Coherence Aspects of Acoustic Wave Transmission Through a Medium with Temperature Fluctuations

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Abstract

The statistical properties of an ultrasonic wave that traverses a medium exhibiting temperature fluctuations are investigated. A heated grid in air is utilized to simulate the random thermal pattern typical of atmospheric and marine environments. The effect of temperature fluctuations on the attenuation of the transmitted pressure field and its mutual coherence is examined.

Experimental results are compared with theoretical estimates based on a solution of the stochastic Helmholtz equation in the parabolic or paraxial approximation. These comparisons suggest that the entire spectrum of the turbulence and the particular form of the incident wave (spherical or collimated) play significant roles.

Introduction

The propagation of sound waves through temperature fluctuations has been investigated under laboratory conditions by several authors e.g. Stone & Mintzer (1), Chotiros & Smith (2), Sederovitz & Favret (3), with a principal emphasis on the statistics of the wave amplitude fluctuations and the related dependence on frequency, range of propagation and turbulence characteristics.

The laboratory experiments employed a water tank in which the random temperature field was produced by a heater array mounted at the bottom. In this arrangement discrete impurities appear, such as minute air bubbles due to thermal degassing, and present an additional and not well controlled parameter, as pointed out by Neubert & Lumley (4). An installation with a grid in air offers advantages and has been chosen ; since anechoïc conditions may be easily realized around the set up continuous wave

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signals may be used as well as the sound pulses which are exclusively generated in a finite tank.

Measurements in the atmosphere and ocean have also been effected by Daigle, Piercy & Embleton (5) and by Urick (6) ; in such circumstances, however, the uncertainties with regard to relevant environmental parameters, namely the velocity and temperature variations, make for difficulty in assessing their individual influences on wave attenuation or mutual coherence. This investigation was undertaken to clarify the latter aspects, based on a well controlled experimental facility. To facilitate the analysis and achieve practical features in the model, it is assumed that the acoustic wave length λ remains small compared with the integral scale L_T of the temperature field which, in turn, is smaller than the range of propagation X i.e. $X >> L_T >> \lambda$.

Theoretical estimates can then be based on the parabolic approximation which is well suited for the present conceptions and experimental conditions ; and the main objective is to contrast theory and experiment with a view to clarifying the influence of the exact shape of the acoustic beam as well as the entire spectrum of the thermal turbulence field.

Theoretical considerations

The propagation of a time harmonic acoustic wave in a random medium characterized by small temperature fluctuations ($T'/T_0 << 1$) and an integral turbulent scale L_T large in comparison with the acoustic wave length λ, is governed by the stochastic Helmholtz equation (Tatarski (7)),

$$(1) \qquad \begin{aligned} &(\Delta + k_o^2 (1 + \varepsilon(\vec{r})))p(\vec{r}) = 0 \\ &\varepsilon(\vec{r}) = - T'(\vec{r})/T_o \end{aligned}$$

where $p(\vec{r})$ denotes the space dependence of the pressure, k_0 the wave number, $\varepsilon(\vec{r})$ the fluctuation in the index of refraction and $\vec{r} = (x,y,z)$. For paraxial transmission along the x-direction, it follows that $p(\vec{r})$ can be expressed as $p(\vec{r}) = U(\vec{r}) \exp (ik_0x)$; after substitution into Eq. 1 and omission of the second derivative $\partial^2 U/\partial x^2$ one obtains,

$$2ik_o \frac{\partial}{\partial x} U(x,\vec{\rho}) + \Delta_{\vec{\rho}} U(x,\vec{\rho}) + k_o \varepsilon(x,\vec{\rho}) U(x,\vec{\rho}) = 0 \quad x \geqslant 0$$

(2)

$$\Delta_{\vec{\rho}} = \partial^2/\partial y^2 + \partial^2/\partial z^2 \quad ; \quad \vec{\rho} = (y,z)$$

and the appropriate initial condition takes the form $U(0,\vec{\rho}) = U_0(\vec{\rho})$ at $x = 0$. The equation (2), of parabolic type, has been extensively studied (Tatarski (7), Ishimaru (8)) and can be solved for the statistical characteristics of the field if one assumes that the fluctuation $\varepsilon(\vec{r})$ is gaussian and delta-correlated in the x direction, i.e. :

$$(3) \quad < \varepsilon(x,\vec{\rho})\ \varepsilon(\varsigma',\vec{\rho}') > = \delta(x - x')\ A(\vec{\rho} - \vec{\rho}')$$

where $A(\vec{\rho}_d = \vec{\rho} - \vec{\rho}')$ is related to the spectrum $\phi_\varepsilon(K)$ by the following two-dimensional Fourier transform

$$(4) \quad A(\vec{\rho}_d) = 2\pi \int \phi_\varepsilon(K) \exp(i\vec{K}.\vec{\rho}_d)\, d\vec{K}$$

For an isotropic turbulence $A(\rho_d)$ can be expressed as :

$$(5) \quad A(\rho_d) = 4\pi^2 \int_0^\infty K J_o(K\rho_d)\ \phi_\varepsilon(K)\, dK$$

In many practical cases $\phi_\varepsilon(K)$ may be approximated by the modified von Karman spectrum :

$$\phi_\varepsilon(K) = 0.033\ C_\varepsilon^2\ (K^2 + 1/L_o^2)^{-11/6} \exp(-K^2/Km^2)$$

(6)

$$C_\varepsilon^2 = 1.91\ (T'/2T_o)^2\ L_o^{-2/3} \quad ; \quad Km = 5.92/\ell_o$$

C_ε^2 is the structure constant, L_0 is the outer scale of turbulence related to the integral scale $L_T (L_0 = 1.339\ L_T)$ and l_0 is the inner scale.

The ensemble average of Eq. 2 provides an equation governing the coherent field $<U(x,\vec{\rho})>$:

$$(7) \quad 2ik_o \frac{\partial}{\partial x} < U(x,\vec{\rho}) > + \Delta_{\vec{\rho}} < U(x,\vec{\rho}) > + k_o^2 < \varepsilon(x,\vec{\rho}) U(x,\vec{\rho}) > = 0$$

Using the Furutsu-Novikov formula (9) to evaluate the term $<\varepsilon U>$ Tatarski transforms Eq. 7 into :

$$(8) \quad (2ik_o \frac{\partial}{\partial x} + \Delta_{\vec{\rho}} + i\frac{k_o^3}{4} A(o)) < U(x,\vec{\rho}) > = 0$$

and the appropriate solution is :

$$< U(x,\vec{\rho}) > = U_o(\vec{\rho}) \exp(-\alpha x)$$

(9)

$$\alpha = k_o^2 \frac{A(o)}{8} = 0.391\ C_\varepsilon^2\ k_o^2\ L_o^{5/3} = k_o^2 \left(\frac{T'}{2T_o}\right)^2 L_T$$

We note that the attenuation of the coherent field depends explicitly on turbulent parameters L_T and T'/T_0 and that the transmitted field U becomes almost incoherent when x is large. In a similar way it can be shown that the mutual coherence function $\Gamma(x, \vec{\rho}_s, \vec{\rho}_d) = \langle U(x, \vec{\rho}_1)\, U^*(x, \vec{\rho}_2) \rangle$ $(\vec{\rho}_d = \vec{\rho}_1 - \vec{\rho}_2$, $\vec{\rho}_s = \frac{1}{2}(\vec{\rho}_1 + \vec{\rho}_2))$ obeys the equation :

$$(10) \quad \left(2ik_o \frac{\partial}{\partial x} + \nabla_{\vec{\rho}_s} \nabla_{\vec{\rho}_d} + i \frac{k_o^3}{2} (A(o) - A(\rho_d))\right) \Gamma(x, \vec{\rho}_s, \vec{\rho}_d) = 0$$

with an initial condition $\Gamma(0, \vec{\rho}_s, \vec{\rho}_d) = \Gamma_0(\vec{\rho}_s, \vec{\rho}_d)$ at x = 0. An exact solution of Eq. 10 is available for both plane and spherical waves ; in the case of a beam the solution assumes a particular simple form at points of observation, in any transverse plane, which are equidistant from the axis (Fante (10)). The useful expressions of the transverse coherence function $\Gamma(x, \rho_d)$ for the present investigation are therefore :

$$(11) \quad \Gamma(x,\rho_d) = \frac{1}{x^2} \exp\left(-4\pi^2 k_o^2 \int_o^x du \int_o^\infty (1 - J_o(K\tfrac{u}{x}\rho_d))\, \phi_\varepsilon(K) K dK\right)$$

for a spherical wave and,

$$(12) \quad \Gamma(x,\vec{\rho}_d) = \frac{w_o^2}{8\pi} \iint_{-\infty}^{+\infty} \exp(-A\rho_d^2 - BK_d^2 + C\vec{K}_d \cdot \vec{\rho}_d - H)\, d\vec{K}_d$$

$$H = \frac{k_o^2}{4} \int_o^x du \int_o^\infty (1 - J_o(K|\vec{\rho}_d - \frac{u}{k_o}\vec{K}_d|))\phi_\varepsilon(K) K dK$$

$A = 1/2w_o^2$, $B = w_o^2 (1 + \alpha^2 x^2)/8$, $C = \alpha x/2$, $\alpha = \lambda/\pi w_o^2$ for a collimated gaussian beam $U_0(\rho) = \exp(-\rho^2/w_0^2)$.

Experimental arrangement

The heated grid as well as the locations of the acoustic transmitter and receiver are sketched in Fig. 1. The grid consists of a biplane arrangement of conductors with a square mesh of 9 cm scale. The overall dimensions of the grid are 1.1 m x 2.2 m and the maximum power consumption equals 32 kW. The grid was placed horizontally in a large anechoïc room (11) and the mixing of the free convection plumes above it generates the turbulent thermal field. The acoustic propagation measurements were made at the height H = 1.75 m which corresponds to 20 times the scale of mesh. The mean temperature rise above ambiant was 27°C and the temperature fluctuations had a rms value of 1.7 10^{-2}. The homogeneity of the thermal field was achieved within 0.5°C excluding 1.5 meshes near the edges. The one dimensional spectrum of T' was measured with a FFT analyser Nicolet 660A, in the range .5-200 Hz with a constant bandwidth of .5 Hz. Frequencies were converted into wave numbers Kl by a Taylor hypothesis based on the mean upward velocity (1.25 m/s measured with a hot wire). The results are given in Fig. 2 and compared with the one dimensional spectrum deduced from the modified von Karman form (Eq. 6) by $F_{T'}(K_1) = \int_{K_1}^{\infty} K\, \phi_\varepsilon(K)\, dK$. The integral scale L_T deduced from the spectrum or by integration of spatial correlation equals 7.6 cm. The inner scale l_0 related to the high frequency cutoff of the spectrum was estimated to be 0.1 cm. The index changes which may be induced by the velocity fluctuations in the x direction prove to be negligible ; indeed an upper limit is given by u'_z/c (u'_z component in the upward direction) and measurements indicate that u'_z/c = 6.10-4.

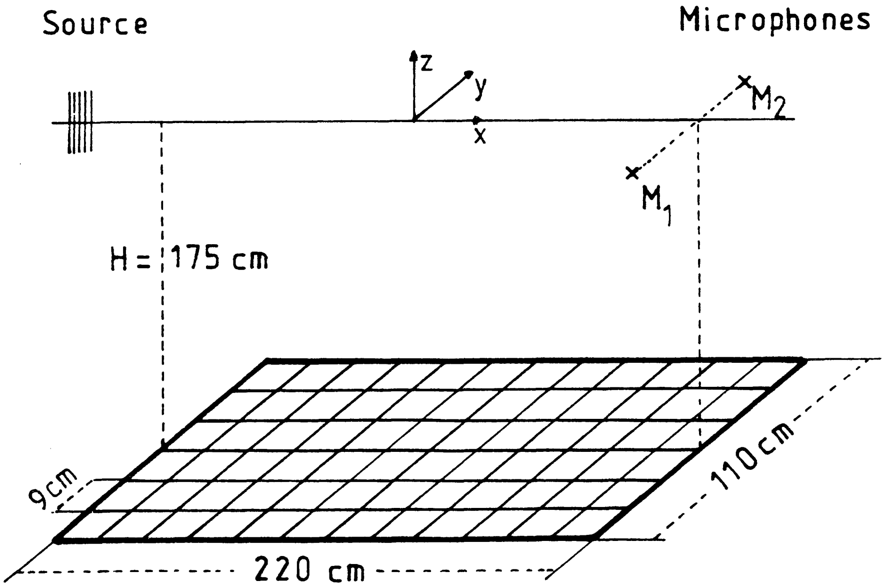

Fig. 1 - Experimental set up.

The spherical acoustic waves were generated by TDK ultrasonic sources (f = 23.4 kHz, 39 kHz) and the collimated beam by a Sell home made transducer whose area is 100 cm^2 (Blanc-Benon

(12)). Frequency is adjustable in the range 20-100 kHz. The transmitted signals were received on 1/4" microphones (Bruel & Kjaer 4135) located in a normal plane to the x-axis. The coherent part was obtained by cross correlating the incident wave with the transmitted wave. The transverse coherence functions were measured over a large range of ρ_d ($0 < \rho_d < 3\ L_T$). A Nicolet 660A FFT analyser as well as a HP 3721A correlator were used in these measurements. The various distances of propagation through turbulence in the range 0.5 m < x < 2.2 m were obtained by a longitudinal displacement of the microphones, the whole area of the grid being kept heated to insure the homogeneity of the thermal field.

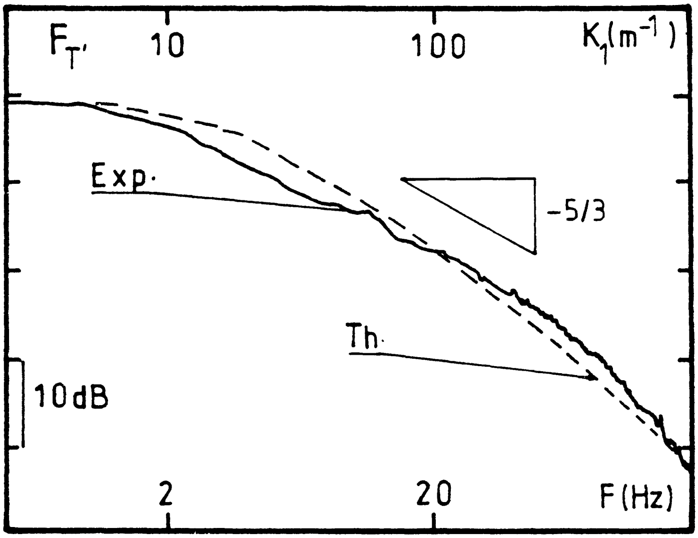

Fig. 2 - Comparison between the experimental (one dimensionnal) spectrum of temperature fluctuations, $F_{T'}$, and the theoretical spectrum deduced from

$$F_{T'}(K_1) = \int_{K_1}^{\infty} K\ \phi_\varepsilon(K)\ dK$$

with the von Karman function $\phi_\varepsilon(K)$.

Results

1. Attenuation of the coherent wave

The experimental results concerning <U> are displayed in Fig.3, which correlates the values of log (<U>/U_0) with αx. An exponential rise in attenuation is closely realized on the range

$0 < \alpha x < 4$; the significant and nearly uniform attenuation on the range $\alpha\pi > 4$ cannot be accounted for with the parabolic approximation.

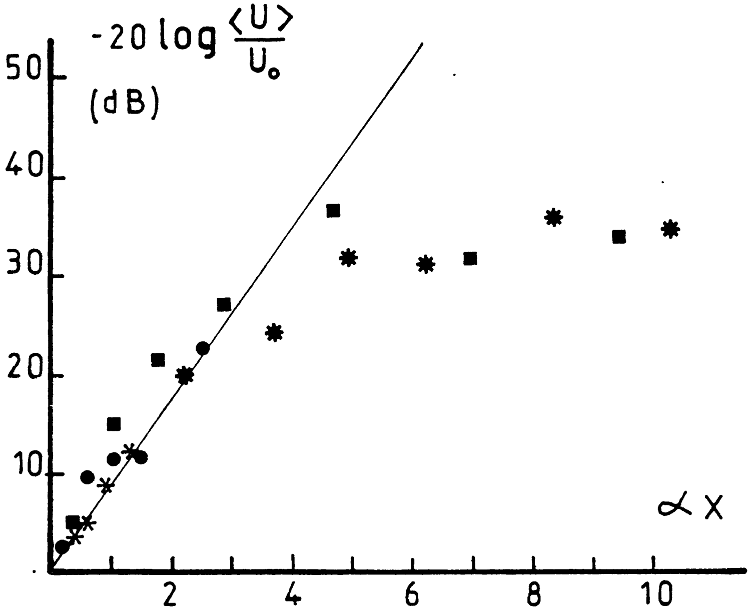

Fig. 3 - Attenuation of the coherent field <U> : a collimated beam (20 kHz < f < 100 kHz) for three ranges of propagation (● x = 54 cm ; ■ x = 103 cm ; ✱ x = 220 cm) and a spherical wave ✳ f = 23.5 kHz for x = 32 cm ; x = 56 cm ; x = 81 cm ; x = 131 cm.

2. Transverse coherence functions

In Fig. 4 the measured values of the coherence function for the spherical wave are plotted in terms of the receiver separation ρ_d for two ranges of propagation (X = 110 cm and X = 220 cm). The solid curves are computed from Eq. 11, employing a modified von Karman spectrum (Eq. 6), and the dotted curves are obtained from Eq. 11 but with a Kolmogorov spectrum $\phi_\varepsilon = 0.033\ C_\varepsilon^2\ K^{-11/3}$. We note that the coherence functions decrease rapidly as the propagation length and frequency increase. The transverse coherence function is accurately described by a von Karman spectrum, which takes into account the outer scale of turbulence.

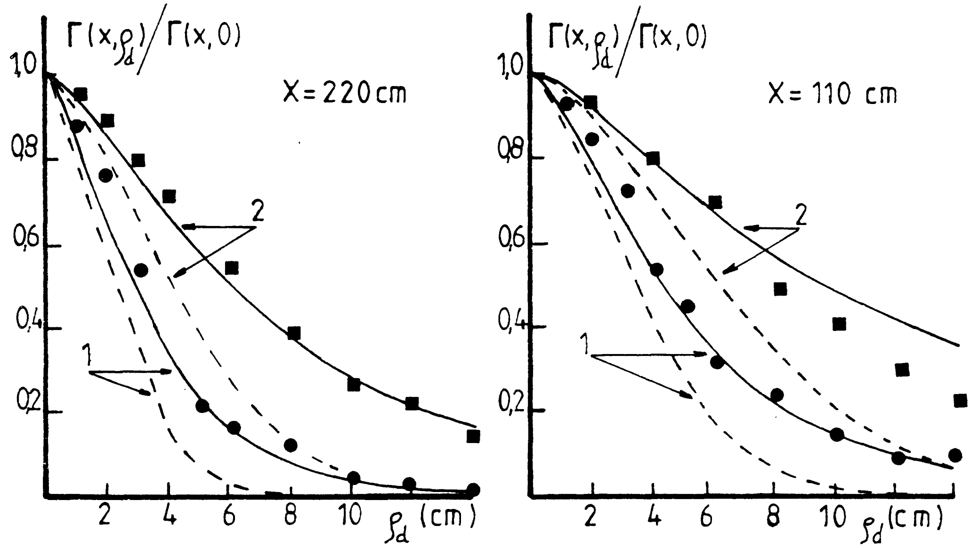

Fig. 4 - Transverse coherence functions for a spherical wave (● f = 39 kHz ; ■ f = 23.5 kHz) Numerical estimates with a modified von Karman spectrum (————) and a Kolmogorov spectrum (----------).

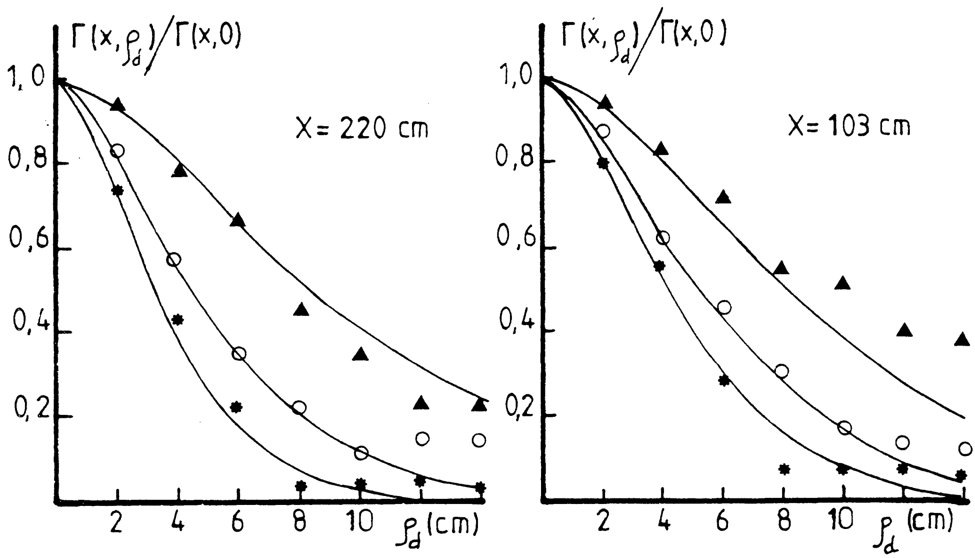

Fig. 5 - Transverse coherence functions for a collimated beam (✱ f = 50 kHz, w_0 = 3.3 cm / ○ f = 39 kHz, w_0 = 4.1 cm / ▲ f = 23.5 kHz, w_0 = 6.6 cm). Numerical estimates with a modified von Karman spectrum (————).

The experimental results for a collimated beam are given in Fig. 5 ; here the solid curves, obtained from Eq. 12 with a modified von Karman spectrum, reveal a good accord with measurements on the range $0 < \rho_d < 1.5\,L_T$. For large values of ρ_d the measured transverse coherence increases slighthly, owing to the fact that the actual beam, with side lobes, differs from the gaussian model which is adapted in the analysis.

Conclusion

The effect of temperature fluctuations on the propagation of acoustic waves is revealed with carefully controlled experiments involving a heated air grid. The relevance of the entire spectrum of turbulence has been demonstrated and, in particular, for the transverse coherence function. It appears that the reduction in coherence cannot be estimated by a single turbulence scale, and that a satisfactory estimate is achieved with a modified von Karman spectrum. The differences which arise in connection with different forms of the incident wave necessitate separate attention.

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References

1. Stone, R.G.; Mintzer, D. ; "Range dependence of acoustic fluctuations in a randomly inhomogeneous medium" J. Acous. Soc. Am. 34, pp. 647-653 (1962).

2. Chotiros, N.P. ; Smith, B.V. ; "Sound amplitude fluctuations due to a temperature microstructure" J. Sound Vib. 64 (1979) 349-369.

3. Sederowitz, W.J. ; Favret, A.G. ; "Covariance between acoustic signals traveling diverging paths in a random medium" J. Acous. Soc. Am. 45 (1969) 386-391.

4. Neubert, J.A. ; Lumley, J.L. ; "The nature of sound fluctuation in the upper ocean II" J. Acous. Soc. Am. 64 (1978) 1148-1158.

5. Daigle, G.A. ; Piercy, J.E. ; Embleton, T.F.W. ; "Line-of-sight propagation through atmospheric turbulence near the ground" J. Acoust. Soc. Am. 74 (1983) 1505-1513.

6. Urick, R.J. ; "Sound propagation in the sea" Defense Advanced Research Projects Agency. US Government Printing Office Washington, 1979.
7. Tatarski, V.I. ; "The effects of turbulent atmosphere on wave propagation" IPST Keter Press Jerusalem 1971.
8. Ishimaru, A. ; "Wave propagation and scattering in random media" Academic Press New-York Vol. 2, 1978.
9. Novikov, E.A. ; "Functionals and the random force method in turbulence theory" Soviets Physics JETP 20-5 (1965) 1290-1294.
10. Fante, R.L. "Mutual coherence function and frequency spectrum of laser beam propagating trough atmospheric turbulence" J. Opt. Soc. Am. 64 (1974), 592-598.
11. Sunyach, M. ; Brunel, B. ; Comte-Bellot, G. ; Performances de la soufflerie anéchoïque à grandes vitesses de l'Ecole Centrale de Lyon" Revue d'Acoustique 73 (1985) 316-330.
12. Blanc-Benon, Ph. ; "Effet d'une turbulence cinématique sur la propagation des ondes acoustiques" Thèse de Docteur Ingénieur, ECL 81-02, 1981.