Homework 1 – Fourier's transform using Matlab

1. Write a Matlab script to compute the Fourier transform $\hat{f}(k_1)$ of

$$f(x_1) = \frac{1}{1+(x_1/l)^2}$$

Validate your result with the analytical solution, and plot the two functions.

Take the inverse Fourier transform to retrieve $f(x_1)$.

Study two cases corresponding to a low and high frequency resolution, and motivate your choice.

Note that this script will be used in the next classroom, and that a tutorial is provided in the next page.

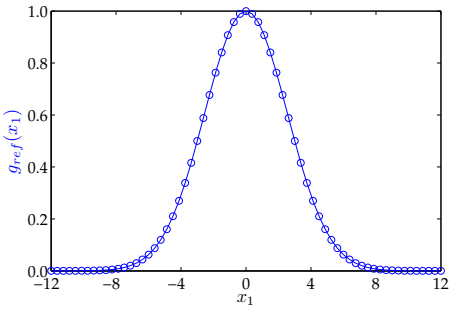
$$\hat{f}(k_1) = \mathscr{F}\left[f(x_1)\right] = \frac{1}{2\pi}\int_{-\infty}^{+\infty} f(x_1)e^{-ik_1x_1}\,dx_1 \qquad f(x_1) = \mathscr{F}^{-1}\left[\hat{f}(k_1)\right] = \int_{-\infty}^{+\infty} \hat{f}(k_1)e^{ik_1x_1}\,dk_1$$

fft	ifft
$\left[\hat{f}_l\right] = \sum_{n=1}^{N} f_n e^{-i2\pi(n-1)(l-1)/N} \qquad 1 \le l \le N$	$[f_n] = \frac{1}{N}\sum_{n=1}^{N} \hat{f}_l e^{i2\pi(n-1)(l-1)/N} \qquad 1 \le n \le N$
$\hat{f}_l = \left[\hat{f}_l\right] \times \frac{dx_1}{2\pi}$	$f_n = [f_n] \times \frac{2\pi}{dx_1} \qquad \left(dk_1 = 2\pi \times \frac{1}{N dx_1}\right)$

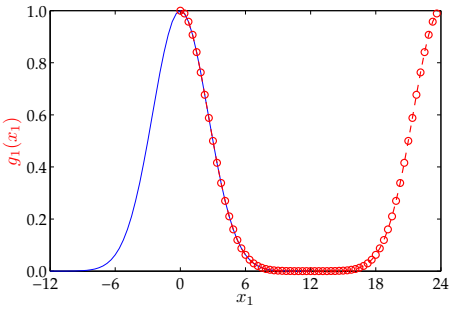
TABLE 1 – FFT using Matlab.

$$f(x_1) = e^{-\ln 2\left(\frac{x_1}{b}\right)^2} \qquad \hat{f}(k_1) = \frac{b}{2\sqrt{\pi \ln 2}} e^{-\frac{(bk_1)^2}{4\ln 2}}$$

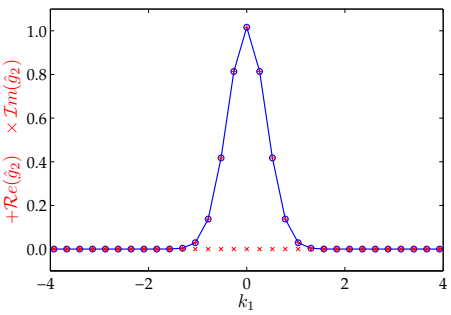
```
%.. Fourier transform of a Gaussian function
nfft = 64;
nf = nfft/2;

x = linspace(-12,12,nfft+1);
nx = length(x);
nx2 = (nx+1)/2;

dx = x(2)-x(1);
dk = 2*pi/(nfft*dx);

%.. analytical functions g and gk
b = 3.;
gref = exp(-log(2.)*(x/b).^2);

kx = -nf*dk:dk:nf*dk;
gkref = b/(2.*sqrt(pi*log(2.))) * exp(-(kx*b).^2/(4*log(2.)));
```



```
%.. Fourier transform
x1 = 0:dx:(nfft-1)*dx;
g1 = fftshift(gref(1:nfft));
```



```
kx1= 0:dk:(nfft-1)*dk;
gk1 = fft(g1) * dx * 1/(2*pi);

kx2 = -nf*dk:dk:(nf-1)*dk;
gk2 = ifftshift(gk1);
```



```
%.. Inverse Fourier transform
gk2 = fftshift(gk2);

gbis = ifft(gk2);
gbis = gbis *2*pi/dx;

xbis = -nf*dx:dx:(nf-1)*dx;
gbis = fftshift(gbis);
```