Design and experimental validation of an array of accelerometers for in-flow acoustic beamforming applications

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The aim of this work is to present an experimental validation of in-flow beamforming using vibration measurements. An antenna of accelerometers is mounted on a thin structure placed in the flow. High wavenumbers of the turbulent boundary layer are naturally filtered out by the structure, such that accelerometers are mainly dominated by the acoustic part of the excitation. An inverse method is used to reconstruct the pressure exciting the structure from vibration measurements, that is then injected in a beamforming code. The experiment shows the ability of the inverse method to reconstruct the acoustic part of the excitation, and validates the possibility to use it for acoustic source localisation.

I. Introduction

The experimental characterization of a surface pressure field exciting a structure in a flow is not an easy task. The difficulty is mainly related to the spatial complexity of the field, whose wavenumber spectrum is very wide. It generally requires the use of high density flush mounted microphone arrays,[1] and/or some hypothesis on the spatial homogeneity of the field statistical properties.[2] The correct estimation of the acoustic part of the field (low wavenumbers) also requires the use of high density arrays, in order to limit aliasing of high wavenumber components. However, some alternative approaches exist to extract the acoustic part without spatial oversampling, based on the use of physical low-pass anti-aliasing filters. A first possibility is to use either surface microphones (typically B&K 4948), that average the parietal pressure over a relatively large area, or microphones protected from the flow by a thin membrane.[3] A second possibility is to estimate the pressure field from measurements of the vibration of a structure excited by the flow[4,5] . In this case, the properties of the wavenumber low-pass filter are determined by the structure properties: the cutoff wavenumber corresponds simply to the natural wavenumber of the structure. The acoustic part of the pressure field is thus recovered using an inverse method known as (C)FAT for (Corrected) Force Analysis Technique[6,7] . The ability of this approach to extract the acoustic part of a turbulent pressure field has been recently studied in the frame of numerical[8] and experimental[9] validations.

On the other hand, the CFAT method has been recently implemented for beamforming applications based on vibration measurements.[10] The parietal acoustic field exciting a baffled plate is identified from vibration measurements (using CFAT), and then used as an input for standard beamforming codes. The approach has been validated experimentally for a pure acoustic load (without flow).

The aim of this work is to report an experimental validation of the CFAT implementation of beamforming in the case of a turbulent excitation mixed with an acoustic load. The purpose is to show experimentally the ability of the approach to extract the acoustic part of the parietal pressure field, and to illustrate the possibility to use this acoustic part as an input for acoustic beamforming.

This paper is constituted of three main parts. The first part is dedicated to the theoretical formulation of the CFAT method, with some original developments concerning the application of CFAT for cross spectral data. The second part concerns the development and validation of a 1D array of accelerometers embedded on a thin beam, including the design of a specific mounting. The last part is dedicated to the experimental setup and results.

II. Theoretical formulation of CFAT for distributed random excitations

A. The CFAT theory

In this section, the CFAT will be briefly reviewed. The aim of CFAT is to recover the pressure field exciting a thin structure (beam, plate) from its vibration response.[7] It is based on the local equation of motion of the structure, expressed as follows (for the beam case)

$$\mathcal{E}I\frac{\partial^4}{\partial x^4}w(x) - \rho S\omega^2 w(x) = p(x), \tag{1}$$

where $\mathcal{E}$, I, ρ, S are the physical parameters of the beam (Young's modulus, area moment of inertia, density and cross section area, respectively), $w(x)$ the transverse displacement of the beam and $p(x)$ the load distribution. The principle of FAT[6] is to recover the right hand side of Eq. (1) from the experimental assessment of the left hand side terms. The fourth order spatial derivative is estimated using a finite difference scheme, using a 5-points window centred on the estimation point:

$$\frac{\partial^4}{\partial x^4}w(x) \approx \delta_\Delta^{4x}(x) = \frac{w_{(x-2\Delta)} - 4w_{(x-\Delta)} + 6w_{(x)} - 4w_{(x+\Delta)} + w_{(x+2\Delta)}}{\Delta^4}, \tag{2}$$

where Δ is the spatial step of the scheme. The wavenumber response of the method is defined as the ratio between the wavenumber spectrum of the identified pressure field $\tilde{p}(x)$ and the one of the true pressure field $E = \mathcal{F}[\tilde{p}(x)]/\mathcal{F}[p(x)]$(see[7] for details). This response is drawn in Fig. 1 (left) for different values of number of points by structural wavelength $n = \lambda_N/\Delta = 2\pi/(k_N\Delta)$, with k_N the natural wavenumber of the structure at the frequency of interest ω (in rad/s):

$$k_N^4 = \frac{\rho S}{\mathcal{E}I}\omega^2. \tag{3}$$

The method acts like a low pass filter, with a cut-off frequency around the natural wavenumber k_N and a slope depending on the value of n. A singularity is also observed around $k = k_N$, where the actual load can be strongly overestimated. This singularity is efficiently corrected using CFAT,[7] a version of FAT including a

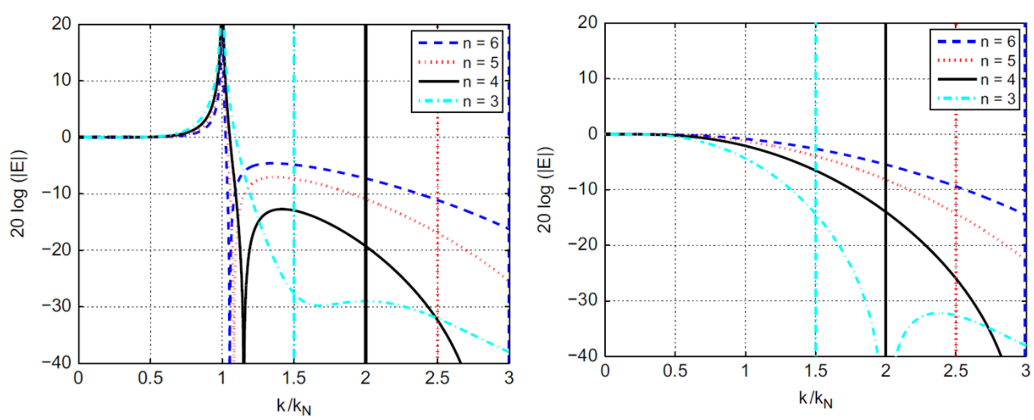

Figure 1. Wavenumber domain response of FAT (left) and CFAT (right) for different values of n (number of points by structural wavelength).[7] Vertical lines indicate Nyquist wavenumbers for each value of n.

correction factor aiming at suppressing the singularity around $k = k_N$ (Fig. 1, right). The CFAT expression of the load at point x is expressed as follows

$$\tilde{p}(x) = \mathcal{E}I\mu^4\delta_\Delta^{4x}(x) - \rho S\omega^2 w(x), \tag{4}$$

where μ^4 is the frequency dependent correction factor:

$$\mu^4 = \frac{\Delta^4 k_N^4}{(2 - 2\cos(k_N \Delta))^2}. \tag{5}$$

The wavenumber response of CFAT (Fig. 1, right), taking into account the correction factor, is finally a simple low-pass filter. This spatial filtering effect will be used in the following as a physical anti-aliasing property for the spatial sampling of the incident pressure field. If this field contains the contribution of a turbulent boundary layer, high wavenumbers above the natural wavenumber of the plate will be efficiently filtered out, thus relaxing the spatial sampling criterion to a value of at least two points by structural wavelength. This gives a high frequency limit to the method :

$$\Delta < \lambda_N/2 \quad \Leftrightarrow \quad \omega < \frac{\pi^2}{\Delta^2}\sqrt{\frac{\mathcal{E}I}{\rho S}}. \tag{6}$$

Unfortunately, there is also a low frequency limit which is due to the estimation of the fourth order spatial derivative using a finite difference scheme. It has been observed[11] and shown[12] that this limit corresponds to a number of measurement points by structural wavelength equal to approximately 4:

$$\Delta > \lambda_N/4 \quad \Leftrightarrow \quad \omega > \frac{\pi^2}{4\Delta^2}\sqrt{\frac{\mathcal{E}I}{\rho S}}. \tag{7}$$

Below this frequency, the measurement noise is more and more amplified, leading to potentially strong overestimations of the pressure field level. To overcome this difficulty, spatial filtering can be applied[6] , or alternatively it is possible to adapt the step of the finite difference scheme as a function of the frequency[12] . This operation can be realized without conducting measurements with different grids, simply by down-sampling the grid to a step $R\Delta$ ($R = 1, 2, 3, ...$). In this case, the low frequency limit can be revised to the following expression:

$$R\Delta > \lambda_N/4 \quad \Leftrightarrow \quad \omega > \frac{\pi^2}{4R^2\Delta^2}\sqrt{\frac{\mathcal{E}I}{\rho S}}. \tag{8}$$

However, This operation has an effect on the length of the grid on which the pressure field can be reconstructed (that constitutes the aperture for beamforming applications), because it is not possible to apply the finite difference scheme on the edges of the measurement grid.

B. Matrix formulation for distributed random excitations

If the 1D displacement field is measured at M points of a regular grid with a step Δ, the load field can be assessed at a number of $M - 4R$ points (excluding the $2R$ points at both ends of the grid, with R the downsampling factor (R=1,2,3)). This can be written in a matrix form :

$$\{p\}_{M-4R} = [\Phi]\,\{w\}_M, \tag{9}$$

where $\{p\}_{M-4R}$ and $\{w\}_M$ are pressure estimation points and displacement measurement points, respectively, sorted as a function of their position x, and where $[\Phi]$ is the CFAT operator (given hereafter for $R = 1$ for the sake of clarity):

$$[\Phi] = \frac{\mu^4 \mathcal{E}I}{\Delta^4}\begin{bmatrix} 1 & 0 & & 0 \\ -4 & 1 & & 0 \\ 6 & -4 & & ... \\ -4 & 6 & & 0 \\ 1 & -4 & ... & 1 \\ 0 & 1 & & -4 \\ 0 & 0 & & 6 \\ ... & ... & & -4 \\ 0 & 0 & & 1 \end{bmatrix}^T - \rho S \omega^2 \begin{bmatrix} 0 & 0 & & 0 \\ 0 & 0 & & 0 \\ 1 & 0 & & ... \\ 0 & 1 & & 0 \\ 0 & 0 & ... & 0 \\ 0 & 0 & & 0 \\ 0 & 0 & & 1 \\ ... & ... & & 0 \\ 0 & 0 & & 0 \end{bmatrix}^T \tag{10}$$

For aeroacoustic applications, Eq. (9) will be formulated in a quadratic form to consider partially correlated pressure and displacement fields:

$$[S_{pp}] = [\Phi]\,[S_{ww}]\,[\Phi]^H\,, \tag{11}$$

where $[S_{pp}]$ and $[S_{ww}]$ stand for cross spectral matrices of pressure and displacement fields, respectively.

III. Development and validation of a 1D array of accelerometers

The CFAT method has been developed for either beam or plate-like structures[7] , involving either line or matrix arrays of accelerometers, respectively. A natural choice for aeroacoustic applications would be the plate, to be able to characterize turbulent pressure fields in two dimensions. However, this would require an important number of accelerometers. Thus, for practical reasons, the beam, equipped with a line array of accelerometers, has been preferred for this first experimental implementation (see Fig. 2). The beam-like

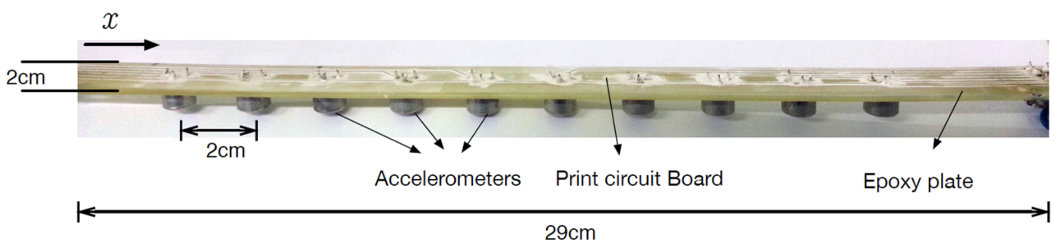

Figure 2. Epoxy beam sensor with 10 embbeded accelerometers.

sensor is a $20 \times 300 \times 1.5$ mm epoxy board, on which 10 small ICP accelerometers are welded. The wiring of sensors is directly printed on the board. The spacing between accelerometers is 20mm. This parameter, as well as the physical properties of the material, defines the frequency bandwidth of the sensor, which is here equal to [500; 3000] Hz. The design of the support of the beam sensor is not straightforward. First it has to be correctly baffled, to minimize acoustic excitation on the backside of the beam. Second, the beam has to be free on its long side edges to satisfy the analytical model on which is based the method. A particular care has to be taken in the design of the support, to try to meet as much as possible these two requirements. Another difficulty of the 1D (beam) approach, as compared to the 2D (plate) one, is that the spatial filtering is not the same in both directions. Along the axis of the beam, the spatial filtering corresponds to the one described in the previous section. In the transversal direction, the displacement of the beam is supposed to be constant (which means neglecting torsional modes, that are potentially contributing to the beam's response in the frequency range of interest). In this direction, the spatial filter is a rectangular window, whose length is equal to the beam width, and whose wavenumber response is also a low pass filter, with a frequency independent response (a cardinal sine). Thus, the device is expected to behave differently when inserted either along stream or span wise directions, when excited by a turbulent boundary layer.

IV. Experimental validation

The beam sensor device is placed in an anechoic wind tunnel, and submitted to a turbulent boundary layer at several flow speeds from 0 to 32m/s. An acoustic source is placed at about 1m of the device, outside the flow (see Fig. 3).

The beam can be mounted either parallel (streamwise) or perpendicular (spanwise) to the flow. A microphone is flush-mounted next to the beam sensor, in front of one of the pressure identification point, so as to compare the pressure identified from the vibration of the beam to a directly measured one.

Some results are presented in Fig. 4. It can be seen that without flow, the acoustic pressure identified with the beam sensor is in very good agreements with the direct microphone measurement on the whole frequency range from 500 to 3000 Hz. With flow, it is seen that the flush mounted microphone is dominated by the contribution of the TBL, the energy of which is about 10 to 15 dB louder than the acoustic source contribution on the whole frequency range. On the other hand, the pressure obtained with the beam sensor corresponds to the measured one without flow. Results with flow on the microphone are equivalent with or without the acoustic source. The pressure identified using the beam sensor with the acoustic source switched

Figure 3. Experimental setup. Wind tunnel in the anechoic room, beam sensor in the spanwise direction (left). Close up view of the device, beam sensor in the streamwise direction (right).

off is much lower, and represent the low wavenumber part of the TBL that is not filtered by the structure. This residual part is almost 25dB lower than the spectrum measured by the microphone.

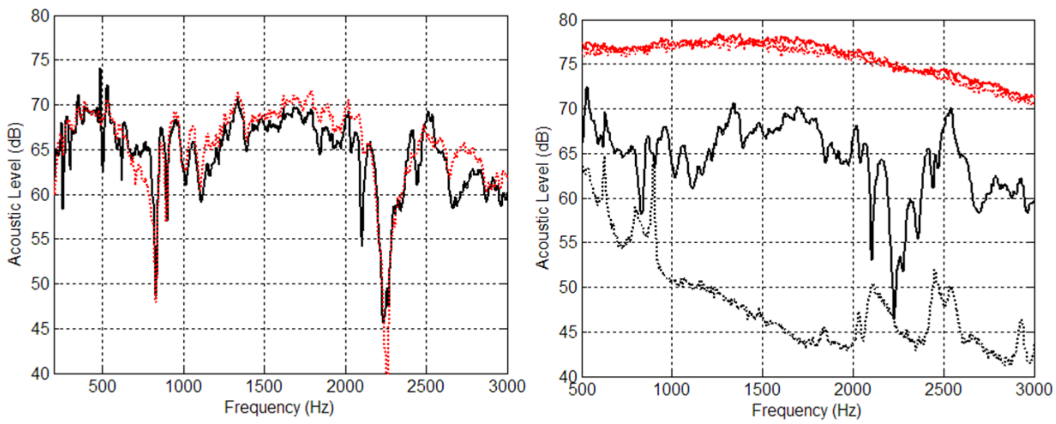

Figure 4. Experimental results: pressure autospectrum at the center of the spanwise device estimated with the beam sensor (black) and with the flush mounted microphone (red). Left: without flow, acoustic source on. Right: with flow $U_\infty = 22m/s$, acoustic source on (solid lines) and off (dotted lines).

The identified parietal pressure is finally used as an input for standard plane wave based beamforming. The results obtained for the stream-wise configuration (Fig. 5), show that the acoustic source is correctly localized, for different flow speeds, from 0 (without flow) to 32m/s. The only noticeable difference is in the angular localisation of the maximum, that is slightly moved when the flow speed increases, because of the convection effects that are not taken into account in the beamforming code.

V. Conclusion

This work presents a first experimental proof of concept of in-flow beamforming based on vibration measurements. The main advantage of this approach as compared to flush mounted microphones is that measurements are much less contaminated by flow noise, because of wavenumber filtering properties of the structure. Another good point is that a wall can be instrumented with a limited intrusivity (no holes are required). However, a special care has to be taken in the conception of the sensing device, in order to determine correctly the targeted frequency range of application, which is moreover limited to few octave intervals.

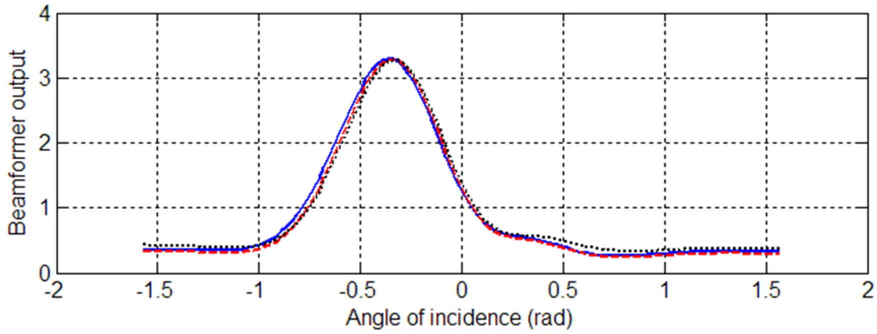

Figure 5. Beamforming results averaged between 1 and 3kHz, for $U_\infty = 0 - 16 - 32 m/s$ (blue-red-black).

Acknowledgments

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