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Oftentimes physics teachers need complex instrumentation to execute experiments in order to show a physical law, but sometimes it is possible to perform a good experiment with simple objects and instruments. Indeed, on occasion we have “in our hands” a simple but efficient “instrument” that can facilitate understanding of physics for our students. Consider, for instance, spherical aberration and the reflection caustic.

Elementary physics textbooks generally state that spherical concave mirrors reflect rays that are parallel to the optical axis to a single point defined as the focus of the mirror. Immediately before or after this sentence, it is usually admitted that this is true only if small aperture mirrors are considered, or it is true in the limit of paraxial approximation (rays parallel and close to the optical axis).

What is the limit of validity of paraxial approximation? How close to the optical axis should the rays be? What does small aperture mean? We describe here a simple, practical demonstration that can help the understanding of the limit of validity of the imaging properties of spherical mirrors, avoiding the confusion students may have between the ideal and real behavior of a spherical mirror.

Observing the Caustic

Take from your finger your ring (assuming you wear one) and put it on a white surface. If sunlight from a window or the light from a single lamp strikes it, you will readily see a bright figure formed on the surface inside the ring like that shown in Fig. 1. What a caustic!

The figure has a shape very similar to the reflection caustic curve we usually see in physics textbooks. It is interesting to investigate how that figure is produced, even without a mathematical approach. We immediately understand that the figure is due to the reflection that occurs at the internal surface of the ring opposite the source. If we assume the internal surface of the ring approximates a small section of a spherical mirror, the bright figure is the reflection caustic of this mirror segment and is due to spherical aberration. But we know the caustic is a surface defined by an envelope of reflected rays, and the question is: Why are we able to see it?

Fig. 1. Demonstration of ring reflection caustic.

Explaining the Reflection Caustic

In the case of a cylindrical mirror like our ring mirror, it is easy to predict the geometry of the reflection caustic surface; it is simply given by a set of identical reflection caustic curves for each section of the cylindrical mirror. In this case the caustic surface is vertical like the axis of the cylindrical mirror, as illustrated in Fig. 2. But in our case we are able to see the caustic because the light source is off the optical axis of the ring mirror. If the source is off the optical axis, the caustic surface is no longer vertical but tilted downward, because the reflected rays are directed downward. Since the caustic is defined as an envelope curve of reflected rays, what we observe in Fig. 1 is not the caustic itself but its projection on the surface lying under the ring. If we assume the internal surface of the ring as a thin section of a spherical mirror, we can say that what we observe as the ring reflection is the caustic curve produced by the aberration of the spherical mirror to which the ring surface belongs.

The reflection caustics can also be frequently observed in cups containing liquids. In this case the caustic is produced by reflection occurring at the internal side of the cup and it is projected on the liquid surface.

Proposed Classroom Activity

My experience with the ring mirror experiment has been very positive. My students are attracted to the activity and say that it has aided their understanding of the aberration and nature of the caustic. Prior to the laboratory session, I give a lecture on the reflection at curved surfaces (spherical, cylindrical) and show by geometrical construction...
that, for a spherical mirror, more than one focal point exists. (I suggest that the caustic surface should not be identified at this stage, because it has, strictly speaking, only a geometrical meaning; it could be perceived by the students as a surface without a specific meaning, provoking a misconception.) After this introductory lecture, I do the following demonstration. Put the ring mirror on a white surface and set an extended light source far enough from the mirror and in such a way that rays incident on the internal surface of the ring can be considered parallel to each other (collimated light beam). The source is set slightly off the optical axis of the reflecting surface of the ring. Students should first observe the resulting caustic and discuss it, trying to find (with the help of the teacher) the relation between what they are observing and what they drew in the previous lecture. Now it is opportune to discuss explicitly the caustic as a geometric construction from which the caustic curve can be identified and constructed. It can also be useful to show the tridimensional shape of the caustic surface. This can be easily accomplished by using a thin book and opening it to obtain the caustic surface of Fig. 3.

A number of observations can be guided by the teacher:

- The paraxial focus can be identified as the vertex of the caustic curve. Note that the surface involved in the reflection is about one-half of the internal surface of the ring, namely the internal surface that is in front of the source.
- Rotations of the ring around its axis do not produce changes of the caustic.
- Changes of the caustic can be observed by tilting the ring or moving the source up and down.
- Changes of the caustic can be observed when the distance from source to ring is varied.
- If characters are traced on the internal surface, reflection images of these characters can be observed and discussed.

Discuss these observations after asking the students what changes they expect for each situation.

Now proceed to show that the real behavior of a spherical mirror can approach the ideal case of paraxial approximation. A simple slit with variable aperture can be made, for example, with two pieces of paper. Initially the aperture of the slit should be larger or equal to the ring diameter. As the size of the slit is reduced (from which we can now define the aperture of the mirror), it can be observed that the size and shape of the caustic changes continuously. For a certain size of the slit (or mirror aperture), the naked eye of the observer cannot distinguish whether the edges of the caustic are curved or straight, as shown in Fig. 4. At this point we can say that, for our naked eyes, that is the upper limit of validity of the paraxial optics. Moreover, we can explain what a small aperture for a mirror means, making sure the students will understand. This last observation can be a way to show how, by reducing the size of the aperture of a mirror, it is possible to reduce also the amount of spherical aberration. I like to ask my class to find other situations in which a caustic can be observed. There are many such situations, including all cylindrical objects such as a glass, a jar, a pan, and even other examples such as a drop of liquid.

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References

3. Note: In fact, if we consider a situation in which we have half a cylinder and the optical rays are parallel to the optical axis of the mirror, the reflecting rays will all lie in planes orthogonal to the axis of the cylinder. In this case, to show the presence of the caustic you should put a small translucent screen along the optical axis to show that no single thin bright line (paraxial focus) is found. Moving the screen along the optical axis will reveal a large bright line that is present along the optical axis, but with no single focus. We would see the line of minimum confusion, which corresponds to the circle of minimum confusion for a spherical mirror.
4. Note: For a spherical mirror, when a source is off the optical axis, the aberration is called coma.