



Imprint of Vortical Structures on the Near-Field Pressure of a Turbulent Jet

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<https://doi.org/10.2514/1.J061010>

The distributions of turbulent scales in a Mach 0.9 isothermal round jet are investigated for the purpose of developing linear surface-based models for the noise source that can be informed by a Reynolds-averaged Navier–Stokes (RANS) solution of the flowfield. The jet is calculated by large-eddy simulation (LES), which enables the computation of two-point space–time correlations throughout the jet and its near-acoustic field. Time, length, and convective-velocity scales are examined on the surface of peak Reynolds stress (SPS), representing the location of the most energetic eddies, and on a “radiator surface” at the boundary between the rotational and irrotational fields. The nature of the space–time correlations is different for axial velocity fluctuations and pressure fluctuations. Velocity-based correlations appear to capture localized turbulent events, whereas pressure-based correlations appear dominated by the interaction of large eddies with the surrounding potential flow. The correlation length scales are larger on the radiator surface than on the SPS, thus indicating that small-scale eddies do not make a significant imprint on the radiator surface. The scales associated with an emulated RANS solution of the flow are compared to the LES-based scales. Simple relationships are inferred that may aid the development of rapid predictive models.

Nomenclature

C	= coefficient
D_j	= nozzle-exit diameter
g	= magnitude of principal component of the Reynolds stress
k	= turbulent kinetic energy
L	= fixed-frame length scale
\hat{L}	= moving-frame length scale
M_j	= jet exit Mach number
p	= pressure
R	= space–time correlation
r_i	= radial location of inner edge of mixing layer
r_{SPS}	= radial location of surface of peak stress
St_D	= Strouhal number based on nozzle diameter
T	= temperature
t	= time
U_c	= convective velocity
U_j	= jet exit velocity
u, v, w	= velocity components in Cartesian coordinate system
\bar{u}_{CL}	= centerline mean velocity
x, r, ϕ	= cylindrical coordinates, with x on jet axis
x, y, z	= Cartesian coordinates, with x on jet axis
δ_θ	= momentum thickness of the mixing layer
ε	= turbulent dissipation
ξ, η, α	= displacements in cylindrical coordinates

ξ, ψ, ζ	= displacements in Cartesian coordinates
ρ	= density
τ	= time shift
Φ	= azimuthal scale
ω	= vorticity vector

I. Introduction

THE turbulent exhaust of jet engines continues to be a significant contributor to aircraft noise. The research effort presented here is motivated by the need of the aerospace community for low-cost predictive models of the jet noise emission in isolated and installed configurations. For these models to have satisfactory turnaround times and become useful design tools, they need to rely on rapid solutions of the flowfield such as Reynolds-averaged Navier–Stokes (RANS) simulations. The focus here is on surface-based models where the noise source is prescribed on a surface surrounding the jet flow, which is commonly located in the linear pressure field [1,2]. This approach has the advantage of enabling predictions of not only propagation but also scattering from airframe surfaces using linear propagation techniques such as the boundary element method [3,4]. In addition, surface-based models could simplify the treatment of complex multistream asymmetric jets that have shown promising noise reductions [5,6].

To inform the surface-based noise source model from the RANS flowfield, one needs to establish physical connections between the dominant turbulent events in the vortical region of the jet and the linear pressure statistics near the edge of the jet where the surface-based model would be defined. The basic elements of this connection are depicted in Fig. 1 and represent the integration of concepts initially proposed in Refs. [6,7]. In a time-averaged sense, the action of the turbulent eddies that dominate sound production is represented by the statistics on the surface of peak Reynolds stress, which is abbreviated here as SPS. The noise source is prescribed as a distribution of random partial fields on a radiator surface at the edge of the jet. Each partial field is an amplitude modulated pressure wave, convected at velocity U_c , having finite axial and azimuthal extent. The convective velocity governs the radiation efficiency of the jet [6,8,9] and is thus deemed the most important quantity to model. Near- and far-field statistics are constructed via stochastic superposition of the partial fields and their sound radiation. An early treatment can be seen in Ref. [10]. The simulated statistics on the

Presented as Paper 2021-1184 at the AIAA SciTech 2021 Forum, Virtual Event, 11–15 and 19–21 January 2021; received 18 June 2021; revision received 22 August 2021; accepted for publication 24 August 2021; published online 29 October 2021. Copyright © 2021 by Andres Adam, Dimitri Papamoschou, and Christophe Bogey. Published by the American Institute of Aeronautics and Astronautics, Inc., with permission. All requests for copying and permission to reprint should be submitted to CCC at www.copyright.com; employ the eISSN 1533-385X to initiate your request. See also AIAA Rights and Permissions www.aiaa.org/randp.

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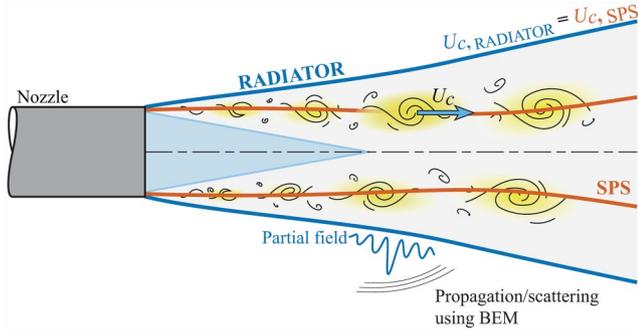


Fig. 1 Basic elements of surface-based noise source modeling (BEM = Boundary Element Method).

radiator surface are expected to match those of the turbulent jet. Quantities of interest include the convective velocity as well as correlation length scales in the axial and azimuthal directions. Although a certain level of empiricism is expected, understanding the processes by which events in the vortical region leave a “footprint” on the radiator surface is essential for the construction of physically meaningful models.

Turbulence scales in jet shear layers have received considerable attention in the literature. One can cite the measurements obtained by hot-wire anemometry of Harper-Bourne [11] and Morris and Zaman [12], which are frequently used to inform jet noise source models. More recently, Proença et al. [13] performed a comprehensive hot-wire investigation in the shear layer of various unheated single-stream jets. Studies have also been performed using particle image velocimetry, such as those by Fleury et al. [14] and Pokora and McGuirk [15], which have provided detailed information on the correlations of the velocity field. However, past works have focused primarily on the jet lip line and centerline, and they provided little information on statistics near the edge of the jet.

Previous works by our group on large-eddy simulation (LES) of multistream jets have studied the distribution of convective velocity U_c and have made some inroads into the modeling issues outlined earlier in this paper [16,17]. However, the complexity of the jet flows did not facilitate a clear understanding of the connection between vortical and near-pressure fields. In this study, we have the opportunity to analyze data from a highly resolved LES of a canonical Mach 0.9 round jet. Using two-point space–time correlations, characteristic length and velocity scales are calculated throughout the jet flow and near-acoustic field. The distribution of these scales on the SPS and on the radiator surface are of particular interest. We thus aim to investigate physical connections between the core of the vortical field and the resulting pressure signature at the edge of the jet, where a linear source model would be prescribed. In addition, we use the LES results to emulate the RANS flowfield and evaluate the potential of using RANS to model the scales relevant to noise generation.

The present paper is organized as follows. The flow conditions and numerical scheme are outlined in Sec. II. The definition of the turbulence scales and the methodology for their calculation are provided in Sec. III. The relevant surfaces analyzed in this study are defined in Sec. IV. The key features of the flowfield are presented in Sec. V. The results of performing cross correlations in the longitudinal direction, including axial length scales, are studied in Sec. VI. The corresponding results of cross correlations in the azimuthal direction are studied in Sec. VII. Preliminary attempts at modeling the turbulence scales based on the time-averaged flow, as would be obtained with RANS, are presented in Sec. VIII. Concluding remarks are finally drawn in Sec. IX.

II. Numerical Flowfield

A. Jet Flow

We consider an isothermal single-stream round jet at a velocity of $U_j = 309$ m/s, a Mach number of $M_j = 0.9$, and a diameter-based Reynolds number of $Re_D = 10^5$. The jet originates from a pipe of

constant diameter D_j and discharges into air with a temperature of $T_a = 293$ K and a pressure of $p_a = 10^5$ Pa. The pipe exit is at $x = 0$, and the pipe wall thickness is $0.053D_j$. This jet has been studied in previous works [18–22].

The pipe flow originates at $x = -D_j$, where a Blasius laminar boundary-layer profile with a thickness of $\delta_{BL} = 0.075D_j$ is imposed for the axial velocity. In addition, radial and azimuthal velocities are set to zero, pressure is equal to its ambient value, and the temperature is resolved by a Crocco–Busemann relation. The jet is initially disturbed by adding random low-level vortical structures uncorrelated in the azimuthal direction at $x = -0.475D_j$ inside the pipe [18]. The forcing strength is empirically set to match the nozzle-exit conditions measured in a tripped jet [23]. The resulting mean axial velocity profile is very similar to the laminar profile imposed at the nozzle inlet, whereas the rms velocity profile show a peak axial turbulence intensity of $u'_e/U_j = 0.0914$.

B. Numerical Methods

The numerical framework for the LES used in this work is identical to that used in previous jet simulations [18,20–22,24–28]. The computational code solves the three-dimensional Navier–Stokes equations in cylindrical coordinates (x, r, ϕ) using low-dissipation and low-dispersion explicit schemes. The singularity at the axis is solved by the method of Mohseni and Colonius [29]. The region near the axis also presents a restriction in time step that is alleviated by calculating the azimuthal derivatives at coarser resolutions than those permitted by the grid [30], yielding an azimuthal resolution of $2\pi/16$. Spatial discretization is performed by fourth-order, eleven-point centered finite differences, and time integration is carried out using a second-order six-stage Runge–Kutta algorithm [31]. The flow variables are filtered every time step by a sixth-order 11-point centered explicit filter [32].

The explicit filtering used in the present simulation has the dual purpose of removing grid-to-grid oscillations and of performing as a subgrid-scale high-order dissipation model for the regions where the mesh grid is not fine enough to capture the smallest scales of turbulence. Filters and noncentered finite differences are also used near the pipe walls and grid boundaries. At the boundaries, a sponge zone combining grid stretching and Laplacian filtering is used to avoid acoustic reflections, in addition to the radiation conditions of Tam and Dong [33]. The effectiveness of this numerical framework has been assessed in previous studies in diverse cases such as subsonic jets, Taylor–Green vortices, and turbulent channel flows [18,34–37], where it has been compared against direct numerical simulations; and the properties of the filtering dissipation have been examined.

C. Simulation Parameters

The simulation presented is a highly resolved LES. Its characteristics are reported in detail in Ref. [20], where a grid-sensitivity study of the jet flow and acoustic fields is carried out, as well as in a very recent paper [22] in which the acoustic tones emerging near the nozzle are analyzed. The mesh grid contains $n_x \times n_\phi \times n_r = 2085 \times 1024 \times 512$ points, and its physical extents are $20D_j$ and $7.5D_j$ in the axial and radial dimensions, respectively. The minimum mesh spacings are equal to $\Delta r = 0.0018D_j$ at $r = D_j/2$ and $\Delta x = 0.0036D_j$ at $x = 0$. The maximal mesh spacing in the jet near field is equal to $\Delta r = 0.0375$, leading to a Strouhal number of $St_D = fD_j/U_j = 5.9$ for an acoustic wave discretized by five points per wavelength, where f is the frequency.

The simulation was performed with an Open Multi-Processing parallelized solver using a time step of $\Delta t = 0.7 \times \Delta r_{\min}/c_0$ to ensure numerical stability. The simulation had an initial transient period of $87.5D_j/U_j$. The simulation time after the transient period is equal to $1250D_j/U_j$. During the stationary time, the density, velocity components, and pressure are recorded at a sampling frequency of $St_D = 12.8$ in eight equidistant azimuthal angles. Their Fourier coefficients in the azimuthal directions, estimated over the full section (x, r) , are also saved for the first nine azimuthal modes. The statistics

are averaged in the azimuthal direction, when possible. The size of the database amounts to approximately 6.5 TB. Previous work on this simulation has reported good agreement with experimental results of similar jets [21].

III. Scales in LES Flowfield

A. Space–Time Correlations

Axial length and velocity scales are computed using two-point space–time correlations. On a meridional plane, the normalized space–time correlation of zero-mean variable f with reference point (x, r) is

$$R_{ff}(x, r; \xi, \eta, \tau) = \frac{\overline{f(x, r, t)f(x + \xi, r + \eta, t + \tau)}}{(\overline{f^2(x, r, t)}\overline{f^2(x + \xi, r + \eta, t)})^{1/2}} \quad (1)$$

where the overline denotes time averaging. The variable f represents the fluctuating axial velocity or pressure, yielding R_{uu} or R_{pp} , respectively.

The space–time correlations are computed using a linear stencil centered around a reference point (x, r) . Even though axial correlations are sought, care must be taken that the stencil does not connect dissimilar regions of the jet. This is of particular relevance to the computation of length scales near the edge of the jet, where the transition from hydrodynamic to acoustic fields occurs within a thin layer that originates at the nozzle lip and is inclined at approximately 9 deg with respect to the jet centerline. A purely axial stencil near this transition layer poses the risk of extending from the acoustic to the hydrodynamic regions. Accordingly, the stencil angle, defined by $\theta = \arctan(\eta/\xi)$, follows the scheme depicted in Fig. 2. At points within the lip line of $r = 0.5D_j$, the displacement is purely axial. Outside the lip line, the stencil aligns with the reference location (x, r) and the nozzle lip, up to a maximum angle of 9 deg. The transition layer is not known a priori and is based on the computation of the convective velocity U_c , which uses purely axial stencils. An example of space–time correlation given the reference point is plotted in Fig. 3. For a given axial separation ξ , the correlation peaks at a time

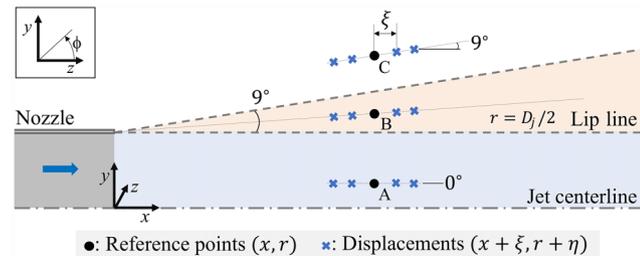


Fig. 2 Examples of stencils used in determination of length scales. Reference points are located (point A) between centerline and lip line, (point B) between lip line and 9 deg edge, and (point C) outside 9 deg edge.

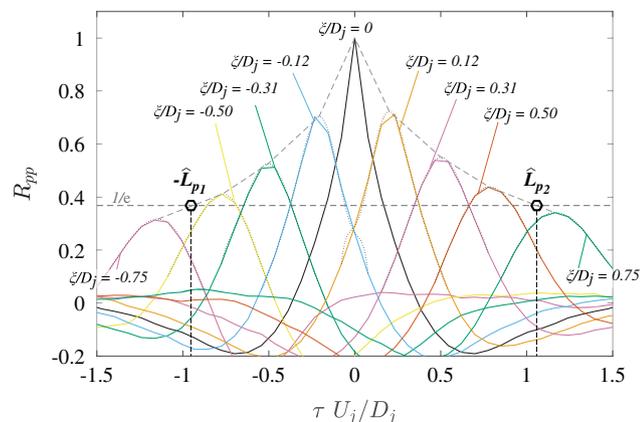


Fig. 3 Space–time correlation R_{pp} in vortical field at $x/D_j = 2.5$ and $r/D_j = 0.5$: fits by seventh-order polynomials (dotted lines), and envelope and coordinates relevant to the calculation of \hat{L}_p (dashed lines).

shift τ_{\max} . To minimize the error due to the discrete form of the correlation, τ_{\max} is determined after fitting a seventh-order polynomial to the correlation curve for each displacement, as shown in the figure.

B. Convective Velocity

In the space–time correlations, each discrete axial separation ξ_i around a reference location (x, r) yields a correlation that peaks at time shift $\tau_{\max,i}$. The associated phase speed is $U_{c_i} = \xi_i/\tau_{\max,i}$. The axial convective velocity at that reference location is calculated as the average of U_{c_i} extracted from the correlations whose peak values exceed 0.4. The convective velocity can be based on velocity or pressure fluctuations, yielding U_{c_u} and U_{c_p} , respectively.

C. Fixed-Frame Length Scales

The fixed-frame length scale is the axial distance L over which events remain correlated at a fixed time. Following Harper-Bourne [11], Morris and Zaman [12], and Dahl [38], the length scale is the displacement ξ at which the correlation R_{ff} decays to $1/e$. Noting that R_{ff} is not an even function of ξ [13], displacements ξ are taken in the negative and positive directions relative to the reference location. This procedure yields two displacements, $\xi = -L_{f_1}$ and $\xi = L_{f_2}$, that satisfy $R_{ff}(x, r; \xi, \eta(\xi), 0) = 1/e$, with η and ξ linked according to procedure outlined in Sec. III.A. The fixed-frame length scale is the average of the two displacements:

$$L_f = \frac{1}{2}(L_{f_1} + L_{f_2}) \quad (2)$$

In practice, the application of Eq. (2) requires interpolation between the discrete data of the simulation. The definition of length scale used here differs from the classical definition as the integral of the correlation curve [39]. However, the two approaches yield similar results because the integral approach often terminates at the first zero crossing of R_{ff} . Comparison of the two approaches, not presented for brevity, shows that the quantitative values differ slightly but the qualitative trends are consistent.

D. Moving-Frame Length Scales

The moving-frame length scale \hat{L} is a measure of the axial distance over which turbulence remains correlated as it convects downstream. It can also be interpreted as the “length of life” of a turbulent eddy, which is the distance traveled until its characteristics are lost through pairing or other mechanisms. Kerhervé et al. [39] noted that this is not a “real” length scale but rather a product of the convective velocity and a time scale. Nevertheless, it is an intrinsic characteristic of the stochastic events of the jet and can be useful for aeroacoustic models that use a Lagrangian frame of reference. The focus now is the envelope of the correlations represented by the dashed line of Fig. 3. Similar to the fixed-frame length scales, displacements $\xi = [-\hat{L}_{f_1}, \hat{L}_{f_2}]$ satisfy $R_{ff}(x, r; \xi, \eta(\xi), \tau_{\max}(\xi)) = 1/e$, as illustrated in the figure. The moving-frame length scale is then defined as

$$\hat{L}_f = \frac{1}{2}(\hat{L}_{f_1} + \hat{L}_{f_2}) \quad (3)$$

The values of ξ corresponding to \hat{L}_{f_1} and \hat{L}_{f_2} are linearly interpolated from the correlation maxima.

E. Azimuthal Scale

The azimuthal scale is defined for zero axial, radial, and time separations. The related two-point correlation is

$$R_{ff}(x, r, \alpha) = \frac{\overline{f(x, r, \phi, t)f(x, r, \phi + \alpha, t)}}{\overline{f^2(x, r, \phi, t)}} \quad (4)$$

where α is the azimuthal separation. This formulation reflects the stationarity in azimuthal angle ϕ expected for an axisymmetric

jet. For a given reference point (x, r, ϕ) , the azimuthal scale Φ_f is defined by

$$R_{ff}(x, r; \alpha = \Phi_f) = 1/e \quad (5)$$

The range of the azimuthal scale is $\Phi_f \in [0, \pi]$, where $\Phi_f = \pi$ indicates that R_{ff} is above the threshold of $1/e$ for the entire azimuthal range. Note that in this work, the azimuthal variations of the flowfield are reconstructed from the first nine Fourier modes.

IV. Relevant Surfaces

A. Surface of Peak Reynolds Stress

In the acoustic analogy model of Ref. [6] for multistream jets, it was surmised that the sound emission is strongly influenced by the dynamics of the outer shear layer. In a time-averaged sense, the action of the eddies in that shear layer is represented by the outermost peak of the Reynolds stress, resulting in the definition of the outer surface of peak Reynolds stress. The magnitude of the principal component of the Reynolds stress, normalized by the mean density, is

$$g = |\langle u'q' \rangle| \quad (6)$$

where u' is the axial velocity fluctuation, and q' is the transverse velocity fluctuation in the direction of the mean velocity gradient. For a single-stream round jet, there is only one peak of g and the mean velocity gradient is in the radial direction. The surface is therefore named SPS. It represents the locus of the most energetic eddies and can be readily detected in RANS simulations by modeling g as explained in Sec. VIII.

B. Radiator Surface

The radiator surface is a surface close to the edge of the jet; outside of which, the propagation of pressure perturbation is governed by the homogeneous linear wave equation. It is on this surface that the noise sources could be modeled in the form of linear partial fields [10]. This model would be informed by turbulence statistics of the vortical field computed by RANS. Given the impact of the convective velocity on the turbulence radiation efficiency, it is desirable that the partial fields on the radiator surface have the same convective-velocity distribution as that of the underlying eddies that dominate noise emission. The specific definition for the radiator surface used in the present work is the surface near the edge of the jet where the p' -based convective velocity matches that on the SPS at the same axial location. This definition is similar to that used in previous works on multistream jets [16,17] and has been shown to enclose well the vortical field of the jet.

V. Overview of the Flowfield

This section presents key features of the LES flowfield that are important for scaling and interpreting the correlations that follow in Secs. VI and VII.

A. Mean Velocity

Isocontours of the normalized mean axial velocity, \bar{u}/U_j , averaged in the azimuthal direction are plotted in Fig. 4a. The axial distribution of the centerline mean velocity, \bar{u}_{CL}/U_j , is plotted in Fig. 4b and

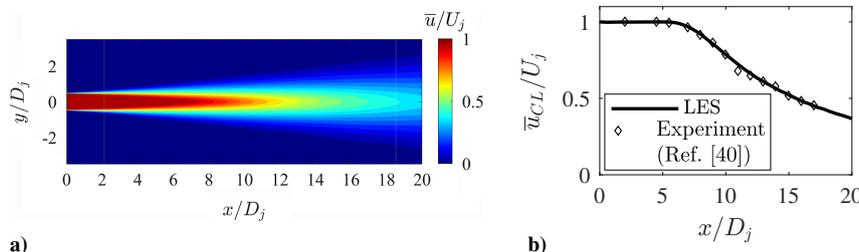


Fig. 4 Distribution of normalized mean axial velocity: a) isocontours on a symmetry plane; and b) axial distribution on jet centerline compared to experiments by Lau et al. [40].

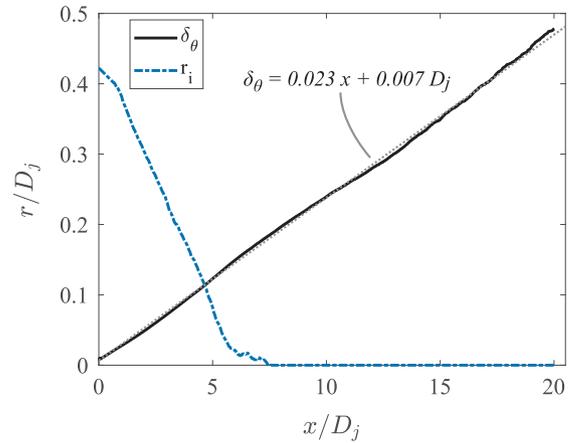


Fig. 5 Axial distributions of momentum thickness δ_θ/D_j (solid black line) and inner edge of shear layer r_i/D_j (dashed blue line). Dotted line indicates linear fit.

shows good agreement with the experimental data of Lau et al. [40]. The end of the potential core, defined here as the point where the centerline mean axial velocity decays to $\bar{u}_{CL} = 0.95U_j$, is located near $x = 7.4D_j$.

The momentum thickness of the jet is relevant to scaling relations that follow in Secs. VI and VII. To maintain the same definition within and downstream of the potential core, a generalization of the shear-layer momentum thickness is used:

$$\delta_\theta = \int_0^\infty \frac{\bar{\rho}(r)\bar{u}(r)}{\bar{\rho}_{CL}\bar{u}_{CL}} \left(1 - \frac{\bar{u}(r)}{\bar{u}_{CL}}\right) dr \quad (7)$$

where the centerline mean conditions are used for normalization. The growth of $\delta_\theta(x)$, plotted in Fig. 5, is well approximated by the linear fit $\delta_\theta/D_j = 0.023x/D_j + 0.007$.

The radial location of the inner edge of the shear layer $r_i(x)$ is also relevant to scaling relations that will be developed. Within the extent of the potential core, the inner edge is defined as the interior locus where the magnitude of the mean vorticity equals $0.05U_j/D_j$; that is,

$$|\bar{\omega}(x, r = r_i)| = 0.05 \frac{U_j}{D_j}, \quad r < D_j/2 \quad (8)$$

Past the potential core, where the annular shear layer has collapsed on itself, we set $r_i = 0$. The axial evolution of r_i is plotted in Fig. 5.

B. Reynolds Stress

Isocontours of the normalized Reynolds-stress magnitude g/U_j^2 are plotted in Fig. 6. The location of the SPS is included in the figure. The distribution of g/U_j^2 creates two distinct lobes that reach a minimum separation around $x/D_j = 9$. Per its definition, the SPS is the locus of the radial maximum of the Reynolds stress. This places the surface very close to the nozzle lip line ($r/D_j = 0.5$) until $x/D_j = 11$. Downstream of this station, the SPS expands at an angle of approximately 4 deg with respect to the jet axis. The maximum

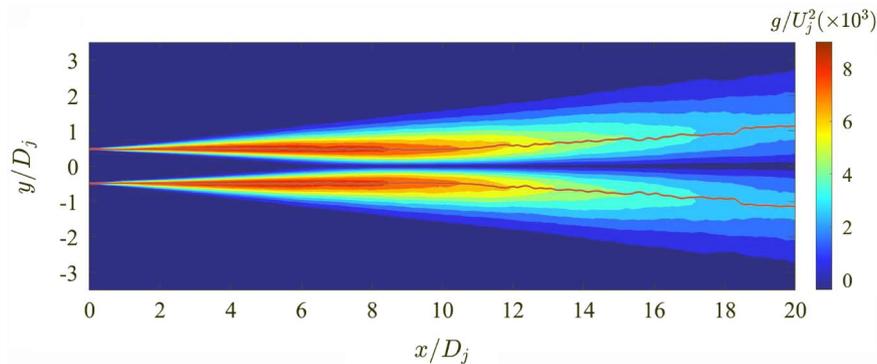


Fig. 6 Isocontours of normalized Reynolds-stress magnitude g/U_j^2 . Red line denotes SPS.

value of the Reynolds stress is $g/U_j^2 = 0.0089$ and occurs near $x/D_j = 5$.

C. Convective Velocity

The $x - r$ distribution of the convective velocity U_{c_p} calculated from space-time correlations of the pressure fluctuation is plotted in Fig. 7. Two distinct regions are noted: an inner region encompassing the jet flow, where U_{c_p} generally follows the trend of the mean axial velocity; and an outer region near the edge of the jet and beyond, where U_{c_p} rapidly increases to supersonic values indicative of the acoustic field. The radiator surface, depicted by the white line, is located at this transition region between hydrodynamic and acoustic fields. Its growth is nearly linear at an angle of 9 deg relative to the jet centerline. The SPS is included in the plot for completeness. The trends can be further quantified by plotting the radial distributions of U_{c_p} and \bar{u} at $x/D_j = 5$ in Fig. 8. The potential core is manifested by the flat regions of the profiles near $r = 0$. Starting from the jet centerline, U_{c_p} initially follows the trend of the mean velocity, reaches a minimum, and then starts rising within the rotational region of the jet. This reversal has also been seen in earlier LES of a single-stream jet [41]. The radiator surface is located slightly outward of the reversal, where U_{c_p} matches that on the SPS. With further increase of the radius, U_{c_p} increases rapidly and attains large values in the acoustic field. The very large, supersonic values of U_{c_p} at large

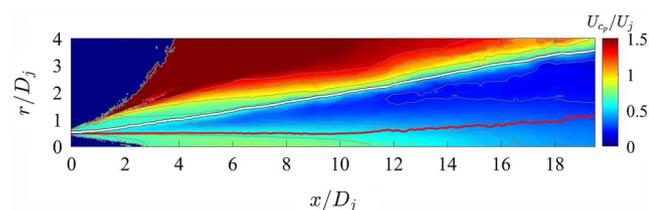


Fig. 7 Isocontours of normalized convective velocity U_{c_p}/U_j on a meridional plane: SPS (red line), and radiator surface (white line).

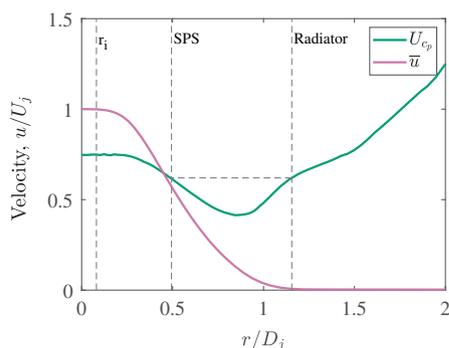


Fig. 8 Radial distributions of U_{c_p} and \bar{u} at $x/D_j = 5$. By definition, U_{c_p} on the radiator surface equals that on the SPS.

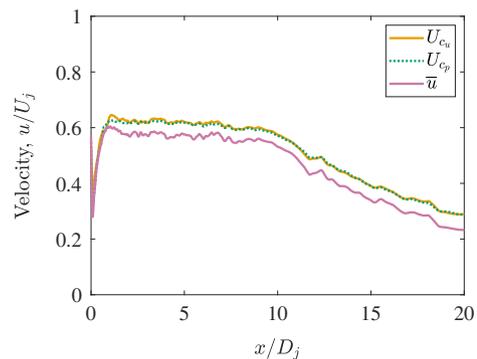


Fig. 9 Axial distributions of U_{c_p} , U_{c_u} , and \bar{u} on the SPS.

radius represent the trace along x of acoustic waves propagating at large angles with respect to the downstream direction.

It is instructive to compare the axial distribution of U_{c_p} and the mean axial velocity \bar{u} on the SPS. As seen in Fig. 9, the two distributions are very close, with \bar{u} being about 8% lower for $x/D_j < 10$. This indicates that the convective velocity can be approximated well by the axial mean velocity on the SPS [6]. Within the potential core, the convective velocity is near $U_{c_p} = 0.6U_j$. For completeness, the figure includes the convective velocity U_{c_u} , which is based on space-time correlations of u' . The distributions of U_{c_u} and U_{c_p} are practically identical. The three curves of Fig. 9 show that, in the region of the most energetic eddies represented by the SPS, the axial transport of velocity and pressure fluctuations involves the same mechanism of convection by the local mean velocity. This is not true in other regions of the jet where significant differences can be found between U_{c_u} and U_{c_p} .

VI. Longitudinal Correlations

A. Two-Dimensional Correlations

Correlations on a meridional plane can give insight on the structure of turbulence and the resulting acoustic field [21]. They also help in understanding the length scale distributions presented in the following sections: Secs. VI.B and VI.C. Space-time correlations R_{uu} and R_{pp} on a meridional plane are plotted in Fig. 10 for three time separations. The reference point is on the SPS at $x/D_j = 3.0$. The normalized time separations are $\tau U_j/D_j = -1.94$ (top row), 0 (middle row), and 1.94 (bottom row). A vertical dashed line indicates the position that corresponds to a downstream convective velocity of $U_c = 0.6U_j$. R_{uu} shows a compact area of correlation, mostly limited to a small positive (red) region that travels along the SPS and a negative (blue) oblong region along the radiator surface. On the other hand, R_{pp} has a wide structure of radially oriented lobes extending from the centerline well into the acoustic field; the main structure comprises a positive lobe (red) flanked by strong negative lobes (blue), resembling a hydrodynamic pressure wave. The structures of R_{uu} and R_{pp} travel at similar convective velocities,

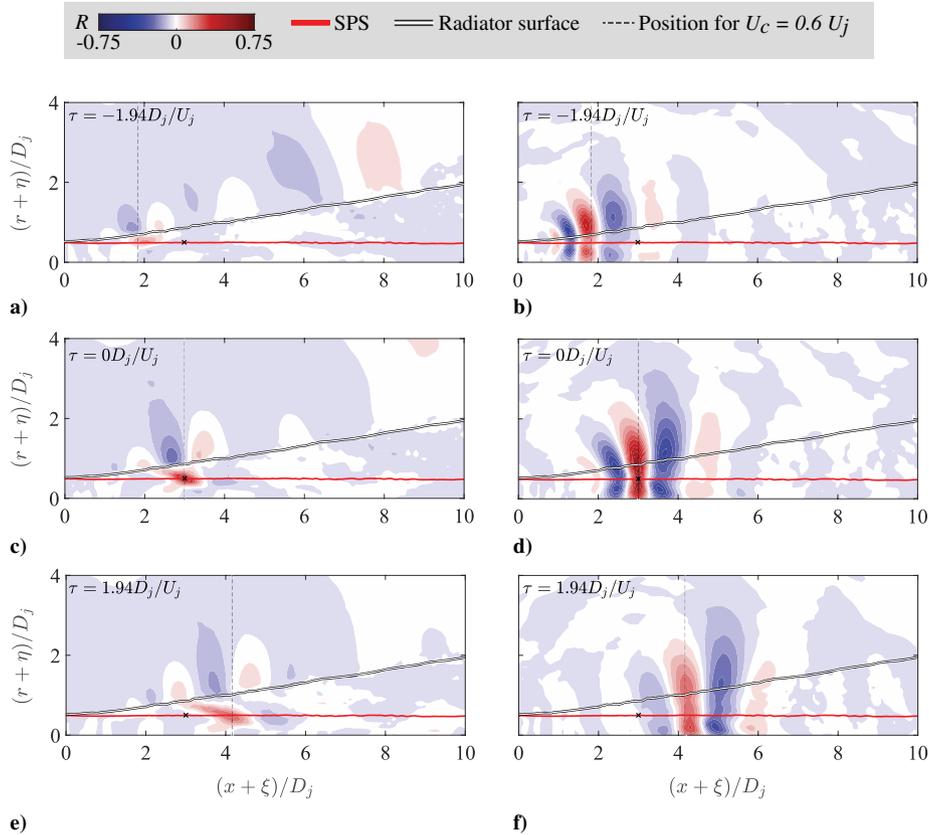


Fig. 10 Contours of R_{uu} (left column) and R_{pp} (right column) with reference point on SPS at $(x, r) = (3, 0.5)D_j$ for three time separations.

but the former is slightly slower and the latter slightly faster than $0.6U_j$.

We now examine the evolution of R_{uu} and R_{pp} with the reference point on the SPS at $x = 7.5D_j$, near the end of the potential core. Figure 11 displays the resulting correlations at time separations of $\tau U_j / D_j = -3.49$ (top row), 0 (middle row), and 3.49 (bottom row). The time progression of R_{uu} shows a compact structure of positive correlation traveling along the SPS. The effects of that structure are felt above the radiator surface as a region of negative correlation. At $\tau = 0$, the peak of the negative correlation is -0.24. The limited size of the positive structure indicates that the u' correlations capture the

local but not global effects of the convecting eddies. The axial velocity fluctuation thus becomes decorrelated as we move from the SPS to the outer edge of the jet. This explains the region of low correlation between positive and negative structures in Figs. 11a, 11c, and 11d.

The time progression of R_{pp} is seen in Figs. 11b, 11d, and 11f. A wave-packet-like structure is evident for all the time separations. At zero time separation, in Fig. 11d, a region of strong positive correlation emanates as a radial beam from the reference point to all across the jet: from the centerline to well outside the radiator surface. It is evident how the footprint of the large-scale structures is imprinted on

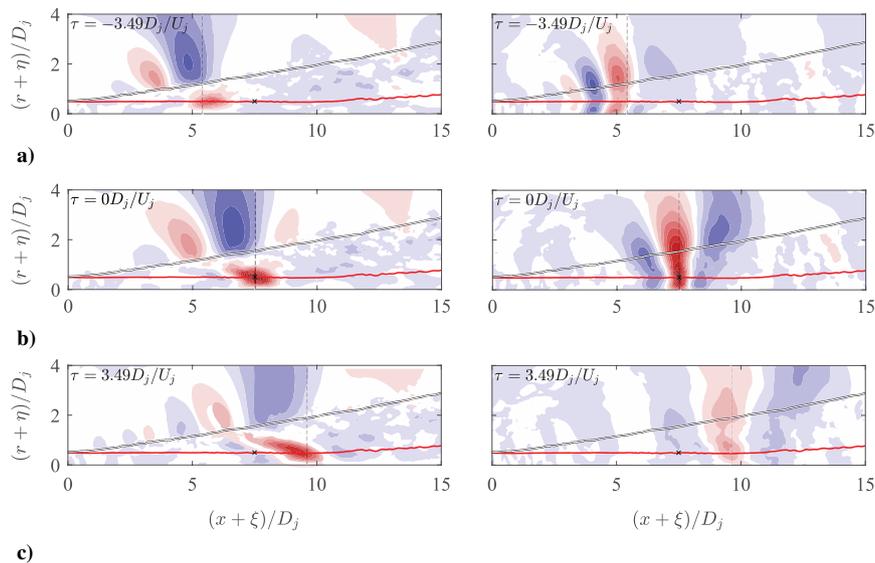


Fig. 11 Contours of R_{uu} (left column) and R_{pp} (right column) with reference point on SPS at $(x, r) = (7.5, 0.5)D_j$ for three time separations. Legend is the same as for Fig. 10.

the radiator surface. For negative and positive time separations, in Figs. 11b and 11f, similar but weaker structures are seen, with a rapid decorrelation for $\tau > 0$. The latter may be explained by the breakdown of the shear-layer structures after the end of the potential core [42]. It is notable that the R_{pp} correlation structures convect fairly uniformly with a speed near $0.6U_j$, whereas the R_{uu} structures are stretched and their convective speed depends on radial location.

The differences in location, shape, and convective velocity of the features of R_{uu} and R_{pp} in Figs. 10 and 11 suggest that the two correlations portray events of different nature. The u' -based correlations reflect compact, localized turbulence events that travel along the SPS. The p' -based correlations appear to reflect interactions of eddies with the potential flow around them, namely, the ambient air and the potential core, which result in large regions of influence. Consequently, pressure-based scales may not capture local events as accurately as velocity-based scales.

B. Fixed-Frame Length Scales

The distributions of fixed-frame length scales defined in Sec. III are now presented. Isocontours of normalized length scales based on axial velocity fluctuation, L_u/D_j , and on pressure fluctuation, L_p/D_j , are plotted in Fig. 12. The radiator surface marks the end of the vortical region, inside of which the scales L generally follow the axial growth of the momentum thickness. At a given axial location and going outward from the inner edge of the shear layer r_i , L_u increases, reaches a local maximum within the jet, and then decreases to a local minimum at the edge of the jet. Outside the jet, L_u rises to high values. The local radial minimum of L_u closely follows the radiator surface. This suggests a loss of coherence of velocity-based events near the edge of the jet. Near the SPS, L_u experiences the strongest radial variation within the jet flow. In Fig. 12b, L_p does not have a local maximum inside the jet in the radial direction. Instead, it is strictly increasing until reaching a maximum around one nozzle diameter outward from the radiator surface. Its maximum radial gradient is found near the radiator surface.

The radial trends of Fig. 12 are interpreted as follows: The inner vortical region of the jet, marked by the SPS, contains the main events of mixing and momentum transport of the mixing layer; see Eq. (6). That region encompasses the largest scales of turbulence and, through energy cascade, small scales as well. The combination of large and small scales drives the value of the length scale L below the actual length of the large scales [12]. As one leaves the SPS toward higher radial locations with lower vorticity, the effects of small eddies are lost faster than those of large eddies, resulting in the increase of scales versus radius seen in Fig. 12. It appears that the pressure-based scales grow radially due to this effect until peaking beyond the radiator surface, into the linear field. Thus, that region of radial maxima is due to the footprint of the large-scale vortical structures inside the jet [43,44]. The velocity-based scales have a different

behavior, whereby they capture the extent of the eddies themselves but not their footprint away from them. As one approaches the outer edge of the jet, the velocity fluctuations disassociate from the core vortical region and become influenced by a combination of acoustic effects and small eddies of secondary acoustic relevance. This complex combination of influences causes the “valley” of low length scale L_u at the edge of the jet, seen in Fig. 12a. This observation is supported by the recent work of Camussi and Bogey [45] on this same jet, which found that the region of the vortical field near the edge of the jet shows high values of intermittency, and thus incoherent behavior. Further away from the jet edge, the acoustic perturbations start to dominate and cause the increase in length scale in the near-acoustic region.

To further quantify the connection between events on the SPS and radiator surface, the axial distributions of length scales L_u and L_p on these surfaces are plotted in Fig. 13. All the curves exhibit a substantially linear growth along the first 15 jet diameters. In addition, the length scales on the SPS acquire a higher value when calculated from velocity fluctuation than from pressure fluctuation. This result challenges the intuitive prediction that velocity scales would be lower due to a richer content of high frequencies, and therefore smaller scales [43]. However, this difference is likely to result from the specific definition of length scale used, based on Eq. (3), where only positive values of the correlation function are considered. Indeed, the correlations in Figs. 10 and 11 at $\tau = 0$ show strong negative lobes of R_{pp} upstream and downstream from the reference point. Therefore, if negative values of correlations were to be considered when defining the axial span of turbulence events, the pressure-based length scale would be longer than the velocity-based scale. Comparing scales based on the same variable, it is evident that those on the radiator surface are consistently larger than those on the SPS, with the pressure-based scale exhibiting the strongest increase in this regard. These results suggest that the signature of the eddies on the radiator surface is more coherent than the effect of the eddies in the vortical field. They are in line with the findings of Arndt et al. [46] that the pressure signal measured at the outer edge of a jet mixing layer is dominated by the large-scale structure of the turbulent flow and is correlated over longer distances than the underlying velocity fluctuations.

To examine the radial evolution of the correlation scales, they are plotted in the “similarity” form of $L/\delta_\theta(x)$ versus $(r - r_i(x))/\delta_\theta(x)$. This form represents the expectation that the correlation scale grows with the momentum thickness δ_θ . The radial variations of the correlation scales at eight axial stations, from $x/D_j = 1$ to 15, are shown in Fig. 14a for L_u and Fig. 14b for L_p . The locations of the centerline, SPS, and radiator surface are marked. The similarity form leads to a fair collapse of the curves, particularly for axial locations $x/D_j \geq 3$ for L_u and $x/D_j \geq 5$ for L_p . The length scales at $x/D_j = 1$ and $x/D_j = 3$ acquire high values near the centerline, possibly indicating the influence of coherent waves within the potential core. Overall, at all axial stations, common trends are found between the SPS (marked

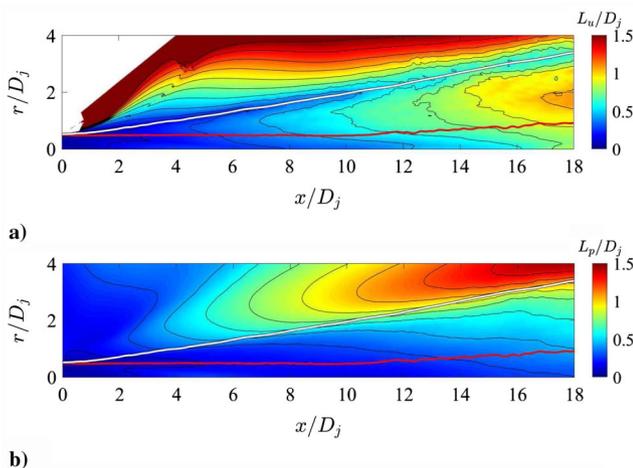


Fig. 12 Isocontours of normalized length scales a) L_u/D_j and b) L_p/D_j ; SPS (red line), and radiator surface (white line).

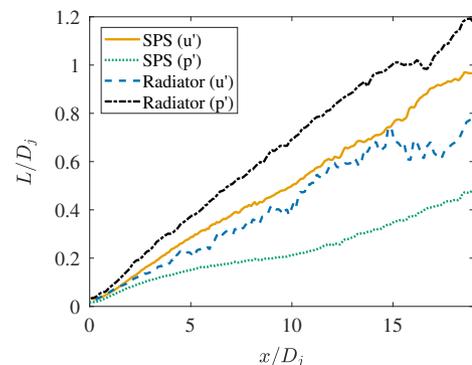


Fig. 13 Axial distribution of correlation scales L on relevant surfaces: L_u on SPS (orange solid line), L_u on radiator surface (blue dashed line), L_p on SPS (green dotted line), and L_p on radiator surface (black dashed-dotted line).

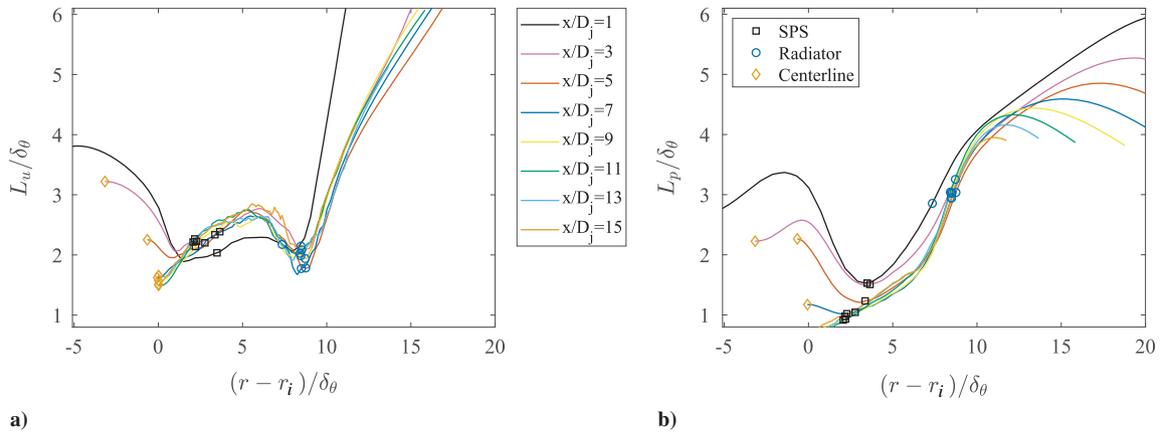


Fig. 14 Plots of normalized length scales L/δ_θ versus normalized radius $(r - r_i)/\delta_\theta$ based on a) axial velocity and b) pressure. Legend applies to both subfigures.

in black squares) and the radiator surface (blue circles) even though the trends differ between L_u and L_p . At high radii of $(r - r_i)/\delta_\theta \geq 12$, the curves diverge as a result of the acoustic propagation in a variety of directions, depending on the axial position considered. The results in Fig. 14 are in agreement with the those by Fleury et al. [14] and Proença et al. [13], in which fixed-frame u' -based correlation scales along the centerline and lip line grow according to the shear-layer width. This consistent scale growth is generalized here for all relevant axial and radial locations, and for both velocity- and pressure-based scales.

C. Moving-Frame Length Scales

The distributions of the normalized moving-frame length scales based on axial velocity fluctuation \hat{L}_u/D_j and pressure fluctuation \hat{L}_p/D_j are presented in Fig. 15. The trends are somewhat similar to those seen for the fixed-frame scales in Fig. 12. With regard to the velocity-based scales, a notable difference is that the radial maxima of \hat{L}_u inside the vortical field in Fig. 15a are more localized and occur inward, closer to the SPS, relative to those of L_u in Fig. 12a. This difference can be explained by examining the convection of the structures in Figs. 10 and 11, where the positive structure of R_{uu} rotates counterclockwise as a result of the convective velocity decreasing with radius within the vortical region. High convective velocity near the jet axis elongates the lifespan of the turbulence, which is reflected by the radial maxima of \hat{L}_u near the SPS. As with the fixed-frame scale distribution of Fig. 12a, \hat{L}_u reaches minimum value near the location of the radiator surface. The pressure-based

scale \hat{L}_p follows the same general trends as \hat{L}_u but with lower values within the vortical region and a weak maximum slightly outside the SPS. Immediately outside the radiator surface, \hat{L}_p rises rapidly to high values.

The axial distributions of the moving-frame length scales on the SPS and radiator surface are plotted in Fig. 16. Contrary to the fixed-frame length scales, where all curves exhibit a linear growth, here, only those based on axial velocity fluctuation increase linearly on the SPS. The pressure-based length scale \hat{L}_p rises rapidly within the first few jet diameters, reaches an apparent plateau, and then resumes growth at a lower rate. On the radiator surface, it increases linearly at a fast rate up to $x/D_j \approx 2.5$, reaches a plateau for $2.5 \leq x/D_j \leq 7.5$, and then grows at a rate similar to that of \hat{L}_u . The difference in the trends of \hat{L}_u and \hat{L}_p is in line with the observations made in Sec. VI.A, where it is discussed that \hat{L}_u follows the growth of the vortical region, whereas \hat{L}_p appears affected by large-scale events that result from interactions of eddies with the potential flow surrounding them. Thus, \hat{L}_p grows faster than the shear layer in the initial few diameters of the jet. The nature of this phenomenon remains under investigation.

The radial variations of \hat{L}_u and \hat{L}_p are plotted in Fig. 17 using a normalization based on δ_θ as in Fig. 14. The curves at locations past the end of the potential core tend to collapse well. The curves at early axial locations are also very close to each other in the case of \hat{L}_u in Fig. 17a, which shows an approximately self-similar behavior, in agreement with the results by Proença et al. [13]. In contrast to \hat{L}_u , the scales \hat{L}_p in Fig. 17b grow rapidly near the nozzle exit and do not exhibit self-similar behavior.

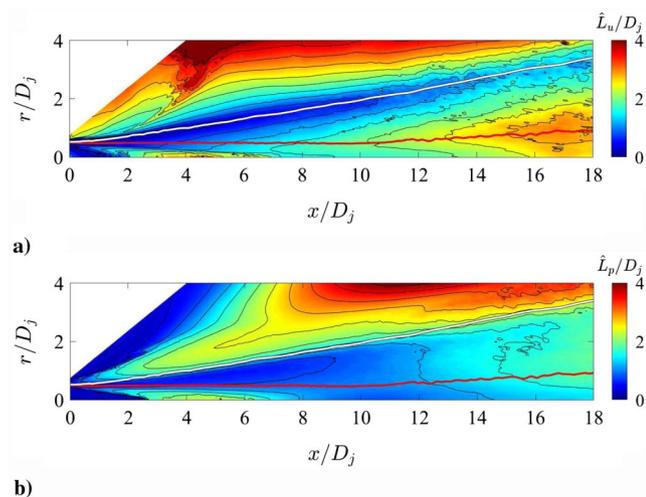


Fig. 15 Isocontours of normalized length scales a) \hat{L}_u/D_j and b) \hat{L}_p/D_j ; SPS (red line), and radiator surface (white line).

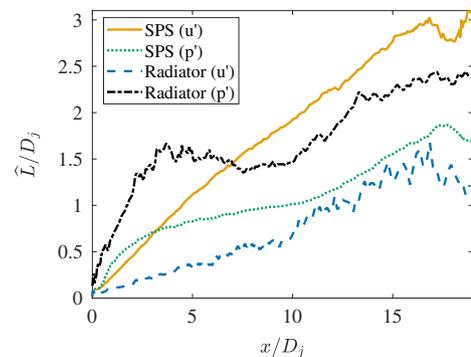


Fig. 16 Axial distribution of length scales \hat{L} on relevant surfaces: \hat{L}_u on SPS (orange solid line), L_u on radiator surface (blue dashed line), \hat{L}_p on SPS (green dotted line), and \hat{L}_p on radiator surface (black dashed-dotted line).

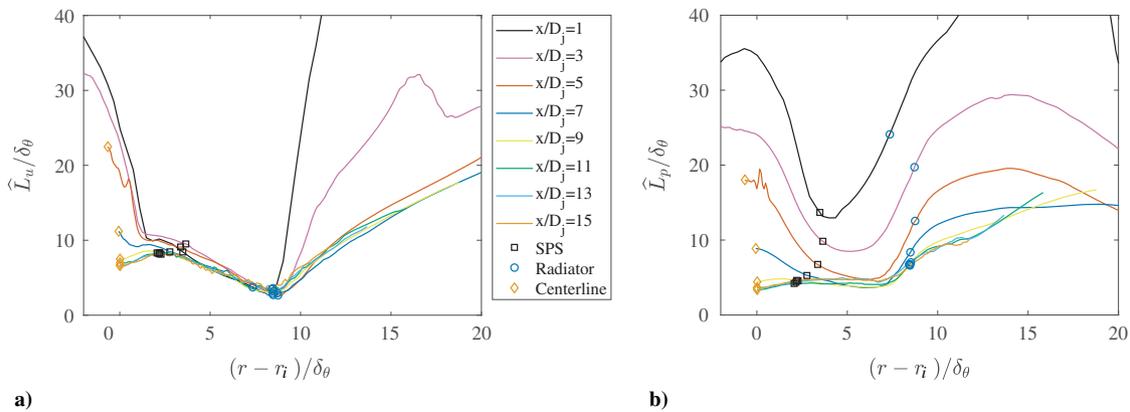


Fig. 17 Plots of normalized moving-frame length scales \hat{L}/δ_θ versus similarity coordinate $(r - r_i)/\delta_\theta$ based on a) axial velocity, and b) pressure. Legend applies to both subfigures.

VII. Azimuthal Correlations

A. Modal Components

The azimuthal composition of the flow is studied by examining the radial variations of the energy of its Fourier modes m . The energy of the first five Fourier modes that compose the u' and p' fluctuations is shown in Fig. 18. These distributions do not significantly depend on their axial location, and so the results are only shown for $x/D_j = 7.5$. The modal energy distribution based on velocity fluctuation in Fig. 18a shows that the vortical region is dominated by modes $m = 2, 3, 4$, whose contributions peak near the location of the SPS. Lower modes such as $m = 0, 1$ contain lower energy inside the vortical region but dominate the linear field past the radiator surface. On the other hand, the modal energies based on pressure fluctuation in Fig. 18b have fairly constant levels between the SPS and the radiator surface, with mode $m = 1$ accounting for 33% of the total modal power. Going from the vortical to the linear field, the contributions of modes 0 and 1 rise and peak at distances $1.0D_j$ and $0.3D_j$ from the radiator surface, respectively. In that region, the two modes represent about 80% of the total power. Continuing outward, the energy of those modes decreases and all modes reach levels between 8 and 28%. This radial variation is consistent with the radiation pattern observed in turbulent jets, where low modes dominate at shallow angles from the jet axis, and all modes have similar energy levels radiating perpendicularly to the jet axis [47].

B. Cross-Sectional Correlations

Two-dimensional correlations performed on a cross section of the jet at $x/D_j = 7.5$ are represented in Fig. 19. The reference point is located on the SPS and marked by a white cross mark. The displacements in the z and y directions are denoted ζ and ψ , respectively.

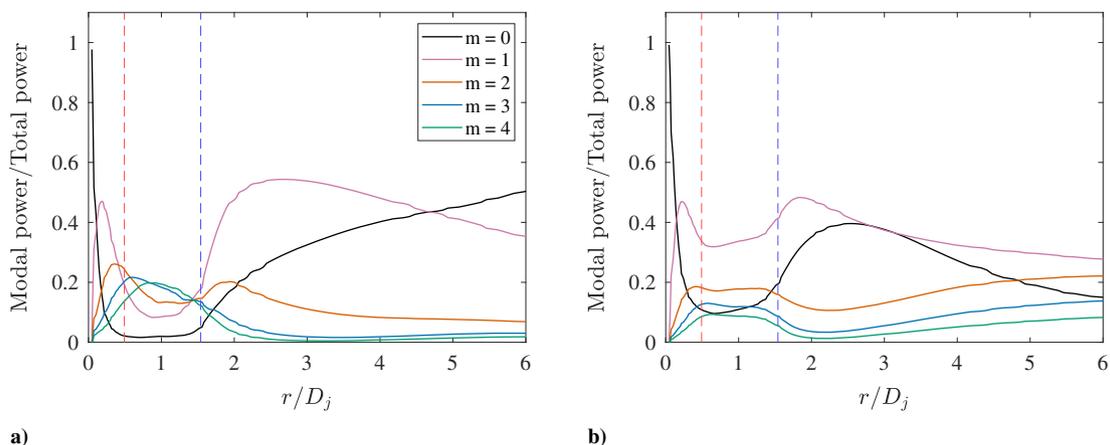


Fig. 18 Radial distribution of modal powers on $x = 7.5D_j$ for first five modes based on a) axial velocity and b) pressure. Vertical lines mark locations of SPS (red) and radiator surface (blue).

Similar to the longitudinal correlations presented in Sec. VI.A, there are notable differences between the distributions of R_{uu} and R_{pp} . The velocity correlations in Fig. 19a show a discrete, oblong shape of limited radial and azimuthal extent flanked by negative sidelobes. The pressure correlations in Fig. 19b exhibit a wider azimuthal extent that remains roughly constant within the vortical field and increases rapidly in the linear field.

C. Azimuthal Scales

Isocontours of azimuthal scales based on axial velocity and pressure fluctuations are plotted in Fig. 20. By definition, there is full azimuthal coherence in the vicinity of the centerline as $r \rightarrow 0$. For both distributions, the azimuthal scale reaches a minimum of around 15 deg for Φ_u and 25 deg for Φ_p inside the vortical layer. With increasing radius, the azimuthal scale rises and becomes large in the acoustic near field. This rise starts earlier for Φ_p . The distributions in Fig. 20 reflect a similar behavior to the fixed-frame length scales L_u and L_p , plotted in Fig. 12, divided by the radius r .

The axial distributions of Φ_u and Φ_p on the SPS and radiator surface are plotted in Fig. 21. All curves display an initial growth followed by saturation. The saturation is consistent with the circumferential length scale and the radius of the surface both growing linearly. The pressure-based azimuthal scales Φ_p display a faster initial growth than the velocity-based scales Φ_u . On the radiator surface, Φ_p show the strongest correlation, which is consistent with the cross-sectional results in Fig. 19b. They grow rapidly within the first three jet diameters, and then they saturate at approximately $\pi/3$. Radial profiles of Φ_u and Φ_p are plotted in Fig. 22 as a function of $(r - r_i)/\delta_\theta$ and for several axial stations. Except for the region very near the nozzle exit ($x/D_j \leq 1$), the profiles for each scale collapse fairly well within the radiator surface.

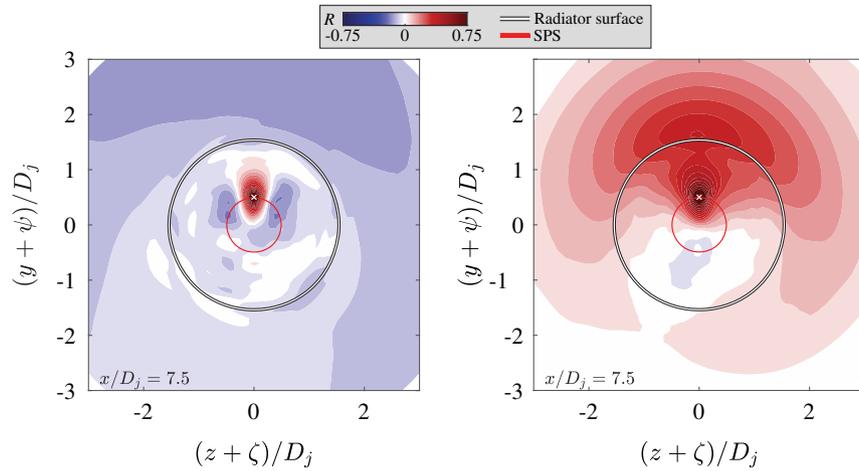


Fig. 19 Contours of a) R_{uu} and b) R_{pp} on the cross-sectional plane $x = 7.5D_j$ with reference point on SPS at $\phi = \pi/2$ (white cross mark). Time shift is zero.

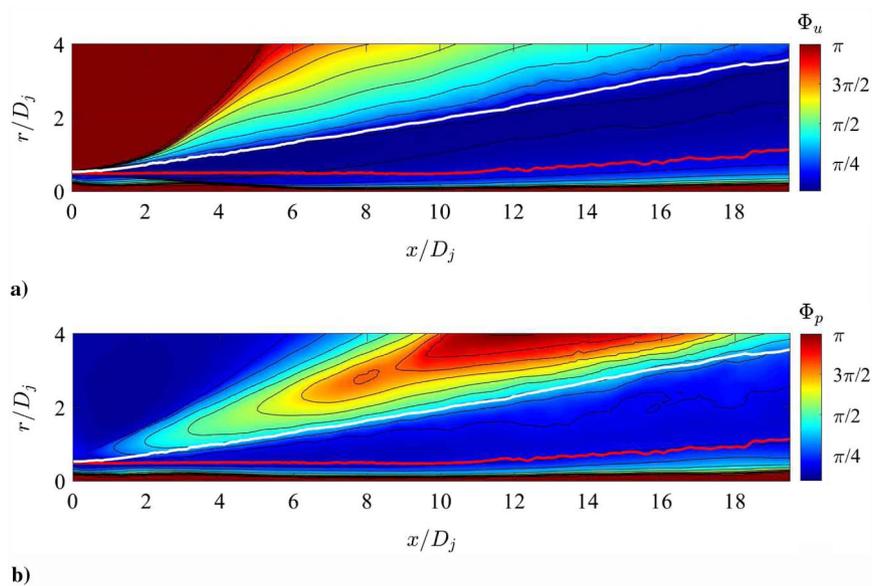


Fig. 20 Isocontours of normalized azimuthal scales a) Φ_u and b) Φ_p , with units of radians: SPS (red line), and radiator surface (white line).

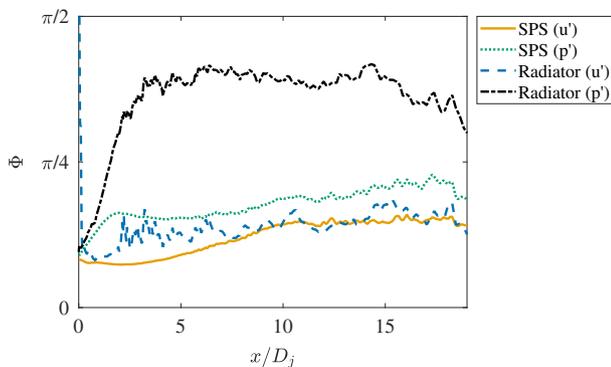


Fig. 21 Axial distribution of azimuthal scales Φ on relevant surfaces: Φ_u on SPS (orange solid line), Φ_u on radiator surface (blue dashed line), Φ_p on SPS (green dotted line), and Φ_p on radiator surface (black dashed-dotted line).

VIII. Modeling Based on the Time-Averaged Flow

The previous sections used the LES flowfield to gain insight into the correlations between the vortical and the near-acoustic fields. The radiator surface is defined to contain the imprint of turbulent eddies

that is crucial for modeling the jet noise source. However, the LES is costly and cannot yet be considered as a design tool. Can the properties on the radiator surface be informed by the time-averaged solution? This question is addressed here at a preliminary level.

A. RANS-Based Statistics

RANS-based statistics are relevant to the low-cost modeling of the noise source proposed in Ref. [6]. Here, we do not use results derived directly from a RANS simulation of the jet but instead use the statistics of the LES fields to emulate a RANS simulation. Although this approach was taken because of the lack of a readily available, equivalent RANS simulation, an advantage of this emulation can be recognized: it allows us to avoid the discrepancies typically obtained by RANS and LES solutions due to the turbulence and subgrid models used. Here, the relations between turbulence structures and their impact on the time-averaged flow are thus evaluated without differences in the mean flowfields. To distinguish our approach from a direct RANS solution, we will refer to it as “emulated RANS” and hereafter refer to it as ERANS.

A comparison of jet flowfields calculated from LES and direct RANS shows similar distributions for the mean velocity, turbulent kinetic energy, and Reynolds stress [16,41]. Importantly, the peak values of the turbulent kinetic energy and Reynolds stress are matched well. A consistent trend is that RANS overpredicts the

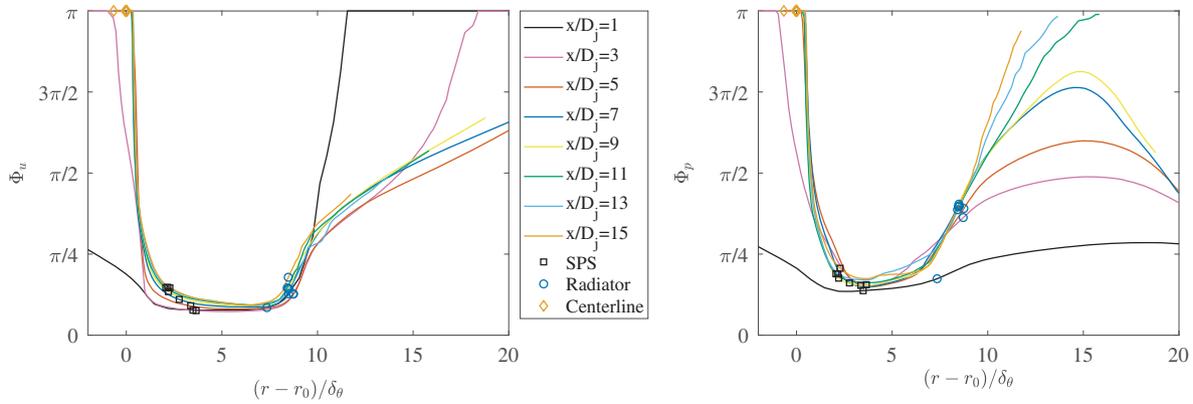


Fig. 22 Azimuthal scales Φ versus similarity coordinate $(r - r_i)/\delta_\theta$ based on a) axial velocity and b) pressure. Legend applies to both subfigures.

length of the potential core relative to the LES by about 10–20%. The same observation applies when comparing RANS to experiments [48] and is a well-known shortcoming of RANS closure models for free shear flows [49]. Because the errors are systematic, they can be accounted for in jet noise prediction methodologies [12,48]. With regard to the ERANS results to be presented here, it is expected that they will capture more faithfully the real mean flow than a direct RANS simulation would. However, a direct RANS solution could be scaled, if necessary, to filter out the effects of the overpredicted potential core length.

The ERANS flowfield comprises the mean velocity vector, mean density, turbulent kinetic energy k , and viscous dissipation ϵ . The principal component of the Reynolds stress, which is the analog to Eq. (6), is modeled as

$$g = \nu_T \left| \frac{\partial \bar{u}}{\partial r} \right| \quad (9)$$

where the turbulent viscosity ν_T is given by the dimensional construct

$$\nu_T = C_\mu \frac{k^2}{\epsilon} \quad (10)$$

The coefficient C_μ was set here at the value of 0.067, as will be discussed in Sec. VIII.C.

Relying again on a dimensional construct, the ERANS-based length scale is

$$L = C_L \frac{k^{3/2}}{\epsilon} \quad (11)$$

A related azimuthal scale is based on the preceding length scale divided by the radial coordinate r of the relevant surface:

$$\Phi = C_\Phi \frac{k^{3/2}}{\epsilon r} \quad (12)$$

In the aforementioned relations, C_L and C_Φ are coefficients determined empirically for the LES; k and ϵ are evaluated on the SPS.

LES allows for the direct computation of the turbulent kinetic energy. It does not directly give the viscous dissipation because the energy dissipation combines viscous and filtering effects [35]. The actual viscous dissipation ϵ must be computed from the budget of turbulent kinetic energy. Following Darisse et al. [50], the dissipation in a turbulent axisymmetric jet is computed as

$$\begin{aligned} \epsilon = & - \left[\bar{u} \frac{\partial k}{\partial x} + \bar{v} \frac{\partial k}{\partial r} \right] - \left[\bar{u}^2 \frac{\partial \bar{u}}{\partial x} + \bar{v}^2 \frac{\partial \bar{v}}{\partial r} + \bar{w}^2 \frac{\bar{v}}{r} + \bar{u}' \bar{v}' \left(\frac{\partial \bar{v}}{\partial x} + \frac{\partial \bar{u}}{\partial r} \right) \right] \\ & - \left[\frac{\partial k \bar{u}'}{\partial x} + \frac{1}{r} \frac{\partial (r k \bar{v}')}{\partial r} \right] - \frac{1}{\rho} \left[\frac{\partial p' \bar{u}'}{\partial x} + \frac{1}{r} \frac{\partial (r p' \bar{v}')}{\partial r} \right] \end{aligned} \quad (13)$$

where $\kappa = (u'^2 + v'^2 + w'^2)/2$ is the instantaneous turbulent kinetic energy. We identify the first term (in brackets) as the advection, the second one as the production, the third one as the turbulent transport, and the fourth one as the transport by pressure fluctuations.

B. Radiator Surface

As the radiator surface marks the transition between the vortical and the linear fields, it is natural to seek a criterion for its location based on the mean vorticity $\bar{\omega}$. The following criterion

$$|\bar{\omega}| \frac{D_j}{U_j} = C_\omega$$

is proposed, with C_ω chosen to get a match with the radiator surface defined in Sec. IV and displayed in Fig. 4. An excellent agreement is found for $C_\omega = 0.03$ in Fig. 23. Note that the location of the ERANS-based radiator surface is fairly insensitive to C_ω as long as C_ω is small. Varying C_ω from 0.01 to 0.10, for instance, results in a change in the angle of radiator surface of ± 1 deg around the actual angle of 9 deg.

C. Convective Velocity

Recall that, by definition, the convective velocity on the radiator surface equals that on the SPS. The distribution of the ERANS-based Reynolds-stress field defined by Eq. (9) is practically identical to that given by LES using Eq. (6), which is displayed in Fig. 6, with $C_\mu = 0.067$ in Eq. (10). This value is somewhat lower than the commonly used $C_\mu = 0.09$, but the value of C_μ does not affect the conclusions that follow. Given the strong similarity of the ERANS- and LES-based Reynolds stress fields, the ERANS-based SPS is very close to the LES-based SPS. Following the concept introduced in Ref. [6], the ERANS-based convective velocity is modeled as the mean axial velocity on the SPS; that is,

$$U_{c,\text{RANS}} = \bar{u}(x, r = r_{\text{SPS}}) \quad (14)$$

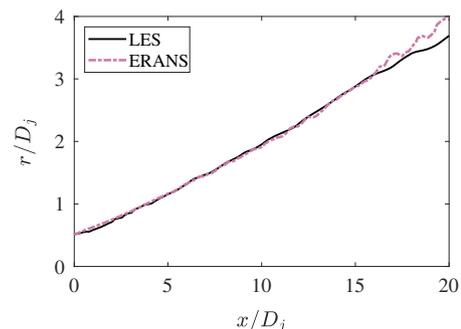


Fig. 23 Approximation of the radiator surface using the criterion $|\bar{\omega}| D_j / U_j = 0.03$.

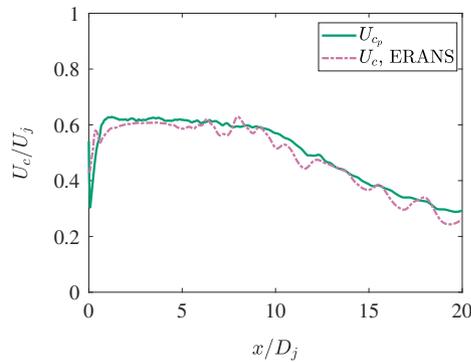


Fig. 24 Comparison of LES- and ERANS-derived convective velocities on the radiator surface.

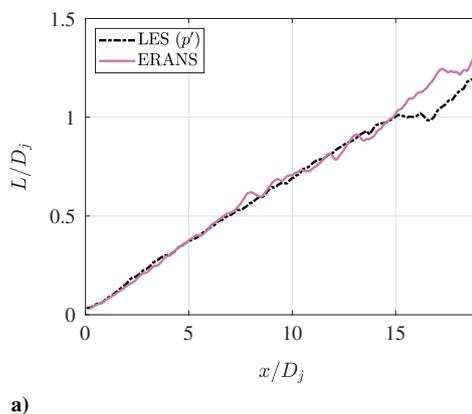
where r_{SPS} denotes the radial location of the ERANS-based SPS. As shown in Fig. 24, a good agreement is obtained between the axial distributions of U_{c_p} on the radiator surface, derived from the space–time correlations of p' , and of $U_{c,\text{ERANS}}$. Note that this agreement does not involve any tunable parameters.

D. Length Scales

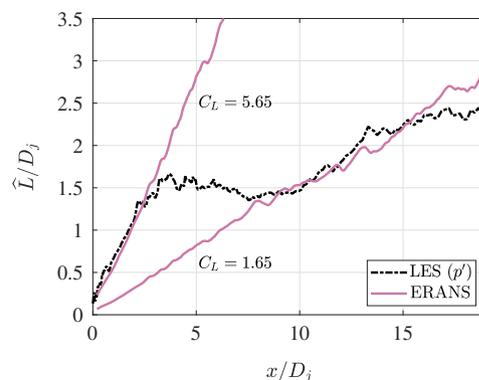
ERANS-based length scales are evaluated using Eq. (11) on the ERANS-derived SPS. They are compared with the LES-derived scales, based on p' , on the radiator surface in Fig. 25. The fixed-frame scale L_p , which displays a linear growth, is matched well by the ERANS scale with $C_L = 0.76$. The trend of the moving-scale scale \hat{L}_p is more complex, as discussed in Sec. VI.C, and thus not amenable to simple scaling. The rapid initial rise can be modeled by the ERANS scale with $C_L = 5.65$ up to about $x/D_j = 2.5$. A plateau of $\hat{L}_p/D_j \approx 1.5$ follows, and an apparently linear growth resumes past the end of the potential core, $x/D_j \geq 7.5$. The latter trend can be matched by the ERANS scale with $C_L = 1.65$. The complex growth for \hat{L}_p suggests different mechanisms for p' generation in the initial region of the jet and in the region past the end of the potential core. A full understanding of the physics is lacking at this point and will be the topic of future research.

E. Azimuthal Scale

The ERANS-based azimuthal scale is calculated from the ERANS length scale divided by the radius of the relevant surface. Here, Eq. (12) is evaluated with k and ε on the SPS, and radius r is set to that of the radiator surface. As shown in Fig. 26, the coefficient $C_\Phi = 2.6$ results in an ERANS-based azimuthal scale that provides a fair approximation to the growth and saturation of the LES-derived azimuthal scale based on R_{pp} .



a)



b)

Fig. 25 Fitting of ERANS-based length scales on radiator surface: a) fixed-frame scale with $C_L = 0.76$, and b) moving-frame scale with $C_L = 5.65$ and $C_L = 1.65$ for initial region and for region past potential core, respectively.

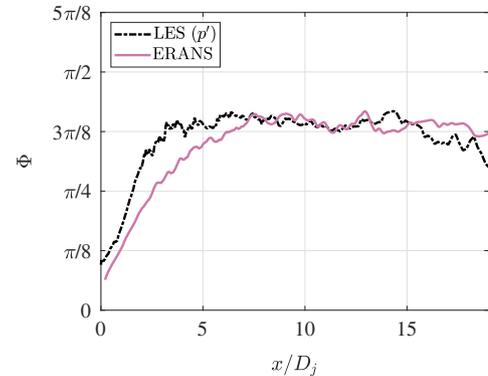


Fig. 26 Fitting of ERANS-based azimuthal scales on the radiator surface with $C_\Phi = 2.60$.

IX. Conclusions

A highly resolved LES of a Mach 0.9 isothermal turbulent jet is used to investigate the connection between the statistics of the vortical field and its pressure signature on the edge of the jet. The representative locations for those regions are the surface of peak Reynolds stress and the radiator surface, respectively. The radiator surface was defined in previous works [17] as the location where the convective velocity equals that on the SPS. Two-point space–time correlations are used to compute the convective velocity and correlation scales in the axial and azimuthal directions across the vortical and very near fields of the jet. The correlations are based on the axial velocity fluctuation u' and pressure fluctuation p' . In the two cases, two definitions of axial scales are considered: the fixed-frame scale L and the moving-frame scale \hat{L} .

The two-dimensional space–time correlations evaluated along a longitudinal and a cross-sectional plane, for reference points on the SPS, help us to understand the phenomena at play and key differences between velocity and pressure fluctuations. On the radial–axial plane, pressure-based correlations exhibit a wave-packet-like pattern that is coherent radially and travels fairly uniformly with the convective velocity U_c on the SPS. On the contrary, the velocity-based correlation lacks strong radial coherence and its convection is strongly influenced by the local mean-flow velocity. These results suggest that velocity-based correlations capture localized turbulent events, whereas the pressure-based correlations are dominated by the interaction of large eddies with the surrounding potential flow. Consequently, the axial and radial distributions of the corresponding length scales follow different trends. The pressure-based correlation scales on the radiator surface are larger than the velocity- or pressure-based scales on the SPS, indicating that fine-scale vortical motions do not significantly imprint the pressure field on the radiator surface. These observations are also applicable to correlations on a cross-sectional plane. There, the velocity-based correlation shows events at the SPS

have a limited range of effect, whereas pressure-based correlations have a wide azimuthal range of influence, which grows notably outside the radiator surface.

Regarding the axial trends, the velocity-based length and azimuthal scales increase largely linearly with downstream distance, thus following the spreading of the vortical region. The same holds for the moving-frame length scales based on velocity. However, the moving-frame length scale based on pressure grows rapidly in the first few jet diameters, saturates, and then grows slowly. This effect is most notable on the radiator surface. The azimuthal scale based on pressure also rises quickly and then saturates.

The radial variations of fixed-frame length scales collapse well on a single curve when the radial coordinate is offset by the radial location of the inner edge of the shear layer and normalized by the momentum thickness. This works particularly well for axial locations of $x \geq 3D_j$. The same scaling is carried out with moving-frame length scales and azimuthal scales with equally good results, with the exception of moving-frame length scales based on pressure whose growth does not follow that of the momentum thickness. A particularly interesting finding is that the radiator surface passes through a valley (local minimum) of velocity-based length scale (fixed or moving-frame), indicating that the velocity fluctuations there are mostly decorrelated from events in the core vortical region.

The potential of extracting relevant velocity and length scales from the time-averaged flowfield is finally evaluated. LES results are used to emulate a RANS solution. The focus is on p' -based scales on the radiator surface. ERANS can satisfactorily predict the convective velocity on the radiator surface without any fitting parameters. The distributions of azimuthal scale and fixed-frame axial length scale are matched approximately by fitting constants to the ERANS-derived scales. The distribution of the moving-frame axial length scale is complex and requires a higher level of empiricism. The geometry of the radiator surface can be accurately reproduced using a criterion based on the magnitude of the mean vorticity.

Acknowledgments

We gratefully acknowledge a Balsells Fellowship that partially supported A. Adam. C. Bogey was partially supported by the LABEX CeLyA (ANR-10-LABX-0060/ANR-16-IDEX-0005). The jet simulations have been performed using High Performance Computing (HPC) resources from the Pole of Modeling and Computing in Engineering and Information Sciences of the University of Lyon and from the Institute of Scientific Informatics Development and Resources under allocation 2020-2a0204 made by Large National High Performance Equipment.

References

- [1] Ffowcs Williams, J., and Hawkins, D., "Sound Generation by Turbulence and Surfaces in Arbitrary Motion," *Philosophical Transactions of the Royal Society of London, Series A: Mathematical and Physical Sciences*, Vol. 264, No. 1151, 1969, pp. 321–342. <https://doi.org/10.1098/rsta.1969.0031>
- [2] Pilon, A., and Lyrintzis, A., "Development of an Improved Kirchhoff Method for Jet Aeroacoustics," *AIAA Journal*, Vol. 36, No. 5, 1998, pp. 783–790. <https://doi.org/10.2514/2.437>
- [3] Papamoschou, D., "Prediction of Jet Noise Shielding," AIAA Paper 2010-0653, 2010. <https://doi.org/10.2514/6.2010-653>
- [4] Piantanida, S., Jaunet, V., Huber, J., Wolf, W., Jordan, P., and Cavalieri, A., "Scattering of Turbulent-Jet Wavepackets by a Swept Trailing Edge," *Journal of the Acoustical Society of America*, Vol. 140, No. 6, 2016, pp. 4350–4359. <https://doi.org/10.1121/1.4971425>
- [5] Henderson, B., Leib, S., and Wernet, M., "Measurements and Predictions of the Noise from Three-Stream Jets," AIAA Paper 2015-3120, 2015. <https://doi.org/10.2514/6.2015-3120>
- [6] Papamoschou, D., "Modelling of Noise Reduction in Complex Multi-stream Jets," *Journal of Fluid Mechanics*, Vol. 834, Nov. 2018, pp. 555–599. <https://doi.org/10.1017/jfm.2017.730>
- [7] Papamoschou, D., Johnson, A. D., and Phong, V., "Aeroacoustics of Three-Stream High-Speed Jets from Coaxial and Asymmetric Nozzles," *Journal of Propulsion and Power*, Vol. 30, No. 4, 2014, pp. 1055–1069. <https://doi.org/10.2514/1.B35130>
- [8] Papamoschou, D., Xiong, J., and Liu, F., "Reduction of Radiation Efficiency in High-Speed Jets," AIAA Paper 2014-2619, 2014. <https://doi.org/10.2514/6.2014-2619>
- [9] Stuber, M., Lowe, K., and Wing, F., "Synthesis of Convection Velocity and Turbulence Measurements in Three-Stream Jets," *Experiments in Fluids*, Vol. 60, No. 83, 2019, pp. 998–1001. <https://doi.org/10.1007/s00348-019-2730-5>
- [10] Papamoschou, D., "On the Connection Between near and far Pressure Fields of a Turbulent Jet," AIAA Paper 2018-1251, 2018. <https://doi.org/10.2514/6.2018-1251>
- [11] Harper-Bourne, M., "Jet Noise Turbulence Measurements," AIAA Paper 2003-3214, 2003. <https://doi.org/10.2514/6.2003-3214>
- [12] Morris, P., and Zaman, K., "Velocity Measurements in Jets with Application to Jet Noise," *Journal of Sound and Vibration*, Vol. 329, No. 4, Feb. 2010, pp. 394–414. <https://doi.org/10.1016/j.jsv.2009.09.024>
- [13] Proença, A., Lawrence, J., and Self, R., "Measurements of the Single-Point and Joint Turbulence Statistics of High Subsonic Jets Using Hot-Wire Anemometry," *Experiments in Fluids*, Vol. 60, No. 4, 2019, Paper 63. <https://doi.org/10.1007/s00348-019-2716-3>
- [14] Fleury, V., Bailly, C., Jondeau, E., Michard, M., and Juvé, D., "Space-Time Correlations in Two Subsonic Jets Using Dual Particle Image Velocimetry Measurements," *AIAA Journal*, Vol. 46, No. 10, 2008, pp. 2498–2509. <https://doi.org/10.2514/1.35561>
- [15] Pokora, C., and McGuirk, J., "Stereo-PIV Measurements of Spatio-Temporal Turbulence Correlations in an Axisymmetric Jet," *Journal of Fluid Mechanics*, Vol. 778, July 2015, pp. 216–252. <https://doi.org/10.1017/jfm.2015.362>
- [16] Adam, A., Papamoschou, D., Xiong, J., and Liu, F., "The Very Near Pressure Field of Three-Stream Jets," AIAA Paper 2018-1739, 2018. <https://doi.org/10.2514/6.2018-1739>
- [17] Adam, A., Papamoschou, D., Xiong, J., and Liu, F., "Vorticity Dynamics and Flow Statistics Near the Edge of High Speed Multi-Stream Jets," AIAA Paper 2019-1303, 2019. <https://doi.org/10.2514/6.2019-1303>
- [18] Bogey, C., Marsden, O., and Bailly, C., "Large-Eddy Simulation of the Flow and Acoustic Fields of a Reynolds Number 10^5 Subsonic Jet with Tripped Exit Boundary Layers," *Physics of Fluids*, Vol. 23, No. 3, 2011, Paper 035104. <https://doi.org/10.1063/1.3555634>
- [19] Bogey, C., Marsden, O., and Bailly, C., "Influence of Initial Turbulence Level on the Flow and Sound Fields of a Subsonic Jet at a Diameter-Based Reynolds Number of 10^5 ," *Journal of Fluid Mechanics*, Vol. 701, May 2012, pp. 352–385. <https://doi.org/10.1017/jfm.2012.162>
- [20] Bogey, C., "Grid Sensitivity of Flow Field and Noise of High-Reynolds-Number Jets Computed by Large-Eddy Simulation," *International Journal of Aeroacoustics*, Vol. 17, Nos. 4–5, 2018, pp. 399–424. <https://doi.org/10.1177/1475472X18778287>
- [21] Bogey, C., "Two-Dimensional Features of Correlations in the Flow and Near Pressure Fields of Mach Number 0.9 Jets," AIAA Paper 2019-0806, 2019. <https://doi.org/10.2514/6.2019-0806>
- [22] Bogey, C., "Acoustic Tones in the Near-Nozzle Region of Jets: Characteristics and Variations Between Mach Numbers 0.5 and 2," *Journal of Fluid Mechanics*, Vol. 921, June 2021, Paper A3. <https://doi.org/10.1017/jfm.2021.426>
- [23] Zaman, K. B. M. Q., "Effect of Initial Condition on Subsonic Jet Noise," *AIAA Journal*, Vol. 23, No. 9, 1985, pp. 1370–1373. <https://doi.org/10.2514/3.9094>
- [24] Bogey, C., and Bailly, C., "Influence of Nozzle-Exit Boundary-Layer Conditions on the Flow and Acoustic Fields of Initially Laminar Jets," *Journal of Fluid Mechanics*, Vol. 663, Nov. 2010, pp. 507–538. <https://doi.org/10.1017/S0022112010003605>
- [25] Bogey, C., Marsden, O., and Bailly, C., "Effects of Moderate Reynolds Numbers on Subsonic Round Jets with Highly Disturbed Nozzle-Exit Boundary Layers," *Physics of Fluids*, Vol. 24, No. 10, 2012, Paper 105107. <https://doi.org/10.1063/1.4757667>
- [26] Bogey, C., and Marsden, O., "Identification of the Effects of the Nozzle-Exit Boundary-Layer Thickness and Its Corresponding Reynolds Number in Initially Highly Disturbed Subsonic Jets," *Physics of Fluids*,

- Vol. 25, No. 5, 2013, Paper 055106.
<https://doi.org/10.1063/1.4807071>
- [27] Bogey, C., and Sabatini, R., "Effects of Nozzle-Exit Boundary-Layer Profile on the Initial Shear-Layer Instability, Flow Field and Noise of Subsonic Jets," *Journal of Fluid Mechanics*, Vol. 876, Aug. 2019, pp. 288–325.
<https://doi.org/10.1017/jfm.2019.546>
- [28] Bogey, C., "Generation of Excess Noise by Jets with Highly Disturbed Laminar Boundary-Layer Profiles," *AIAA Journal*, Vol. 59, No. 2, 2021, pp. 569–579.
<https://doi.org/10.2514/1.J059610>
- [29] Mohseni, K., and Colonius, T., "Numerical Treatment of Polar Coordinate Singularities," *Journal of Computational Physics*, Vol. 157, No. 2, 2000, pp. 787–795.
<https://doi.org/10.1006/jcph.1999.6382>
- [30] Bogey, C., de Cacqueray, N., and Bailly, C., "Finite Differences for Coarse Azimuthal Discretization and for Reduction of Effective Resolution Near Origin of Cylindrical Flow Equations," *Journal of Computational Physics*, Vol. 230, No. 4, 2011, pp. 1134–1146.
<https://doi.org/10.1016/j.jcp.2010.10.031>
- [31] Bogey, C., and Bailly, C., "A Family of Low Dispersive and Low Dissipative Explicit Schemes for Flow and Noise Computations," *Journal of Computational Physics*, Vol. 194, No. 1, 2004, pp. 194–214.
<https://doi.org/10.1016/j.jcp.2003.09.003>
- [32] Bogey, C., de Cacqueray, N., and Bailly, C., "A Shock-Capturing Methodology Based on Adaptive Spatial Filtering for High-Order Non-Linear Computations," *Journal of Computational Physics*, Vol. 228, No. 5, 2009, pp. 1447–1465.
<https://doi.org/10.1016/j.jcp.2008.10.042>
- [33] Tam, C. K., and Dong, Z., "Radiation and Outflow Boundary Conditions for Direct Computation of Acoustic and Flow Disturbances in a Non-uniform Mean Flow," *Journal of Computational Acoustics*, Vol. 4, No. 2, 1996, pp. 175–201.
<https://doi.org/10.1142/S0218396X96000040>
- [34] Bogey, C., and Bailly, C., "Large Eddy Simulations of Transitional Round Jets: Influence of the Reynolds Number on Flow Development and Energy Dissipation," *Physics of Fluids*, Vol. 18, No. 6, 2006, Paper 065101.
<https://doi.org/10.1063/1.2204060>
- [35] Bogey, C., and Bailly, C., "Turbulence and Energy Budget in a Self-Preserving Round Jet: Direct Evaluation Using Large Eddy Simulation," *Journal of Fluid Mechanics*, Vol. 627, May 2009, pp. 129–160.
<https://doi.org/10.1017/S0022112009005801>
- [36] Fauconnier, D., Bogey, C., and Dick, E., "On the Performance of Relaxation Filtering for Large-Eddy Simulation," *Journal of Turbulence*, Vol. 14, No. 1, 2013, pp. 22–49.
<https://doi.org/10.1080/14685248.2012.740567>
- [37] Kremer, F., and Bogey, C., "Large-Eddy Simulation of Turbulent Channel Flow Using Relaxation Filtering: Resolution Requirement and Reynolds Number Effects," *Computers and Fluids*, Vol. 116, Aug. 2015, pp. 17–28.
<https://doi.org/10.1016/j.compfluid.2015.03.026>
- [38] Dahl, M., "Turbulent Statistics from Time-Resolved PIV Measurements of a Jet Using Empirical Mode Decomposition," AIAA Paper 2012-2297, 2012.
<https://doi.org/10.2514/6.2012-2297>
- [39] Kerhervé, F., Fitzpatrick, J., and Jordan, P., "The Frequency Dependence of Jet Turbulence for Noise Source Modelling," *Journal of Sound and Vibration*, Vol. 296, No. 1, 2006, pp. 209–225.
<https://doi.org/10.1016/j.jsv.2006.02.012>
- [40] Lau, J., Morris, P., and Fisher, M., "Measurements in Subsonic and Supersonic Free Jets Using a Laser Velocimeter," *Journal of Fluid Mechanics*, Vol. 93, No. 1, 1979, pp. 1–27.
- [41] Papamoschou, D., Xiong, J., and Liu, F., "Towards a Low-Cost Wavepacket Model of the Jet Noise Source," AIAA Paper 2015-1006, 2015.
<https://doi.org/10.2514/6.2015-1006>
- [42] Violato, D., and Scarano, F., "Three-Dimensional Vortex Analysis and Aeroacoustic Source Characterization of Jet Core Breakdown," *Physics of Fluids*, Vol. 25, No. 1, 2013, Paper 015112.
<https://doi.org/10.1063/1.4773444>
- [43] Picard, C., and Delville, J., "Pressure Velocity Coupling in a Subsonic Round Jet," *International Journal of Heat and Fluid Flow*, Vol. 21, No. 3, 2000, pp. 359–364.
[https://doi.org/10.1016/S0142-727X\(00\)00021-7](https://doi.org/10.1016/S0142-727X(00)00021-7)
- [44] Coiffet, F., Jordan, P., Delville, J., Gervais, Y., and Ricaud, F., "Coherent Structures in Subsonic Jets: A Quasi-Irrotational Source Mechanism?" *International Journal of Aeroacoustics*, Vol. 5, No. 1, 2006, pp. 67–89.
<https://doi.org/10.1260/147547206775220407>
- [45] Camussi, R., and Bogey, C., "Intermittent Statistics of the 0-Mode Pressure Fluctuations in the Near Field of Mach 0.9 Circular Jets at Low and High Reynolds Numbers," *Theoretical and Computational Fluid Dynamics*, Vol. 35, No. 2, 2021, pp. 229–247.
<https://doi.org/10.1007/s00162-020-00553-9>
- [46] Arndt, R., Long, D., and Glauser, M., "The Proper Orthogonal Decomposition of Pressure Fluctuations Surrounding a Turbulent Jet," *Journal of Fluid Mechanics*, Vol. 340, June 1997, pp. 1–33.
<https://doi.org/10.1017/S0022112097005089>
- [47] Cavalieri, A. V. G., Jordan, P., Colonius, T., and Gervais, Y., "Axisymmetric Superdirectivity in Subsonic jets," *Journal of Fluid Mechanics*, Vol. 704, July 2012, pp. 388–420.
<https://doi.org/10.1017/jfm.2012.247>
- [48] Bridges, J., "Rapid Prediction of Installed Jet Noise from RANS," AIAA Paper 2019-2732, 2019.
<https://doi.org/10.2514/6.2019-2732>
- [49] Georgiadis, N., and DeBonis, J., "Navier–Stokes Analysis Methods for Turbulent Jet Flows with Application to Aircraft Exhaust Nozzles," *Progress in Aerospace Sciences*, Vol. 42, Nos. 5–6, July–Aug. 2006, pp. 378–418.
<https://doi.org/10.1016/j.paerosci.2006.12.001>
- [50] Darisse, A., Lemay, J., and Benaïssa, A., "Budgets of Turbulent Kinetic Energy, Reynolds Stresses, Variance of Temperature Fluctuations and Turbulent Heat Fluxes in a Round Jet," *Journal of Fluid Mechanics*, Vol. 774, June 2015, p. 95–142.
<https://doi.org/10.1017/jfm.2015.245>

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 Associate Editor