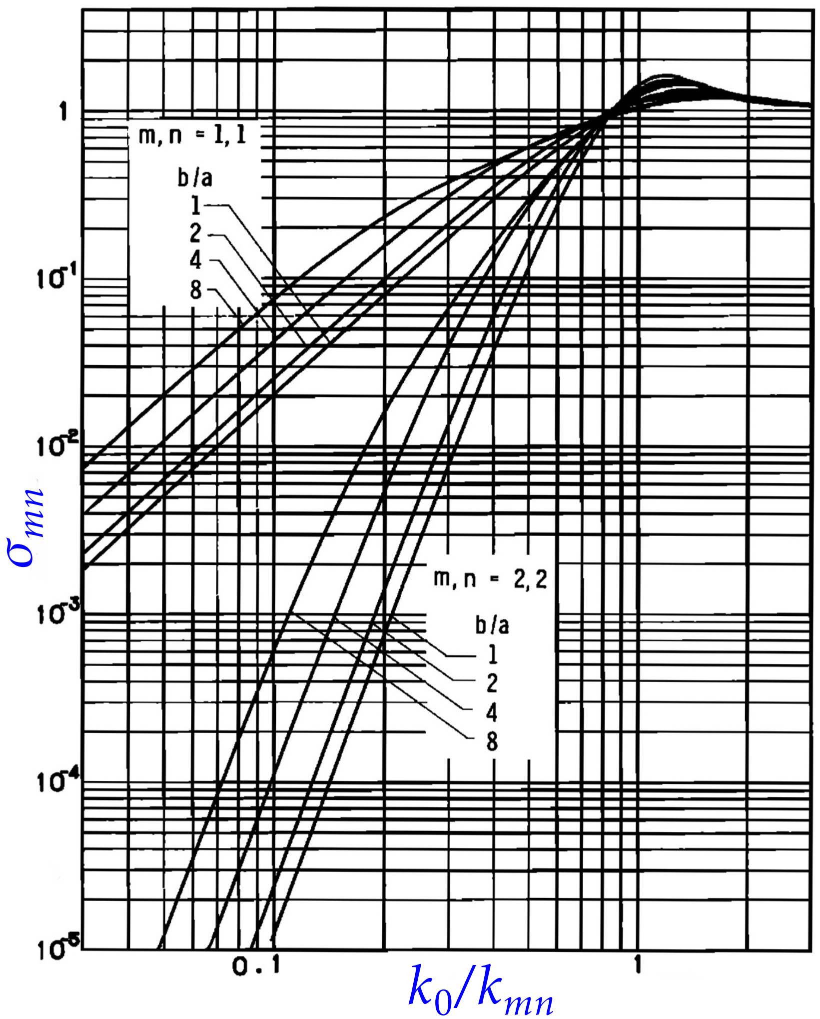
The power spectral density associated with each mode (m, n) is

$$S_{mn}(\omega) = \frac{1}{(\omega_{mn}^2 - \omega^2)^2 + \beta_s^2 \omega_{mn}^2 \omega^2}$$

(basically an oscillator, refer to the first lecture)

Turbulence-induced plate vibrations

Radiation efficiency of the modes (1,1) and (2,2) for various aspect ratio b/a of the plate



Optimal efficiency for $k_0 \simeq k_{mn}$, corresponding to the concept of spatial coincidence (caution : the frequency changes with k_{mn})

For $k_0 \gg k_{mn}$, $\sigma_{mn} \to 1$

For $k_0 \ll k_{mn}$, rapid decrease of σ_{mn}

(Wallace, 1972, *J. Acoust. Soc. Am.*)

Turbulence-induced plate vibrations

Vibroacoustics response in Fourier space

Homogeneous and stationnary turbulence

$$R_{pp}(\boldsymbol{r},\tau) = \overline{p'(\boldsymbol{x}_\perp,t)p'(\boldsymbol{x}_\perp+\boldsymbol{r},t+\tau)}$$

Cross power spectral density

$$R_{pp}(\boldsymbol{r},\omega) = \mathcal{F}\left\{R_{pp}(\boldsymbol{r},\tau)\right\}$$

Wavenumber-frequency spectrum $\Phi_{pp}(\boldsymbol{k},\omega)$

$$\Phi_{pp}(\boldsymbol{k},\omega) = \frac{1}{(2\pi)^2}\iint R_{pp}(\boldsymbol{r},\omega)e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}d\boldsymbol{r} \qquad \boldsymbol{k} = (k_1,k_2)$$

$k_0 \sim \omega$ acoustic wavenumber

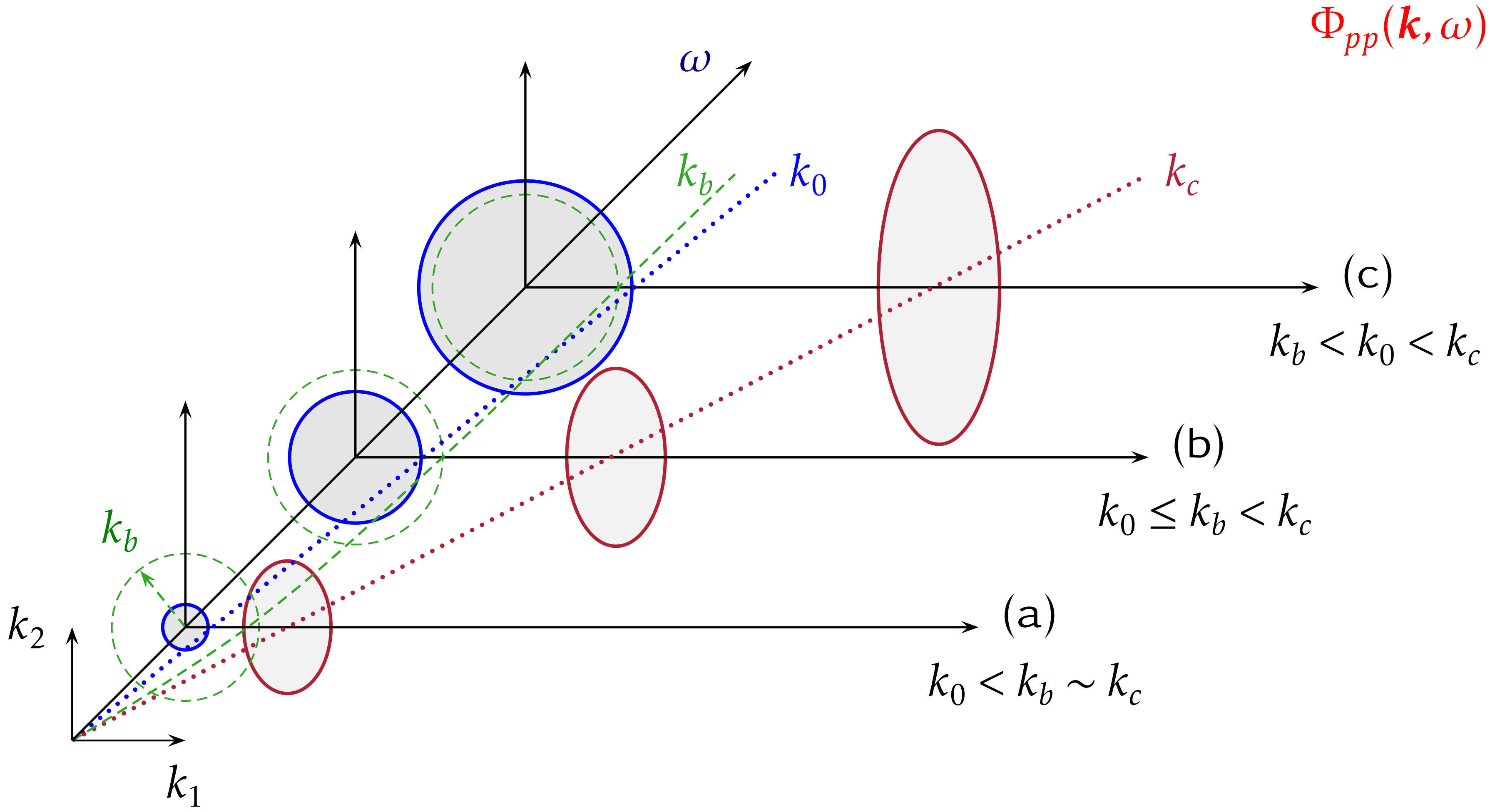
$k_b \sim \omega^{1/2}$ bending wavenumber

$k_c = \omega/U_c = k_0/M_c$ $(M_c = U_c/c_0, U_c \simeq 0.7U_\infty)$ convective wavenumber

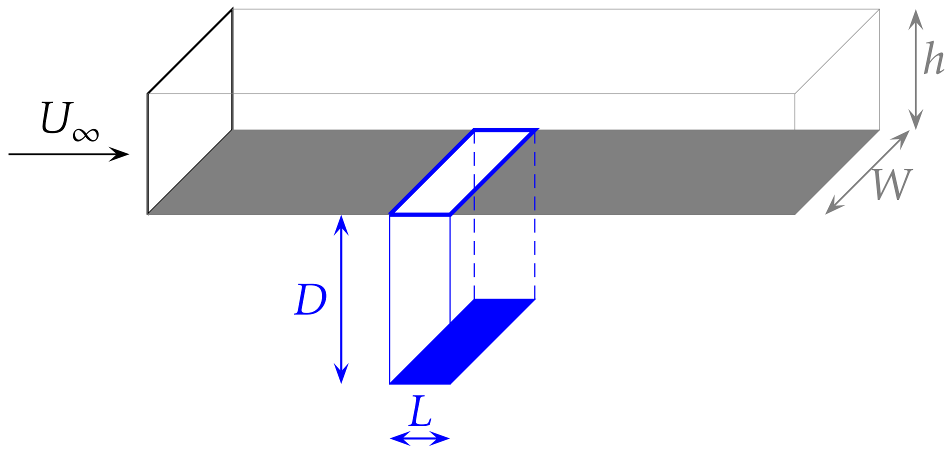
Turbulence-induced plate vibrations

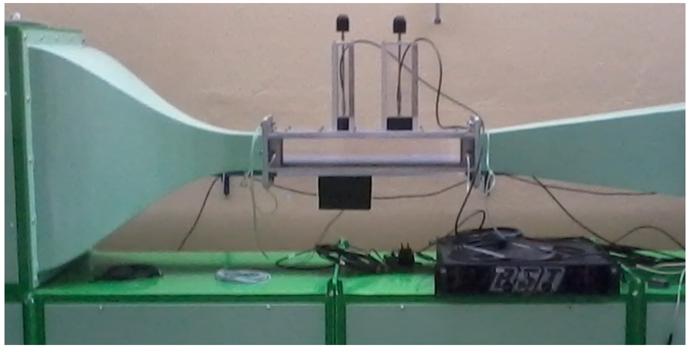
Vibroacoustics response in Fourier space

(for a low Mach number TBL, $M_c \ll 1$)



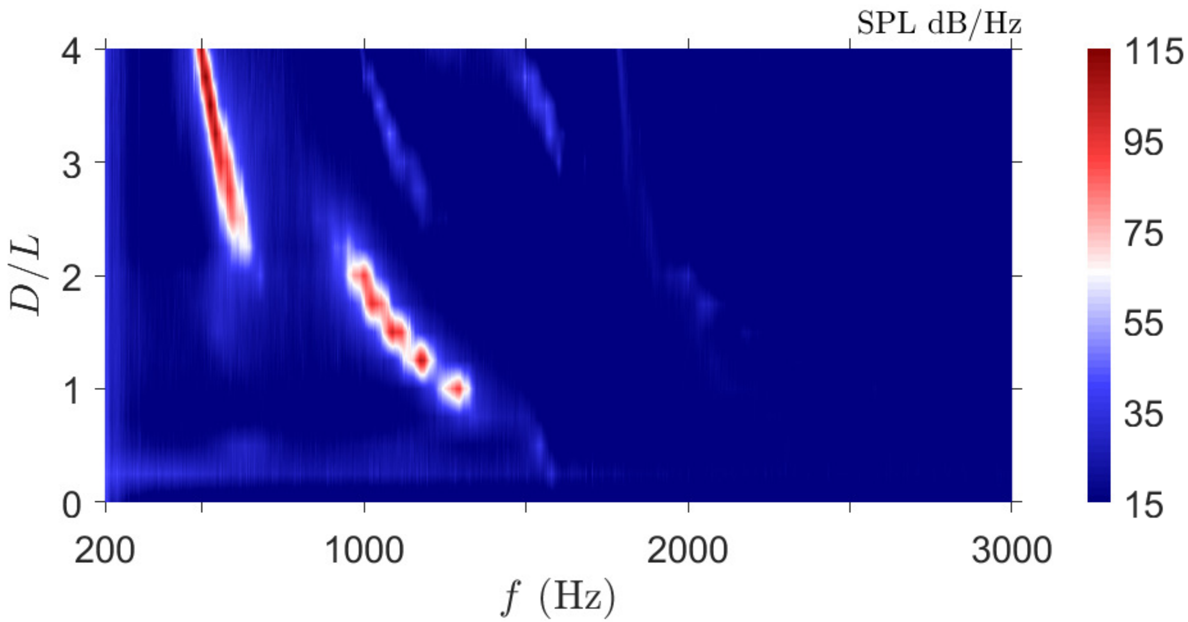
Flow-induced cavity oscillations





SPL measured inside the cavity

$$\begin{cases} L = 0.04 \text{ m} \\ h = 0.1 \text{ m} \\ U_\infty = 52.2 \text{ m.s}^{-1} \\ 0 \leq D \leq 4L \end{cases}$$



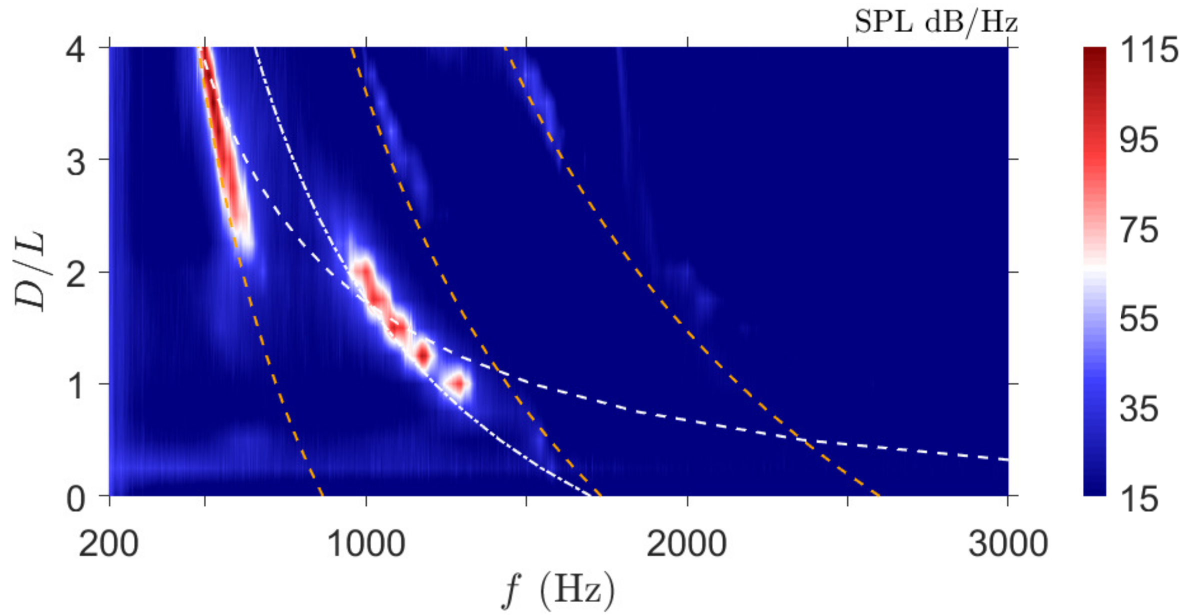
Flow-induced cavity oscillations

Cavity depth modes $- - -\ n = 0$
(cavity+end-correction)

$$f = (2n+1)\frac{c_\infty}{4(D+0.41L)}$$

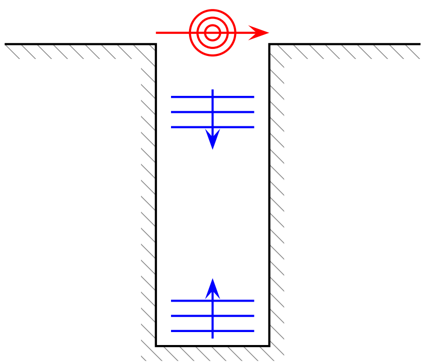
Acoustic modes $-\cdot-\cdot-\ n = 1$
(cavity+channel height)

$$f = n\frac{c_\infty}{2(h+D)}$$



Aeroacoustic feedback loop
$- - -\ n = 1, 2, 3$

$$f = \frac{n}{L/U_c + 2D/c_\infty}$$

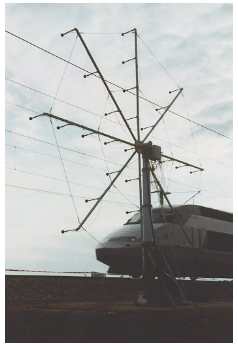


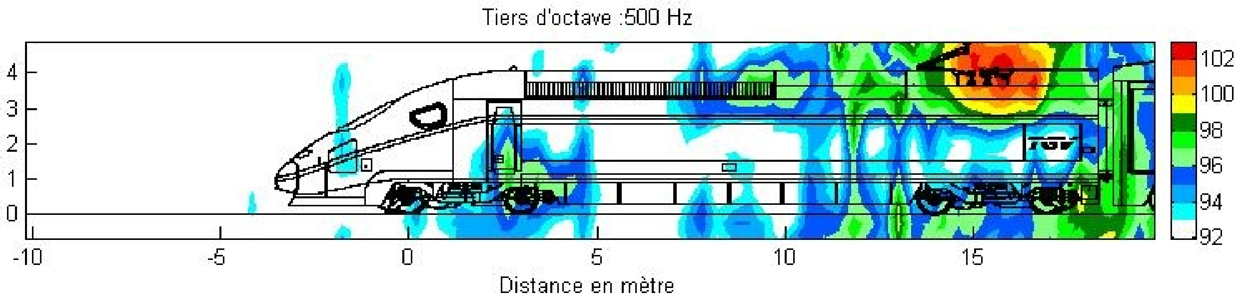
Convection of turbulent structures within the shear layer, which impact at the downstream corner, generation of acoustic waves (depth modes) and excitation of the shear layer upstream.

Modelling : frequency selection + gain $\rightsquigarrow$ whistle

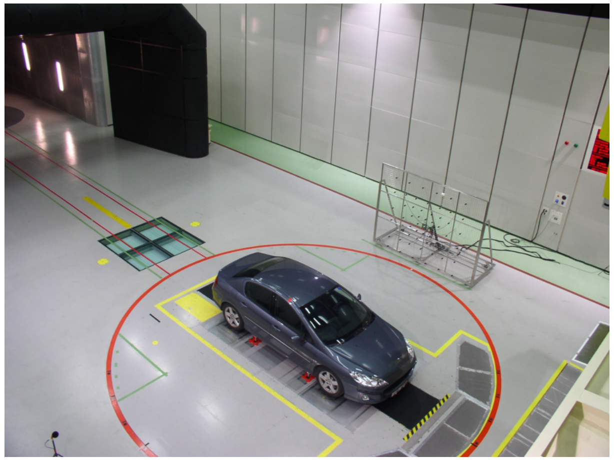
Flow-induced cavity oscillations

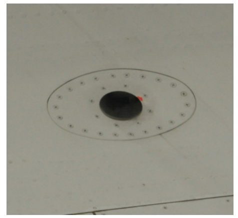
A long-lasting problem in transport





TGV - high speed train (rear engine) - courtesy of SNCF





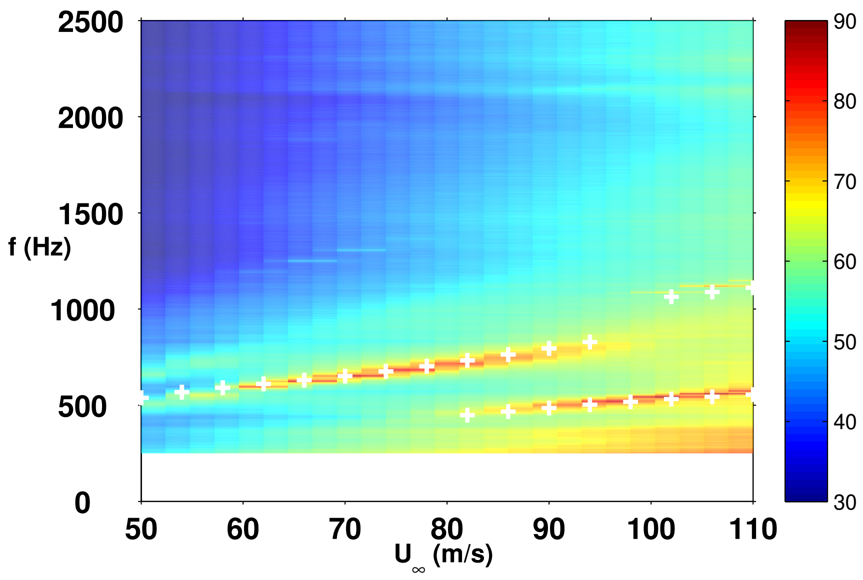
fuel pressure relief vent on a A319

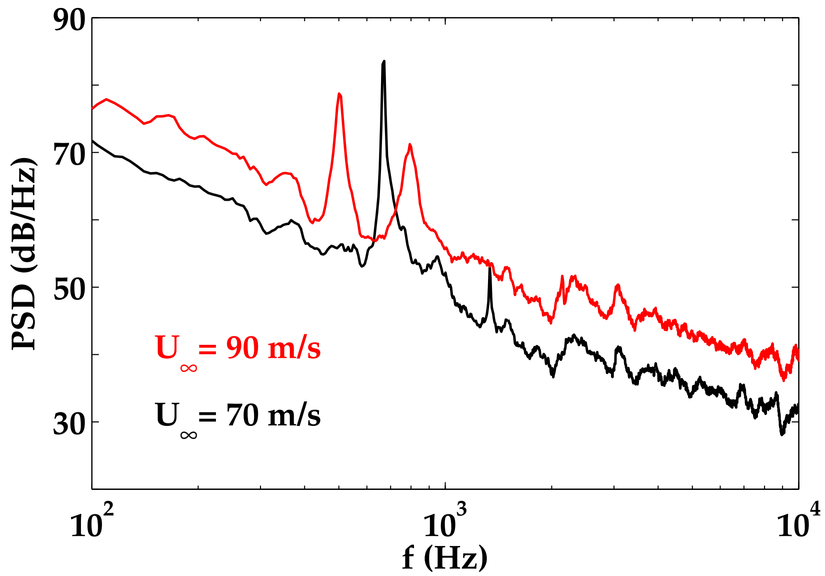
Courtesy of Jan Delf (DLR), refer to *AIAA Paper 2002-2470*

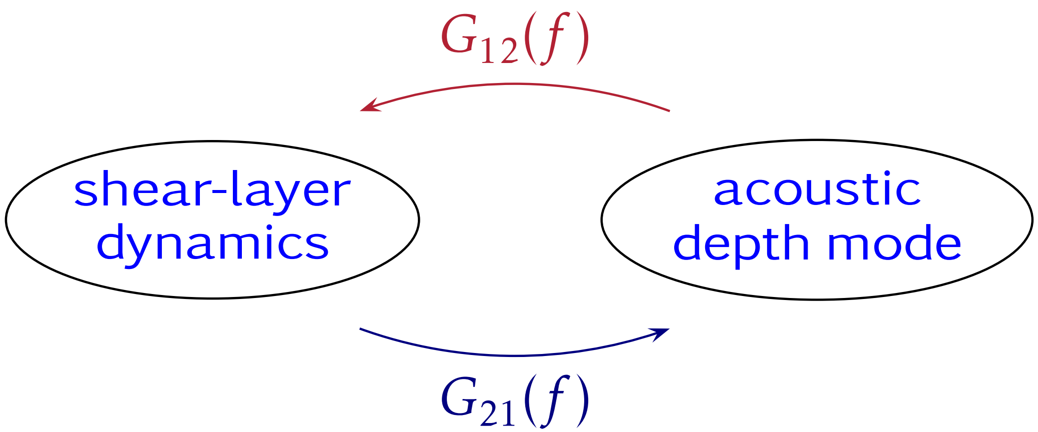
Cavity noise

Measured acoustic spectra at 1 m above a cylindrical cavity

Diameter $D = 10$ cm, depth $H = D$, flow speed $50 \leq U_\infty \leq 110$ m/s TR-PIV 







$$\mathfrak{Im}\left(G_{12} \times G_{21}\right) = 0$$

$\Longrightarrow$ resonance frequencies +

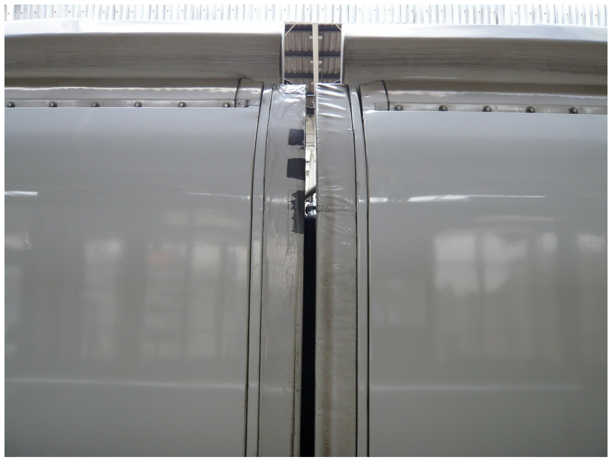
Marsden *et al.*, 2012, *J. Sound Vib.*, **331**

Elder, 1978, *J. Acoust. Soc. Am.*, **63**(3)

Cavity noise

Shinkansen (Tokyo – Sendai, july '08)

E2 series – 275 km/h
E4 series – 240 km/h



aerodynamic noise generated by intercoach spacing

Cavity noise

New Hayabusa Shinkensan train ('the bullet train')

THE DAILY YOMIURI

SUNDAY, MARCH 6, 2011

THE DAILY YOMIURI (2011)

PRICE ¥120 (¥2,650 per calendar month)—tax included

The Yomiuri Shimbun

Fast trainspotting

Train fans snap pictures as the new Hayabusa bullet train on the Tohoku Shinkansen line leaves Tokyo Station on its maiden trip to Shin-Aomori Station on Saturday morning. The train, the first entirely new model introduced by East Japan Railway Co. since 1997, achieved speeds of up to 300 kph, 25 kph faster and quieter than the conventional Hayate superexpress. It connects Tokyo and Shin-Aomori in three hours and 10 minutes. (Related story on Page 2)

7% annual growth eyed in new 5-yr plan

BEIJING (Reuters)—China is on course for another five years of robust growth although inflation threatens social stability and must be tamed, Premier Wen Jiabao said Saturday.

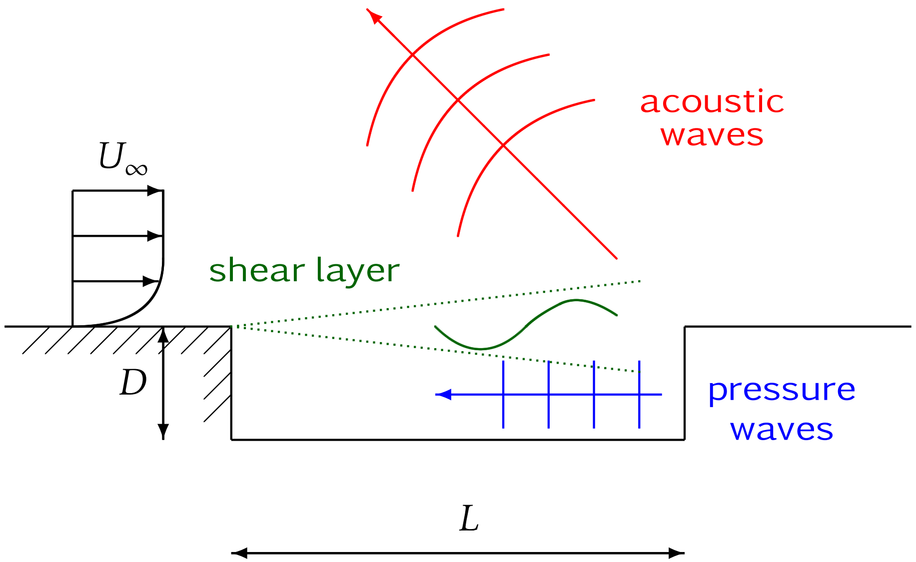
In China's version of a "State of the Union" address to the annual parliament session, Wen said the top priority this year was to curb price rises that were making life difficult for ordinary people.

faster
and
QUIETER !

Cavity noise

Flow-induced cavity oscillations : aeroacoustic feedback loop
for shallow cavities (at rather high Mach numbers)

$L/U_c + L/c = n/f$



Rossiter formula (1964)

$$\mathrm{St} = \frac{fL}{U_\infty} = \frac{n - \alpha}{M + \dfrac{1}{\kappa}}$$

f frequency
L length
U_∞ free stream velocity
n number of vortices
α phase lag
$\kappa = U_c/U_\infty$, U_c convection velocity
$M = U_\infty/c$

- No information about amplitude or mode selection (L/δ_θ)
- Acoustic resonance can superimpose (longitudinal or depth mode)

Concluding remarks

...

To summarize

Fluid-structure interaction

- Flow : lift and drag defined for a steady flow, buoyancy force (gravity), added mass
- Body oscillator : an elementary model to understand physics (a specific mode in a modal approach)
- Scales and waves in solid and fluid, dimensionless parameters
- Distinction between instability (galloping) and resonance (vortex shedding)
- Vibroacoustics : acoustic radiation of structures