TIME-DOMAIN SIMULATIONS OF A FLOW DUCT WITH EXTENDED-REACTING ACOUSTIC LINERS

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ABSTRACT

A finite-difference time-domain (FDTD) approach is proposed to model extended-reacting liners under a grazing mean flow. It is based on an equivalent fluid model of the material together with the auxiliary differential equation method (ADE). The methodology is applicable to any liner material amenable to an equivalent fluid description, such as rigid-frame porous materials and metamaterials. It has been validated against a semi-analytical solution in a 1D test case and against experimental measurements in a duct.

1. INTRODUCTION

A number of semi-analytical and numerical models have been proposed in the literature to describe sound attenuation in ducts by porous materials, including the boundary element method [1], the finite element method [2], and the mode-matching method [3]. Sound propagation in rigid-frame porous materials can be tackled using an equivalent fluid model [4–6]. Here we propose a time-domain approach, which is the natural approach to study transient propagation problems. Various techniques have been proposed to avoid the expensive computation of the convolution integrals that appear in the time-domain equations [7,8]. Here we use the additional differential equation method (ADE) coupled with partial fraction expansions of the effective density and the effective compressibility. As opposed to other approaches, ADE allows to maintain a high-order accuracy in time [7]. Furthermore, the solution can be obtained with high accuracy at a low computational cost, using high order finite-difference schemes and low-storage high-order temporal schemes [9] (FDTD).

Others examples of sound-absorbing materials that can be described by an equivalent fluid approach are double-porosity materials [10], and locally-resonant acoustic metamaterials [11]. Time-domain simulations of sound propagation on prototypical metamaterial models have been recently shown to be well-posed [12].

The configuration considered in this work is shown in Fig. 1. The main duct has a side cavity filled with sound-absorbing material. In the main duct the linearized Euler equations are applied, with a mean flow profile having zero velocity on the rigid walls as well as on the air-material interface.

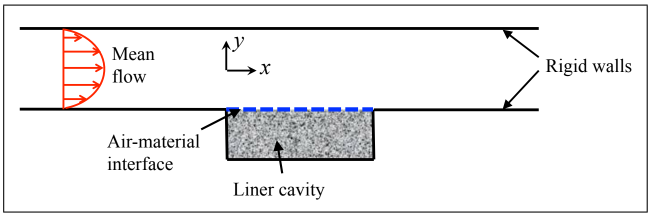

Figure 1. Sketch of the geometry of the 2D lined duct.

2. PROPAGATION EQUATIONS IN THE LINER MATERIAL

Sound propagation inside the material is described using an equivalent fluid, characterized by the effective density, ρ_e and the effective compressibility, C_e:

$$\begin{aligned} j\omega\rho_e(\omega)\hat{u} + \frac{\partial \hat{p}}{\partial x} &= 0, \\ j\omega\rho_e(\omega)\hat{v} + \frac{\partial \hat{p}}{\partial y} &= 0, \\ j\omega C_e(\omega)\hat{p} + \frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{v}}{\partial y} &= 0. \end{aligned} \tag{1}$$

The effective density and compressibility are expressed in terms of a partial fraction expansion. For the density we have (an analogous expression applies to the effective compressibility):

$$\rho_e(\omega) = \rho_{e\infty} + \sum_{k=1}^{Nr\rho} \frac{A_{\rho k}}{\lambda_{\rho k} - j\omega}$$
$$+ \sum_{l=1}^{Ni\rho} \left(\frac{B_{\rho l} + \mathrm{j}C_{\rho l}}{\alpha_{\rho l} + j\beta_{\rho l} - j\omega} + \frac{B_{\rho l} - \mathrm{j}C_{\rho l}}{\alpha_{\rho l} - j\beta_{\rho l} - j\omega} \right). \tag{2}$$

Introducing the partial fraction expansions into Eqs. (1) and taking the inverse Fourier transform leads to:

$$\rho_{e\infty}\frac{\partial u}{\partial t} + \sum_k A_{\rho k}\phi^x_{\rho k} + 2\sum_l \left(B_{\rho l}\psi^{xr}_{\rho l} + C_{\rho l}\psi^{xi}_{\rho l}\right)$$
$$+ \left(\sum_k A_{\rho k} + 2\sum_l B_{\rho l}\right) u + \frac{\partial p}{\partial x} = 0, \tag{3}$$

$$\rho_{e\infty}\frac{\partial v}{\partial t} + \sum_k A_{\rho k}\phi^y_{\rho k} + 2\sum_l \left(B_{\rho l}\psi^{yr}_{\rho l} + C_{\rho l}\psi^{yi}_{\rho l}\right)$$
$$+ \left(\sum_k A_{\rho k} + 2\sum_l B_{\rho l}\right) v + \frac{\partial p}{\partial y} = 0, \tag{4}$$

$$C_{e\infty}\frac{\partial p}{\partial t}+\sum_k A_{Ck}\phi_{Ck}+2\sum_l\left(B_{Cl}\psi^r_{Cl}+C_{Cl}\psi^i_{Cl}\right)$$
$$+\left(\sum_k A_{Ck}+2\sum_l B_{Cl}\right)p+\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=0, \quad (5)$$

where the auxiliary functions $\phi^x_{\rho k},\phi^y_{\rho k},\phi_{Ck}$ come from the real poles, and $\psi^{xr}_{\rho l},\psi^{yr}_{\rho l},\psi^{xi}_{\rho l},\psi^{yi}_{\rho l},\psi^r_{Cl},\psi^i_{Cl}$ come from the complex conjugate poles. All of them are defined as convolution integrals which are, each of them, equivalent to a partial differential equation. For example, the partial differential equation corresponding to the axial component of the kth real pole of the effective density is

$$\frac{\partial\phi^x_{\rho k}}{\partial t}+\lambda_{\rho k}\phi^x_{\rho k}+\lambda_{\rho k}u = 0, \quad k=1,...,Nr\rho \quad (6)$$

The ensemble of partial differential equations is discretized using high-order spatial and temporal schemes [9].

3. MODEL FOR THE AIR-MATERIAL INTERFACE

The air-material interface is treated through the characteristic variables. The characteristics traveling towards the boundary from the air side and the material side are, respectively:

$$q^i_a = p_a-\rho_0 c_0 v_a, \quad (7)$$
$$q^i_m = p_m+\rho_{e\infty}c_{e\infty}v_m. \quad (8)$$

The characteristics traveling away from the interface must then be determined from the ones traveling to the interface. The presence of a resistive screen at the interface causes a pressure jump, $p_a-p_m=R_{sh}v$, while the normal velocity is continuous, $v_a=v_m=v$. The characteristics traveling away from the interface are in this case:

$$q^o_a=\frac{R_{sh}-\rho_0c_0+\rho_{e\infty}c_{e\infty}}{R_{sh}+\rho_0c_0+\rho_{e\infty}c_{e\infty}}(q^i_a-q^i_m)+q^i_m, \quad (9)$$
$$q^o_m=\frac{R_{sh}+\rho_0c_0-\rho_{e\infty}c_{e\infty}}{R_{sh}+\rho_0c_0+\rho_{e\infty}c_{e\infty}}(q^i_m-q^i_a)+q^i_a. \quad (10)$$

4. VALIDATION AND PERSPECTIVES

The FDTD algorithm has been first validated for the 1D case against a semi-analytical solution, in which case the porous layer is equivalent to a surface impedance. In a second stage, it has been validated against experimental results in a lined duct. Figure 2 shows the measured transmission and reflection coefficients and the FDTD predictions. The agreement is excellent.

Current work and future perspectives are focused on the case of a mean flow, and to the case of metamaterial-based liners.

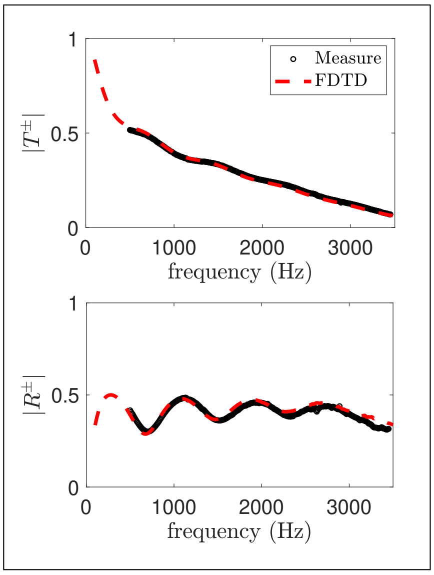

Figure 2. Comparison of the transmission and reflection coefficients of the FDTD code with the experimental results of Aurégan and Singh [13] for a Nickel-Chrome alloy foam ($\phi = 0.99, \alpha_\infty = 1.17, \Lambda = 1.0\cdot10^{-4}$ m, $\Lambda' = 2.4\cdot10^{-4}$ m), corresponding to a cavity length of 200 mm and a depth of 25 mm.

5. ACKNOWLEDGMENT

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