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Technical Notes

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Decrease of the Effective Reynolds Number with Eddy-Viscosity Subgrid-Scale Modeling

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Introduction

IN large-eddy simulation (LES) of turbulent flows, the fine scales affected by viscous diffusion are usually not resolved. It is therefore generally accepted that artificial damping is required to dissipate the turbulent kinetic energy. The classical approach consists of subgrid modelings based on eddy viscosity and developed from physical considerations to represent the subgrid-stress tensor, the most famous one being the Smagorinsky model later improved by dynamical formulations.[1] However, the use of eddy viscosity in LES modelings still raises fundamental questions. For instance, eddy-viscosity modelings might dissipate the turbulent energy through a wide range of scales up to the larger ones, which should be dissipation-free at high Reynolds numbers.[2] More fundamentally, because eddy viscosity has the same functional form as the molecular viscosity it is difficult to define the effective Reynolds number of the simulated flows.[3]

Alternatives to the eddy-viscosity approach have therefore been proposed using filtering for modeling the effects of the subgrid scales properly, by minimizing the amount of dissipation on the larger resolved scales. One way[4,5] consists of using low-dissipative schemes for time and space discretization, while explicitly applying a compact/selective filter to the flow variables with the aim of removing only the wave numbers located near the grid cutoff wave number. In this case, the energy is only diffused when it is transferred from the larger scales to the smaller scales discretized by the mesh grid. This LES methodology was recently successfully applied to isotropic turbulence, channel flows, and jets.[4–7] Visbal and Rizzetta[4] obtained for instance better results using compact filtering alone than with Smagorinsky models for isotropic turbulence.

A Mach number $M = u_j/c_0 = 0.9$, round jet at a high Reynolds number $Re_D = u_j D/\nu = 4 \times 10^5$ jet was first simulated by the authors of the present Note using selective filtering alone[5]; u_j is the jet exit velocity, c_0 the speed of sound in the ambient medium, D the jet diameter, and ν the kinematic molecular viscosity. The computed flow and sound fields were found to be in agreement with measurements at high Reynolds numbers. They were also shown not to depend on the filtering procedure. The same jet was also simulated using the dynamic Smagorinsky model.[8] Significant discrepancies were observed with respect to the results obtained with the filtering alone. The objective of the current Note is to show that they can be attributed to the decrease of the effective Reynolds number of the jet in the LES using the eddy-viscosity modeling. To support this, some flow and noise features dependent on the Reynolds number are investigated. They are specifically compared to experimental and numerical data for Mach 0.9 jets at low and high Reynolds numbers.

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Numerical Simulations

The specifications of the simulations reported in the present Note are given in Table 1. Initial conditions are defined for isothermal round jets with centerline velocities and diameters yielding a fixed Mach number $M = 0.9$ and varying Reynolds numbers. LESsf and LESdsm denote the simulations of the high Reynolds number $Re_D = 4 \times 10^5$ jet performed using the selective filtering alone or in combination with the dynamic Smagorinsky model (DSM). Note that the former simulation is referred to as LESac or also as LESsf and the latter as LESdsm in recent papers,[5,8] where details about the implementation of the DSM can moreover be found. LESre1 and LESre2 simulations are carried out using the filtering alone, for jets at the lower Reynolds numbers of $Re_D = 10^4$ and $Re_D = 5 \times 10^3$, respectively. These two values were chosen to flank the effective Reynolds number expected in LESdsm. Indeed, as shown in Fig. 1 the eddy viscosity ν_t in LESdsm is of the order of 50 times the

Table 1 Initial conditions and subgrid modelings in the different simulations

Reference	M	Re_D	sgs modeling[a]
LESsf	0.9	4×10^5	sf[b]
LESdsm	0.9	4×10^5	sf + dsm[c]
LESre1	0.9	10^4	sf
LESre2	0.9	5×10^3	sf

[a]sgs is used for subgrid scale.
[b]sf for selective filtering.
[c]dsm for dynamic Smagorinsky model.

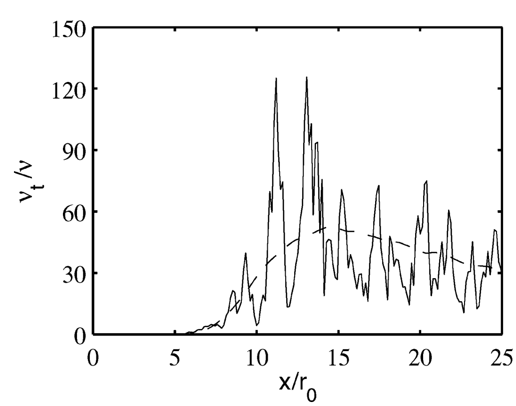

Fig. 1 Centerline profiles of the ratio ν_t/ν between eddy and molecular viscosities in the LESdsm simulation: ——, instantaneous and – – –, time-averaged profiles.

molecular viscocity, which could lead to an effective jet Reynolds number of $\widetilde{Re}_D = u_j D/\nu_t \simeq 8 \times 10^3$.

The numerical algorithm and parameters are identical to those of LESsf simulation described in earlier references.[5,8] The filtered compressible Navier–Stokes equations are solved using explicit low dispersive and low dissipative schemes.[9] Thirteen-point finite differences are used for spatial discretization while a six-stage Runge–Kutta algorithm is applied for time integration. Grid-to-grid oscillations are removed thanks to an explicit filtering that is optimized to damp the scales discretized by less than four grid points without affecting the larger scales. The filtering is applied explicitly to the density, momentum, and pressure variables, every two iterations, sequentially in the Cartesian directions, as reported in detail in a longer paper.[5] In LESsf, LESre1, and LESre2, it requires about 20% of the total computational time. Note also that in LESdsm the simulation time is increased by 25% as a result of the dynamic Smagorinsky procedure. To compute the noise, nonreflective boundary conditions are implemented.[10] The computational domain is discretized by a 12.5 million point Cartesian grid with 15 points in the jet radius r_0. The flow is computed up to an axial distance $x = 25r_0$. The sound field is calculated up to $x = 30r_0$ and, radially, up to $r = 15r_0$ from the jet axis, and resolved for Strouhal numbers $Sr = fD/u_j < 2$, where f is the frequency.

In all simulations, mean axial velocity at the jet inflow boundary is defined by a hyperbolic-tangent profile with a ratio $\delta_\theta / r_0 = 0.05$ between the shear-layer momentum thickness δ_θ and the jet radius. Mean radial and azimuthal velocities are set to zero, pressure is set to the ambient pressure, and the density profile is obtained from a Crocco–Buseman relation. To seed the turbulence, small random disturbances[11] are added to velocity profiles in the shear layer following the procedure used in the LESsf simulation.[5]

Results

Snapshots of the turbulent jets and of their radiated sounds, as well as flow and noise properties provided by the simulations, are reported in related papers.[8,12] In the present Note, our attention is focused on three flow features that appear to depend notably on the effective Reynolds number of the computed jets: the centerline mean axial velocity, turbulence intensity on the jet axis, and the sound spectrum in the sideline direction from the jet.

The centerline mean axial velocities u_c obtained from the LES and from measurements[13,14] for two Mach $M = 0.9$ jets at Reynolds numbers $Re_D = 3.6 \times 10^3$ and $Re_D = 10^6$ are presented in Fig. 2. The data are shifted in the axial direction to match the different core lengths and thus to compare properly the velocity decays after the potential core. A good agreement is observed between the velocity decays from LESsf and from the experimental high-Reynolds-number jet. Moreover, as the Reynolds number decreases in LESre1 and LESre2, the velocity decay is more rapid and moves closer to

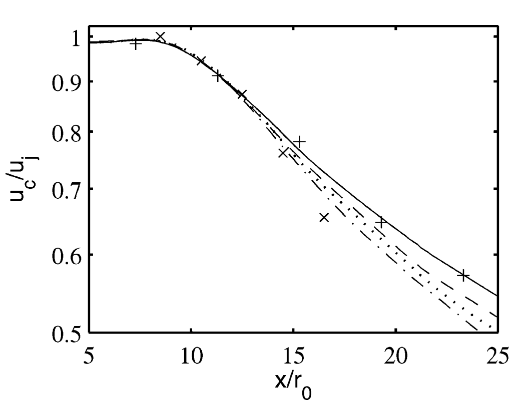

Fig. 2 Profiles of the mean centerline velocity u_c/u_j in ——, LESsf; . . . ., LESdsm; – – –, LESre1; and -·-·- LESre2 simulations. Measurements: ×, Stromberg et al.[13] ($M = 0.9$, $Re_D = 3.6 \times 10^3$) and +, Lau et al.[14] ($M = 0.9$, $Re_D = 10^6$). The LESdsm, LESre1, LESre2, and experimental profiles are shifted in the axial direction with respect to the LESsf profile to yield identical core lengths.

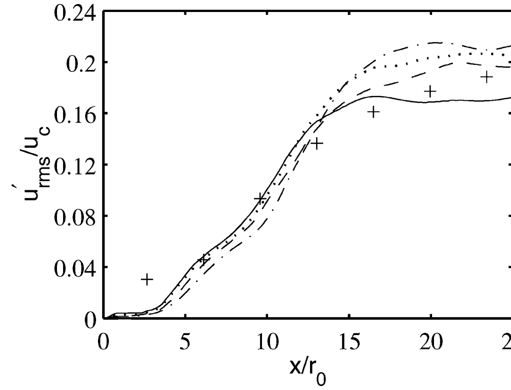

Fig. 3 Centerline profiles of the turbulence intensity $u'_{\rm rms}/u_c$ in ——, LESsf; . . . ., LESdsm; – – –, LESre1; and -·-·-, LESre2 simulations. Measurements: +, Arakeri et al.[15] ($M = 0.9$, $Re_D = 5 \times 10^5$), shifted in the axial direction for the comparison.

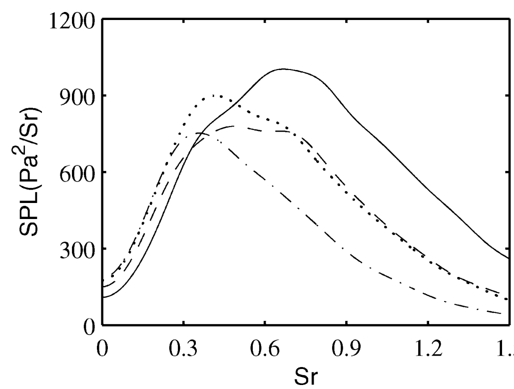

Fig. 4 Sound-pressure spectra as a function of Strouhal number $Sr = fD/u_j$, in linear scales, obtained in the sideline direction at $x = 11r_0$ and $r = 15r_0$ from ——, LESsf; . . . ., LESdsm; – – –, LESre1; and -·-·-, LESre2 simulations.

that measured for the $Re_D = 3.6 \times 10^3$ jet. These results suggest that the effective Reynolds numbers of the jets computed using the selective filtering alone are preserved, that is, that they correspond to the Re_D value given by the initial conditions. This does not seem to be the case when using the eddy-viscosity modeling. The velocity decay from LESdsm indeed stands between those from LESre1 and LESre2. This indicates that the effective Reynolds number in LESdsm is decreased and agrees with the $\widetilde{Re}_D \simeq 8 \times 10^3$ value estimated from the eddy viscosity instead of the $Re_D = 4 \times 10^5$ value given by the initial conditions.

The centerline profiles of the turbulence intensity $u'_{\rm rms}/u_c$, where u' is the fluctuating axial velocity, are now presented in Fig. 3. The profile from LESsf agrees fairly well with experimental data for a $M = 0.9$ jet at high Reynolds number.[15] In LESre1 and LESre2 at low Reynolds numbers, the increase of the turbulence intensities is more rapid. As for the velocity decay, the profile from LESdsm is found between those from LESre1 and LESre2. Note that in jets the self-similarity region is reached when $u'_{\rm rms}/u_c \simeq 0.25$ on the jet axis.[16] This region is therefore not observed in the present simulations for $x \leq 25r_0$, even if its establishment can be expected to occur at a shorter axial distance at lower Reynolds numbers. In LESsf (Ref. 5) it was also noticed still not to be obtained at $x = 60r_0$, in agreement with experimental observations at high Reynolds numbers.[16] These results suggest that the effective flow Reynolds number can be preserved using the filtering alone, but decreased in LESdsm by the eddy-viscosity model.

The pressure field radiated in the sideline direction from the jet is finally investigated. It is usually associated with the jet fine-scale turbulence[17] and is therefore closely dependent on the Reynolds number.[12] The sound spectra obtained at $x = 11r_0$ and $r = 15r_0$ are shown in Fig. 4. As the Reynolds number decreases in LESre1 and

LESre2, a part of the high-frequency components of the spectra disappears with respect to LESsf. Consequently the frequency peak moves to lower Strouhal numbers, and the overall sound pressure level decreases. The sound spectrum from LESdsm is similar to those at low Reynolds numbers, in accordance with the decrease of the effective Reynolds number using eddy viscosity.

Conclusions

The present simulations of jets illustrate the effects of subgrid modeling based on eddy viscosity in LES. The effective flow Reynolds number is found to be dramatically decreased using the dynamic Smagorinsky model, whereas it seems to be preserved and to correspond to the initial jet conditions using the selective filtering alone. This basic deficiency of the eddy-viscosity subgrid modeling can question its use for the study of free shear flows where the Reynolds number is a key parameter, as for instance for the investigation of jet noise.[12]

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