ABSTRACT

Large Eddy Simulations (LES) of round free jets at Mach number \(M = 0.9\) with Reynolds numbers over the range \(2.5 \times 10^3 \leq \text{Re}_D \leq 4 \times 10^5\) are performed using explicit selective/high-order filtering with or without Dynamic Smagorinsky Model (DSM). Flow features as well as turbulent kinetic energy budgets in the turbulent jets are reported. Using filtering alone, the results are independent of the filtering strength, and the effects of the Reynolds number on jet development are successfully calculated, indicating that the effective Reynolds number of the computed \(\textit{nows} corresponds to the Reynolds number given by the initial conditions. The contributions of filtering and molecular viscosity to the dissipation are also presented. Using DSM, the jet development shows that the effective flow Reynolds number is artificially decreased, and a large part of the dissipation is supplied by the eddy viscosity, in the same way as a significant part of the dissipation is provided by molecular viscosity at low Reynolds numbers. Moreover the DSM results are not appreciably modified when subgrid-scale kinetic energy is combined with eddy-viscosity.

INTRODUCTION

In simulations, important issues need to be addressed to ensure that the solutions are physically correct and that they are not artifacts of the computational procedure. This is particularly the case for the numerical dissipation, whose effects are to be minimized so that they do not exceed the filtering strength and do not govern the flows. In Large Eddy Simulation (LES) where only the turbulent scales discretized by the grid are calculated, this point is particularly crucial. The scales affected by viscous diffusion are indeed lacking. An artificial damping is therefore required to dissipate the turbulent kinetic energy, and in practice to enforce stability.

An eddy viscosity is generally introduced in LES to account for the dissipative effects of the subgrid scales. The widely-used Dynamic Smagorinsky model (DSM) is based on this hypothesis. In this model, especially developed by Moin et al. (1991) for compressible turbulence, the eddy viscosity is expressed from physical considerations and its amplitude is estimated from the computed scales. However the use of eddy viscosity in LES still raises important questions. For instance, eddy-viscosity models might dissipate the turbulent energy excessively, through a wide range of turbulent scales up to the larger ones, at high Reynolds numbers (Bogey and Bailly, 2004b). Moreover, since eddy viscosity has the same functional form as molecular viscosity, it is difficult to define the effective Reynolds number of the simulated flows (Domaradski and Yee, 2000; Bogey and Bailly, 2005c).

Alternatives to the eddy-viscosity approach, based on filtering for modelling the effects of the subgrid scales, have therefore been proposed. One way consists in using low-dissipative schemes for time and space discretization, while explicitly applying a compact/selective filter to the flow variables with the aim of removing only the wave numbers located near the grid cutoff wave number. In this case, the amount of dissipation on the larger resolved scales is minimized and the energy is only diffused when it is transferred to the smaller scales discretized by the mesh grid. This LES methodology was recently successfully applied to isotropic turbulence, channel flows and jets (Visbal and Rizzetta, 2002; Rizzetta et al., 2003; Mathew et al., 2003; Bogey and Bailly, 2005a). Visbal and Rizetta (2002) obtained for instance better results using filtering alone than with Smagorinsky models for isotropic turbulence.

In the present study, Large Eddy Simulations of round free jets at Mach number \(M = u_j/c_0 = 0.9\) (\(u_j\) is the jet exit velocity, \(c_0\) the speed of sound in the ambient medium) are reported. The simulations are performed with low-dissipative numerical schemes (Bogey and Bailly, 2004a), using explicit selective/high-order filtering with or without Dynamic Smagorinsky Model. A jet at a high Reynolds number \(\text{Re}_D = u_j D/\nu = 4 \times 10^5\) (\(D\) is the jet diameter, \(\nu\) the kinematic molecular viscosity) was first simulated by the authors (Bogey and Bailly, 2005a) using filtering alone. The computed flow and sound fields were found to be in agreement with measurements at high Reynolds numbers. This jet at \(\text{Re}_D = u_j D/\nu = 4 \times 10^5\) is now simulated using filtering of different strengths alone or in combination with DSM, with the aim of studying the influence of the filtering and DSM on the jet flow. Jets with lower Reynolds numbers, over the
range $2.5 \times 10^3 \leq \text{Re}_D \leq 10^4$, are also computed using filtering alone. The motivation is here to reproduce the effects of the Reynolds number on jet development, which could indicate that the effective Reynolds number of the simulated flows corresponds to the expected $\text{Re}_D = u_c D / \nu$ given by the jet initial conditions, and is therefore preserved by the filtering methodology. Moreover the magnitudes of the dissipation mechanisms involved in the present LES using different subgrid modellings and at various Reynolds numbers will be investigated. With this in view, kinetic energy budgets are calculated, and the dissipation rates due to molecular viscosity, filtering and DSM are presented and compared.

The present paper is organized as follows. First the parameters of the different simulations are described. Flow features obtained from the LES are then shown. The kinetic energy budgets are subsequently calculated, and the contributions of molecular viscosity, explicit filtering and DSM to the dissipation are presented. Finally concluding remarks are drawn.

SIMULATIONS

The specifications of the simulations are given in Table 1. Initial conditions are defined for isothermal round jets with centerline velocities and diameters yielding a fixed Mach number $M = 0.9$ and various Reynolds numbers. LESsf, LESsf2, LESdsm and LESdsm2 jets are at the high Reynolds number of $\text{Re}_D = 4 \times 10^4$, while LESre1, LESre2 and LESre3 jets are at the lower Reynolds numbers of $\text{Re}_D = 10^4$, $\text{Re}_D = 5 \times 10^3$ and $\text{Re}_D = 2.5 \times 10^3$, respectively.

The numerical algorithm is that of the reference simulation LESsf, described in detail and referred to as LESac in earlier papers (Bogey and Bailly, 2005a, 2005b). The filtered compressible Navier-Stokes equations are solved using explicit low dispersive and low dissipative schemes developed in Bogey and Bailly (2004a). Thirteen-point finite differences are used for spatial discretization while a six-stage Runge-Kutta algorithm is applied for time integration. Grid-to-grid oscillations are removed thanks to an explicit filtering which is optimized to damp the scales discretized by less than four grid points without affecting the larger scales. The filtering is applied explicitly to the density, momentum and pressure variables, every two or three iterations, sequentially in the Cartesian directions. The computational domain is discretized by a 12.5 million point Cartesian grid with 15 points in the jet radius $r_0$. The flow is computed up to an axial distance $x = 25 r_0$.

In all simulations, mean axial velocity at the jet inflow boundary is defined by a hyperbolic-tangent profile with a ratio $\delta_y / r_0 = 0.05$ between the shear-layer momentum thickness $\delta_y$ and the jet radius. Mean radial and azimuthal velocities are set to zero, pressure is set to the ambient pressure, and the density profile is obtained from a Crocco-Busemann relation. Small random disturbances are added to velocity profiles in the shear layer in order to seed the turbulence following exactly the procedure used in the LESsf simulation. Note that the effects of the inflow conditions and forcing are investigated in detail in Bogey and Bailly (2005b).

In LESsf and LESsf2, the selective filtering is used alone, every second and third iteration respectively, in order to study the influence of the filtering strength on results. In LESdsm and LESdsm2, filtering is applied as in LESsf but in combination with the Dynamic Smagorinsky Model (DSM). An eddy viscosity $\nu_t$ and a subgrid-scale kinetic energy $k_{sgs}$ are introduced to approximate the deviatoric and isotropic parts of the subgrid stress tensor $T_{ij}$, then expressed as

$$T_{ij} = \tilde{\nu}_t \tilde{u}_i \tilde{u}_j - 2 \mu_t \delta_{ij} - (2/3) \tilde{\nu}_t k_{sgs} \delta_{ij},$$

where the superscript $D$ denote the deviatoric part, the bar and tilde indicate the LES filterings for compressible turbulence, see Moin et al. (1991). The eddy viscosity is parametrized using the Smagorinsky model as $\nu_t = C_l \Delta^2 \tilde{S}$ and the SGS kinetic energy as $k_{sgs} = C_s \Delta^2 \tilde{S}^2$ where $\Delta = (\Delta x \Delta y \Delta z)^{1/3}$ and $\tilde{S} = (2 S_{ij} S_{ij})^{1/2}$. The coefficients $C_l$ and $C_s$ are computed using the dynamic procedure proposed by Lilly (1992). The subgrid scale kinetic energy $k_{sgs}$ is used only in LESdsm2 simulation. Moreover, in both LESdsm and LESdsm2, the eddy viscosity $\nu_t$ is of the order of fifty times the molecular viscosity, as shown in Bogey and Bailly (2004b, 2005c), which might lead to an effective jet Reynolds number of $\text{Re}_D = u_c D / \nu_t \approx 8 \times 10^3$.

In the low-Reynolds-number simulations LESre1, LESre2 and LESre3, the filtering is used alone as in LESsf. Note finally that the two Reynolds numbers in LESre1 and LESre2, $\text{Re}_D = 10^4$ and $\text{Re}_D = 5 \times 10^3$, were chosen to flank the effective Reynolds number $\text{Re}_D$ expected in LESdsm and LESdsm2.

FLOW PROPERTIES

Snapshots of vorticity are presented in Figure 1 for the simulations LESsf and LESdsm at high $\text{Re}_D$, and for the simulations LESre1, LESre2 and LESre3 at low $\text{Re}_D$. The turbulent development of the jets appears to depend appreciably on the forcing mechanisms involved in the present LES using different subgrid modellings and at various Reynolds numbers will be investigated. With this in view, kinetic energy budgets are calculated, and the dissipation rates due to molecular viscosity, filtering and DSM are presented and compared.

The present paper is organized as follows. First the parameters of the different simulations are described. Flow features obtained from the LES are then shown. The kinetic energy budgets are subsequently calculated, and the contributions of molecular viscosity, explicit filtering and DSM to the dissipation are presented. Finally concluding remarks are drawn.

<table>
<thead>
<tr>
<th>$\text{Re}_D$</th>
<th>filtering freq.</th>
<th>DSM</th>
<th>$x_c / r_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LESsf</td>
<td>$4 \times 10^4$</td>
<td>$2 \Delta t$</td>
<td>-</td>
</tr>
<tr>
<td>LESsf2</td>
<td>$4 \times 10^4$</td>
<td>$3 \Delta t$</td>
<td>-</td>
</tr>
<tr>
<td>LESdfsm</td>
<td>$4 \times 10^4$</td>
<td>$2 \Delta t$</td>
<td>$\nu_t$</td>
</tr>
<tr>
<td>LESdsm2</td>
<td>$4 \times 10^4$</td>
<td>$2 \Delta t$</td>
<td>$\nu_t + k_{sgs}$</td>
</tr>
<tr>
<td>LESre1</td>
<td>$10^4$</td>
<td>$2 \Delta t$</td>
<td>-</td>
</tr>
<tr>
<td>LESre2</td>
<td>$5 \times 10^3$</td>
<td>$2 \Delta t$</td>
<td>-</td>
</tr>
<tr>
<td>LESre3</td>
<td>$2.5 \times 10^3$</td>
<td>$2 \Delta t$</td>
<td>-</td>
</tr>
</tbody>
</table>
related to the dynamic procedure which strongly reduces the magnitude of the eddy viscosity in the transitional shear layer. However, there seems to be less fine scales in LESdsm than in LESsf. The turbulent field from LESdsm resembles more that from LESre1, obtained at a lower Reynolds number.

The profiles of centerline mean axial velocity are presented in Figure 2. They are shifted axially to yield identical core lengths \( x_c \) in order to compare properly the velocity decays after the potential core.

The profiles from the simulations at \( Re_D = 4 \times 10^5 \) are plotted in Figure 2(a). They clearly show that the velocity decay is more rapid in the LES using DSM. Moreover the profiles from LESsf and LESsf2 simulations, where filtering is applied alone but at a different frequency, are very close. In the same way, using DSM with and without \( k_{sgs} \), the results do not differ appreciably.

The velocity profiles from LESsf and the LES at low Reynolds numbers are displayed in Figure 2(b). The velocity decay after the potential core is found to be faster as the Reynolds number decreases. This behaviour is in accordance with experimental data. The velocity decay of the LESre3 jet at \( Re_D = 2500 \) is indeed similar to that measured for a jet at \( Re_D = 3600 \) by Stromberg et al. (1980). A good agreement is also observed between the results from the simulated high-Reynolds-number jet and from the experimental, initially laminar jet at \( Re_D = 5 \times 10^5 \) of Arakeri et al. (2003). Note finally that the velocity decay from LESdsm would stand between those from LESre1 and LESre2.

The centerline profiles of the turbulent axial velocity \( u'_{rms} \) are now presented in Figure 3(a) for the simulations at \( Re_D = 4 \times 10^5 \) and in Figure 3(b) for the simulations at low Reynolds numbers. Figure 3(a) shows that the peak of \( u'_{rms} \) is higher using DSM. As for the velocity decay in Figure 2(a), there is...
also no significant effect of the filtering frequency nor of the subgrid-scale kinetic energy on the turbulence intensity.

In Figure 3(b), the amplitude of the peak of $u_{rms}^'$ increases as the Reynolds number decreases. This trend is supported by Direct Numerical Simulation (DNS) and experimental data. The profile from the LESre3 jet at Re$_D$ = 2500 compares indeed favorably with that from the DNS jet at Re$_D$ = 3600 computed by Freund (2001), while the profile from the LESsf jet at high Re$_D$ agrees fairly well with that from the experimental jet at Re$_D$ = 5 x 10$^5$ of Arakeri et al. (2003). Note also here that the $u_{rms}^'$ peak from LESdsms would be between those from LESre1 and LESre2.

ENERGY BALANCE AND DISSIPATION

The budget of the turbulent kinetic energy is calculated in order to determine the contributions of the explicit filtering, molecular viscosity, and DSM to the energy dissipation in the present simulations. The contribution of the filtering is evaluated from the balance of all the other terms of the energy budget, calculated explicitly from the LES fields. The admissibility of doing so is justified by the very low dissipation of the time-integration algorithm. The maximum amount of dissipation provided by the optimized Runge-Kutta algorithm, obtained by the higher angular frequency resolved, is indeed only of 3 x 10$^{-5}$ per iteration. The equation for the filtered turbulent kinetic energy budget is derived from the filtered Navier-Stokes equations given in Bogey and Bailly (2005a):

$$0 = -\frac{\partial}{\partial x_j}(\langle \rho \rangle k) + \langle \rho u_i' u_j' \rangle \frac{\partial |u_i|}{\partial x_j} - \frac{1}{2} \frac{\partial}{\partial x_j} \langle \rho u_i'^2 u_j'^2 \rangle$$

advection production turb. diffusion

$-\langle \tau_{ij} \frac{\partial u_i'}{\partial x_j} \rangle - \langle \tau_{ij} \frac{\partial u_j'}{\partial x_j} \rangle + \frac{\partial}{\partial x_j} \langle u_i' \tau_{ij} \rangle + \frac{\partial}{\partial x_j} \langle u_j' T_{ij} \rangle$

visc. dissip.

$-\frac{\partial}{\partial x_i} \langle p' u_i' \rangle + \langle p' \frac{\partial u_i'}{\partial x_i} \rangle - \langle u_i' \rangle \frac{\partial \langle \rho \rangle}{\partial x_i} + D_{\text{filter}}$

pressure-velocity

where the turbulent kinetic energy is $\langle k \rangle = \langle \rho u_i'^2 / 2 \rangle$. Density, velocity and pressure are represented by $\rho$, $u_i$ and $p$, the turbulent and subgrid stress tensors by $\tau_{ij}$ and $T_{ij}$. The tilde and overbar indicate Favre and Reynolds grid-filtered quantities, and the prime fluctuating quantities. Statistical averaging is denoted by $\langle \rangle$, and $|u_i| = \langle \rho u_i'^2 \rangle / \langle \rho \rangle$. Except for the filtering dissipation $D_{\text{filter}}$, all the terms are evaluated explicitly from the LES data. The main ones are those corresponding to the meanflow advection, to production, to viscous dissipation, to turbulence diffusion, to the pressure-velocity term, and, possibly, to the DSM contribution.

The kinetic energy budget at $x = 20r_0$ is shown in Figure 4 for the high-Re$_D$ simulation LESsf using filtering alone. We can first notice that the viscous dissipation is negligible, and that the dissipation is thus only due to filtering. The present curves agree fairly well with corresponding experimental curves of the literature, see for instance Panchapakesan and Lumley (1993). However they cannot be compared directly because the location $x = 20r_0$ is far from the self-similarity region (Wyganski and Fiedler, 1969), where the measurements are done. Note also that, in the energy budget issued from the LES, the pressure-velocity term is calculated, whereas this term is neglected in Panchapakesan and Lumley (1993).

Figure 3: Centerline profiles of the $u_{rms}$ fluctuating axial velocity. (a) LESsf, LESsf2, LESdsm, LESdsm2. (b) LESsf, LESre1, LESre2, LESre3, $\delta_{0.5}$ measurements of Arakeri et al. (2003) shifted by -6.5$\delta_0$ in the axial direction (M=0.9, Re$_D$=5x10$^5$), $\triangle$ DNS results of Freund (2001) shifted by -1.8$\delta_0$ (M=0.9, Re$_D$=3600).

Figure 4: Kinetic energy budget at $x = 20r_0$ in LESsf: advection, production, viscous dissipation, filtering dissipation, turbulence diffusion, pressure-velocity term. Curves are normalized by $\rho_0 u_0^2 \delta_{0.5}$ ($\rho_0$ and $u_0$: centerline mean density and axial velocity, $\delta_{0.5}$: jet half-width).
We now focus on the dissipation mechanisms involved in the LES, and more especially on their magnitudes evaluated from the energy budgets. The contributions to energy dissipation at \( x = 20r_0 \) are thus represented in Figure 5(a) for LESsf and LESsf2, and in Figure 5(b) for LESdsms. In both cases, the effects of molecular viscosity are negligible, as expected, because of the high Reynolds number \( Re_D = 4 \times 10^5 \) involved.

In Figure 5(a), using selective filtering alone, energy dissipation is found to be provided only by the filtering. The dissipation rates are moreover very close in LESsf and LESsf2 where the explicit filtering is applied every second and third iteration, respectively. This demonstrates clearly that the dissipation rate in the present LES is independent of the filtering strength and, consequently, is determined only by physical mechanisms. This important feature is undoubtedly due to the high selectivity of the filtering which does not appreciably affect the scales discretized by more than four grid points.

In Figure 5(b), using DSM, energy dissipation is shown to be ensured both by the filtering and by the eddy viscosity. Since eddy viscosity affects all the resolved scales as molecular viscosity (Bogey and Bailly, 2004b), energy is dissipated through the smaller resolved scales, but also through the larger ones which should be dissipation-free at \( Re_D = 4 \times 10^5 \). We can also observe that contribution of the subgrid-scale kinetic energy is negligible, as suggested by Erlebacher et al. (1992).

Finally, the dissipation rates due to molecular viscosity and filtering at \( x = 20r_0 \) are presented in Figure 6 for the simulations LESre1, LESre2 and LESre3 using filtering alone. In these LES at low Reynolds numbers, the effects of the molecular viscosity are significant. Moreover, the lower the Reynolds number, the larger the part of the dissipation that is provided by the viscosity. At \( x = 20r_0 \) on the jet axis, the part of the viscous dissipation indeed represents 37% of the total dissipation in LESre1 at \( Re_D = 10^4 \), 55% in LESre2 at \( Re_D = 5 \times 10^3 \) up to 79% in LESre3 at \( Re_D = 2.5 \times 10^3 \). This behaviour results from the fact that, as the Reynolds number decreases, the dissipation scales are larger and tend to be discretized by the grid. At a sufficiently low Reynolds number, all dissipation scales would then be resolved accurately and the dissipation due to the selective filtering would become negligible.
CONCLUSIONS

In this work, Mach 0.9, round free jets at different Reynolds numbers have been simulated using low dissipative schemes. In order to take into account the effects of the subgrid scales, a selective filtering was applied explicitly with and without dynamic Smagorinsky model. The flow developments have been compared. Kinetic energy budgets have also been calculated and the magnitudes of the different dissipation mechanisms have been shown. The main results are the following:

- In the LES using the filtering alone, flow features and dissipation rates are independent of the filtering strength, because of the high selectivity of the filter. At high Reynolds number when dissipation scales are not discretized, energy dissipation is mainly ensured by the filtering and, as the Reynolds number decreases, the contribution of molecular viscosity becomes larger.

- The influence of the Reynolds number on the jet development is well reproduced in the LES using the filtering alone: As the Reynolds number decreases, the jet develops more rapidly after the potential core, with higher turbulence intensity. This point shows that the effective Reynolds number of the simulated jets agrees well with the Reynolds number $Re_D = u_j D / \nu$ given by the initial conditions using this LES approach.

- In the LES using DSM, a significant part of the energy is dissipated by the eddy viscosity. Since eddy viscosity has the same functional form as molecular viscosity, the effective flow Reynolds number is artificially decreased and is about $Re_D = u_j D / \nu_e$. This point is strongly supported by the flow development obtained from the LES using DSM which are similar to those observed at lower Reynolds numbers.

- The use of a subgrid-scale kinetic energy in DSM has no significant influence on the flow properties nor on the kinetic energy budget.

It is hoped that the present results would be of interest to point out the special features of the explicit selective filtering and of the subgrid-scale models based on eddy-viscosity. They show the crucial importance of the additional dissipations used in LES to take into account the effects of the subgrid scales. Thus eddy-viscosity models appear not to be recommended to study flow phenomena, such as jet noise (Boge y and Bailly, 2004c), where the Reynolds number is a key parameter. In that case, the explicit filtering approach may be one of the appropriate way.

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