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STATISTICAL PROPERTIES OF ACOUSTIC WAVES PROPAGATING THROUGH TURBULENT THERMAL FIELDS

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Abstract

Measurements are obtained for the statistical properties of ultrasonic waves which propagate through a turbulent temperature field. Two heated grids in air are utilized to generate a spatially random thermal pattern typical of atmospheric or oceanic environments. Incident acoustic waves are emitted with spherical or piston-like transducers.

Experimental values for the second order coherence function are theoretically predicted using the stochastic Helmholtz equation in the parabolic approximation and a modified von Karman spectrum. The importance of the outer scale of turbulence is pointed out. Results are also given for the scintillation index and the probability distribution of the intensity.

Introduction

Acoustic wave propagation in a turbulent medium undergoes chaotic changes of phase and amplitude (intensity). Measurements of acoustic change in the atmosphere and ocean have been obtained ; but because the details of the velocity and temperature variations in these media were unknown, the influence of these parameters on the sound characteristics, such as the mutual coherence function or the intensity fluctuations, were not obtainable.

The propagation of sound waves through turbulent velocity fields has already been investigated under laboratory conditions by several authors (Candel, Julienne and Juliand [(1)], Blanc-Benon [(2)], Guedel [(3)]). For temperature fields, Stone and Mintzer [(4)] made experiments with a water tank in which a random temperature field was produced by a heater array placed at the bottom. But in this arrangement, minute air bubbles were generated. As pointed out by Numbert and Lumley [(5)] the appearance of these discrete impurities presented an additional and not well controlled parameter. An installation with a grid in air eliminates this problem and has been chosen in this work. To simulate atmospheric or oceanic conditions of acoustic propagation, it is assumed that the acoustic wave length λ remains small compared to the integral length scale L of the temperature field, which, in turn, is smaller than the range of propagation x, i.e. $x \gg L \gg \lambda$.

Experimental arrangement

The heated grids as well as the locations of the acoustic transmitter and receiver are sketched in Fig. 1. Each grid consists of a plane arrangement of conductors with square mesh of 9 cm. The two grids are placed horizontally in a large anechoic room (10 m x 7 m x 8 m) and the mixing of the free convection plumes above them generates the turbulent thermal field. The second grid is shifted above the first grid to change the size of the heating cells (M = 9 cm ; M = 4.5 cm). The overall dimensions of the system are 1.1 m x 2.2 m and the maximum power consumption of each grid is 32 Kw.

The acoustic propagation measurements were made at the height H = 1.75 m corresponding to 20 or 40 times the mesh size M. With one heated grid, the mean temperature rise above ambient was 27° C and the relative temperature fluctuation $T'/\bar{T}$ had an rms value of $1.7\ 10^{-2}$. The statistical uniformity of the thermal field was achieved within 0.5° C excluding 1.5 meshes near the edges. The one dimensional spectrum $F_{T'}(K_1)$ of T' was measured with a FFT analyser Nicolet 660A, in the range 0.5 - 200 Hz with a constant bandwith of 0.5 Hz. Frequencies were converted into wave numbers K_1 by a Taylor hypothesis based on the mean upward velocity (1.25 m/s measured with a hot wire). Results are given in Fig. 2 and compared with the one-dimensional spectrum deduced from the modified von Karman form $\phi_\varepsilon(K)$ by :

$$F_{T'}(K_1) = \int_{K_1}^{\infty} K\, \phi_\varepsilon(K)\, dK \qquad (1)$$

$$\phi_\varepsilon(K) = 0.033\, C_\varepsilon^2\, (K^2 + 1/L_0^2)^{-\frac{11}{6}} \cdot \exp(-K^2/K_m^2)$$

$$C_\varepsilon^2 = 1.91\, (T'/2\bar{T})^2\, L_0^{-2/3} \quad ; \quad K_m = 5.92/\ell_0$$

C_ε^2 is the structure constant, L_0 is the outer scale of turbulence related to the integral scale L_T ($L_0 = 1.339\ L_T$), ℓ_0 is the inner scale, and $\phi_\varepsilon(K)$ is the spectrum of the fluctuation $\epsilon(r)$ in the index of refraction. The integral scale L_T deduced by integration of the spatial correlation curve is 7.6 cm. The inner scale ℓ_0 related to the high frequency cutoff was estimated to be 0.1 cm. The index changes which may be induced by velocity fluctuations in the x direction prove to be negligible ; indeed an upper limit is given by u'_z/c_0 (u'_z is the fluctuating component in the upward direction) and measurements indicates that $u'_z/c_0 = 6.10^{-4}$. The same measurements were also made with the two heated grids. The mean temperature rise was 35° C and the rms value of $T'/\bar{T}$ can reach $2.5\ 10^{-2}$. The mean velocity in the upward direction was 1.5 m/s and the integral scale L_T was 5 cm. Additionnal details are given in Blanc-Benon [(6)].

The spherical acoustic waves were generated by TDK ultrasonic sources (f = 23.5 kHz ; 31 kHz ; 39 kHz ; 75 kHz), and the collimated beam by a home-made piston-like transducer using the Sell technique. The area is 100 cm^2 and the frequency is adjustable in the

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range 20-100 kHz (Blanc-Benon [1]). The transmitted signals were received on 1/4" microphones (Brüel & Kjaër 4135) located in a plane normal to the x-axis.

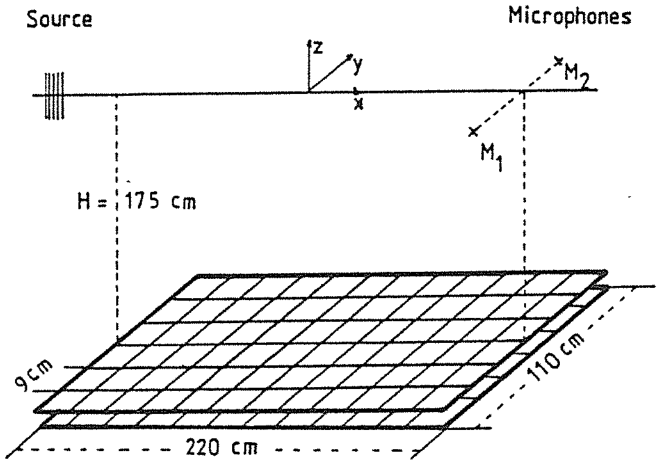

Fig. 1 - Experimental set-up.

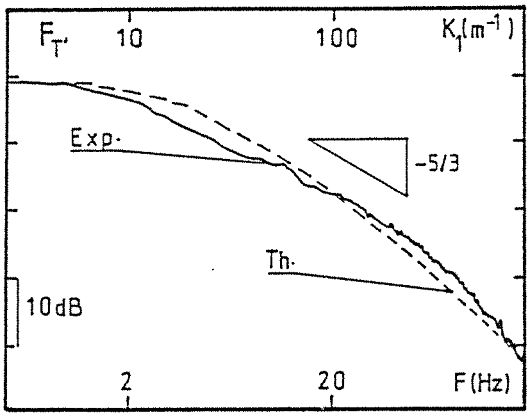

Fig. 2 - Comparison between the experimental (one-dimensional) spectrum of temperature fluctuations, $F_{T'}$, and the theoretical spectrum deduced from Eq. 1 with the von Karman function.

Theoretical considerations

The propagation of a time-harmonic acoustic wave in a random medium characterized by small temperature fluctuations ($T'/\bar{T} \ll 1$) and an integral turbulent scale L_T large in comparison with the acoustic wavelength λ, is governed by the stochastic Helmholtz equation (Tatarski [7]) :

$$\left[\Delta + k_0^2 \left(1 + \mathcal{E}(\vec{r})\right)\right] P(\vec{r}) = 0. \qquad (2)$$

$$\mathcal{E}(\vec{r}) = -T'(\vec{r})/\bar{T}$$

where $P(\vec{r})$ denotes the space dependence of the pressure, K_0 the wave number, $\mathcal{E}(\vec{r})$ the fluctuations in the index of refraction, and $\vec{r} = (x, y, z)$. For paraxial transmission along the x-direction, $P(\vec{r})$ can the be expressed as $P(\vec{r}) = U(\vec{r})\exp(ik_0 x)$; after subtitution into Eq. 2 and omission of the second derivative $\partial^2 U/\partial x^2$ one obtains the parabolic equation :

$$2ik_0 \frac{\partial}{\partial x} U(x,\vec{\rho}) + \Delta_{\vec{\rho}} U(x,\vec{\rho}) + k_0 \mathcal{E}(x,\vec{\rho}) U(x,\vec{\rho}) = 0 \qquad x \geqslant 0 \qquad (3)$$

$$\Delta_{\vec{\rho}} \equiv \partial^2/\partial y^2 + \partial^2/\partial z^2 \ ; \quad \vec{\rho} \equiv (y, z)$$

and the appropriate initial condition takes the form $U(0, \vec{\rho}) = U_0(\vec{\rho})$ at $x = 0$. Equation 3 has been extensively studied (Tatarski [7], Ishimaru [8]) and can be solved accurately for the first two moments of the field (average field, mutual coherence function) if one assumes that the fluctuation $\mathcal{E}(\vec{r})$ is gaussian and delta-correlated in the x direction, i.e. :

$$\langle \mathcal{E}(x,\vec{\rho})\, \mathcal{E}(x',\vec{\rho}') \rangle = \delta(x-x')\, A(\vec{\rho}-\vec{\rho}') \qquad (4)$$

where $A(\vec{\rho}_d = \vec{\rho}-\vec{\rho}')$ is related to the spectrum $\Phi_\mathcal{E}(K)$ by the following two-dimensional Fourier transform :

$$A(\vec{\rho}_d) = 2\pi \iint_{-\infty}^{+\infty} \Phi_\mathcal{E}(\vec{K}).\exp(i\vec{K}.\vec{\rho}_d)\, d\vec{K} \qquad (5)$$

For isotropic turbulence $A(\vec{\rho}_d)$ can be expressed as :

$$A(\vec{\rho}_d) \equiv A(\rho_d) = 4\pi^2 \int_0^\infty K J_0(K.\rho_d)\, \Phi_\mathcal{E}(K)\, dK \qquad (6)$$

The expression of the mutual coherence function useful for the present investigation is therefore :

$$\Gamma(x,\vec{\rho}_s,\vec{\rho}_d) = \iint_{-\infty}^{+\infty} \tilde{\Gamma}\left(0,\vec{K}_d,\vec{\rho}_d - \frac{x}{k_0}\vec{K}_d\right). \exp\left(i\vec{K}_d.\vec{\rho}_s - H(x,\vec{K}_d,\vec{\rho}_d)\right) d\vec{K}_d \qquad (7)$$

$$H(x,\vec{K}_d,\vec{\rho}_d) = \frac{k_0^2}{4} \int_0^x \left(A(0) - A\left(\vec{\rho}_d - \frac{z}{k_0}\vec{K}_d\right)\right) dz$$

$$\tilde{\Gamma}(0,\vec{K}_d,\vec{\rho}_d) = \frac{1}{4\pi^2} \iint_{-\infty}^{+\infty} \Gamma(0,\vec{\rho}_s,\vec{\rho}_d)\exp(-i\vec{K}_d.\vec{\rho}_s)\, d\vec{\rho}_s$$

with

$$\vec{\rho}_s = \frac{1}{2}(\vec{\rho}_1 + \vec{\rho}_2) \ ; \quad \vec{\rho}_d = \vec{\rho}_1 - \vec{\rho}_2$$

For a spherical wave Eq. 7 can be simplified to the form :

$$\Gamma(x,\vec{\rho}_s,\vec{\rho}_d) = \frac{1}{x^2}\cdot \exp\left(i\frac{k_o}{x}\vec{\rho}_s\cdot\vec{\rho}_d - H\right) \tag{8}$$

$$H = \frac{k_o^2}{4}\int_0^x \left(A(0) - A\left(\frac{z}{x}\vec{\rho}_d\right)\right) dz$$

For the fourth-order coherence function, which gives information about the intensity fluctuations there is no exact solution. Several authors approximate this moment by asymptotic methods (Gochelashvili and Shishov (9), Zavorotni, Klyatskin and Tatarski (10), Whitman and Beran (11)) or numerical methods (Macaskill (12), Tur (13), Uscinski (14)). For weak fluctuations the variance of the log-amplitude is very often predicted ; for strong fluctuations it is common to estimate the normalized variance of irradiance function $\sigma_I^2 = (\langle I^2\rangle - \langle I\rangle^2)/\langle I\rangle^2$. Even though no exact solution of the fourth-order coherence equation exists, a useful approach is made by Gracheva, Gurvich, Kaskarov and Pokasov (15) who suggested that σ_I^2 depends on only one parameter x/ℓ, where ℓ appears as a longitudinal scale $(0.033\, C_\varepsilon^2\, \pi\, k_o^{7/6})^{-6/11}$. For comparison with previous experiments in laser propagation through the atmosphere, it is more convenient to use the parameter β_o^2 which is the estimate of σ_I^2 in Rytov's method. We note that β_o^2 is a function of the incident acoustic wave, and can be expressed simply only for plane waves or spherical waves with a Kolmogorov spectrum for $\phi_\varepsilon(K)$.

Results

Transverse coherence functions

In Fig. 3 the measured values of the coherence function for the spherical wave are plotted in terms of the receiver separation ρ_d for two ranges of propagation (x = 122 cm, 155 cm). The solid curves are computed from Eq. 8, employing a modified von Karman spectrum (Eq. 1), and dotted curves are obtained from Eq. 8 but with a Kolmogorov spectrum $0.033\, C_\varepsilon^2 K^{-11/6}$ We note that the coherence functions decrease rapidly as the propagation length and frequency increase. The transverse coherence function is accurately described by a von Karman spectrum which takes into account the outer scale of turbulence.

The experimental results for a piston-like source are given in Fig. 4. The solid curves are obtained from Eq. 7 using a two-dimensional FFT algorithm with a modified von Karman spectrum (Blanc-Benon, Chaize and Juvé (16)). We note good agreement with the experimental data. In this numerical evaluation of the mutual coherence function $\Gamma(x,\vec{\rho}_s,\vec{\rho}_d)$ we can also calculate the intensity repartition $\langle I(x,\vec{\rho}_s)\rangle = \Gamma(x,\vec{\rho}_s,0)$. In Fig. 5 the measurements of <I> are plotted for two ranges of propagation (x = 90 cm ; x = 160 cm) and one frequency (f = 50 kHz). We observe that the transverse intensity repartition is very well predicted even in the region where lateral lobes appear.

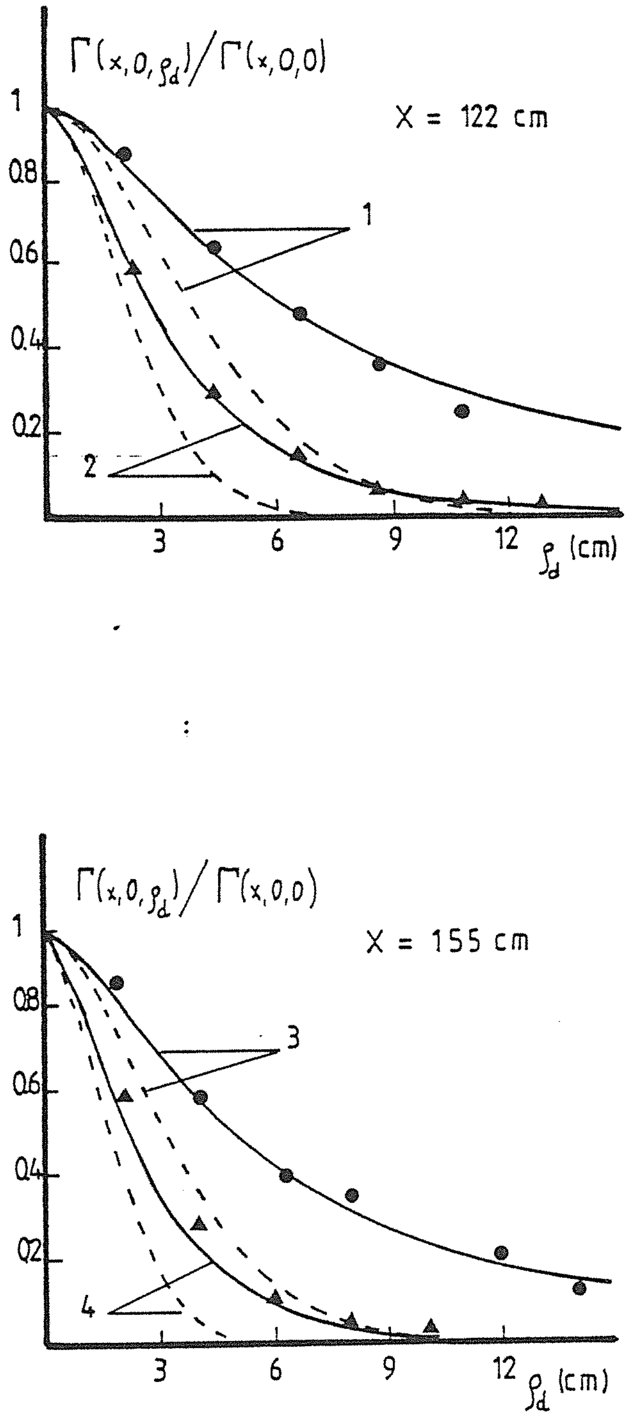

Fig. 3 - Transverse coherence functions for a spherical wave (● f = 23.5 kHz ; ▲ f = 39 kHz). Numerical estimates with a modified von Karman spectrum (——)and a Kolmogorov spectrum (----) with L_T = 5 cm ; $T'/\overline{T}$ = 0.025.

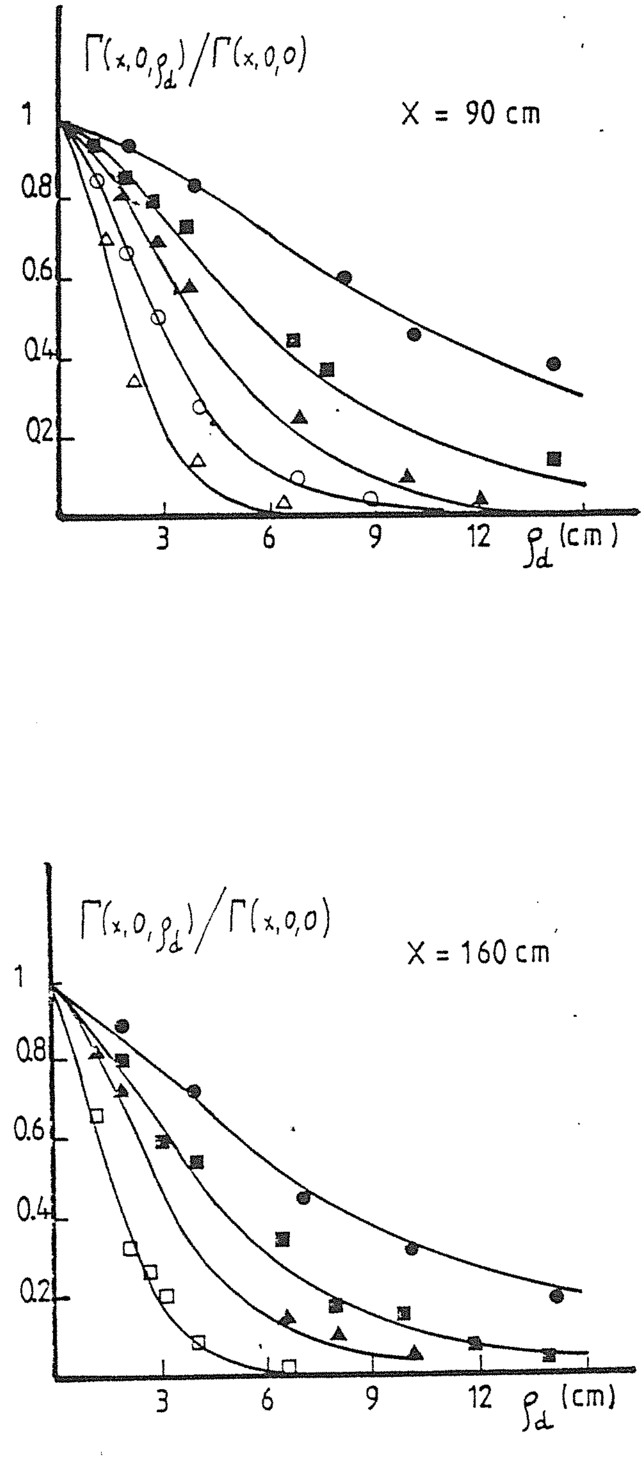

Fig. 4 - Transverse coherence function for a piston-like source (● f = 20 kHz ; ■ f = 30 kHz ; ▲ f = 40 kHz ; ○ f = 50 kHz ; □ f = 60 kHz ; △ f = 70 kHz). Numerical estimates with a two-dimensional FFT algorithm and a modified von Karman spectrum ($L_T = 5$ cm ; $T'/\overline{T} = 0.025$) (——).

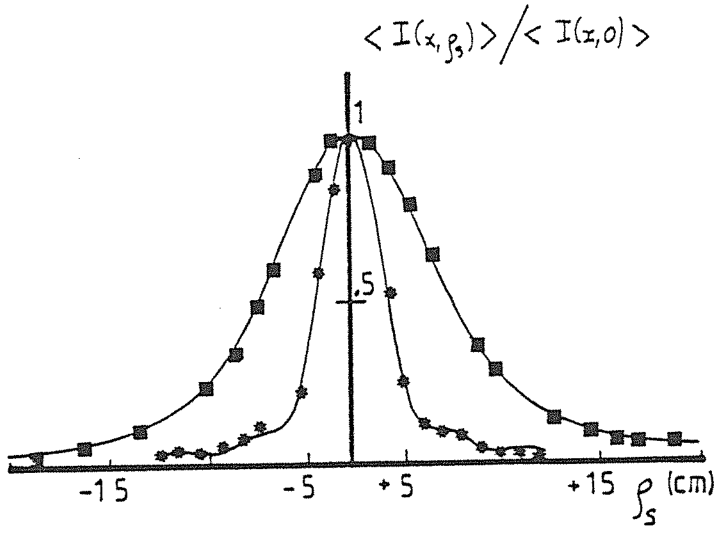

Fig. 5 - Transverse intensity repartitions for a piston-like transducer (■ x = 160 cm ; ✱ x = 90 cm). Numerical estimates with a two-dimensional FFT algorithm (——).

<u>The normalized variance</u>

The normalized variance $\sigma_I^2 = (\langle I^2\rangle - \langle I\rangle^2)/\langle I\rangle^2$ is obtained by a digital treatment of the acoustic pressure signal. In a first step, the pressure signal is heterodyned to a frequency of 5 kHz. Then using a digital acquisition DIFA TR10 30 (10 bit) interfaced to a PDP 11-23 processor, the pressure field is recorded with a sample time of 40 µs. Typically we calculate the different moments of the intensity $\langle (I - \langle I\rangle)^n \rangle$ with 98 blocks of 8192 samples (n is the order of the moment, < > indicates an ensemble average).

In Fig. 6 the measured values of σ_I for a spherical wave are plotted in terms of β_0 calculated with Rytov's method :

$$\beta_0 = \left(0.56\, C_\varepsilon^2\, k_0^{7/6}\, x^{11/6}\right)^{1/2} \tag{9}$$

Different frequencies have been used (f = 23.5 kHz ; 39 kHz ; 75 kHz), and the distance of propagation in the turbulent medium is in the range 0.7 m < x < 2.2 m. The characteristics of the turbulent thermal fields are $L_T = 7.6$ cm, $T'/\overline{T} = 0.017$ and $L_T = 5$ cm, $T'/\overline{T} = 0.025$. The solid curve represents the solution for the large fluctuations ($\beta_0 >> 1$) suggested by Gochelashvili and Shishov [9] :

$$\sigma_I^2 = 1. + 1.9\,(\beta_0^2)^{-2/5} \tag{10}$$

For the weak fluctuations ($\beta_o << 1$) we indicate a linear increase of σ_I as predicted by Rytov's method (dotted curve). When the path of propagation x, the temperature fluctuation $T'/\bar{T}$, or the frequency f increases, the normalized variance σ_I tends to saturate at a level slightly above 1.

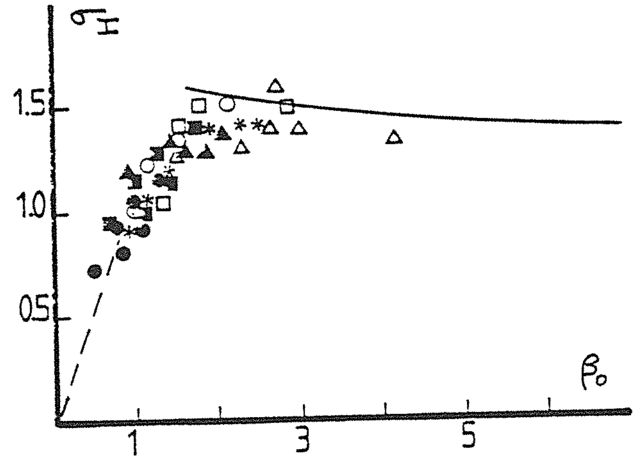

Fig. 6 - Scintillation index σ_I of spherical wave, versus β_o.

a/ L_T = 7.6 cm ; $T'/\bar{T}$ = 0.017
● f = 23.5 kHz ; ▲ f = 75 kHz ; ■ f = 39 kHz.

b/ L_T = 5 cm ; $T'/\bar{T}$ = 0.025
○ f = 23.5 kHz ; * f = 31 kHz
□ f = 39 kHz ; △ f = 75 kHz

—— curve $1 + 1.9\,(\beta_o^2)^{-2/5}$.

Fig. 7 shows the experimental values of σ_I for a piston-like source. For comparison with previous analysis we use the parameter β_o calculated by Tatarski [7] in the plane wave case :

$$\sigma_I^2 = 0.65 \int_0^\infty t^{-8/3}(1-\cos t)\left(1-\exp(0.9\,\beta_o^2\,t^{5/6})\right)dt \quad (11)$$

$$\beta_o = \left(1.23\; C_t^2\; k_o^{7/6}\; x^{11/6}\right)^{1/2}$$

Experimental data are obtained for seven frequencies (f = 20 kHz ; 30 kHz ; 40 kHz ; 50 kHz ; 60 kHz ; 70 kHz ; 80 kHz), several paths of propagation in the range 0.7 m - 2.2 m, and the same turbulent thermal fields as before. We observe that σ_I increases until $\beta_o = 2$ and then we note a dependence on the parameter β_o similar to the plane wave case but with a different saturation level. To explain this dependence of σ_I on β_o it would be necessary to take into account the effect of the outer scale of turbulence as has been done by Whitman and Beran [11] for a plane wave.

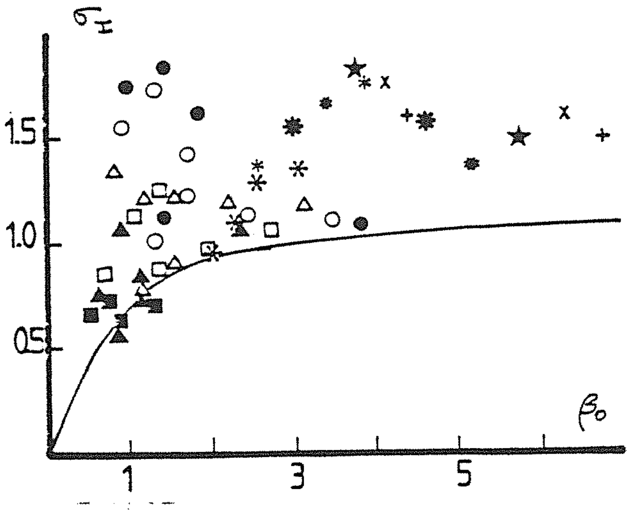

Fig. 7 - Experimental scintillation index σ_I of piston-like source versus β_o.

a/ L_T = 7.6 cm ; $T'/\bar{T}$ = 1.7 10^{-2}

f ■ 20 kHz ; ▲ 30 kHz ; □ 40 kHz
△ 50 kHz ; ○ 60 kHz ; ● 70 kHz

b/ L_T = 5 cm ; $T'/\bar{T}$ = 0.025

f ✳ 20 kHz ; * 30 kHz ; ✸ 40 kHz
● 50 kHz ; ★ 60 kHz ; × 70 kHz ; + 80 kHz.

Preliminary measurements of the intensity probability distribution

There is considerable theoretical and experimental interest in the probability distribution of the normalized intensity I/<I> (Strohbehn, Wang and Speck [17], Furutsu [18]). For weak fluctuations the application of the central limit theorem leads to a log-normal distribution for the intensity I (Tatarski [19]). For the saturation region a Rayleigh distribution in amplitude is often proposed and leads to an exponential distribution for the intensity. In this paper we introduce a generalized gamma distribution that varies smoothly from log-normal to exponential as a function of the parameters b and k :

$$W(I) = \left(b\,\mu^k/\Gamma(k)\right) I^{bk-1}\, e^{-\mu I^b} \quad (12)$$

$$\mu = \left(\Gamma(k+1/b)/\Gamma(k)\right)^b$$

Γ Gamma function

The parameters b and k can be deduced from the measurements of the moments $m_2 = \langle I^2\rangle/\langle I\rangle^2$ and $m_3 = \langle I^3\rangle/\langle I\rangle^3$. These are obtained by solving the two non linear equations :

$$m_2 = \frac{\Gamma(k)\,.\,\Gamma(k+2/b)}{\Gamma^2(k+1/b)} \tag{13}$$

$$m_3 = \frac{\Gamma^2(k)\,.\,\Gamma(k+3/b)}{\Gamma^3(k+1/b)}$$

In Fig. 8 experimental histograms of the normalized intensity $I/\langle I\rangle$ are plotted for a spherical wave at the frequency of 23.5 kHz and for four distances of propagation in the turbulent field (x = 71.5 cm ; 135 cm ; 177.5 cm ; 220 cm). The solid curves correspond to the distributions calculated from Eq. 12 with the estimates b and k deduced from the solution of Eq. 13. We note a good agreement between measurement and prediction. Similar results were also observed in the case of acoustic propagation through a turbulent velocity medium (Blanc-Benon [2]). To justify the fitting of the data set by a generalized gamma distribution we used a Kolmogorov-Smirnov "goodness-of-fit" test. We can display the results of this goodness test by plotting the theoretical cumulative distribution function F(I) and the associated upper and lower 90 % confidence bands, $F^*(I) + \Delta_\alpha$ and $F^*(I) - \Delta_\alpha$, where F^* is the empirical cumulative distribution. An example is given in Fig. 9.

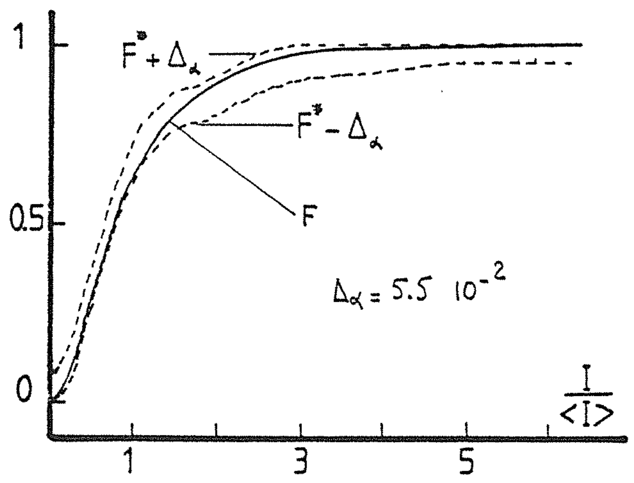

Fig. 9 - Cumulative distributions for generalized gamma F and upper and lower 90 % confidence functions $F^* + \Delta_\alpha$, $F^* - \Delta_\alpha$ associated to the experimental function F^*.

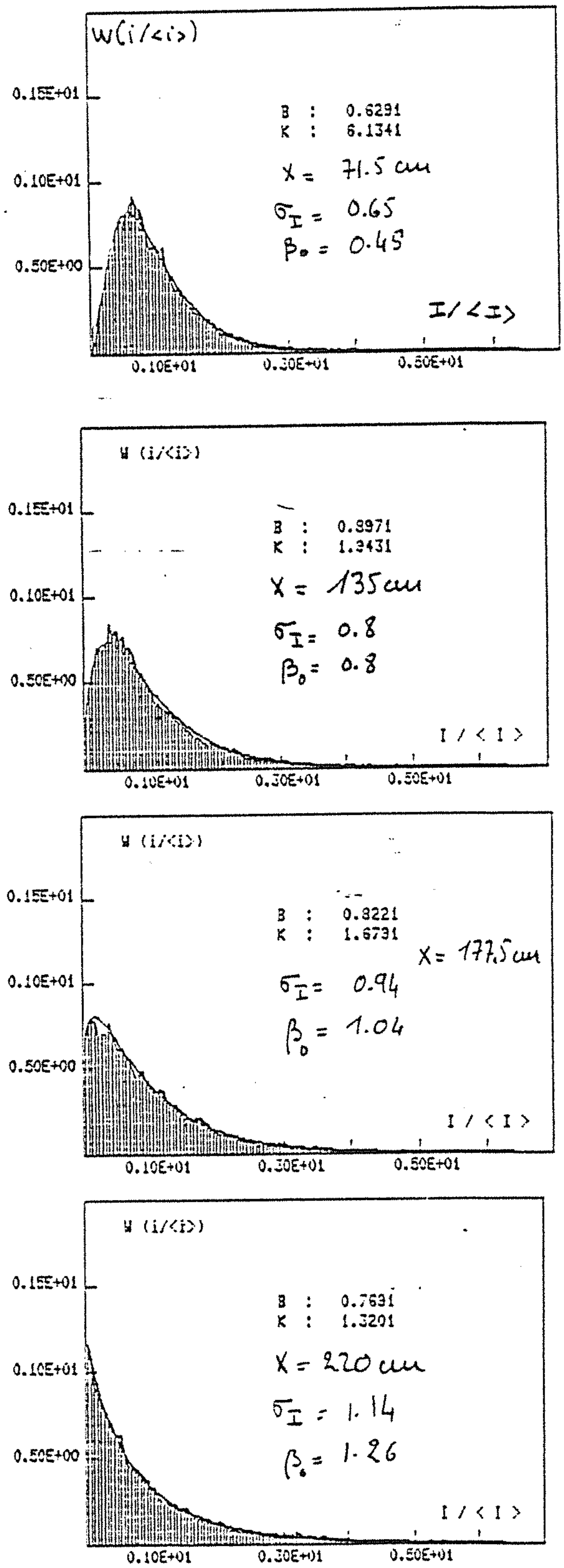

Fig. 8 - Evolution of the probability distribution of the normalized intensity $I/\langle I\rangle$, for a spherical wave (23.5 kHz) with four ranges x and L_T = 7.6 cm, $T'/\overline{T}$ = 0.017.

Conclusions

The effect of temperature fluctuations on the propagation of acoustic waves has been investigated with well controlled experiments involving a heated air grid. The importance of the entire spectrum of turbulence has been demonstrated for the transverse coherence function. It appears that a satisfactory theoretical estimate may be obtained for different incident waves (spherical or piston like sources) using a modified von Karman spectrum in the calculations based on the parabolic approximation of the Helmholtz equation. Also we have presented results for the scintillation index and the probability distribution of the intensity for which a generalized gamma distribution is proposed. Finally, we infer from the experimental data that it will be important to take into account the effect of the outer scale of turbulence in the calculation of the fourth-order coherence function.

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