Journal of Computational Acoustics, Vol. 16, No. 2 (2008) 137–162 © IMACS



INVESTIGATION OF 3D ACOUSTICAL EFFECTS USING A MULTIPROCESSING PARABOLIC EQUATION BASED ALGORITHM

K. CASTOR

Laboratoire de Détection et de Géophysique, Département Analyse Surveillance, Environnement, Commissariat à l'énergie Atomique BP 12, FR-91680 Bruyères-le-Châtel, France kaelig.castor@cea.fr

F. STURM

Laboratoire de Mécanique des Fluides et d'Acoustique, UMR CNRS 5509 École Centrale de Lyon, 36, avenue Guy de Collongue FR-69134 Ecully Cedex, France frederic.sturm@ec-lyon.fr

> Received 20 November 2006 Revised 5 April 2007

A parallelized algorithm based on an existing 3D wide-angle parabolic equation model is developed to perform numerical simulations of underwater acoustic propagation on a massively parallel computer. The parallelization method used is a suitable two-level procedure: A frequency decomposition and a spatial decomposition of the calculations, which are respectively dedicated to reduce CPU times for broadband and cw signal propagation. The high-performance of the parallelized algorithm is examined for the 3D extension of the classical ASA wedge benchmark. CPU times are reported and both speedup and efficiency are analyzed. An investigation of significant 3D effects at higher frequencies and at longer propagation ranges than in earlier works [F. Sturm, J. Acoust. Soc. Am. 117(3) (2005) 1058–1079] is performed with reasonable CPU times by using the new parallel algorithm. Further, the feasability of the procedure applied to a realistic oceanic environment problem involving both real sound speed profiles and bathymetry data sets is also illustrated.

Keywords: Sound propagation modeling; parabolic equation; azimuthal coupling; parallel processing; high-performance computing.

1. Introduction

In some underwater acoustic propagation problems, horizontal refraction effects are weak enough to allow 2D models to predict sound propagation accurately. However, it is well known that for some particular oceanic environments involving bathymetric slopes and/or horizontal sound speed gradients, significant 3D effects can be observed.¹⁻⁷ It is then necessary to use full 3D models to account for coupling of the propagating acoustical energy from one vertical plane to another. Among these models, 3D parabolic equation (PE) based models have proven to give good results.^{8–17} Their main drawback is that they can be highly computationally time consuming, even when harmonic sources emitting at very low frequencies are considered. Investigations of most 3D benchmark problems using PE models are thus often limited to source frequencies of few tens of Hertz only (e.g., 25 Hz for the classical 3D wedge benchmark problem). This issue is all the more crucial as one is concerned with the study of broadband and/or long-range propagation problems.

The aim of this work is to solve realistic underwater acoustic propagation problems that were hardly reachable in reasonable CPU times until now. For that purpose, a parallelized algorithm based on an existing 3D parabolic equation based model 3DWAPE^{15–17} is developed and numerical simulations are carried out using the parallelized version of the code on a massively parallel computer providing a high computational efficiency. The Message-Passing Interface (MPI) communication library is used in the parallel algorithm which is based on two principal levels dedicated to reduce CPU times for broadband and cw signal propagations, respectively. The paper is organized as follows: Section 2 deals with computational complexities and communication issues associated to N × 2D and 3D computations at a single frequency or for a broadband source. The parallelization strategy is detailed in Sec. 3.1 and the computational performances are analyzed in Sec. 3.2. Finally, Sec. 4 shows that the new parallel algorithm allows to investigate significant 3D effects at higher frequencies and at longer propagation ranges for the ASA 3D wedge benchmark (Sec. 4.1) and for a realistic oceanic environment (Sec. 4.2).

2. Computational Complexity Analysis of 3DWAPE

Generally speaking, the computational complexity deals with the resources being required during computation to solve a given problem. The most common resources are *time* (how many steps it takes to solve such a problem) and *space* (how much memory it takes). Here, we only consider the time complexity which is given by the number of steps that an algorithm takes to solve a problem, as a function of the size of the input data. Because CPU time of an algorithm can be directly related to its computational complexity, it is necessary to analyze the complexity to optimize an algorithm. Indeed, an analysis of complexity gives the required number of operations, and then, can provide an *a priori* relationship between CPU time and the number of processors used. Ideally, for a parallel computation, complexity and CPU time are inversely proportional to the number of processors used. However, in practical cases, it is not true when the number of processors used increases since idle and communication times between processors occurring in a parallel computation are non-negligible, and hence tend to deteriorate the efficiency of the parallel algorithm.

Let us analyze now the computational complexity of the 3D PE based model 3DWAPE.^{15–17} This model solves the problem of acoustic waves emanating from an isotropic (omnidirectional) point source, and propagating in general 3D oceanic environments. The propagation domain consists of a multilayered waveguide composed of a water column overlying one or several fluid sediment layers. Cylindrical coordinates are used with r, θ , z,

representing respectively the horizontal range from the source, the azimuthal (bearing) angle, and the depth (increasing downwards) below the ocean surface. No cylindrical symmetry is assumed on the geometry of the waveguide. Hence, the geometry of each layer is fully 3D. The point source is located at r = 0 and $z = z_S$. Its time dependence can be either transient (broadband source) or harmonic (cw source). The 4D (i.e., three spatial dimensions and time) problem of broadband pulse propagation is solved using an approach based on Fourier synthesis of frequency-domain solutions (discussed in more detail later on).

In the case of a harmonic point source emitting at frequency f, the model solves the 3D acoustic problem in the frequency domain using a parabolic equation based approach. The acoustic field $\psi = \psi(r, \theta, z; \omega)$ (with $\omega = 2\pi f$) is calculated as the solution of the following outgoing equation

$$\frac{\partial \psi}{\partial r}(r,\theta,z;\omega) = \mathrm{i}k_0 \left(\sum_{k=1}^{n_{\mathrm{p}}} \frac{a_{k,n_{\mathrm{p}}}\mathcal{X}}{\mathcal{I} + b_{k,n_{\mathrm{p}}}\mathcal{X}} + \frac{\frac{1}{2}\mathcal{Y}}{\mathcal{I} + \frac{1}{4}\mathcal{Y}} \right) \psi(r,\theta,z;\omega)$$
(1)

for $0 \leq r \leq r_{\text{max}}$, $0 \leq \theta \leq 2\pi$, $0 \leq z \leq z_{\text{max}}$, and the initial condition $\psi(r = 0, \theta, z; \omega) = \psi^{(0)}(\theta, z; \omega)$, with $\psi^{(0)}$ a given complex-valued function simulating a point source at r = 0 and $z = z_S$. In Eq. (1), n_p is the number of Padé terms, $k_0 = \omega/c_0$, with c_0 a reference sound speed, \mathcal{I} is the identity operator, and \mathcal{X}, \mathcal{Y} are operators defined by

$$\mathcal{X} = (n_{\alpha}^2 - 1)\mathcal{I} + \frac{\rho}{k_0^2}\partial_z(\rho^{-1}\partial_z) \quad \text{and} \quad \mathcal{Y} = \frac{1}{k_0^2 r^2}\partial_\theta^2, \tag{2}$$

with $n_{\alpha}(r, \theta, z) = (c_0/c(r, \theta, z))(1 + i\eta\alpha)$ the complex (to account for lossy layers) index of refraction, α the attenuation expressed in decibels per wavelength, $\eta = 1/(40\pi \log_{10} e)$, and ρ the density. The operator \mathcal{Y} handles the azimuthal diffraction. By neglecting this term in Eq. (1), but conserving the azimuthal dependence in $n_{\alpha}(r, \theta, z)$, the 3D model becomes a N × 2D model, or pseudo 3D (i.e. without azimuthal coupling). The 3D parabolic equation given by Eq. (1) is a paraxial approximation of the 3-D Helmholtz equation. It has a verywide-angle capability in depth and a wide-angle capability in azimuth, but does not have a wide-angle capability as referring to the discussion in Ref. 17 [Sec. II.C]. Regarding the energy-conservation issues inherent in one-way PE models, the method used to compute the correct outgoing field is based on a single-scatter formalism and is similar to the one used in Refs. 18 and 19. The acoustic field ψ is related to the frequency-domain acoustic pressure $\hat{P} = \hat{P}(r, \theta, z; \omega)$ by

$$\widehat{P}(r,\theta,z;\omega) = H_0^{(1)}(k_0 r)\psi(r,\theta,z;\omega),$$

where $H_0^{(1)}$ denotes the zeroth-order Hankel function of the first kind. The acoustic field ψ satisfies a pressure-release boundary condition at the ocean surface z = 0, an outgoing radiation condition at infinity, a 2π -periodicity condition in azimuth, and appropriate boundary conditions at each interface. In addition, to simulate a bottom half-space, an increasing attenuation coefficient is introduced in the lower part of the domain to prevent unwanted

reflections from the pressure-release imposed boundary condition at the maximum computation depth z_{max} . As an alternative, one could replace advantageously the absorbing subbottom with the perfectly matched layer^{20–22} (PML) absorber as in Ref. 23. The PML technique is currently not implemented in 3DWAPE. The angular frequency ω is treated as a parameter, the complex-valued field ψ being sought as a function of the spatial variables r, θ , and z, only.

The numerical method used to solve the initial and boundary value problem described above is a marching algorithm similar to an alternating direction implicit (ADI) method for parabolic partial differential equations. Let $\{r_0, r_1, \ldots, r_{L-1}, r_L\}$ be a uniform partition of $[0, r_{\text{max}}]$ such that $r_0 = 0$ and $r_L = r_{\text{max}}$, and Δr denote the constant increment in range: $r_n = n\Delta r, 0 \leq n \leq L$. Given the 3D field ψ at the discrete range r_n, ψ is obtained at the next discrete range r_{n+1} in two steps. By letting

$$\mu_{\pm}^{(k)} = b_{k,n_{\rm p}} \pm \frac{{\rm i}k_0 \Delta r}{2} a_{k,n_{\rm p}}, \quad 1 \le k \le n_{\rm p},$$

the first step consists in computing $n_{\rm p}$ intermediate fields denoted $u^{(1)}(\theta, z), u^{(2)}(\theta, z), \ldots, u^{(n_{\rm p})}(\theta, z)$, by solving

step 1
$$\begin{cases} [\mathcal{I} + \mu_{-}^{(1)} \mathcal{X}^{n+\frac{1}{2}}] u^{(1)}(\theta, z) = [\mathcal{I} + \mu_{+}^{(1)} \mathcal{X}^{n+\frac{1}{2}}] u^{(0)}(\theta, z), \\ [\mathcal{I} + \mu_{-}^{(2)} \mathcal{X}^{n+\frac{1}{2}}] u^{(2)}(\theta, z) = [\mathcal{I} + \mu_{+}^{(2)} \mathcal{X}^{n+\frac{1}{2}}] u^{(1)}(\theta, z), \\ \vdots \\ [\mathcal{I} + \mu_{-}^{(n_{\mathrm{p}})} \mathcal{X}^{n+\frac{1}{2}}] u^{(n_{\mathrm{p}})}(\theta, z) = [\mathcal{I} + \mu_{+}^{(n_{\mathrm{p}})} \mathcal{X}^{n+\frac{1}{2}}] u^{(n_{\mathrm{p}}-1)}(\theta, z) \end{cases}$$

for $0 \le \theta \le 2\pi$ and $0 \le z \le z_{\max}$, where $u^{(0)}(\theta, z) = \psi(r_n, \theta, z; \omega)$. The second step consists in computing $u^{(n_p+1)}(\theta, z)$ from the last intermediate field $u^{(n_p)}(\theta, z)$ obtained in step 1, by solving

step 2
$$\left[\mathcal{I} - \frac{\mathrm{i}k_0\Delta r}{4}\mathcal{Y}^{n+\frac{1}{2}}\right]u^{(n_{\mathrm{p}}+1)}(\theta, z) = \left[\mathcal{I} + \frac{\mathrm{i}k_0\Delta r}{4}\mathcal{Y}^{n+\frac{1}{2}}\right]u^{(n_{\mathrm{p}})}(\theta, z),$$

where $u^{(n_p+1)}(\theta, z) = \psi(r_{n+1}, \theta, z; \omega)$. In the case of a N×2D approach, step 2 is ignored and $u^{(n_p)}(\theta, z) = \psi(r_{n+1}, \theta, z; \omega)$. For a 3D computation, both step 1 and step 2 must be considered.

Let $\{\theta_1, \theta_2, \ldots, \theta_M, \theta_{M+1}\}$ be a uniform partition of the azimuthal interval $[0, 2\pi]$ such that $\theta_1 = 0$ and $\theta_{M+1} = 2\pi$, and $\{z_0, z_1, \ldots, z_N, z_{N+1}\}$ be a uniform partition of the depth interval $[0, z_{\max}]$ such that $z_0 = 0$ and $z_{N+1} = z_{\max}$. Let $\Delta\theta$, Δz be the increments, respectively in azimuth ($\theta_i = (i-1)\Delta\theta$, $1 \le i \le M+1$) and in depth ($z_j = j\Delta z$, $0 \le j \le N+1$). For $0 \le k \le n_p + 1$, we denote

$$u_{i,j}^{(k)} = u^{(k)}(\theta_i, z_j), \quad 1 \le i \le M, \quad 1 \le j \le N.$$

The discretization in the depth direction is done using a piecewise-linear finite-element (FE) method. Hence, at each discrete range, step 1 is replaced by step 1':

step 1'
$$\begin{cases} [I_N + \mu_-^{(1)} X_N^{n+\frac{1}{2}}] \mathbf{u}_i^{(1)} = [I_N + \mu_+^{(1)} X_N^{n+\frac{1}{2}}] \mathbf{u}_i^{(0)}, & 1 \le i \le M, \\ [I_N + \mu_-^{(2)} X_N^{n+\frac{1}{2}}] \mathbf{u}_i^{(2)} = [I_N + \mu_+^{(2)} X_N^{n+\frac{1}{2}}] \mathbf{u}_i^{(1)}, & 1 \le i \le M, \\ \vdots \\ [I_N + \mu_-^{(n_p)} X_N^{n+\frac{1}{2}}] \mathbf{u}_i^{(n_p)} = [I_N + \mu_+^{(n_p)} X_N^{n+\frac{1}{2}}] \mathbf{u}_i^{(n_p-1)}, & 1 \le i \le M, \end{cases}$$

where I_N and $X_N^{n+\frac{1}{2}}$ are square matrices of order N, I_N is the identity matrix, $X_N^{n+\frac{1}{2}}$ comes from the FE discretization of the differential operator $\mathcal{X}^{n+\frac{1}{2}}$, and

$$\mathbf{u}_{i}^{(k)} = [u_{i,1}^{(k)}, u_{i,2}^{(k)}, \dots, u_{i,N}^{(k)}]^{T}, \quad 1 \le i \le M, \quad 0 \le k \le n_{\mathrm{p}}.$$

Solving step 1' requires the inversion for $1 \leq k \leq n_p$, of M algebraic linear systems of order N (with tridiagonal matrices). These inversions correspond to the calculation of the intermediate fields at successively adjacent azimuths $\theta_1, \theta_2, \ldots, \theta_M$, as shown in Fig. 1(a). Each linear system is solved using a Gaussian algorithm optimized for tridiagonal matrices, the number of elementary arithmetic operations required to invert each of these linear systems being of the order of N. Since each inversion must be repeated for each term of the Padé series expansion, the complexity of step 1 is in $\mathcal{O}(n_p M N)$.

The discretization in the azimuthal direction is done using a $(2\ell + 1)$ -point stencil finitedifference (FD) scheme. This scheme, which corresponds to a higher-order centered FD scheme and can be seen as an extension of the more classical 2nd-order FD scheme ($\ell = 1$), allows one to reduce the required number of points in the azimuthal direction while still



Fig. 1. Resolution schemes for the $N \times 2D$ part (a) and the azimuthal coupling part (b).

obtaining accurate solutions.¹⁶ At each range step, step 2 is replaced by step 2':

step 2'
$$\left[\mathbf{I}_M - \frac{\mathbf{i}k_0\Delta r}{4}\mathbf{Y}_M^{n+\frac{1}{2}}\right]\widetilde{\mathbf{u}}_j^{(n_{\mathrm{p}}+1)} = \left[\mathbf{I}_M + \frac{\mathbf{i}k_0\Delta r}{4}\mathbf{Y}_M^{n+\frac{1}{2}}\right]\widetilde{\mathbf{u}}_j^{(n_{\mathrm{p}})}, \quad 1 \le j \le N,$$

where

$$\widetilde{\mathbf{u}}_{j}^{(k)} = [u_{1,j}^{(k)}, u_{2,j}^{(k)}, \dots, u_{M,j}^{(k)}]^{T}, \quad k \in \{n_{\mathrm{p}}, n_{\mathrm{p}}+1\}, \quad 1 \le j \le N.$$

Here, I_M denotes the identity matrix of order M and $Y_M^{n+\frac{1}{2}}$ is an Mth order square matrix which comes from the FD discretization of the differential operator $\mathcal{Y}^{n+\frac{1}{2}}$. It is to be noted that $Y_M^{n+\frac{1}{2}}$ is $(2\ell+1)$ -diagonal matrix with entries in the upper right and lower left corners to account for the continuity condition in the azimuthal direction. Solving for the azimuthal coupling part (step 2') thus requires the inversion of N algebraic linear systems of order M. As shown in Fig. 1(b), the inversion of these N linear systems now corresponds to the calculation of the acoustic field at successive fixed depths z_1, z_2, \ldots, z_N . As the matrices depend only on the discretization in range, an LU decomposition followed by a forwardand back-substitution is used to solve the N linear systems at each range step. This can be directly performed at a work load in $\mathcal{O}(\ell^2 M)$ and $\mathcal{O}(\ell NM)$, respectively. The complexity of step 2' is thus in $\mathcal{O}(\ell^2 M + \ell MN) \approx \mathcal{O}(\ell MN)$, since N is large compared to ℓ .

Finally, the total computational complexity for solving 3D problems can be expressed as the sum of the computational complexities of step 1' and step 2', times the number of range steps L (recall that step 1' and step 2' must be repeated for each discrete range r_n , $1 \leq n \leq L$). It thus depends linearly on L, M, and N. Suppose that a 3D computation is performed and that the frequency f of the harmonic point source is multiplied by a given factor C. Then, the three spatial increments Δr , $\Delta \theta$, Δz (respectively L, M, N) must be reduced (respectively multiplied) by the same factor C. Due to the linear complexity of the algorithm with respect to L, M, N, the CPU time will thus be multiplied accordingly by C^3 . For instance, by doubling f, the CPU time is multiplied by a factor of $2^3 = 8$ (instead of a factor of $2^2 = 4$ for an equivalent N × 2D computation). Suppose now that, instead of doubling f, the maximum computation range r_{\max} is doubled. We still assume here that a 3D computation is performed. Then, by using the same value of the range increment, the number of points in range (L) is doubled. Besides, to maintain the necessary arclength between adjacent azimuthal angles, the number of points used in azimuth $\left(M\right)$ must be doubled as well. Hence, the CPU time is accordingly multiplied by a factor of $2^2 = 4$ (instead of a factor of 2 when a N × 2D computation is desired). Notice that when doubling both the frequency and the maximum computation range, CPU time will be multiplied by a factor of $2^3 \times 2^2 = 32$ for a full 3D computation, instead of a factor of $2^2 \times 2 = 8$ for its equivalent N × 2D computation. (It will be shown in Sec. 3.2) that the study of the 3D wedge benchmark problem at higher frequencies, requires to increase the maximum propagation range in order not to miss some of 3D effects present in the waveguide.)

Assuming now a broadband source, solving a pulse propagation problem with the Fourier synthesis approach requires to decompose the source pulse using a Fourier transform, then to select a frequency spacing and solve the 3D propagation problem for each discrete frequency within a frequency-band of interest, and finally to perform inverse Fourier transforms of the frequency-domain solutions to obtain the time signal at any given receiver:

$$P(r,\theta,z;t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \widehat{\mathcal{S}}(\omega) H_0^{(1)}(k_0 r) \psi(r,\theta,z;\omega) \mathrm{e}^{-\mathrm{i}\omega t} \,\mathrm{d}\omega\,,\tag{3}$$

where $\widehat{\mathcal{S}}(\omega)$ is the source spectrum, and $\psi(r, \theta, z; -\omega) = \overline{\psi(r, \theta, z; \omega)}$ so that the timedomain acoustic pressure $P = P(r, \theta, z; t)$ is real-valued. As explained before, solving the 3D propagation problem for each discrete frequency is performed by 3DWAPE using the parabolic approach.

In summary, the computational complexity analysis of 3DWAPE gives an indication about the CPU time required for a calculation, and shows that the algorithm can be naturally parallelized. The relevant parallelization strategy is indeed straightforward and consists of two stages: First, since Fourier synthesis is used to solve a pulse propagation problem, a broadband computation can be handled by distributing the calculations at each frequency on different processors. Second, since the model uses a PE approach coupled with an operator splitting technique that allows the separation of the marching algorithm into two successive steps (referred to in this paper as the N \times 2D and the azimuthal coupling steps), the calculations at each single frequency can be accelerated by distributing all the required matrix inversions on different processors. The parallelization strategy is detailed in the following section.

3. Multiprocessor Implementation

3.1. Parallelization strategy

We describe here each parallelization algorithm when they are used separately and/or simultaneously. For each case, some elementary examples are given to illustrate the parallelization method. In the following, two classical parameters, namely the *speedup* and the *efficiency*, are used to characterize the computational performances of a parallel calculation. Suppose p processors are used. The *speedup* is defined as the time to complete an algorithm with only one processor, divided by the time to complete the same algorithm with p processors. The *efficiency* is defined as the ratio of the speedup over p.

3.1.1. First parallelization algorithm: Frequency decomposition (FD)

The first parallelization algorithm is specially dedicated to handle efficiently broadbandsignal propagation. Computing the time signal at a given receiver requires to perform the following steps: First, the source pulse is decomposed using a Fourier transform. Then, the 3D propagation problem is solved independently at each frequency. These uncoupled frequency-domain calculations are performed in parallel using different processors. In practice, it often happens that the number of discrete frequencies, N_{freq} , is larger than the number of processors available, p. In that particular case, more than one discrete frequency are handled by the same processor. This implies that the total number of frequencies be first divided into g frequency groups $\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_g$, with g equal to the number of processors used. All the discrete frequencies within the same frequency group are then handled sequentially by the same processor. Recall that several numerical parameter values (e.g., Δr , $\Delta \theta, \Delta z$) depend on the acoustic wavelength and that higher frequencies are more CPU time consuming than lower frequencies. In order to reduce idle time and consequently optimize the first parallelization algorithm, a cyclical repartition of all the discrete frequencies is used to equilibrate the workload per processor. For example, suppose that the source spectrum is sampled using 256 discrete frequencies ($N_{\text{freq}} = 256$) denoted f_k , $1 \leq k \leq 256$, with $f_k < f_{k'}$ if k < k', and that 64 processors are available (p = 64). The total number of frequencies is divided into 64 frequency groups (g = p):

$$\mathcal{F}_1 = \{f_1, f_{65}, f_{129}, f_{193}\}, \ \mathcal{F}_2 = \{f_2, f_{66}, f_{130}, f_{194}\}, \dots, \ \mathcal{F}_{64} = \{f_{64}, f_{128}, f_{192}, f_{256}\}.$$

Here, each frequency group is composed of 4 elements. An illustration of this cyclical repartition is given in Fig. 2. Each frequency belonging to \mathcal{F}_k is handled by the same processor. Hence, each processor handles both lower and higher frequencies within the frequency bandwidth, and, on average, is allocated about the same computational load. At the end of all the calculations, the frequency-domain solutions at the desired receiver position are collected by only one processor in order to perform an inverse Fourier transform. Note that the communications between processors occur only at the beginning and at the end of the whole process. Communication times are negligible and, hence, will not deteriorate significantly the performances of this first parallelization algorithm.

However, one should keep in mind that the cyclical repartition of all the frequencies, which is used to reduce idle time, is all the more inefficient as the (average) cardinal number of each frequency group is small. The most unfavorable situation corresponds to the case for which each frequency group is composed of only one single discrete frequency. In our example with $N_{\rm freq} = 256$, this would happen if 256 processors were used, which would lead



Fig. 2. Illustration of the cyclical repartition (first parallelization algorithm) of the 256 discrete frequencies f_k , $1 \le k \le 256$, into 64 frequency groups $\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_{64}$.

to 256 frequency groups (g = p): $\mathcal{F}_1 = \{f_1\}, \mathcal{F}_2 = \{f_2\}, \mathcal{F}_3 = \{f_3\}, \dots, \mathcal{F}_{256} = \{f_{256}\}$. Idle time would not be negligible in this case. Thus, for a fixed number of discrete frequencies, an efficiency loss is expected when increasing the number of processors.

3.1.2. Second parallelization algorithm: Spatial decomposition (SD)

The second parallelization algorithm is based on a spatial decomposition of the 3D PE calculations. It is thus dedicated to accelerate the calculations at one single frequency. Suppose the acoustic field is known at a given discrete range r_n . As explained in Sec. 2, the computation of the solution at the next discrete range $r_n + \Delta r$ is achieved in two successive steps corresponding to the N × 2D part and the azimuthal coupling part. The first step requires inverting M algebraic linear systems (see Fig. 1(a)). The parallelization strategy consists in distributing these inversions on different processors. Once this first step is accomplished, the results of each single processor are re-distributed on the other processors to get prepared for the azimuthal coupling part. The same parallelization strategy is then used to invert the N linear systems of the second step (see Fig. 1(b)). Once more, the results need to be re-distributed between processors before starting the computation at the next discrete range. It should be noted that, unlike the first parallelization algorithm, no cyclical repartition of the grid points in azimuth (for the N × 2D part) or in depth (for the azimuthal coupling part) is needed here.

For example, suppose that a single-frequency calculation requires 360 azimuthal points (M = 360) and 600 depth points (N = 600), and that 3 processors are used (p = 3). Then, for this second parallelization algorithm, the spatial domain is first decomposed into 3 groups of azimuth denoted Θ_1 , Θ_2 , Θ_3 . All the azimuths which belong to the same azimuthal group are then handled by the same processor. Once this N × 2D step is achieved, the azimuthal coupling is performed by applying the same procedure in the depth direction. The spatial domain is decomposed into 3 groups of depth denoted Z_1 , Z_2 , Z_3 . All the depths within the same depth group are then handled by the same processor. This example is summarized in Table 1.

Interprocessor communications are not negligible and can lead to significant efficiency losses. Indeed, to better understand this, suppose p processors $P_0, P_1, \ldots, P_{p-1}$ are used and let $k \in \{0, 1, \ldots, p-1\}$. Let also $\theta_{i_{start}}, \theta_{i_{start}+1}, \ldots, \theta_{i_{end}}$ denote the successive azimuths handled by the processor P_k for the N × 2D part, and $z_{j_{start}}, z_{j_{start}+1}, \ldots, z_{j_{end}}$ denote the successive depths handled by the same processor P_k for the azimuthal coupling part

Table 1. Illustration of the spatial segmentation of the calculation domain occurring in the second parallelization algorithm at a single frequency. Example with 360 discrete points in azimuth, 600 discrete points in depth, and 3 processors (P_0 , P_1 , P_2).

Processor ID	P ₀	P ₁	P_2
$N \times 2D$ part	$\Theta_1 = \{\theta_1, \dots, \theta_{120}\}$	$\Theta_2 = \{\theta_{121}, \dots, \theta_{240}\}$	$\Theta_3 = \{\theta_{241}, \dots, \theta_{360}\}$
Azimuthal coupling part	$\mathcal{Z}_1 = \{z_1, \dots, z_{200}\}$	$\mathcal{Z}_2 = \{z_{201}, \dots, z_{400}\}$	$\mathcal{Z}_3 = \{z_{401}, \dots, z_{600}\}$



Fig. 3. Communications (before starting the azimuthal coupling part of the second parallelization algorithm) for one processor P_k with $k \in \{0, 1, \ldots, p-1\}$. The azimuths (depths) handled by P_k for the N×2D part (resp. azimuthal coupling part) are denoted $\theta_{i_{\text{start}}}, \ldots, \theta_{i_{\text{end}}}$ (resp. $z_{j_{\text{start}}}, \ldots, z_{j_{\text{end}}}$). Black squares indicate data to be sent by P_k to the other processors. Grey squares indicate data to be received by P_k from the other processors.

(see Fig. 3). Then, at the end of the first step, P_k must send (and receive) blocks of data to (resp. from) all the other processors.

- Blocks of data sent by P_k correspond to results obtained by P_k at grid points (θ_i, z_j) with $i_{\text{start}} \leq i \leq i_{\text{end}}$ and with $1 \leq j < j_{\text{start}}$ (data sent to P_1, \ldots, P_{k-1}) and $j_{\text{end}} < j \leq N$ (data sent to P_{k+1}, \ldots, P_{p-1}). These grid points are indicated in Fig. 3 by squares filled in black.
- Blocks of data received by P_k correspond to results obtained by the other processors at grid points (θ_i, z_j) with $j_{\text{start}} \leq j \leq j_{\text{end}}$ and with $1 \leq i < i_{\text{start}}$ (data received from P_1, \ldots, P_{k-1}) and $i_{\text{end}} < i \leq M$ (data received from P_{k+1}, \ldots, P_{p-1}). They are indicated in Fig. 3 by squares filled in grey.

The reverse communication process need to be achieved at the end of the second step, before starting the computation at the next discrete range. Interprocessor communications have to be repeated at every step in range. Therefore, for this second parallelization algorithm, the efficiency may be limited due to non-negligible communication times.

3.1.3. Combination of both parallelization algorithms

When performing broadband computations, both parallelization algorithms can be combined. In this case, the strategy is to allocate more than one processor when performing each single-frequency run. Suppose for example that the sampling of the source spectrum leads to 256 discrete frequencies ($N_{\rm freq} = 256$) and that $1024 = 4 \times 256$ processors are available (p = 1024). Then, each of the 256 uncoupled frequency-domain calculations is

performed using a subset of 4 processors. It should be noted that, as already pointed out in Sec. 3.1.1, it seldom happens in practice that the number of processors available exceeds the number of discrete frequencies. However, the combination of both parallelization algorithms is still achievable when $p \leq N_{\text{freq}}$. In that particular case, the total number of frequencies need to be first divided into g frequency groups $\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_q$, and a cyclical repartition of the frequencies is used to reduce idle time occurring in the first parallelization algorithm. The number q of frequency groups must be less than the number p of processors. Note that q = p means that only the first parallelization algorithm is used. Then, the set of processors is accordingly portioned into g processor groups denoted $\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_q$. For each $k \in \{1, 2, \ldots, g\}$, a frequency group \mathcal{F}_k is associated to a processor group \mathcal{P}_k . For each successive frequency of \mathcal{F}_k , all the processors of \mathcal{P}_k are used simultaneously to handle the second parallelization algorithm. An example is given in Fig. 4 to clarify the method: 64 processors are used (p = 64) to handle all the calculations; the sampling of the broadband source uses 256 frequencies $(N_{\text{freq}} = 256)$. The 64 processors are denoted P_0, P_1, \ldots, P_{63} . For the first parallelization algorithm, the frequencies are portioned into 16 frequency groups (g = 16). Each frequency group is then composed of 16 distinct frequencies since $g \times 16 = N_{\text{freq}}$. (Notice that the particular case of 64 frequency groups (i.e., g = 64) has been given as an example in Sec. 3.1.1 and illustrated in Fig. 2.) As shown in Fig. 4, a cyclical repartition of the frequencies is performed. Therefore, the frequency and processor groups are given by

$$\begin{cases} \mathcal{F}_1 = \{f_{1+16(k-1)}, 1 \le k \le 16\}, & \mathcal{P}_1 = \{P_0, P_1, P_2, P_3\}, \\ \mathcal{F}_2 = \{f_{2+16(k-1)}, 1 \le k \le 16\}, & \mathcal{P}_2 = \{P_4, P_5, P_6, P_7\}, \\ \vdots \\ \mathcal{F}_{16} = \{f_{16+16(k-1)}, 1 \le k \le 16\}, & \mathcal{P}_{16} = \{P_{60}, P_{61}, P_{62}, P_{63}\} \end{cases}$$

Then, for each single-frequency calculation, 4 processors are allocated to the second parallelization algorithm. For instance, processors P_0 , P_1 , P_2 , P_3 are used to solve the acoustic problem at frequency f_1 . In the meantime, processors P_4 , P_5 , P_6 , P_7 are used to solve the acoustic problem at frequency f_2 ,..., and processors P_{60} , P_{61} , P_{62} , P_{63} at frequency f_{16} .



Fig. 4. Illustration of the cyclical repartition (first parallelization algorithm) of the 256 discrete frequencies f_i , $1 \le i \le 256$, into 16 frequency groups $\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_{16}$.

3.2. Computational performances

The first and second parallelization algorithms presented in Secs. 3.1.1 and 3.1.2, respectively, have been tested and validated on several 3D benchmarks.²⁴ It should be noted that the performances of the combined algorithm presented in Sec. 3.1.3 have not been analyzed in this paper. Hence, all the parallelized computations presented hereafter were obtained using either the first parallelization algorithm (frequency decomposition) or the second parallelization algorithm (spatial decomposition). In this section, some results of the high-performance computing are presented for the 3D extension of the original ASA wedge benchmark,²⁵ whose configuration is described in detail in Refs. 14 and 16. This original 3D benchmark has been selected among others due to its computational cost that is particularly well suited for the study of the performances of both parallelization algorithms. Indeed, in order to be able to show the evolution of the computational performances when increasing the number of processors, and allow a relevant analysis of both efficiency and speedup, CPU times corresponding to calculations with the largest number of processors available (here 64 processors) should not be too low. On the other hand, calculations performed using only one processor should not be too long (i.e., less than a couple of days) for convenience. A good compromise has been found between these two requirements, considering the present 3D test case.

A broadband sound pulse centered at 25 Hz with a bandwidth of 40 Hz covering the band 5–45 Hz, is considered. The source spectrum is discretized using a frequency sampling of 0.1429 Hz, leading to 281 discrete frequencies ($N_{\rm freq} = 281$). For this test case, following the results reported in Ref. 17, an accurate 3D calculation at 25 Hz can be achieved using M = 3240 azimuthal points (with a 8th-order FD scheme in azimuth), N = 500 depth points, a range step of 10 m, and two Padé terms in depth ($n_{\rm p} = 2$). The maximum depth of the computational grid is equal to 1000 m. The parallel machine used is a HP SC45 cluster with 214 nodes, each of which contains 4 processors running at 1.25 GHz and 4 GB of RAM.

Table 2 presents the CPU times corresponding to full 3D computations for a broadband pulse, obtained using up to 64 processors and for a maximum propagation range of 16 km.

Table 2. 3D ASA wedge benchmark results for the propagation of a broadband sound pulse (first parallelization algorithm: frequency decomposition) with a 25 Hz central frequency and a 40 Hz bandwidth.

Number of Processors	4D CPU Time	4D Speedup	4D Efficiency
1	1 day 10 h $59{\rm mn}~53{\rm s}$	1.00	1.00
2	$17\mathrm{h}$ $18\mathrm{mn}$ $59\mathrm{s}$	2.02	1.01
4	$9\mathrm{h}$ $33\mathrm{mn}$ $32\mathrm{s}$	3.66	0.92
8	4 h 42 mn 7 s	7.44	0.93
16	$2 \mathrm{h} 26 \mathrm{mn} 30 \mathrm{s}$	14.33	0.90
32	$1\mathrm{h}13\mathrm{mn}45\mathrm{s}$	28.48	0.89
64	$43\mathrm{mn}$ $4\mathrm{s}$	48.77	0.76

Number of Processors	$\begin{array}{c} {\rm N}\times 2{\rm D}\\ {\rm CPU} \ {\rm Time} \end{array}$	$\begin{array}{l} N\times 2D\\ Speedup \end{array}$	$N \times 2D$ Efficiency	3D CPU Time	3D Speedup	3D Efficiency
1	$20\mathrm{mn}~46\mathrm{s}$	1.00	1.00	$52\mathrm{mn}$ $2\mathrm{s}$	1.00	1.00
2	$9\mathrm{mn}~59\mathrm{s}$	2.08	1.04	$22\mathrm{mn}$ 4s	2.36	1.18
4	$4\mathrm{mn}~50\mathrm{s}$	4.30	1.07	$9\mathrm{mn}~57\mathrm{s}$	5.23	1.31
8	$2\mathrm{mn}~24\mathrm{s}$	8.67	1.08	$6\mathrm{mn}~42\mathrm{s}$	7.77	0.97
16	$1\mathrm{mn}~21\mathrm{s}$	15.44	0.97	$4\mathrm{mn}$ $5\mathrm{s}$	12.75	0.80
32	$47\mathrm{s}$	26.35	0.82	$2\mathrm{mn}~23\mathrm{s}$	21.90	0.68
64	$34\mathrm{s}$	36.54	0.57	$1\mathrm{mn}~40\mathrm{s}$	31.17	0.49

Table 3. 3D ASA wedge benchmark results at a single frequency (second parallelization algorithm: spatial decomposition) f = 25 Hz.

Table 3 presents CPU times corresponding to $N \times 2D$ and 3D calculations, both for a cw source emitting at 25 Hz, obtained using up to 64 processors. The maximum computation range is 25 km. Though it is not necessary to use such a large number of points in azimuth when performing $N \times 2D$ computation (usually, including 360 azimuthal points is reasonable to re-construct a horizontal plot of the $N \times 2D$ field), the same number of azimuthal points (M = 3240) was used for both N × 2D and 3D calculations, in order to estimate the computational time cost that can be attributed only to the azimuthal coupling part. Indeed, the $N \times 2D$ results (CPU time, speedup, efficiency) shown in Table 3 correspond to computations performed by simply discarding the 3D parallelized algorithm in the azimuthal coupling part. By doing that, interprocessor communications are still present at the end of the $N \times 2D$ part at each range step. This explains the (otherwise surprising) low efficiency values in the azimuthally uncoupled N \times 2D results for p = 32 or p = 64. It should be noted also that all the results of CPU times presented here are not averaged. Hence, their values can change slightly from one computation to another, leading to efficiency values that may exceed 1. Good performances for both parallelization algorithms are obtained. For instance, the use of 64 processors in parallel allows to solve the 4D propagation benchmark problem in less than 1 h (instead of approximately one day and a half by using only one single processor). As expected, the first parallelization algorithm (Table 2) provides a better efficiency than the second one (Table 3) since fewer communications between processors are required. Indeed, with 64 processors, an efficiency of 76% is obtained with the first parallelization algorithm, whereas the efficiency of the second parallelization algorithm is around 50%. However, we observe that the efficiency of the first parallelization algorithm (Table 2) deteriorates when the number p of processors increases. This can be explained easily by the fact that the number of discrete frequencies handled by the same processor diminishes when p becomes larger (see the discussion at the end of Sec. 3.1.1).

Table 3 shows that performing a 3D calculation at a single frequency of 25 Hz with only one processor takes about 1 h of CPU time. By doubling the frequency, the numbers of points in depth, in range, and in azimuth must all be doubled. According to the computational complexity analysis given in Sec. 2, the CPU time is then multiplied by $2^3 = 8$. We thus expect to have CPU times around 8 and 64 h for a cw calculation at 50 and 100 Hz, respectively. These computational costs forbid completely broadband calculations with a source pulse centered at these frequencies. However, Table 3 shows that a parallel 3D calculation at 25 Hz using 64 processors takes less than 2 min. Then, assuming a good efficiency for higher frequencies, the same cw calculation at 100 Hz would take about 2 or 3 h, which is now more reasonable than with the use of only one single processor, and would allow broadband computations for a pulse with a central frequency of 100 Hz with more acceptable CPU times.

4. Investigation of 3D Acoustical Effects

4.1. 3D wedge benchmark at higher frequencies

The parallelized 3D PE algorithm allows us now to investigate the azimuthal coupling effects at higher frequencies and at longer propagation ranges with more reasonable computational times. In this section, we present some results obtained for the 3D wedge benchmark with a cw source emitting at different frequencies: 25, 50, 75, and 100 Hz. For each frequency, the maximum computation range is 80 km instead of 25 km as in the original problem. A Padé-1 approximation $(n_p = 1)$ is used in depth here, since no significant differences have been observed with the solution using a Padé-2 approximation. All the parameters defining the computational grid, need to be reduced as a fraction of the acoustic wavelength when increasing frequency. $N \times 2D$ and 3D computations were performed using $\Delta r = \lambda/6$ and $\Delta z = \lambda/60$, where λ denotes the acoustic wavelength. For the 3D computations, the adjustment of the number M of points in azimuth is particularly important to get a numerical solution that describes accurately all the 3D acoustical effects present in the waveguide. For each frequency, a convergence study has been required to determine M. For instance, a 25 Hz computation required the use of M = 5760 points in the azimuthal direction. Here, convergence is considered to be reached when no significant variation is observed along the $\theta = 90^{\circ}$ azimuth, when doubling the number of azimuthal points.

CPU times for all the calculations when increasing M are reported in Fig. 5. They are consistent with the CPU-time behavior predicted by the computational complexity analysis performed in Sec. 2. For instance, at a given frequency, the CPU time increases (almost) linearly with respect to M. For each frequency, an 8th-order FD scheme was used in azimuth. It should be noted that the use of such a high-order FD scheme permitted a significant reduction of the number of points in the azimuthal direction. Indeed, for a cw source emitting at 25 Hz, a 3D PE run using the classical 2nd-order FD scheme requires 23040 points in azimuth for a maximum computation range of 25 km (results reported in Ref. 16). Note that at 25 Hz, there is no energy propagating across-slope at ranges greater than approximately 40 km (see the upper right subplot of Fig. 6). Thus, a similar 3D run using the 2nd-order FD scheme would require $36864 = 40/25 \times 23040$ points in azimuth for a maximum computation range of 40 km. Here, at the same frequency, using a 8th-order scheme, convergence was reached using 5760 azimuthal points for a maximum computation range of 80 km. Since there is no energy propagating after 40 km, a 3D PE run using the same 8th-order FD scheme would thus require the same number of points in azimuth



Fig. 5. CPU times for the 3D ASA wedge benchmark at different cw source frequencies using 64 processors (second parallelization algorithm: spatial decomposition). For each frequency, the number of azimuthal points is first set to 360 and is then constantly doubled until convergence is reached.

(i.e., M = 5760) for a maximum computation range of 40 km, which is well below the M = 36864 azimuthal points needed by the second-order FD scheme.

Recall that the number of points used in azimuth is the same for all discrete ranges between 0 and $r_{\text{max}} = 80 \text{ km}$. As an evidence, for each frequency, the azimuthal mesh is oversampled at short range. Using as in Ref. 26 an azimuthal increment that depends on range would be preferable since it would certainly allow a reduction of CPU times. Besides, recall that due to the 1/r-term in the azimuthal coupling operator (see Eq. (2)), the problem has a singularity at r = 0. Numerical simulations showed that using too many points in azimuth can lead to numerical problems like arithmetic overflow. Adapting the size of the azimuthal increment with range would also permit to avoid this overflow problem at short range. This option is currently not implemented in the parallelized version of the code. Here, the computations were performed using double-precision arithmetic to avoid the numerical overflow at the vicinity of the source.

Vertical slices of the transmission loss (TL = $-20 \log_{10}(|\psi(r, \theta, z; \omega)|/\sqrt{r})$ with $\omega = 2\pi f$) in the cross-slope direction ($\theta = 90^{\circ}$) are shown in Fig. 6 for the N × 2D and the 3D solutions corresponding to f = 25 Hz, f = 50 Hz, f = 75 Hz, and f = 100 Hz. For a better comparison between the 2D and 3D cases, TL-vs-range curves corresponding to the crossslope direction and to a receiver depth of 30 m are displayed in Fig. 7. The 2D and 3D PE marching algorithms were initialized at r = 0 using a Greene's source.²⁷ For each frequency, the 2D field exhibits for all ranges the interference pattern of all the propagating modes initially present at the source. There are 3, 6, 9, 13 propagating modes at 25, 50, 75, 100 Hz, respectively. Their horizontal wavenumbers and grazing angles are given in Table 4. For



Fig. 6. Transmission loss (in dB re 1 m, vertical slices at a fixed azimuth of 90°) corresponding to 2D (left subplots) and 3D (right subplots) calculations at (from top to bottom) 25, 50, 75, and 100 Hz. The cut-off ranges for mode 1, mode 2, and mode 3, are marked by the vertical bold solid lines on each 3D subplot.

the 3D solutions, the interference patterns are modified during propagation. These changes are due to the horizontal refraction of each propagating mode. These horizontal refraction effects appear at shorter ranges for higher modes than for lower modes since their grazing angles are higher. Indeed, at 25 Hz, across-slope, one can identify successively the cut-off ranges of mode 3, mode 2, and mode 1, at approximately 11, 16, and 40 km. These cut-off ranges are also present at higher frequencies and can be seen on each 3D subplot of Fig. 6. For convenience, the cut-off ranges of only modes 1, 2, 3 are marked on each subplot. Note that, looking at Fig. 6, the cut-off range of mode 1 is present before 80 km at 25 and 50 Hz, but not at 75 and 100 Hz.

In order to observe more precisely the horizontal refraction effects, the 3D PE algorithm can be initialized by each individual propagating mode. We performed all the calculations

153



Fig. 7. Transmission loss (in dB re 1 m) for a receiver at a depth of 30 m and fixed azimuth of 90° , corresponding to N × 2D (dashed lines) and 3D (solid lines) calculations at (from top to bottom) 25, 50, 75, and 100 Hz.

for each propagating mode at each of the four frequency values but only one example is presented here. It corresponds to an excitation of mode 3. Figure 8 shows the vertical slices of the transmission loss in the cross-slope direction corresponding to 3D PE calculations. Figure 9 displays the modal-ray diagrams of mode 3 in the horizontal plane for different frequencies. These modal-ray paths were calculated using the method given in Ref. 28. A higher frequency source leads to a further excursion of the acoustic field in the up-slope

	$25\mathrm{Hz}$		$50\mathrm{Hz}$		$75\mathrm{Hz}$		$100\mathrm{Hz}$	
Mode m	$k_{r,m}$ [rad/m]	ϑ_m [deg]						
1	0.103824	7.49	0.208929	3.99	0.313803	2.72	0.418605	2.07
2	0.101052	15.20	0.207388	8.02	0.312731	5.46	0.417783	4.14
3	0.096225	23.23	0.204783	12.10	0.310934	8.21	0.416409	6.22
4			0.201061	16.26	0.308396	10.99	0.414475	8.31
5			0.196163	20.50	0.305093	13.79	0.411972	10.41
6			0.190048	24.84	0.301000	16.64	0.408889	12.53
7					0.296085	19.52	0.405209	14.67
8					0.290319	22.46	0.400918	16.83
9					0.283690	25.44	0.395996	19.02
10							0.390424	21.24
11							0.384185	23.48
12							0.377276	25.75
13							0.369879	27.99

Table 4. Modal information for frequencies used in the 3D wedge benchmark and for a water-depth of 200 m ($k_{r,m}$: Horizontal wavenumber of mode m; ϑ_m : Grazing angle of the up- and down-going plane waves associated to mode m). Note that ϑ_m are less than the critical grazing angle $\vartheta_c \approx 28.07^{\circ}$.

direction. We observe an overall good agreement between the two sets of subplots. Indeed, one can observe in both modal-ray and 3D PE solutions a succession (across-slope) of three distinct zones, denoted zones I, II, and III, corresponding to one single modal-ray arrival (zone I), multiple modal-ray arrivals (zone II), followed by a shadow zone (zone III) for which there is no modal-ray arrival. It is to be noted that zone II is hardly distinguishable at 25 Hz. Zones I, II, and III, are delimited by vertical dashed lines on Fig. 8 (vertical slices) and by black dots in the cross-slope direction in Fig. 9 (top view). In Fig. 8, the transition between zones II and III exhibits a higher intensity zone (caustic). We also observe in Fig. 8 that zone II starts roughly at the same distance from the source (except for 25 Hz). This behavior is in accordance with the modal-ray paths in the horizontal plane shown in Fig. 9. Besides, we observe on both figures that the width (in the cross-slope direction) of the multiple modal-ray arrival zone increases with frequency, and so the onset of the shadow zone is accordingly shifted in range.

These 3D effects have already been demonstrated.^{29–31} For instance, in Ref. 30, analytical expressions describing modal-ray paths and shadow zone boundaries were derived for several bottom geometries (e.g., wedge-shaped duct, ridge, conical seamount, circular basin) using a ray approach, and used to show a shadow zone with a hyperbolic boundary for each propagating mode. Note that for mode 3, the shadow zone starts before the maximum computation range (80 km) at each of the 4 frequencies considered and can thus be clearly observed on each subplot of Fig. 9. Finally, it is worth noting that the 3D PE calculations predict for each frequency the presence of mode 2 across-slope (evident in the shadow zone of mode 3, see each subplot of Fig. 8), whereas the initial field only included mode 3. This effect is a mode-coupling phenomena occurring during down-slope propagation of mode 3.



Fig. 8. Transmission loss (in dB re 1 m, vertical slices at a fixed azimuth of 90°) for an excitation of mode 3, corresponding to 3D calculations at 25, 50, 75, and 100 Hz. The three distinct zones I, II, III, discussed in the text, are delimited by vertical dashed lines on each subplot.



Fig. 9. Modal ray diagrams (top view) obtained for mode 3 at 25, 50, 75, and 100 Hz. The three distinct zones I, II, and II, are marked by black circles on the horizontal axis in the cross-slope direction.

Of course, the presence of mode 2 cannot be in the modal-ray solutions based on adiabatic mode theory.

4.2. Computations in a realistic environment

The aim is now to show that the use of the newly developed parallelized PE model is not restricted to benchmark problems, and that realistic numerical simulations are possible using some classical geophysical models for the ocean bathymetry and the sound speed profiles. The feasibility of the procedure is illustrated here by focusing on an example in the Mediterranean sea close to the East coast of Corsica (see Fig. 10 for two distinct views of the bathymetry). The point source is located at a latitude of $+42.5^{\circ}$ and a longitude of $+9.7^{\circ}$. The source depth is 30 m. In the calculations, the Smith and Sandwell data set³² was used providing an average sampling of 2' (i.e., approximately 1.4 km in longitude and $1.8 \,\mathrm{km}$ in latitude in this region). The GDEM-V data set³³ with a resolution of 30' was used to include 16 (very similar) sound speed profiles (SSP) in the region of interest. Note that for this realistic acoustical problem, the 3D acoustical effects are mainly attributed to bathymetric variations and not to the SSP spatial dependence. The sound speed profiles were not interpolated in range (for a detailed profile interpolation procedure, see for instance in Ref. 13). Notice that some preliminary results concerning this example were already published for a source frequency of 15 Hz in Ref. 34. Results for a cw source emitting at 15, 45, and 60 Hz are reported here. The maximum computation range is 50 km. Each calculation was carried out initializing the PE algorithm at r = 0 with a Greene's source, and using a Padé-2 approximation $(n_p = 2)$ in depth. A fourth-order FD scheme in azimuth was used since the eighth-order FD scheme used for the 3D wedge benchmark (studied in Sec. 4.1) is only implemented for an oceanic environment with a symmetry about the 0° -azimuth vertical plane. The incremental steps in range and in depth are, respectively,



Fig. 10. Maps of the region of interest in the Mediterranean sea (East coast of Corsica). The point source denoted S is represented on both subplots.

Table 5. CPU times for the realistic example in the Mediterranean sea at different cw source frequencies using 64 processors (second parallelization algorithm: spatial decomposition). The number of azimuthal points is doubled until the convergence is reached.

M	$15\mathrm{Hz}$	$45\mathrm{Hz}$	$60\mathrm{Hz}$
360	$37\mathrm{s}$	$2\mathrm{mn}~26\mathrm{s}$	$29\mathrm{mn}~31\mathrm{s}$
720	$43\mathrm{s}$	$3\mathrm{mn}$ 1s	$42\mathrm{mn}$ $4\mathrm{s}$
1440	$50\mathrm{s}$	$4\mathrm{mn}~29\mathrm{s}$	$1\mathrm{h}\ 2\mathrm{mn}\ 39\mathrm{s}$
2880	$1\mathrm{mn}$ $2\mathrm{s}$	$7\mathrm{mn}~20\mathrm{s}$	$1\mathrm{h}~22\mathrm{mn}~57\mathrm{s}$
5760	$1\mathrm{mn}~41\mathrm{s}$	$13\mathrm{mn}$ $10\mathrm{s}$	$2\mathrm{h}~20\mathrm{mn}~28\mathrm{s}$
11520		$25\mathrm{mn}~24\mathrm{s}$	4 h 18 mn 43 s

 $\lambda/5$ and $\lambda/20$, with λ the acoustic wavelength. As already explained in Sec. 4.1, the number of azimuthal points has been determined by a convergence study. The azimuthal grid spacing was thus gradually reduced (starting with $\Delta \theta = 1^{\circ}$, or equivalently with M = 360) until the numerical solution started to stabilize. The CPU times for all the calculations when increasing M are reported in Table 5. For each frequency, the highest value of M in Table 5 corresponds to the value for which convergence was reached. Figure 11 shows grey-scale



Fig. 11. Transmission loss (in dB re 1 m, vertical slices for a fixed azimuth of 90°) corresponding to N × 2D (top) and 3D (bottom) calculations at 15 Hz.



Fig. 12. Transmission loss (in dB re 1 m) for a receiver at a depth of 30 m and at a fixed azimuth of 90° , corresponding to N × 2D (dashed line) and 3D (solid line) calculations at 15 Hz.

images of the transmission loss (vertical slice at fixed azimuth $\theta = 90^{\circ}$, i.e., along the East coast of Corsica, see Fig. 10) corresponding to N × 2D and 3D calculations at 15 Hz.

For the same azimuth, Fig. 12 displays the corresponding TL-vs-range curves at a receiver depth of 30 m. The interference patterns reveal the presence of only few propagating modes along that specific direction. Note that the water-depth is (weakly) range-dependent along the $\theta = 90^{\circ}$ azimuth. The number of propagating modes is thus allowed to change (slightly) from one distance to another along that azimuthal direction. By comparing the N × 2D and 3D solutions displayed in both Figs. 11 and 12, one can observe that the modal structure of the 3D field is strongly modified along the East coast of Corsica, and shows the presence of typical horizontal refraction effects. Indeed, the N × 2D solution shows approximately the same number of propagating modes at all ranges along the $\theta = 90^{\circ}$ azimuthal direction, whereas the 3D solution exhibits evident modal shadowing effects at $r \approx 25$ km and $r \approx 45$ km (due to horizontal deviation of the propagating energy) along the $\theta = 90^{\circ}$ azimuth. It should be noted that these 3D effects, observed here for a realistic three-dimensional oceanic environment, are very similar to the ones described in detail for the 3D ASA wedge benchmark problem at 25 Hz, although the environmental parameters considered are different.

We also illustrate the feasibility of the procedure at a higher frequency of 60 Hz by showing grey-scale images of the transmission loss corresponding to $N \times 2D$ and 3D calculations (Fig. 13) along a fixed azimuth of 240°. The bathymetry along that azimuthal direction can be seen in Fig. 10. Figure 14 displays for the same azimuth the corresponding TL-vs-range curves at a receiver depth of 30 m. At 60 Hz, significant 3D effects are also clearly present. For instance, one can observe in Fig. 14 the disappearance of one propagating mode at a distance of approximately 25 km, due to horizontal refraction effects. This (again typical) modal shadowing effect can also be observed in Fig. 13 by comparing the



Fig. 13. Transmission loss (in dB re 1 m, vertical slices at a fixed azimuth of 240°) corresponding to N × 2D (top) and 3D (bottom) calculations at 60 Hz.



Fig. 14. Transmission loss (in dB re 1 m) for a receiver at a depth of 30 m and at a fixed azimuth of 240° (right plot), corresponding to N × 2D (dashed line) and 3D (solid line) calculations at 60 Hz.

 $N \times 2D$ and 3D interference patterns in the water column at ranges between approximately 25 and 30 km, leading to different mode cut-off effects during up-slope propagation in the penetrable bottom between 30 and 40 km-ranges. Indeed, the N × 2D solution shows three well-defined successive beams in the bottom (corresponding to modal energy being radiated into the bottom), whereas the full 3D solution shows only two beams.

5. Conclusion

Sound propagation modeling in 3D and/or 4D using 3D PE based model were often limited by computational time issues in the past. To overcome this difficulty, an existing 3D PE code has been implemented in a multiprocessor environment. The parallelization strategy chosen consists of a suitable two-level procedure that includes frequency and spatial decompositions of the 3D PE calculations. The parallelized algorithm has been validated and the computational performances have been analyzed for the 3D ASA wedge benchmark. With the first parallelization algorithm, it has been shown that broadband signal calculations can be efficiently accelerated by distributing independently on different processors the calculations for each frequency. As expected, good computational performances have been obtained in this case since only few communications between processors were needed. The second parallelization algorithm consists in accelerating the calculations at one single frequency by distributing on different processors all the required matrix inversions. It has been shown that this second algorithm provides also good results although interprocessors communications are no longer negligible.

The use of the parallelized 3D PE code has facilitated a preliminary investigation of 3D acoustical effects at higher source frequencies (and also at a longer propagation range) for the 3D wedge benchmark. When increasing frequency, it has been observed that for each propagating mode, the modal deviation due to the 3D wedge predicted using full 3D PE calculations, is in agreement with adiabatic modal-ray path calculations. Furthermore, an interesting result concerning modal coupling phenomena occurring during down-slope propagation has been emphasized. This result has been illustrated in this paper with the presence across-slope of mode 2 in the shadow zone of mode 3, whereas only mode 3 was excited at the source location. This mode coupling effect has been only pointed out here. It certainly needs to be analyzed in detail in future works (e.g., using the spectral decomposition of the PE fields as in Ref. 35). In this paper, we only focused on the 3D acoustical effects corresponding to harmonic point sources. Hence, our concern will be also to focus on broadband signal propagation associated with a study of modal dispersion as in Ref. 17. Reasonable (or at least accessible) 4D CPU times are expected.

It has been shown that parallel computations can overcome CPU time limitations, thus making possible the analysis of 3D acoustical effects for several different propagation scenarios with higher signal frequencies and/or propagation distances. It has been shown that it is also possible to use the parallelized version of the code in more realistic configurations including geophysical data: An example has been presented in this paper. Modal deviation and shadowing effects due to bathymetric slopes have been clearly observed in our example in the Mediterranean Sea for two distinct azimuths and two different frequencies. The analysis of 3D effects in other realistic oceanic environments including different varying bathymetries and/or sound speed profiles, and also broadband source pulses can now be addressed.

Acknowledgments

The authors would like to express their thanks to Olivier Bertrand for his technical contribution to the parallelization of the code. The authors gratefully acknowledge the pertinent comments and suggestions of the two anonymous reviewers.

References

- S. Glegg, G. Deane and I. House, Comparison between theory and model scale measurements of the three-dimensional sound propagation in a shear supporting penetrable wedge, J. Acoust. Soc. Am. 94(4) (1993) 2334–2342.
- 2. A. Tolstoy, 3-D propagation issues and models, J. Comp. Acoust. 4(3) (1996) 243-271.
- C. Chen, J. T. Lin and D. Lee, Acoustic three-dimensional effects around the Taiwan strait: Computational results, J. Comp. Acoust. 7(1) (1999) 15–26.
- G. B. Deane, The penetrable wedge as a three-dimensional benchmark, J. Acoust. Soc. Am. 103(6) (1998) 2989.
- R. Doolittle, A. Tolstoy and M. Buckingham, Experimental confirmation of horizontal refraction of CW acoustic radiation from a point source in a wedge-shaped ocean, J. Acoust. Soc. Am. 83 (1988) 2117–2125.
- G. B. Deane and M. J. Buckingham, An analysis of the three-dimensional sound field in a penetrable wedge with a stratified fluid or elastic basement, J. Acoust. Soc. Am. 93(4) (1993) 1319–1328.
- E. Westwood, Broadband modeling of the three-dimensional penetrable wedge, J. Acoust. Soc. Am. 92(4) (1992) 2212–2222.
- 8. M. D. Collins and S. A. Ching-Bing, A three-dimensional parabolic equation model that includes the effects of rough boundaries, J. Acoust. Soc. Am. 87(3) (1990) 1104–1109.
- D. Lee, G. Botseas and W. L. Siegmann, Examination of three-dimensional effects using a propagation model with azimuth-coupling capability (FOR3D), J. Acoust. Soc. Am. 91(6) (1992) 3192–3202.
- K. B. Smith, A three-dimensional propagation algorithm using finite azimuthal aperture, J. Acoust. Soc. Am. 106(6) (1999) 3231–3239.
- C. F. Chen, Y.-T. Lin and D. Lee, A three-dimensional azimuthal wide-angle model, J. Comp. Acoust. 7(4) (1999) 269–288.
- J. I. Arvelo and A. P. Rosenberg, Three-dimensional effects on sound propagation and matchedfield processor, J. Comp. Acoust. 9(1) (2001) 17–39.
- G. H. Brooke, D. J. Thomson and G. R. Ebbeson, PECan: A Canadian parabolic equation model for underwater sound propagation, J. Comp. Acoust. 9 (2001) 69–100.
- J. A. Fawcett, Modeling three-dimensional propagation in an oceanic wedge using parabolic equation methods, J. Acoust. Soc. Am. 93(5) (1993) 2627–2632.
- F. Sturm, Numerical simulations with 3DWAPE considering shallow water range-dependent environments, J. Acoust. Soc. Am. 105(5) (2001) 2334–2335.
- F. Sturm and J. A. Fawcett, On the use of higher-order azimuthal schemes in 3-D PE modelling, J. Acoust. Soc. Am. 113(6) (2003) 3134–3145.

- F. Sturm, Numerical study of broadband sound pulse propagation in three-dimensional oceanic waveguides, J. Acoust. Soc. Am. 117(3) (2005) 1058–1079.
- G. H. Brooke and D. J. Thomson, A single-scatter formalism for improving PE calculations in range-dependent media, in *NRL PE WORKSHOP II (proceedings of the second parabolic equation workshop)*, eds. S. A. Chin-Bing, D. B. King, J. A. Davies and R. B. Evans (Naval Research Laboratory, 1993), pp. 126–144.
- M. D. Collins and R. B. Evans, A two way parabolic equation for acoustic backscattering in the ocean, J. Acoust. Soc. Am. 91(3) (1992) 1357–1368.
- J. P. Berenger, A perfectly matched layer for the absorption of electromagnetic waves, J. Comput. Phys. 114 (1994) 185–200.
- W. C. Chew and Q.-H. Liu, Perfectly matched layers for elastodynamics: A new absorbing boundary condition, J. Comput. Acoust. 4 (1996) 341–359.
- Q.-H. Liu and J. Tao, The perfectly matched layer for acoustic waves in absorptive media, J. Acoust. Soc. Am. 102(4) (1997) 2072–2082.
- D. Yevick and D. J. Thomson, Impedance-matched absorbers for finite-difference parabolic equation algorithms, J. Acoust. Soc. Am. 107(3) (2000) 1226–1234.
- K. Castor, F. Sturm and P. F. Piserchia, Acoustical propagation modeling using the threedimensional parabolic equation based code 3DWAPE within a multiprocessing environment, J. Acoust. Soc. Am. 116(4) (2004) 2549–2550.
- F. Jensen and C. Ferla, Numerical solutions of range-dependent benchmark problems in ocean acoustics, J. Acoust. Soc. Am. 87(4) (1990) 1499–1510.
- 26. F. Sturm and N. A. Kampanis, Accurate treatment of a general sloping interface in a finite-element 3-D narrow-angle PE model, *J. Comput. Acous.* (to appear).
- R. R. Greene, The rational approximation to the acoustic wave equation with bottom interaction, J. Acoust. Soc. Am. 76 (1984) 1764–1773.
- H. Weinberg and R. Burridge, Horizontal ray-theory for ocean acoustics, J. Acoust. Soc. Am. 55 (1974) 63–79.
- C. H. Harrison, Three-dimensional ray paths in basins, troughs, and near seamounts by use of ray invariants, J. Acoust. Soc. Am. 62(6) (1977) 1382–1388.
- C. H. Harrison, Acoustic shadow zones in the horizontal plane, J. Acoust. Soc. Am. 65(1) (1979) 56-61.
- C. H. Harrison, Wave solutions in three-dimensional ocean environments, J. Acoust. Soc. Am. 93(4) (1993) 1826–1840.
- W. H. Smith and D. T. Sandwell, Global seafloor topography from satellite altimetry and ship depth soundings, *Science* 277 (1997) 1956–1962.
- W. J. Teague, M. J. Carron and P. J. Hogan, A comparison between the generalized digital environmental model and Levitus climatologies, J. Geophys. Res. 95 (1990) 7167–7183.
- K. Castor, F. Sturm and P. F. Piserchia, Analysis of 3D acoustical effects in a realistic oceanic environment, Proc. Int. Conf. Underwater Acoustic Measurements: Technologies & Results, Heraklion, Crete, Greece, 28th June–1st July 2005.
- F. B. Jensen and H. Schmidt, Spectral decomposition of PE fields in a wedge-shaped ocean, in *Progress in Underwater Acoustics*, ed. H. M. Merklinger (Plenum Press, New York, 1987), pp. 557–564.