ASSESSMENT OF LARGE-EDDY SIMULATION BASED ON RELAXATION FILTERING: APPLICATION TO THE TAYLOR-GREEN VORTEX

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ABSTRACT

Results obtained for a Taylor-Green vortex at a Reynolds number of 3000, using Large-Eddy Simulation (LES) based on Relaxation Filtering (RF), are presented in order to assess the quality of the RF-LES methodology. The RF is applied every time step to the velocity components, using a standard filters of orders $k \geq 4$ at a fixed strength σ, to relax subgrid energy from scales at wave numbers close to the grid cut-off wave number. Various combinations of k and σ are considered, for k ranging from 4 to 14 and σ from 0.15 to 1. Error landscapes are obtained by comparing the 64^3 LES results, filtered in post-processing to an effective resolution of four points per wavelength, to 384^3 Direct Numerical Simulation data, filtered at identical resolution. For filters of order $k \leq 6$, the LES accuracy is found to be rather poor and varies significantly with the filtering strength σ. However, for higher order filters, *i.e.* for $k > 6$, the accuracy is good and nearly independent of the strength σ.

INTRODUCTION

In Large-Eddy Simulation (LES) of a turbulent flow, the most significant scales of motion, i.e. the largest and most significant scales of motion in the energy-containing range and inertial range, are resolved in order to obtain a statistically sufficiently accurate prediction of the flow. Since the small scales in the dissipation range are not resolved, their effects must be accounted for by an artificial dissipation mechanism, in order to avoid a pile-up of energy at the cut-off wavenumber imposed by the computational grid. This is usually done by replacing the residual stress tensor in the filtered Navier-Stokes equations with an eddy-viscosity model, or by applying dissipative numerical discretization schemes for the convective terms as in Implicit LES methods. We refer to the reviews by Lesieur and Métais (1996), Grinstein and Fureby (2002), and Domaradzki (2010), and to the books by Geurts (2004) and Sagaut (2005). The amount of dissipation, as well as its spectral distribution, may, however, be difficult to control in these methods as pointed out in Domaradzki et al. (2000, 2002, 2003) and Bogey and Bailly (2005, 2006b). This has led to the development of alternative LES methodologies relying on high-order dissipation mechanisms, such as hyper-viscosity models (Passot and Pouquet 1988, Dantinne et al. 1998) or the relaxation term in the Approximate Deconvolution Model (Stolz et al. 2001).

In recent years, an LES approach based on a Relaxation Filtering (RF) to account for the subgrid dissipation, has been proposed, and applied successfully to various flow configurations by Visbal and Rizzetta (2002), Rizzetta et al. (2003), Mathew et al. (2003) and Bogey et al. (2006a, 2009, 2011), among others. In order to relax the turbulent energy from the small scales at wave numbers close to the grid cut-off wavenumber, a low-pass filter is applied to the components of the velocity field, every n^{th} time step in each Cartesian direction, as follows

$$\tilde{u}(x,t) = u(x,t) - \sigma \sum_{j=-n}^{n} d_j u(x_j,t) \tag{1}$$

where $\tilde{u}$ and u denote respectively the filtered and unfiltered variables, and d_j represents the weighting coefficients that determine the dissipative contribution of the $(2n+1)$-point symmetric filter. The filtering strength σ is between 0 and 1. To obtain the necessary energy dissipation, criteria could be developed to adjust dynamically the filtering frequency and strength to the flow features, *e.g.* in Tantikul and Domaradzki (2010). For practical reasons, however, the filtering is usually applied every time step at a constant strength σ (typically $\sigma \simeq 1$). The results of the RF procedure depend in this case on the shape of the filter and the filtering strength. Since the selected filter must provide sufficient dissipation to the smallest resolved scales while leaving the largest scales mostly unaffected, the influence of the filter-shape, determined by the filter-order, is expected to be dominant, whereas the influence of the filtering strength σ should be minimal. In particular, the amount of energy dissipated by the RF should be nearly independent of σ. Such behaviour has been found previously by Bogey and Bailly (2006, 2006b) in LES of turbulent jets, in which the dissipation rates were nearly unchanged when decreasing the RF frequency, which is equivalent to reducing σ for a fixed filtering frequency. Nevertheless, there is still a need for systematic quantitative assessments of the validity of the RF-LES approach, as pointed out in recent work of Berland et al. (2011) on mixing layers.

For this purpose, the RF-LES methodology is applied in the present paper to the Taylor-Green vortex case, which has been solved over the last years in a series of studies on LES methods, *e.g.* by Drikakis et al. (2006), Fauconnier et al. (2009), Chandy and Frankel (2009) and Johnsen et al. (2010). The Taylor-Green vortex is at a Reynolds number of $\mathrm{Re} = 3000$, which is large enough so that natural transition into small-scale turbulence occurs. The reference solution is obtained from a Direct Numerical Simulation (DNS) on a 384^3 computational grid, and compared to the DNS results on a 256^3 grid as in Brachet *et al.* (1983). Furthermore, 36 RF-LESs of the Taylor-Green vortex are performed on a 64^3 computational grid, using various combinations of standard filters of order k and filtering strength σ. In order to assess the LES accuracy, comparisons are made with the DNS data. Energy spectra, as well as time evolutions of the dissipation rate, kinetic energy and integral length scales, based on the data filtered in post-processing to an effective resolution of 32^3, are considered. A two-parameter study is also performed, in which the discrepancies between the LES and DNS solutions filtered at identical resolution are represented as function of the filtering order and strength in an error-landscape framework, following Meyers et al. (2007) for example. This will allow us to investigate the sensitivity of the LES results to the RF parameters. More importantly, we aim to examine the *a priori* expectations that the results should not vary significantly provided that the order of the filter is sufficiently high.

SIMULATION PARAMETERS
Direct Numerical Simulations

The Taylor-Green vortex at Reynolds number $\mathrm{Re} = 1/\nu = 3000$ is first computed by DNS on a computational grid of 384^3 nodes yielding a maximum wave number $\kappa_{max} = 192$, using a pseudo-spectral solver (Fauconnier et al. 2009) combined with (anisotropic) $2/3$-de-aliasing, and a six-stage low-dissipation Runge-Kutta time stepping (Bogey and Bailly 2004). The time step is $\Delta t = 0.005$, yielding a maximum CFL number of 0.3. The flow is simulated up to $t = 20$, requiring 4000 time steps. The DNS is performed on 96 cores, and the total simulation time is about 300 hours.

As illustrations of the DNS results, energy spectra are represented in figure 1 at increasing times t. The broadening of the spectra during the transition from a well-organized large-scale flow to a developed turbulence characterized by a wide range of small-scale structures can be observed. The time evolutions of the kinetic energy and dissipation rate are displayed in figure 2. The kinetic energy is decaying, as expected in the absence of external forcing. The dissipation rate is seen to reach a peak around $t = 9$, time at which the energy spectrum also tends to have a small $\kappa^{-5/3}$ initial range.

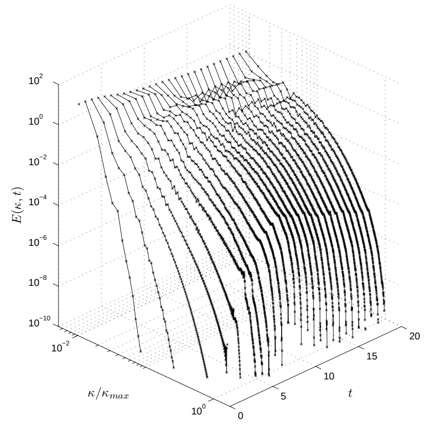

Figure 1. DNS of a Taylor-Green vortex at a Reynolds number of 3000: time evolution of energy spectra.

The 384^3 DNS results are found in figure 2 to compare well with the solutions determined by Brachet et al. (1983) using a 256^3 DNS for the same flow. However, the kinetic energy is slightly higher and the dissipation rate is visibly lower in the 384^3 DNS. To discuss the origin of these discrepancies, the results obtained from a 256^3 DNS, carried out using our pseudo-spectral solver, are shown in figure 2. They nearly collapse with the data of Brachet et al. (1983). This agreement demonstrates the validity of our DNS solver while suggesting that a 256^3 node resolution may not be fully sufficient for the DNS of the Taylor-Green vortex at $\mathrm{Re} = 3000$. Therefore, the 384^3 DNS results will be used as reference solutions in the following sections.

To perform relevant comparisons with the LES, *i.e.* over the same wave-number range, the DNS data are filtered in post-processing to an effective resolution of 32^3 computational nodes, using a sharp Fourier filter. The time evolutions of the kinetic energy and dissipation rate thus obtained (hereafter referred to as the resolved kinetic energy and dissipation rate) are represented in figure 2 as dotted lines. The resolved kinetic energy is close to the total kinetic energy, whereas the resolved dissipation rate is very small with respect to the total dissipation rate. This simply indicates that most of the energy-containing scales here are at wave numbers lower than $\kappa = 16$, whereas most of the energy-dissipating scales are at higher wave numbers.

Large-Eddy Simulations

For the Taylor-Green vortex at $\mathrm{Re} = 3000$, 36 LES computations are performed, each using a different standard explicit filter with a different filtering strength σ. The Relaxation Filtering of the velocity components is applied every time step. The different LESs are determined by the 6×6 combinations of filter orders $k = 4, 6, 8, 10, 12, 14$ with the filtering strengths $\sigma = 0.15, 0.2, 0.4, 0.6, 0.8, 1$, as reported in table 1. Each LES is performed on a 64^3 computational grid, leading to a grid cut-off wave number of $\kappa_{max} = 32$, using the same pseudo-spectral solver as the DNS. Dealiasing is not applied explicitely, because the relaxation filtering is expected to take this into account. To minimize numerical errors, spectral differentiation is used to evaluate spatial derivatives, and a six-

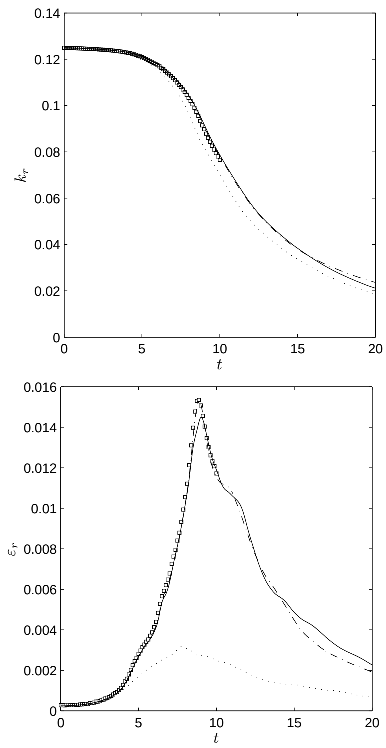

Figure 2. Time evolution of the dissipation rate (top) and kinetic energy (bottom) for a Taylor-Green vortex at Re $= 3000$: $\square$ 256^3 DNS by Brachet et al. (1983), present ——— 384^3 and — · — · 256^3 DNS, ······ present 384^3 DNS low-pass filtered to an effective resolution of 32^3.

stage low-dissipation Runge-Kutta algorithm is implemented for time integration with a time step $\Delta t = 0.025$, yielding a maximum CFL number of 0.25. Each simulation is performed on 16 cores and takes about 4 hours.

For relevant comparisons with the DNS results, as mentioned previously, the LES data are low-pass filtered in post-processing to an effective resolution of 32^3 nodes. The quality of the LES is thus examined for the turbulent scales at wave numbers $\kappa \leq 16$, which are discretized at least by four points per wavelength.

Table 1. Relaxation Filtering parameters in the LES of the Taylor-Green vortex: filter order k and filtering strength σ.

$\sigma \setminus k$	4	6	8	10	12	14
0.15	×	×	×	×	×	×
0.2	×	×	×	×	×	×
0.4	×	×	×	×	×	×
0.6	×	×	×	×	×	×
0.8	×	×	×	×	×	×
1	×	×	×	×	×	×

RESULTS

Energy spectra

Energy spectra $E(\kappa)$ obtained by RF-LES using filters of order 4, 8 and 14, at time $t = 9$ when the peak dissipation rate is reached, are shown in figure 3. They are compared to the filtered DNS spectrum. The LES spectra determined using the 4th-order filter are seen to vary strongly with the filtering strength σ, and especially to differ from the DNS spectrum. More precisely, the energy components are underestimated for the small scales at wave numbers $0.1\kappa_{max} \leq \kappa \leq \kappa_{max}$, whereas they are overestimated for the large scales at $\kappa \leq 0.1\kappa_{max}$, discretized by more than 20 points per wavelength. The 4th-order filtering therefore appears both to excessively damp the turbulent fine scales, and to affect the larger scales of the flow. Fortunately, these unwanted effects gradually disappear when filters of higher order are used. For the filter at order $k = 8$ for instance, the LES spectra are nearly independent of the filtering strength, and they correspond well to the DNS spectrum up to $\kappa \simeq \kappa_{max}/2$, that is around 4 points per wavelength. A similar agreement between LES and DNS spectra is noticed for the filter at order $k = 14$.

Time evolutions of turbulence quantities

To characterize the turbulence features in the LES during the flow transition from the initial Taylor-Green vortex to isotropic small-scale structures, the time evolutions of the resolved kinetic energy $k_r(t)$ and dissipation rate $\varepsilon_r(t)$, calculated as

$$\varepsilon_r(t) = \frac{1}{8\pi^3}\int_0^{\kappa_{max}} 2\nu\kappa^2 E_{LES}(\kappa,t)d\kappa \qquad (2)$$

and

$$k_r(t) = \frac{1}{8\pi^3}\int_0^{\kappa_{max}} E_{LES}(\kappa,t)d\kappa \qquad (3)$$

are considered from $t = 0$ to $t = 20$. They are represented respectively in figures 4 and 5, for relaxation filters of order 4, 8 and 14. As expected from the energy spectra, the dependency on the filtering strength is strong in case of the filter of order 4. The excessive damping of large and small scales due to the low-order Relaxation Filtering results in a significant underestimation of the kinetic energy with respect to the DNS reference solution. The resolved dissipation rate is also much lower than that in the DNS, indicating that the large scales at wave numbers $\kappa \leq 16$ are mainly dissipated by the Relaxation Filtering rather than by molecular viscosity. As a consequence, their dynamics are most probably controlled by the subgrid dissipation model, and the effective Reynolds number of the flow may be artificially lowered.

In the LES using filters of order 8 and 14, the decay of the resolved kinetic energy agrees satisfactorily with that from the DNS. More interestingly, the resolved dissipation rates do not vary much with the filtering strength, and they are very similar to the DNS data. These results demonstrate that the Relaxation Filtering has here a quite limited impact on the large resolved scales. A small overestimation of $\varepsilon_r(t)$ can finally be noted for the filter at order 14, which may suggest that the subgrid dissipation is slightly insufficient in this case.

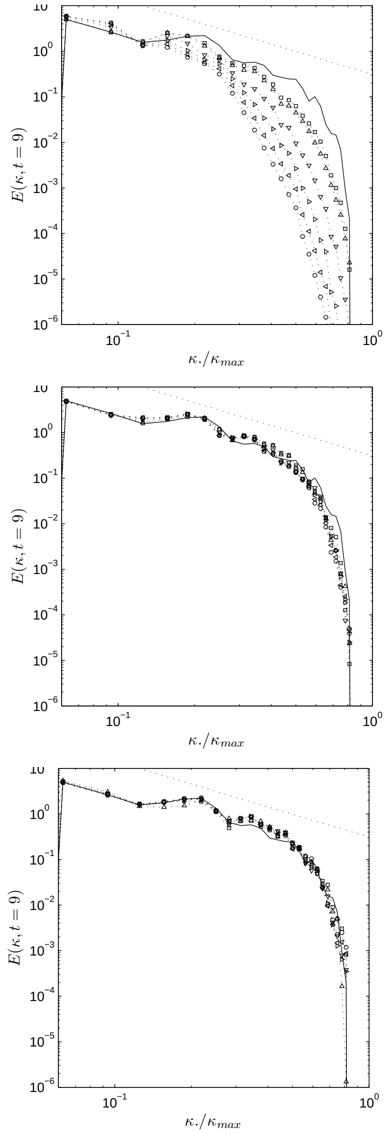

Figure 3. Energy spectra obtained at $t = 9$ in ——— the DNS, and in the LES using RF of order 4, 8 and 14 (from top to bottom) at strength $\square$ $\sigma = 0.15$, $\triangle$ $\sigma = 0.2$, $\triangledown$ $\sigma = 0.4$, $\triangleright$ $\sigma = 0.6$, $\triangleleft$ $\sigma = 0.8$, and $\circ$ $\sigma = 1$. The dotted lines represent a $\kappa^{-5/3}$ decay.

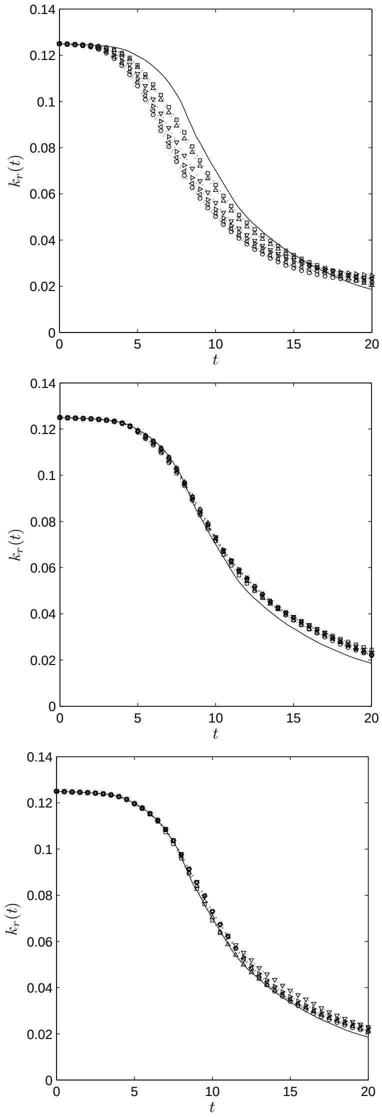

Figure 4. Resolved kinetic energy obtained in ——— the DNS, and in the LES using RF of order 4, 8 and 14 (from top to bottom) at strength $\square$ $\sigma = 0.15$, $\triangle$ $\sigma = 0.2$, $\triangledown$ $\sigma = 0.4$, $\triangleright$ $\sigma = 0.6$, $\triangleleft$ $\sigma = 0.8$, and $\circ$ $\sigma = 1$.

Error landscapes

To quantify the LES accuracy, and its sensitivity to the Relaxation Filtering parameters, error landscapes are computed as function of the filtering order k and strength σ. They show the norm of the differences between DNS and LES results, for the resolved longitudinal integral length scale, kinetic energy, and dissipation rate. These error norms are defined respectively as

$$\Delta L_{11}(t) = \frac{3\pi}{4} \left| \iiint_0^{\kappa_{max}} \kappa^{-1} \left[\frac{E_{DNS}(\kappa,t)}{k_{DNS}(t)} - \frac{E_{LES}(\kappa,t)}{k_{LES}(t)} \right] d\kappa \right| \tag{4}$$

$$\Delta k(t) = \frac{1}{8\pi^3} \left| \iiint_0^{\kappa_{max}} \left[E_{DNS}(\kappa,t) - E_{LES}(\kappa,t) \right] d\kappa \right| \tag{5}$$

and

$$\Delta\varepsilon(t) = \frac{1}{8\pi^3} \left| \iiint_0^{\kappa_{max}} 2\nu\kappa^2 \left[E_{DNS}(\kappa,t) - E_{LES}(\kappa,t) \right] d\kappa \right| \tag{6}$$

Figure 6 shows the error landscapes for ΔL_{11}, Δk and $\Delta\varepsilon$ at time $t = 9$, *i.e.* at maximum dissipation rate, as function of the filtering order and strength. The use of low-order filters leads to errors that depend on the filtering strength. For the filter of order 4, the errors are significant for $\sigma = 0.15$, and become higher when increasing σ. By replacing the 4th-order filter by the 6th-order filter, the errors are strongly reduced, but they are still significant for $\sigma \geq 0.6$, *i.e.* for practical values

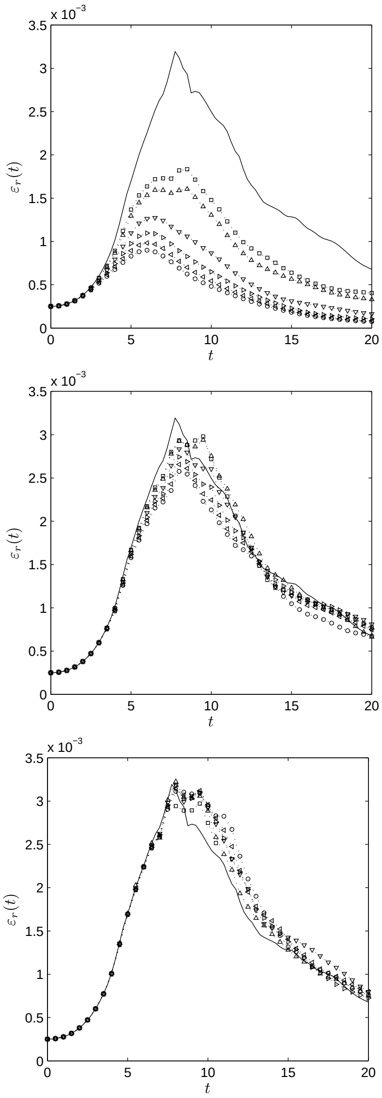

Figure 5. Resolved dissipation rate obtained in ——— the DNS, and in the LES using RF of order 4, 8 and 14 (from top to bottom) at strength □ $\sigma = 0.15$, △ $\sigma = 0.2$, ▽ $\sigma = 0.4$, ▷ $\sigma = 0.6$, ◁ $\sigma = 0.8$, and ○ $\sigma = 1$.

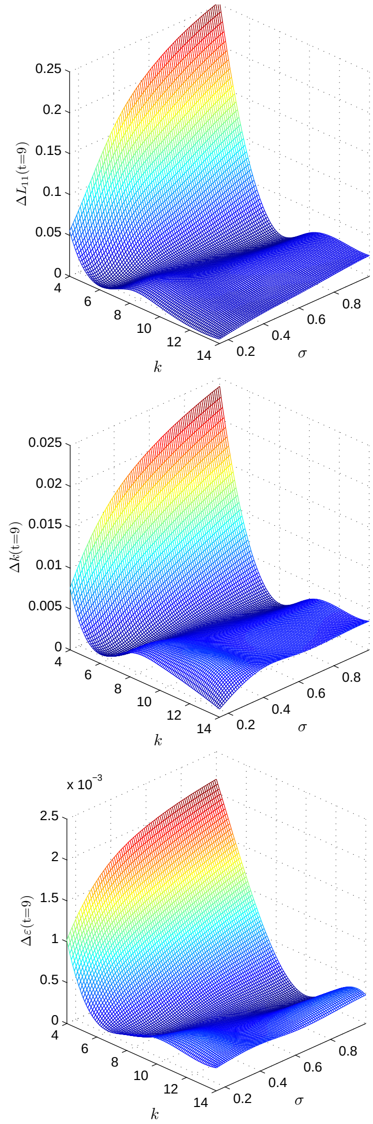

Figure 6. Error landscapes as function of filtering order k and strength σ for the resolved longitudinal integral length scale, kinetic energy and dissipation rate, from top to bottom, at $t = 9$.

of filtering strength. Finally, when the filter order is larger or equal to $k = 8$, good LES accuracy is obtained whatever σ between 0.15 and 1. Therefore the value of the filtering strength does not appear as a crucial parameter in this case.

Remark that the error landscapes exhibit optimal error-valleys, located at $8 \leq k \leq 10$, that depend only weakly on the filtering strength but strongly on the filter order. Consequently, the use of 8th or 10th-order filters seems optimal in the present LES. The use of high-order filters in the Relaxation Filtering procedure leads to a slight reduction of the accuracy. Since the high-order filters are only effective near the grid-cutoff wave number, the reduced accuracy stems most likely from the increased contribution of aliasing errors.

CONCLUSION

In this work, a quantitative assessment of the LES method based on Relaxation Filtering has been conducted by solving the Taylor-Green vortex test case at a Reynolds number of 3000. Comparisons with DNS data show that the accuracy of the LES results depends essentially on the choice of a sufficiently sharp filter. Considering only standard filters in the present study, good agreement with the DNS solutions is in particular found for filter orders $k \geq 8$, nearly independently of the filtering strength. The use of filters of order 8 or 10 appears also optimal.

In further work, a comparative investigation of the contributions of molecular viscosity and subgrid dissipation in spectral space, as done for instance in Bogey et al. (2011), and an analysis on the influence of the numerical discretization and de-aliasing will be carried out. The latter may lead again to error landscapes, since a two-parameter study could be performed for various combinations of the filter order and the order of the spatial discretization.

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