Ray theory analysis and modelling of the secondary sonic boom propagation for realistic atmosphere conditions.

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Abstract. The shock waves generated by a supersonic aircraft are reflected in the upper part of the atmosphere. Back to the ground, they are indirect sonic booms called secondary sonic booms. The recorded signals of secondary sonic booms show a low amplitude and a low frequency. They sound like rumbling noises due to amplitude bursts. These signals strongly depend on the atmospheric conditions, in particular to the amplitude and to the direction of the wind in the stratopause. In the present work, the propagation of secondary sonic booms is studied using realistic atmospheric models up to the thermosphere. The secondary carpet position is investigated by solving temporal ray equations. An amplitude equation including nonlinearity, absorption and relaxation by various chemical species is coupled to the ray solver to get the secondary boom signature at the ground level. Multipath arrivals are directly linked to wind field or 3D inhomogeneities.

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INTRODUCTION

The atmospheric sound speed profile creates waveguides for the shock waves generated by a supersonic aeroplane. The upward shock waves is reflected back to the ground by the temperature gradients in the stratopause or in the thermosphere. The resulting noise disturbance is called secondary sonic boom. It is merely an infrasonic signal and it sounds like a rumble noise associated to bursts [1, 2]. In the present work, the propagation of secondary sonic booms is studied using realistic atmospheric models up to the thermosphere. The secondary carpet position is investigated by solving temporal ray equations. An amplitude equation including nonlinearity, absorption and relaxation by various chemical species is coupled to the ray solver to obtain the secondary boom signature at the ground level. The predicted signatures are compared to recorded signals of secondary sonic booms. A good agreement is found for the amplitude and for the time duration. The bursts seem to be related to multipath arrivals due to direct and indirect secondary sonic boom. The rumbling noise can be interpreted as the effect of finer structures of the atmosphere as gravity waves.

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PREDICTION METHOD

The prediction method is based at least on two assumptions. The first one is that the atmosphere varies over length scales greater than the actual length of the supersonic boom. The second assumption is that the shocks are only weak shocks, *i.e.* the shock pressure amplitude is less than a few percent of the underlying pressure field. The model is derived from the generalized Navier-Stokes equations including earth rotation and from the state equations for the different molecular species of the atmosphere to get relaxation effects. From these equations, a two-step asymptotic development can be conducted. At the first step, a ODE system of six equations is obtained. This system provides the shock wave trajectory called boom rays in this work. The boom rays start at the aeroplane position at a given time τ and are parametrized by their launching angle θ, the azimuthal angle around the aeroplane. The boom rays are similar to the acoustical rays due to the weak shock assumption. The main difference is that their launching polar angle ϕ is fixed by the relation $\phi = \arccos(1/M)$ where M is the Mach number. At a given time, all the shock waves generated by the aeroplane along its trajectory are located on a surface called the Mach surface. The second step of the development leads to an amplitude equation which must be solved along each ray. This equation is used to models the deformation of the shock wave during its propagation into the inhomogeneous atmosphere. It is a nonlinear paraxial equation that includes dispersion, absorption and relaxation effects. This equation is no more valid when the associated ray goes through a caustic and a Hilbert transform is then applied to the shock wave to simulate the crossing of the caustic.

RESULTS FOR SECONDARY BOOMS

The initial conditions are typical of a Concorde flighing at a Mach 2 speed. The aircraft is located at a latitude of around 45 degres North for summer atmospheric conditions. As the aircraft altitude during a supersonic flight is around 15*km*, two waveguides are allowed to the shock rays; the stratospheric waveguide between an altitude of few kilometers up to the stratosphere (~ 50*km*) and the thermospheric waveguide between the ground level and the thermosphere (above 100*km*)(see Fig.1).

The figure 2 shows the Mach surface and the primary and secondary carpet position for this configuration. Actually, the whole part of the secondary, the direct and the indirect one, are created by thermospheric rays which are reflected at an altitude of 120*km*. Due to their long travel into the thermosphere where absorption and relaxation are the main effects, a low amplitude wave is expected. The shock rays that are restrained to the waveguide 1 never reach the ground. The secondary Mach carpet is between 250*km* and 300*km* beyond the plane (here at the origin). The interval of times between the arrivals of the direct (first secondary carpet) and the indirect (second secondary carpet) shock ray is about 2 minutes for a ground observer at the vertical of the aircraft trajectory. This is a first occurence of multipath arrivals.

The secondary sonic booms which travel in the upper part of the atmosphere before being reflected back to the ground are more dependent on the atmospheric structure and the wind velocity in altitude. Depending of the value of the maximum wind velocity

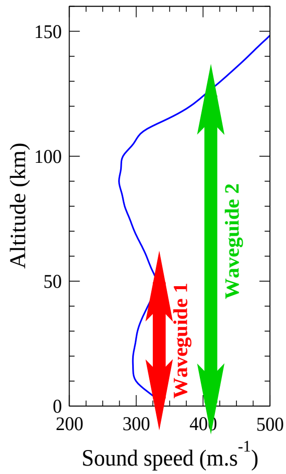

FIGURE 1. Sound speed profile.

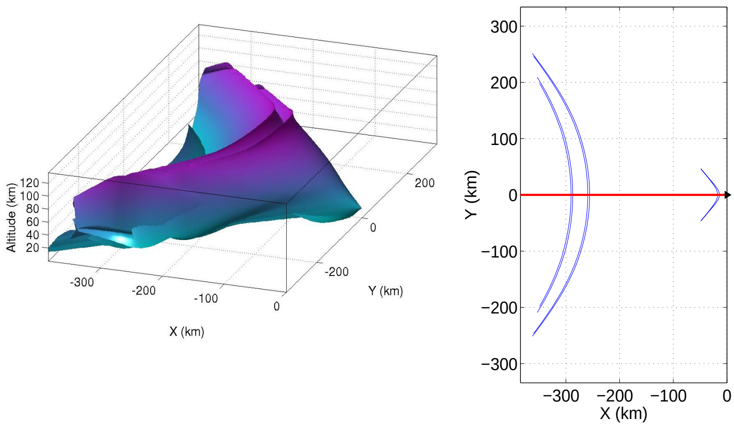

FIGURE 2. Evolution of the Mach surface. Primary and secondary boom carpets at ground .

at the stratopause altitude, the secondary sonic boom is reflected in the thermosphere or in the stratopause and this leads to two different kinds of secondary sonic boom. Considering the mean atmospheric sound speed, two waveguides exist for the boom rays (see Fig. 1). The thermosphere is the upper boundary of the first waveguide. Most of the boom rays trapped in this waveguide reach the ground. As a large part of the boom ray is at altitudes (over 50 km) where absorption and relaxation are the dominant effects, the secondary boom signature is an infrasonic signal (cf. figure 3). Due to its low frequency, this wave can travel over very long distances in the atmosphere. The peak pressure amplitude is around 0.1*Pa* and the frequency is around 0.05*Hz*. These values are in accordance with long distance measurements of secondary boom [3, 4].

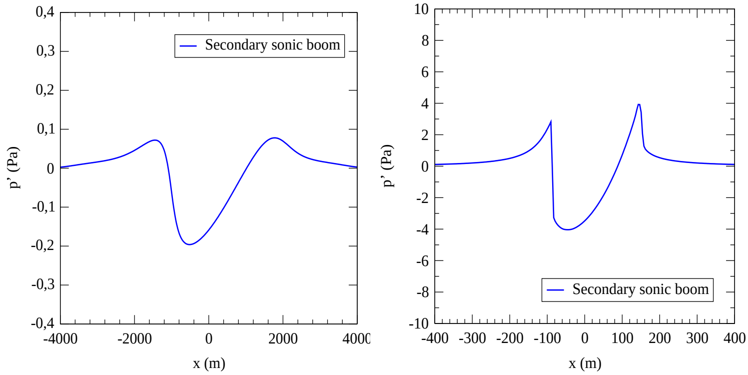

FIGURE 3. Secondary boom signatures : comparison between thermospheric ray (left graph) and stratospheric ray (right).

The second waveguide is between the stratopause and the troposphere. As the maximum sound speed in the stratopause is less than the sound speed at ground level, boom rays trapped in this waveguide reach the ground only if the shock wave is appropriately convected by wind at the reflection altitude (around $50km$). The resulting secondary sonic boom signature has a higher amplitude than the thermospheric one with a peak pressure amplitude of $4Pa$ (cf. figure 3). Its frequency is also much higher ($0.5Hz$) and shocks persist. These values are also in agreement with some measurements of secondary sonic boom [5].

INFLUENCE OF GRAVITY WAVES

The model presented here has been applied to realistic atmospheric conditions. The atmospheric fields are provided by the IAP (Leibniz-Institut für Atmosphärenphysik, Universität Rostock). They include mean pressure, density, temperature and wind data at a given latitude ($69°N$) up to an altitude of $150km$. They are discretized every $10km$ along the latitude and every $100m$ along the altitude. In addition to these mean values, the IAP provides also simulated data of atmospheric gravity waves. They correspond to finer length scale inhomogeneities of the atmosphere and their influence is investigated by adding them to the mean atmospheric fields. These data have to be interpolated over the whole propagation domain. This is performed by using third-order polynomials as continuity has to be preserved up to the second spatial derivatives of the fields.

The primary influence of the gravity waves on the secondary sonic booms concerns the secondary carpets. The figure 4 compares the secondary sonic boom carpets when the atmospheric data do or do not contain gravity waves.

In figure 4 the left plot is obtained when gravity waves are not included to atmospheric data. The leftmost carpet corresponds to the primary boom. The two carpet patches at a medium distance from the aeroplane ($x \sim 150km$) are created by boom rays reflected by the stratopause. The last carpets ($x \sim 300km$) are due to the thermospheric boom rays. Each carpet is composed of a direct and an indirect part, the indirect part being due to the rays reflected back to the atmosphere from the primary carpet position. The distance between the direct and the indirect secondary carpet is around 20*km*. This is in agreeement with the time duration between the bursts of secondary sonic boom (around 30*s*). The carpets are smooth and a clear difference appears between each one.

In figure 4 the right plot is obtained when gravity waves are included. The primary carpet is not influenced by the gravity waves. The gravity wave influence appears on the secondary carpet geometry. The secondary carpets look more complex and the direct and indirect secondary carpets cannot be distinguished anymore. A greater number of rays reaches a given earth location. It may be expected that it is the main reason of the rumble noise.

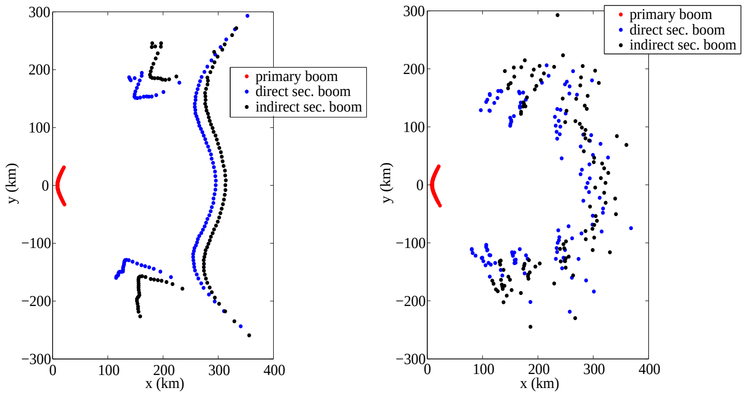

FIGURE 4. Secondary boom carpets for realistic atmosphere (left) ; influence of gravity waves (right).

CONCLUSION

The present results show that the signature model predicts well the amplitude and the frequency of the secondary sonic booms. The rumble noise and the bursts may be explained by multipath arrivals of secondary sonic booms. The bursts seems to be linked to the arrival of direct and indirect rays and the rumble noise to the presence of gravity waves or other fine scale effects in the atmosphere.

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