# THE RADIATED FIELD OF CIRCULAR ARRAYS OF SOURCES NEAR A SCATTERING CYLINDER


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## ABSTRACT

The present study addresses analytically the diffraction of circular arrays of phased sources by a cylindrical obstacle at very low frequencies. The interest is the tonal marine propeller noise at the first harmonics of the blade-passing frequency. At the very low Mach numbers of interest, the expectedly dominant sources of propeller noise are the lift fluctuations on the blades, radiating as rotating dipoles. At a given frequency the field is a superposition of spinning modes. Each mode is exactly reproduced by a set of fixed dipoles of similar amplitude but radiating at different emission times, called source-mode. The modes are used to model the near-field scattering by a cylindrical body at short distance, with the exact tailored Green's function in two dimensions. The configuration is acoustically compact, i.e the sizes of the cylinder and of the source circle, as well as the distance, are small compared to the wavelength. The diffraction is found to induce a strong amplification with respect to what the source mode radiation would be in free field. This regime is explained by an asymptotic analysis. The study stresses that marine propeller noise cannot be predicted in free-field and that installation on a ship is the most crucial aspect.


## 1. INTRODUCTION

The generation and radiation of acoustic waves in water by ships is a topic of wide interest which involves complex physical mechanisms. Firstly noise of vibroacoustic nature is emitted because mechanical vibrations are transmitted to the hull through the driving shaft. Secondly the propellers are known to generate hydrodynamic noise. The latter is generally the most significant contribution to the total noise. It is a matter of concern when addressing the acoustic discretion of ships or the protection of aquatic wild life. This is why nowadays marine propeller design has to both improve the hydrodynamic efficiency and reduce the emitted sound. For this the involved mechanisms must be identified and dedicated prediction models that could be introduced in optimization design loops are needed.

The most crucial aspects of marine propellers when dealing with the associated noise are the so-called installation effects, which are twofold. Typically the propellers of marine vehicles are always mounted in the rear part of the ship hull, therefore they are partly embedded in a non-homogeneous and turbulent wake and/or interact with the turbulent boundary layer developing along the whole length of the hull. Random and periodic fluctuations are induced on the blades, leading to increased, respectively broadband and tonal, acoustic signatures. This is referred to as the hydrodynamic installation effect : any deviation from a pure axisymmetric flow around the propeller axis produces noise. Secondly the mounting of propellers in close vicinity of the hull makes the sound radiated from the blades scattered by the hull in such a way that the radiating properties of the sources are strongly modified. This is especially pronounced for the sources of the tonal noise, known to result from interferences between isolated-blade contributions. As a result the noise of an installed propeller can dramatically differ from what the free-field noise would be, even considering the same sources. This second effect is called the acoustic installation effect. It is what the present work is dealing with.

Both installation effects are usually considered independently. The noise that would be radiated by a propeller in free field but with the real flow distortions, thus the true sources, is predicted relying on Ffowcs Williams & Hawking's formulation of the acoustic analogy [1, 2]. The rotating blades and their accompanying unsteady flows are formally replaced by equivalent sources that are assumed to radiate in a uniform and stagnant unbounded medium. The wave equation of the analogy is therefore solved with the standard free-field Green's function. At very low Mach numbers, the passage of the propeller blades through an inhomogeneous flow generates fluctuating forces on the blades acting as equivalent acoustic dipoles. This so-called loading noise is the dominant contribution to the total sound. As long as the tonal noise at harmonics of the Blade-Passing Frequency (BPF) is considered, each tone of the loading noise is a sum of spinning radiation modes. A mode is a diverging pressure wave combined with an azimuthally periodic pattern spinning at some phase speed, forced by the dipole source strength.

In the presence of surrounding surfaces another Green's function must be used or the wave equation of the analogy solved numerically with additional boundary conditions imposed on the surfaces, depending on the geometry of the latter. This has been the basis of hybrid methods developed in hydroacoustics to estimate the noise nearby

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the hull of a ship [3, 4]. In the present work, diffraction is addressed assuming a rigid cylinder instead of a true hull geometry (Fig. 1), in order to identify key mechanisms with a simple mathematical background. The exact tailored Green's function of the cylinder for the Helmholtz equation is used to this end, the problem being stated in the frequency domain.

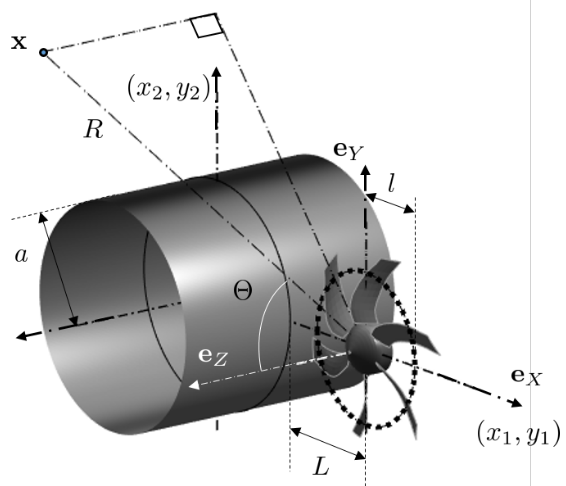



**Figure 1**: Spherical coordinates for the three-dimensional formulation of rotor noise. Subsequent developments refer to the rotor-disk plane $(\mathbf{e}_X, \mathbf{e}_Y)$.

Because diffraction is a matter of compared source-to-obstacle distance and frequency, it differs for all angular positions of a blade element seen as source along its circle. This causes imbalance in the partial cancellations which determine the radiation efficiency of a spinning mode. In order to take this exactly into account in the study, each spinning mode is reproduced with an equivalent circular distribution of stationary and phased dipoles called source-mode [5, 6]. The source-modes must be assumed in the very vicinity of the cylinder to be representative of the hull of a marine vessel. Furthermore their tangential phase speeds are essentially much lower than the speed of sound in water. In this case of low Mach and Helmholtz numbers, the region including the blades and the hull cross-section is acoustically compact. This justifies that a compact approximation of the Green's function is used, in the sense introduced by Howe [7]. This particular regime is known to produce a more or less pronounced amplification of sources of high equivalent polar orders. Its assessment is therefore crucial. This is achieved here with a two-dimensional model to provide a first insight into the physics of amplification. The study is aimed at pointing out that the diffraction generates a strong amplification compared to what the source-mode radiation would be in free field.

Some theoretical background of tonal rotating-blade noise for compact blades and the notion of associated source-modes are introduced in section 2. The analytical expressions of the scattered sound field based on the two-dimensional tailored Green's function of the rigid cylinder for the Helmholtz equation are derived in section 3. Results of a test case representative of marine applications are presented in section 4. The aim is to highlight the amplification caused by cylinder scattering by relying on the analytical Green's function and its asymptotic compact form for very low frequencies. The effect of the source-to-cylinder distance and other key features are then discussed.

## 2. TONAL-NOISE FORMULATION

The present study focuses on the tonal noise of propellers operating at low Mach numbers. Ffowcs Williams and Hawkings's formulation of the acoustic analogy [1, 2] states that the periodic hydrodynamic forces on the blades are the sources of the sound in this case, at least as long as sound radiation is considered in free space. A short reminder is given in this part to illustrate basic properties before addressing installed configurations.

### 2.1 Rotating dipole noise formula

Tonal rotor noise is usually formulated in the frequency domain in the far-field, in a way suited to highlight modulation by the azimuthal flow distortions and interference properties between blades. The mathematical solution is derived assuming a single acoustically compact blade element rotating at constant angular speed $\Omega$ on the circle of radius $l$ in a quiescent propagation medium. For a complete prediction the result should be summed over all elements of a discretized blade, but for the compact blades of marine propellers, a single element carrying the instantaneous integrated force is enough, located at some averaged radius $l$. The complex-valued far-field acoustic pressure at the multiple of order $\mu$ of the BPF reads :

$$p_{\mu B}(\mathbf{x}) = \frac{\mathrm{i}\, k_{\mu B} B}{4\pi R}\, \mathrm{e}^{\mathrm{i}\, k_{\mu B} R} \sum_{s=-\infty}^{\infty} F_s\, \mathrm{e}^{\mathrm{i}\,(\mu B - s)(\phi - \pi/2)} \tag{1}$$
$$\times \mathrm{J}_{\mu B - s}\left(\mu B\, M\, \sin\Theta\right) \left[\cos\gamma\, \cos\Theta - \frac{(\mu B - s)\, \sin\gamma}{\mu B\, M}\right]$$

with the notations defined in Fig. 1, where $M = \Omega l / c_0$ is the tangential Mach number of the element, $k_{\mu B} = \mu B \Omega / c_0$ is the acoustic wavenumber and $\gamma$ the angle of the lift force on the blade element with respect to the rotational axis. $F_s$ are the Fourier coefficients of the dipole strength.

In the plane of the rotor disk considered later on for two-dimensional diffraction studies, $\Theta = \pi/2$ and the formula reduces to :

$$p_{\mu B}(\mathbf{x}) = \frac{\mathrm{i}\, k_{\mu B} B}{4\pi R}\, \mathrm{e}^{\mathrm{i}\, k_{\mu B} R}\, \sin\gamma \times$$
$$\sum_{s=-\infty}^{\infty} F_s\, \mathrm{e}^{\mathrm{i}(\phi - \pi/2)}\, n\, \frac{\mathrm{J}_n\left(\mu B\, M\right)}{\mu B\, M} \tag{2}$$

with $n = \mu B - s$, the number of lobes.

As a result the symmetric mode associated with the Bessel function $\mathrm{J}_0$ and the BLH of order $s = \mu B$ gives a zero contribution. Furthermore for marine propellers the tangential Mach number is very small, so that for moderate blade numbers and BPF harmonic orders the argument of the Bessel functions is small. The limited form for small arguments reads

$$\mathrm{J}_n\left(\mu B\, M\right) \sim \frac{1}{n!}\left(\frac{\mu B\, M}{2}\right)^n ,$$

so that the ratio

$$\frac{\mathrm{J}_n\left(\mu B\, M\right)}{\mu B\, M} \sim \frac{1}{2^n\, n!}\left(\mu B\, M\right)^{n-1}$$

becomes negligible for $n \geq 2$ whereas it is $1/2$ for $n = \pm 1$. This makes the mode order $1$ the only expected significant contribution. These special aspects of vanishing Mach numbers will be re-addressed in section 4. Equation (1) describes tonal propeller noise in the far field. A more general formulation is required to investigate acoustic installation effects. Indeed acoustic scaterring by surrounding solid surfaces located at arbitrary distances also involves the near field of the sources. A dedicated formalism, valid for arbitrary source and observer locations, is described in the next section.

### 2.2 Source-mode expansion

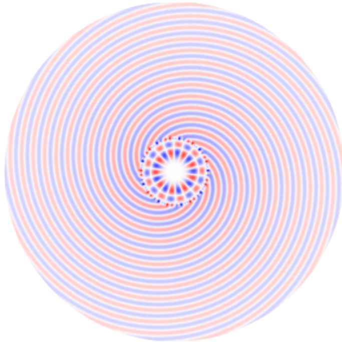
If the time dependence $\mathrm{e}^{-\mathrm{i}\,\omega t}$ is re-introduced, Eq-(1) stresses that at a given frequency $\omega = \mu B\,\Omega$ the sound field is an infinite sum of elementary wave components called radiating spinning modes. Indeed each term of order $s$ has $n = \mu B - s$ lobes spinning at the angular speed $\mu B\omega/n$. Its field can be exactly reproduced by a distribution of stationary dipoles on the circle of radius $l$, with constant amplitude $F_s$ and the phase or time shift able to simulate the rotation. Such a source circle is called a source-mode. Its use is convenient, for instance, to account for the presence of a uniform mean flow [6]. But its main interest is that it gives a representation of the sound field valid at every point in space and not only in the far field, as pointed by Roger *et al.* [5, 8, 9]. This allows to take into account the near-field terms involed in the scattering mechanism by surrounding obstacles.

The point dipole of a source-mode at angle $\alpha$ has the same instantaneous strength $F(\alpha, t)$ as the dipole at angle $\alpha = 0$ but with a time delay $\Delta t = (\mu B - s)\alpha/\omega$ :

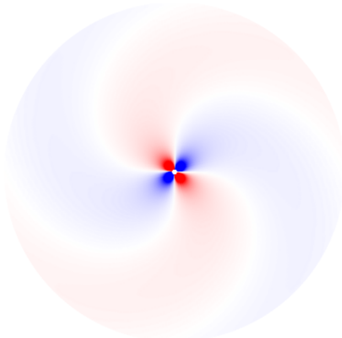
$$\forall \alpha \in [0, 2\pi], F(\alpha, t) = F\left(0, t - \frac{(\mu B - s)\alpha}{\omega}\right) \quad (3)$$

This defines a spinning pattern of angular phase speed $\Omega_s = \omega/n$ with $n = \mu B - s$. For the mode of order $n$ associated with the blade-loading harmonic of order $s$, $F(\alpha, t) = F_s \mathrm{e}^{-\mathrm{i}\,\mu B\,\Omega t}$ with $F_s = F\,\mathrm{e}^{\mathrm{i}\,n\alpha}$.

Instantaneous sound-pressure patterns of isolated modes are plotted in Figs (2a) and (2b) for $\Theta = \pi/2$ for two extreme configurations. The first one, Fig-(2a), is the high-frequency case of a mode $n = 9$ typical of propeller noise in aeronautical applications, for which the source circle is substantially larger than the wavelength. $n$ spiral



(a) mode $n = 9$, $k1 = 8.9$. Larger white circle corresponding to the source circle.



(b) mode $n = 2$, $k1 = 0.34$ . Small white dot standing for the source circle.

**Figure 2**: Instantaneous pressure patterns of spinning radiation modes in non-compact (a) and compact (b) configurations.

wavefronts spinning in the counterclockwise direction are clearly identified, with a rather progressive attenuation imposed by the cylindrical spreading. The second case, Fig-2b, corresponds to a mode $n = 2$. Now the size of the source circle is much smaller than the acoustic wavelength, which means that the configuration is compact, more representative of marine propellers. The spiral wavefronts have a rapidly decreasing amplitude. For both tests the dipole axes are tangent to the source-mode circle.

By linear superposition, the total sound field of a propeller can always be reconstructed by summing individual source-mode fields. This makes source-modes well suited for the physical understanding of free-field rotating-blade noise, and for the assessment of the radiating efficiency of the blade-loading harmonics $F_s$. In the following section, the radiation of a single source-mode is addressed in the presence of a scattering cylinder of circular cross-section.

## 3. THEORETICAL MODEL OF SOUND SCATTERING BY A RIGID CYLINDER

### 3.1 Tailored Green's function

An alternative to Green's formula when solving a problem of acoustics in the presence of solid boundaries is to consider a Green's function tailored to the geometry. In the present study, the sources are assumed very close to the surface of a rigid cylinder of circular cross-section. The tailored Green's function, $G_t'$, solution of the homogeneous Helmholtz equation, is determined by adding the free-field Green's function, $G_i'$, a second term, $G_s'$, such that the total Green's function $G_t' = G_i' + G_s'$ verifies the rigid-wall boundary condition on the cylinder surface.

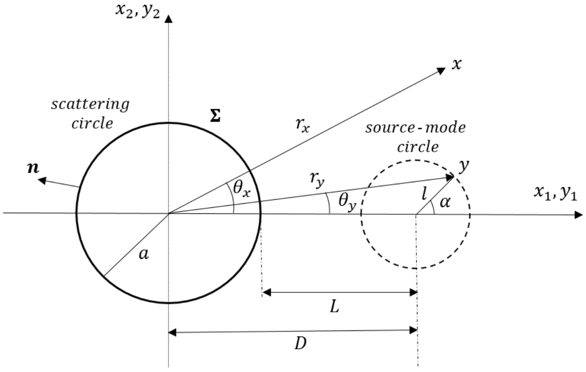



**Figure 3**: Polar coordinates and main notations used in the present formulations.

In the present, preliminary two-dimensional analysis, the free-field Green's function as given by Morse & Ingard [10] in polar coordinates with origin at the center of the circular cross-section, and as sketched in Fig-(3) is used. If $\mathbf{x} = (r_x, \theta_x)$ and $\mathbf{y} = (r_y, \theta_y)$ stand for the observer and source points, respectively, it reads

$$G_i'(\mathbf{x}|\mathbf{y}, \omega) = \frac{1}{4i} \sum_{m=0}^{+\infty} \epsilon_m \cos\left[m(\theta_y - \theta_x)\right] \times \begin{cases} \mathrm{H}_m^{(1)}(kr_x)\,\mathrm{J}_m(kr_y), r_y \leq r_x \\ \mathrm{J}_m(kr_x)\,\mathrm{H}_m^{(1)}(kr_y), r_y \geq r_x \end{cases} \quad (4)$$

where $\epsilon_m = 1$ for $m = 0$ and $\epsilon_m = 2$ for $m > 0$, $k = \mu B \Omega / c_0$ being the acoustic wavenumber. $\mathrm{J}_m$ and $\mathrm{H}_m^{(1)}$ are the Bessel and Hankel functions of order $m$ and of the first kind. The presence of the cylinder of radius $a$ is accounted for by the scattered part of the tailored Green's function, written as :

$$G_s'(\mathbf{x}|\mathbf{y}, \omega) = \frac{1}{4i} \sum_{m=0}^{+\infty} \epsilon_m B_m \mathrm{H}_m^{(1)}(kr_y) \cos\left[m(\theta_y - \theta_x)\right] \quad (5)$$

The rigidity condition $\partial G_t' / \partial n = 0$ at $r_y = a$ yields :

$$B_m = -\frac{\mathrm{J}_m'(ka)}{\mathrm{H}_m^{(1)\prime}(ka)}\,\mathrm{H}_m^{(1)}(kr_x)$$

with

$$\frac{\mathrm{J}_m'(ka)}{\mathrm{H}_m^{(1)\prime}(ka)} = \frac{\mathrm{J}_{m-1}(ka) - \mathrm{J}_{m+1}(ka)}{\mathrm{H}_{m-1}^{(1)}(ka) - \mathrm{H}_{m+1}^{(1)}(ka)}, \; m \geq 1,$$

$$\frac{\mathrm{J}_0'(ka)}{\mathrm{H}_0^{(1)\prime}(ka)} = \frac{\mathrm{J}_1(ka)}{\mathrm{H}_1^{(1)}(ka)}.$$

### 3.2 Application to source-mode scattering

When the cylinder is considered as a noise scatterer, the total field, $p_{tot}'$, sum of the free-field and scattered pressures $p_i'$ and $p_s'$, is calculated from the aforementioned Green's function. This is considered in this section for an arbitrary source-mode of dipoles. Therefore the first derivative of the tailored Green's function with respect to the source coordinates is needed and the sound contribution of a single point dipole of angle $\alpha$ on the mode circle is expressed as :

$$p_{tot}' = \mathbf{F}_s \cdot \nabla(G_i' + G_s') \quad (6)$$

where $\mathbf{F}_s = F_s\,\mathbf{e}_T$ if $\mathbf{e}_T$ is the tangential unit vector and $F_s = F\,\mathrm{e}^{\mathrm{i}\,n\alpha}$ the dipole strength. The derivations of $\nabla(G_i' + G_s')$ are not detailed here for conciseness. They are similar to those provided by Gloerfelt *et al.* [11].

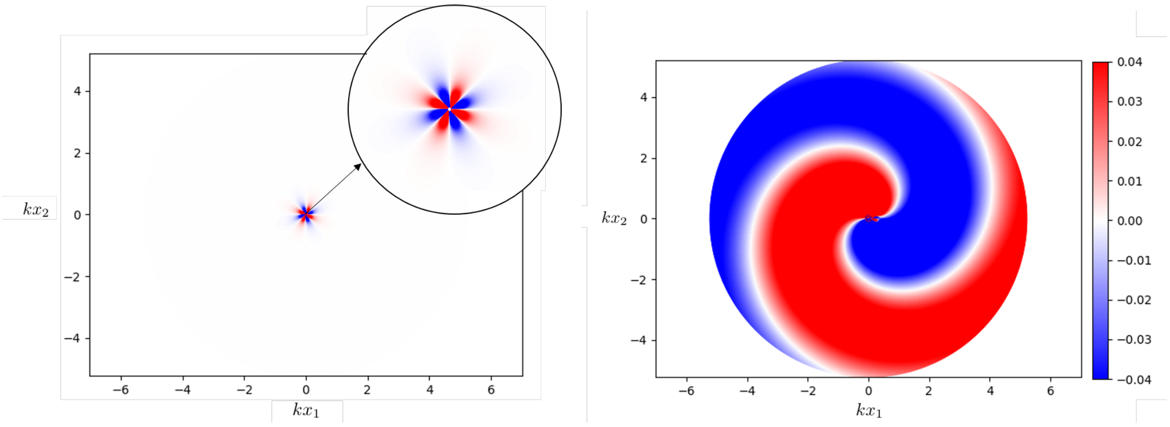
Two specific aspects are addressed in the present work, namely the scattering of complete circles of modal sources, on the one hand, and the scattering of these sources by a cylinder in close vicinity of the circle, on the other hand. Practically the scattering must be calculated for each point dipole of a source-mode and the total field is obtained through an integral over the circle. A similar approach has been detailed by Roger & Moreau [12] in the investigation of the scattering of fan/propeller noise in the air by the edge of a rigid half-plane.

## 4. NUMERICAL RESULTS

### 4.1 Description of the test configurations

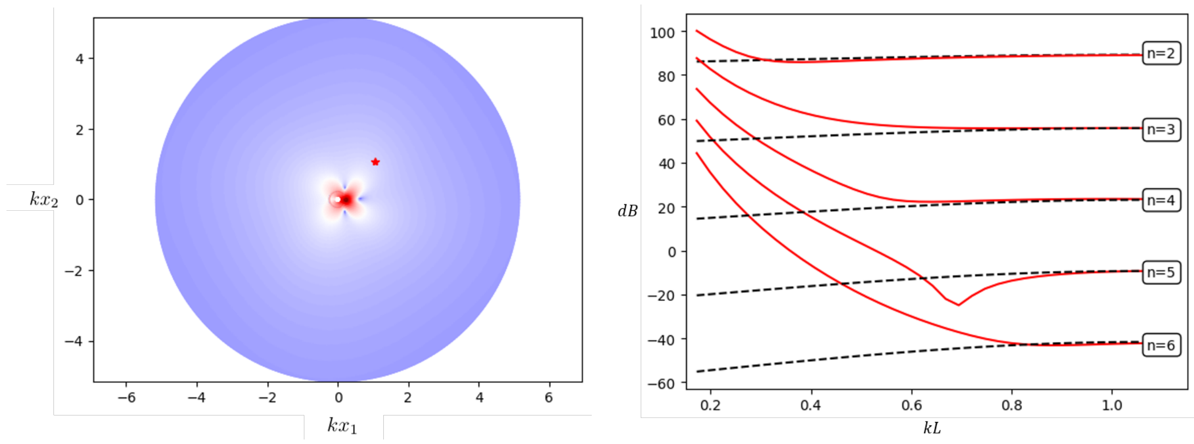
The sources of tonal rotating-blade noise are not directly addressed here but they are known to result from mean-flow deviations from axisymmetry that cannot be avoided in practice. The blade-loading harmonics they generate are associated with spinning modes of radiation, each of which is reproduced by an equivalent source-mode. The source-modes are considered individually to highlight the mechanism of diffraction by a cylinder and their field is computed analytically from Eq-(6). The dipole axes are assumed tangent to the considered source-mode circle, thus with zero radial component, which is a reasonable assumption for an axial-flow propeller as long as blade sweep is neglected. Similar developments could be made for a radial component. The source-mode, of radius $l$, is discretized as a circular array of phased dipoles, in such a way that the results do not depend on the discretization step. Its center is located at a distance $L$ along the $x$-axis from the edge of the cylinder of radius $a$. The notations are summarised in Fig-(3).

Though the formalism is valid for arbitrary values of the parameters, the present paper is dedicated to configurations representative of marine propellers installed on a ship hull. Therefore the distance between the cylinder and the sources, the radius of the cylinder and the radius of




(a) Free-field (b) Total-field

**Figure 4**: Instantaneous pressure field of a circular array of phased dipoles in free-field (a) and in the presence of a scattering cylinder (b) for the mode $n = 4$. Source circle and scattering circle too small to be visible on the plots. $kl = 0.025$, $kL = 0.17$




(a) (b)

**Figure 5**: (a)- Amplitude map in arbitrary decibel scale ; mode $n = 4$, $kl = 0.025$, $kL = 0.17$. (b)- SPL of the free (- - -) and total scattered fields (—) for various modes as functions of $kL$.

the source-modes are all assumed much smaller than the acoustic wavelength. More precisely these compactness conditions read $ka \ll 1$, $kl \ll 1$ and $kL \ll 1$ (Fig. 3).

### 4.2 Test-case evaluation

Test-case results obtained with the exact analytical model are shown in Fig-(4) depicting the instantaneous-pressure maps for a source-circle of radius $l$ in free-field in Fig-(4a) and in the presence of a scattering cylinder in Fig-(4b). The parameters are $ka = 0.025$, $kl = 0.097$, $kL = 0.07$ and the mode order is $n = 4$. Figure (4a) illustrates the wavefront pattern in free field. A zoom is made to identify the pattern because the mode radius is much smaller than the frame of the plot. The source-mode only radiates an evanescent wave : pressure fluctuations concentrate close to the source circle but rapidly vanish away from it. As a result the spinning lobes are unable to feature any visible spiral pattern. This corresponds to a Bessel function approaching zero in the formulation of section 2.1, Eq-(1). Once placed close to the scattering circle the source-mode becomes very efficient because it is dramatically amplified by the effect of the compact Green's function. The total field has a very different radiation pattern with one lobe spinning in the clockwise direction, whereas the free-field mode is spinning in the counterclockwise direction. This behavior highlighted in Fig-(4b) corresponds to the mode $n = -1$. The same has been observed for all other tested higher-order modes, not shown here for the sake of conciseness. Whatever the free-field mode of order $n > 1$ is, the scattered field exhibits the same mode -1 with inversion of the spinning direction. This property is imposed by the asymptotic behavior of the Green's function in the investigated compact regime.

Figure (5) illustrates the effect of the scattering on the amplitude of the radiated field of a mode, for various distances $L$. The amplitude map of the mode $n = 4$ is first plotted as an example in Fig-(5a) for $kL = 0.17$. The source circle is featured by the small white dot at the center. The four 'pseudo-directivity' lobes visible in the near field actually fade out at larger distances, where the amplitude is strongly decreased. The figure also shows the position of an arbitrary observer marked by an asterisk (*) at the coordinates $\theta_x = 45°$ and $kr_x = 1.40$. This fixed point is selected for the analysis of the effect of $kL$, moving the source position horizontally along the $x$-axis from $kL = 0.17$ to $kL = 1.1$, which remains in a range of very low values. This is reported in Fig-(5b) where the SPL (Sound Pressure Level) profiles of both the free field (dashed black lines) and the total field (red plain lines) are plotted as a function of $kL$, also varying the mode order. The results are twofold. Firstly, the free-field pressure amplitude is found to decrease as the mode order increases for the same assumed dipole strength. It must be kept in mind that because of the compactness the field is anyway evanescent. The dotted black lines have a positive slope because the source-to-observer distance decreases as $kL$ increases in this test. Secondly a much higher sound level is produced at the fixed observer's location in the presence of the cylinder and for lower values of $kL$. This amplification, defined as the amplitude ratio between the total and free-field amplitudes or as the SPL difference, dramatically increases as the mode order increases. It defines the asymptotic regime of diffraction resulting from the compact form of the Green's function. Furthermore the extent of the $kL$ range in which amplification occurs is also an increasing function of the mode order, roughly from $kL = 0.3$ to $kL = 0.8$ from the mode 2 to the mode 6 in the present test. The dip observed around $kL = 0.7$ for the mode $n = 5$ corresponds to a local near-field cancellation that takes place exactly at observer's location, similar to the blue dots between the pseudo-lobes in Fig-(5a). As the center of the source-mode is removed away from the cylinder, the SPL of the total field coincides with the direct field, which indicates that the scattered field becomes negligible. The main outcome is that all higher-order modes $n$ give a negligible free-field radiation and that they experience a strong amplification in the compact limit. Moreover higher-order modes must be considered as more compact in the sense that the limit value of $kL$ below which they are amplified increases. This could be different for others sets of parameters.

### 4.3 Formulation of tailored Green's function in the asymptotic regime

#### *4.3.1 Case of higher order modes*

The specific parameter ranges of installed marine propellers make the main results expected from an asymptotic analysis, shortly outlined in this section to interprete the exact calculations. The tailored Green's function is developped in its compact regime by using asymptotic forms of Hankel and Bessel functions given by Abramowitz & Stegun [13]. Consider an observer in the acoustic far field and distributed sources over a circle remaining close to the scattering cylinder, the latter being acoustically compact, so that $kr_x \gg 1\,, kr_y \ll 1\,, ka \ll 1$. The scattering sound pressure radiated by the point dipole in the asymptotic regime for $n > 1$ is obtained by only retaining the leading orders in the derivatives of the Green's function, $kr_y$ and $ka$ being assumed of the same order of magnitude. Again the derivations are not detailed. The asymptotic acoustic pressure reads :

$$p_s(r_x,\theta_x) \sim \frac{\pi\,k}{4}\,(n-1)\,\left(\frac{a}{D}\right)^2\,\left(\frac{l}{D}\right)^{n-1} \times$$

$$\mathrm{e}^{-\mathrm{i}\,\theta_x}\,\sqrt{\frac{2}{\pi\,kr_x}}\,\mathrm{e}^{\mathrm{i}\,kr_x-\pi/2-\pi/4} \tag{7}$$

The most important feature is that, whatever the order $n$ of the source mode is, the asymptotic scattering by the compact Green's function of the cylinder generates the mode $-1$, thus with a single lobe and spinning in the opposite direction as shown in Fig-(4). The factor $(n-1)\,(l/D)^{n-1}$ only determines the amplitude of the radiated field, decreasing as $n$ increases. With the same level of approximations the derivatives of the free-field Green's function for $n > 1$ lead to a zero direct sound field, which means that estimating $p_i(r_x,\theta_x)$ would require to pursue the development to the nearest higher-order term.

#### *4.3.2 Specific cases*

Equation (7) holds for $n > 1$. Special developments are required for the mode $n = 1$, leading to

$$p_s(r_x,\theta_x) \sim -\mathrm{i}\,\frac{\pi\,k}{4}\,\left(\frac{a}{D}\right)^2\,\mathrm{e}^{-\mathrm{i}\,\theta_x}\,... \tag{8}$$

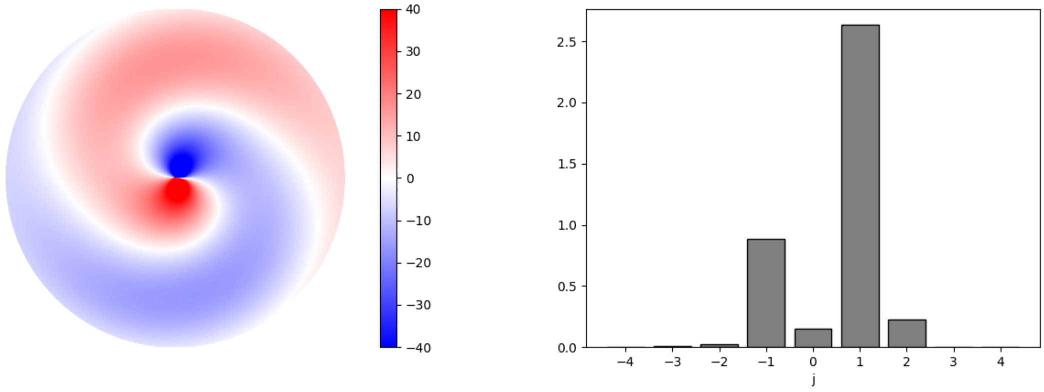
$$\times\sqrt{\frac{2}{\pi\,kr_x}}\,\mathrm{e}^{\mathrm{i}\,kr_x-\pi/2-\pi/4}$$

Again the mode $-1$ is found in the scattered field, but the direct field, derived following the same principles, is nolonger zero. Its leading-order approximation reads

$$p_i(r_x,\theta_x) \sim \mathrm{i}\,\frac{\pi\,k}{4}\,\mathrm{e}^{\mathrm{i}\,\theta_x}\,\sqrt{\frac{2}{\pi\,kr_x}}\,\mathrm{e}^{\mathrm{i}\,kr_x-\pi/2-\pi/4} \tag{9}$$

which corresponds to a significant radiation. The asymptotic scattering causes no amplification in this case, unlike for higher-order modes, since the factor $a/D$ is smaller than 1. Equation (8) is a minor perturbation of the direct field, Eq. (9). This result can be expected from the special structure of the mode $n = 1$. For this mode, diametrically opposite dipole sources are in phase opposition, which means that they point in the same direction. They double each other in amplitude because the diameter is acoustically compact. The total source mode is equivalent to a spinning point dipole. This is confirmed by the test reported in Fig. (6). The total instantaneous acoustic pressure of the mode $n = 1$ as predicted with the exact tailored Green's function is plotted in Fig. (6a), showing that the counterclockwise spinning phase imposed at source is preserved. A modal expansion of this field is next shown in Fig. (6b). For this a circular cut of the total instantaneous pressure at some radius $r_x$ near the outer radius is expanded as spinning modes according to the definition

$$p_t(r_x,\theta_x) = \sum_{j=-\infty}^{\infty} P_j\,\mathrm{e}^{\mathrm{i}\,j\theta_x}\,, \tag{10}$$




(a) (b)

**Figure 6**: (a)- Instantaneous pressure field of a circular array of phased dipoles in the presence of a scattering cylinder for the mode $n = 1$ (Fig. 6a). (b)- Amplitudes of the coefficients $P_j$ of a modal expansion of the field (arbitrary scale); negligible values outside the harmonic range $[-4, 4]$.

$$P_j = \frac{1}{2\pi} \int_0^{2\pi} p_t(r_x, \theta_x) \, \mathrm{e}^{-\mathrm{i}\, j\theta_x} \, \mathrm{d}\theta_x \, ,$$

$j$ standing for the mode orders involved in the exact solution. The mode $n = 1$ clearly dominates the total field, with a secondary contribution of the mode $n = -1$ that remains undiscernable in the total field map in Fig-(6a). This confirms that there is no amplification, unlike the previously discussed case of higher-order modes. The amplitude ratio of about 1/3 between the scattered and direct modes $n = -1$ and $n = 1$, respectively, is well in accordance with the far-field estimate $(a/D)^2$ from Eqs. (8) and (9).

The analysis reveals that the modes $n = \pm 1$ are the most efficient ones for marine propellers, both in free field and in the presence of a hull and that they have only a moderate amount of scattering. These modes are probably efficiently excited at the lowest frequencies for installed propellers with quite small blade numbers, because the orders of possibly contributing blade-loading harmonics $F_s$ would be $s = \mu B - 1$ and $s = \mu B + 1$.

A similar discussion can be made about the symmetric source mode $n = 0$ for which diametrically opposite dipoles now point in opposite directions because they are in phase. They cancel each other so that the expected radiation of the mode is zero in the compact limit. This can be verified easily by repeating the previous analysis in the case $n = 0$ : both the asymptotic free field and the asymptotic scattered field are zero. As a result, the symmetric mode is generated by interaction with the distortion harmonic of order $s = \mu B$ at any BPF order $m$ but it should not be an issue, unlike in the case of aircraft propellers.

The main aspect ignored in the two-dimensional model is the axial dipole strength, which would lead for the mode $n = 0$ to efficient radiation along the axis $\mathbf{e}_Z$. In practice, this will not affect the previous analysis for arbitrary modes. The behaviour seen in two dimensions gives reliable indication of what would happen in three dimensions for $n \geq 1$.

## 5. CONCLUSION

A simple, two-dimensional analytical formulation based on the Green's function of the rigid cylinder for the Helmholtz equation and on the notion of source-modes has been implemented to simulate components of the tonal noise radiated by an installed marine propeller. The source-modes are a consequence of the operation of the propeller in the mean-flow distortion around the hull of a ship. In view of the extremely low Mach numbers in marine applications and of the very low Helmholtz numbers based on the size of a domain encompassing the sources and the scattering cylinder cross-section, an asymptotic regime of diffraction is encountered. This is why an asymptotic formulation has been compared to the exact calculations to interprete the results.

The analytical approach provides a detailed physical analysis, mode by mode. Quite generally, higher-order modes with a number of azimuthal periods or lobes $n > 1$ are found to only generate evanescent waves in free field whereas they experience a very strong amplification in the presence of the cylinder. In fact, the amplification results from the interaction of the near field of the sources with the cylinder. The most spectacular result is that the amplification generates the radiating mode $n = -1$, whatever the mode order $n$ is, with inversion of the phase rotation. In contrast the mode $n = 1$ is already very efficient in free field because of its compactness and only generates a very moderate amount of diffraction. The symmetric mode $n = 0$ radiates negligible sound, both in free-field and in the presence of the cylinder, according to the two-dimensional model. The results suggest that the vicinity of marine propellers to the hull could result in a dramatic acoustic installation effect that cannot be neglected and should be taken into account at the early design stage. The results are found very sensitive to all involved parameters, such as mode order, relative distance to the cylinder and so on. The simple tools proposed in this work are well suited to investigate primary effects in a very fast way.

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