ON THE THEORY OF MOTION IN THE UNIVERSE

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Professor Geoffrey M. Lilley
School of Engineering Sciences
University of Southampton

Opening Remarks

Today we honor Professor Genevieve Comte-Bellot and remember her life-time's research into the structure of turbulence and more recently in aeroacoustics.

It is a privilege to be invited to speak at this gathering of friends and to speak on a subject that evolved from my experimental work in aeroacoustics which began in 1948. In my search of a theory for the noise generated by turbulent motion I first referred to published work on electromagnetic radiation and was confronted with Einstein's work on the vacuum universe and relativity. The propagation of light possessed properties different from sound and yet they both satisfied the same wave equation in a medium at rest. But whereas sound propagation was Galilean invariant, light propagation was Lorentzian invariant. Light transmission was possible in a void and the speed of light was absolute, whereas no sound existed in a void and its speed was a property of the thermodynamics of the medium. It was nearly 50 years later Sir James Lighthill admitted he had followed a similar frustrating path before deciding to introduce his new and independent theory of aerodynamic noise.

In my work on acoustics in the early 1950's I continued to retain an interest in electromagnetic radiation and relativity and to explore where fluid mechanics has a role to play in cosmology. We used the ripple tank analogy to study transonic wave motion, which later was suggested by Ardavan to represent transluminal motion. In the 1960's we studied the instability of the sun's nebula disc in the derivation of the orbits of the planets. We also used a convective tank to model the turbulent convective motion in the sun's photosphere. My interest in cosmology was further extended from lectures I attended given by Fred Hoyle on the steady state Universe and by C.C.Lin, who used linear instability theory to derive a structure for spiral galaxies. In the mid-1980's I found Ya'Zeldovich and Novikov were using fluid dynamics to study motion in the early Universe, with similar results to my own unpublished work, while Sakharov suggested the cosmic soup was comprised of shock waves in random motion at the speed of light.

In the mid-90's I was fortunate to meet Kenneth Thornhill after his extensive work on ethereal cosmology had been refused publication. He showed me where my work on a model for dark matter needed correction. I later had discussions with Sir James Lighthill and received his encouragement to study in depth a model Universe based on Euler's fluid dynamics equations including detailed comparison with current experimental data. An earlier version of this paper was delivered in 2000 and published in 2003. It included, with Thornhill's permission, a simplified version of his theory and conclusions.

Further Notes

- Modern cosmology is based on Einstein's postulate of a vacuum universe leading to relativity. The hot big-bang chemistry evolved from nuclear physics and quantum mechanics. The vacuum universe postulate led to a linearisation of Maxwell's equations of electromagnetic radiation and opened up the opportunity to the world of physics to derive analytical solutions and explore the corresponding simplified physics. The escape from the nonlinear Euler equations proved a gigantic step. The contrast with fluid dynamics was enormous, where Prandtl had shown that problems in nature could only be solved through a knowledge of boundary layer theory, involving friction, fluid viscosity, vorticity, increase in entropy, and nonlinear structural development. Examples include turbulence, boundary layer theory, bird flight, and aerodynamic noise. Was Einstein's simplification justified? We are led to comparison with measurement and experimental data.
- The near exponential growth in applications of electronics and materials has not been influenced by this postulate of a vacuum universe, involving a void in all intergalactic space. It was Zwicky in 1933 who first questioned that there must be more mass in the universe than is derived from the measured illumination from stars and galaxies. Present cosmological theories based on the vacuum universe and discuss dark energy and dark matter but none of these theories gives answers that reflect that the mass of dark matter in the universe exceeds the mass of the visible universe by more than an order of magnitude. Moreover recent measurements show the universe is accelerating outwards, while the theories suggest the opposite.
- In fluid dynamics the appearance of singularities are normally found to be aphysical and arise from imperfections in the mathematical models used. Often singularities are a result of solving linearised approximations to the exact Navier-Stokes equations. Singularities never arise in measurements. Thus in cosmology the void in intergalactic space, the absolute speed of light, the light catastrophe at a speed ration of unity, associated with the Lorentzian transformation, all involve physical phenomena and to each of these we can show fluid dynamics provides a treatment in the shape of introducing a rarefied exotic gas to occupy all intergalactic space, thus avoiding all these contradictions.
- An important part of this study is to explain why this model of a rarefied exotic gas, which we refer to as dark matter,
 has yet to be observed and measured. The study shows the speed of light is no longer an absolute constant but a
 function of the thermodynamics of the dark matter in intergalactic space. Speeds of particulate in excess of the speed
 of light are possible. Motion in the universe is both subluminal, transluminal and superluminal.
- This study is basically an outer solution of a model of the expanding universe, including the present epoch and the future. It includes estimates of the structure of the early universe near the commencement of the expansion and the formation of stars and galaxies. A future development would involve the matching between this outer solution and the inner solution based on the nuclear physics and quantum universe model of the aftermath of a hot big-bang. The results using this model of dark matter, giving the structure of the universe throughout a linear expansion of the universe are calibrated against three measured quantities in the present epoch, namely, the relic black-body radiation temperature in intergalactic space, the speed of light, and the Hubble constant. Using these quantities we find the size of the universe in the present epoch and its total mass. Other results include the velocity and acceleration of its outer boundary, as well as the average pressure and density of the intergalactic medium in the present epoch.
- The universe in the present epoch appears as an enormous globe containing tens of billions of galaxies moving away from our galaxy as tiny, yet massive, dots, at relative speeds comparable with the speed of light. They appear as ships floating on an ocean of dark matter moving at the same speed.

PRESENTATION OUTLINE

Introduction

Vacuum Universe and Dark Matter

- **Covariance of Wave Motion**
- **Michelson-Morley**
- **Properties of Intergalactic Space**
- + Thornhill's model of Dark Matter
- **The Expanding Universe**
- **Conclusions**



INTRODUCTION

Lord Kelvin wrote - 'When you can measure something what you are speaking about, and can express it in numbers, you know something about it - but when you cannot measure it, your knowledge is of a meagre and unsatisfactory kind.'

Dirac made the statement 'The equations of fluid motion are the most complex of all the partial differential equations of mathematical physics and since they are nonlinear, and therefore in general there are no analytical solutions, it becomes impossible to understand the physics of fluid motion by studying the theory of fluid dynamics alone. In unsteady motion such as turbulence it is only possible to study the statistics of the motion and this hides the true physics.'

Properties of Intergalactic Space

- **Einstein's** thought experiments leading to relativity
- Prandtl's experiment on relative motion leading to the theory of the boundary layer
- + **Hubble's** work leading to the expanding universe
- Olber's Paradox
- The measurements of **Penzias** and **Wilson** of the black body radiation temperature in intergalactic space in the present epoch of 2.7(K). Our model of dark matter is assumed to have the same temperature in the present epoch. The speed of light as measured in the present epoch is 3 x 10⁸ m/s.

Thornhill's model of dark matter

- Thornhill's speculation was that dark matter must have been in existence before ordinary atomic and subatomic matter were created, since a medium had to exist to create the necessary electric and magnetic fields.
- + Its particulate has a directed motion as well as a random motion with the speed of light as its average speed.
- Thermodynamic properties of dark matter are those of a standard perfect gas extended to include a Fermi gas.
- \rightarrow Dark matter has six degree of freedom, an adiabatic index of $\gamma = 4/3$ and satisfies Planck's energy distribution.
- → This model of dark matter, has a temperature in the present epoch of 2.7(K), and a partial pressure of 10⁻¹⁴(Pa).

Early Universe

- The mass of Dark Matter is many times the mass of the visible matter in the Universe.
- The result is a sea of Dark Matter expanding at a very high radial speed.
- The galaxies within the Universe are like relatively small concentrated islands of high mass or as ships floating on an ocean of Dark Matter, which fills all the Universe.
- + The presence of Dark Matter in galaxies
- is unimportant since its density is so minute as compared with ordinary matter. On the scale of the Universe the heavy matter in the galaxies behaves like Brownian motion studied by Einstein in 1905.

The Euler Equations for Dark Matter

- The Conservation Equations of Mass, Momentum and Energy for radial motion are
- $> \partial \mathbf{u}/\partial t + \mathbf{u}\partial \mathbf{u}/\partial r + \mathbf{\rho}^{-1}\partial \mathbf{p}/\partial r = 0 : \mathbf{p}/\mathbf{\rho}^{\gamma} = constant$
- These equations for an expanding Universe involve a system of spherical coordinates in an inertial frame at rest. The self-similar equations have $\eta = r/R(t)$ and $u = \eta dR/dt$. All gravitational forces are absent.
- Any fluid or mass in acceleration has in its rest-frame a force in the opposite direction to the acceleration. This centripetal force will be directed to the Universe origin. Thornhill proposes this to be a physical theory for gravitation. The result is

$$d^{2} R/d t^{2} (t_{a}) = M G_{a}/R_{a}^{2} = 6 R_{0} c_{0}^{2}/R_{a}^{2}$$

where M is the total mass of the Universe, on the assumption that most of the mass is near the outer boundary. The positive boundary acceleration is derived from the negative pressure gradient.

The Properties of Dark Matter

The exotic gas, as proposed by Thornhill, satisfies the Maxwell statistics of kinetic theory and the Planck energy distribution of black body radiation. This exotic gas has two fundamental properties namely

$$c^2 = \gamma p/\rho$$
 : $p = (8 \pi^5/45) (kT)^4 / (ch)^3$: $p \sim \rho^{4/3}$

- The exotic gas has a $\gamma = 4/3$. This is also found for a photon gas, and a relativistic degenerate electron gas, where the latter is the limiting case of a Fermi gas.
- ➤ It is assumed that this universe, filled with dark matter, and all visible matter, is expanding into an external void.

THE ACCELERATING UNIVERSE

$$M = (64/315) \Re_a R_a^3 = (64/315) \Re_0 R_0^3$$

$$ho_a = 4.3342 \text{ x} 10^5 \sqrt{p_a/\rho_a} \qquad R_0 = 4.3342 \text{ x} 10^5 \sqrt{p_0/\rho_0}$$

$$d^2 R/t^2 (t_a) = M G_a/R_a^2 = 6 R_0 c_0^2/R(t)^2$$

- > Thornhill used the <u>method of characteristics</u> to calculate the space-time properties of light transmission in the expanding Dark Matter Universe.
- These results led to physical results in good agreement with astronomical observations concerning <u>Hubble's Law</u>, the <u>red shift</u>, <u>refraction</u>, <u>aberration</u> and the observed law of <u>gravitation</u>, all as obtained analytically by Thornhill.

The Dark Matter Universe Model Results

$$> p \sim (1 - \eta^2)^4 / R^4 : r \sim (1 - \eta^2)^3 / R^3 : T \sim (1 - \eta^2) / R$$

$$Arr H_0 \sim 1/t_a : M \sim 10^{57} (kg) : M_{visible} \sim 10^{53} (kg)$$

$$ightharpoonup c_0 = 2x10^{11} (\text{m/s})$$
 : $c_a = 3 \times 10^8 (\text{m/s})$

$$\rightarrow$$
 $t_0 = 3x10^4 \text{ (years)}$: $t_a = 1.3x10^{10} \text{ (years)}$

$$Arr R_0 = 7x10^{23} \text{ (m)}$$
 : $R_a = 2.5x10^{29} \text{ (m)}$

$$V = U_0 = 0 \text{ (m/s)}$$
 : $U_a = 7x10^{11} \text{ (m/s)}$

$$Arr$$
 (Acceleration) : $A_0 = 0.3 \text{ (m/s}^2)$: $A_a = 2x10^{-12} \text{ (m/s}^2)$

$$T_0 = 10^6 (K)$$
 : $T_a = 2.7 (K)$

ADDITIONAL RESULTS FROM THE DARK MATTER MODEL

Pressure $p_a = 1.34 \times 10^{-14} (Pa)$ $p_0 = 0.34 \times 10^9 (Pa)$

Density $\rho_a = 1.98 \times 10^{-31} (kg/m^3) \rho_0 = 1.25 \times 10^{-14} (kg/m^3)$

Temperature $T_a = 2.7(K) T_0 = 1.08 \times 10^6(K)$

Volume of Universe (Present epoch) = $6.78 \times 10^{88} (m^3)$

Mass of our galaxy = 2×10^{41} (kg) No. of galaxies ~ 10^{11}

Mass of Dark Matter Universe = $2.0 \times 10^{57} (kg)$

CONCLUSIONS 1

- All current work on COSMOLOGY recognizes that the mass of the Universe exceeds that of the measured visible matter by a large factor. The missing mass is referred to as Dark Matter.
- > The duality of light transmission through corpuscular motion or by wave propagation is not in doubt except in the limiting case of wave motion in a void.
- ➤ The model of Dark Matter, following Thornhill, leads to an exact self-similar solution to the Euler Equations for the asymptotic properties of a spherically expanding Universe dominated by the flow of Dark Matter.
- > The solution is independent of conditions at the start of the expansion. Its boundary with an external void in the present epoch maintains its positive acceleration.
- > Galaxies appear as isolated points of mass moving with the radial motion of the Dark Matter, like ships on a near infinite ocean.

CONCLUSIONS 11

- Our Universe is dominated by the flow of Dark Matter.
- The density of Dark Matter in the present epoch is very rarefied. The number density is order 400/cm³.
- ➤ The partial pressure of 10⁻¹⁴(Pa) above that of a perfect vacuum is beyond the limits of measurement in the present epoch.
- Thornhill used the <u>method of characteristics</u> to calculate the space-time properties of light wave transmission in the expanding Dark Matter Universe.
- These results led to physical results in good agreement with astronomical observations concerning <u>Hubble's Law</u>, the <u>red shift</u>, <u>refraction</u>, <u>aberration</u> and the observed law of <u>gravitation</u>.
- > The galaxies appear in this model as isolated blobs of mass moving with the radial motion of the Dark Matter. They appear like ships on a near infinite ocean or our view of the Night Sky.

THE EARLY UNIVERSE AND IN THE PRESENT EPOCH



Foreword I

Matter and energy constitute the structure of our Universe but as yet science alone has failed to explain its origin. Maybe it will remain a mystery to all mankind for ever.

- Models of the early Universe have described a possible sequence of events starting from an assumed hot big-bang and followed by a stage of thermal equilibrium at temperatures greater than 10°(K). Within a few minutes of the primordial fireball the production of protons and neutrons, using the methods for modeling thermonuclear explosions and nucleosynthesis, can explain the composition of the Universe as 75% hydrogen and 25% helium-4, plus tiny traces of other elements. This is satisfactory except the creation of electricity and magnetism has never been explained, and yet electrical charge had to be present before particles could come into existence.
- In quantum physics the standard model requires the existence of the Higgs boson, a particle whose life-span is 10⁻⁴⁵s. But even when this is proven to exist, which then confirms the accuracy of the standard model, it will not have explained the existence of all the mass in the Universe.

Ref. Coles P 'Cosmology' OXFORD 2001.

Foreword II

- Einstein introduced the postulate that all interstellar and intergalactic space is a void. This was based on the observation that no medium throughout the space has been measured, and light emitted from galaxies billions of years ago has suffered no interference on its passage to earth. Einstein's Universe is a Vacuum Universe.
- The Universe to remain in stable equilibrium requires a total mass far greater than all the measured visible mass. The missing mass is referred to as Dark Matter. If it is a particulate it must be an exotic gas with a particle size smaller than all known matter in the Universe. Our study suggests reasons why it has not so far been measured.
- Recent astronomical data is suggesting our Universe is accelerating and not decelerating in accordance with theories of cosmology based on the Vacuum Universe. Such an acceleration could be explained by the presence of Dark Matter, and would question many of the results from Einstein's theory of relativity.

Foreword III

- > The Vacuum Universe led to the result that the speed of light is absolute and relativity is Lorentzian and not Galilean as in fluid dynamics.
- Einstein claimed that the laws of fluid dynamics were not laws of Nature, similar to the laws of electro-magnet radiation in free-space. In fluid mechanics the equations are nonlinear and there are no analytic solutions. Numerical and experimental solutions lead to averaged results in which all the physics is lost.
- Einstein's approximation of the Vacuum Universe in effect reduced the equations of electromagnetic radiation to linear equations enabling them to be solved exactly giving analytical results as in other problems of mathematical physics.

Foreword IV

- Modern works on cosmology rarely refer to some of the triumphs of fluid dynamics such as the work of C.C.Lin using the results of linear instability in fluids to determine the structure of the spiral galaxy, and the formation of our solar planetary system from the fluid instability of the initial nebula disc.
- Here we consider a model of Dark Matter, which satisfies all the laws of physics, thermodynamics and fluid dynamics, for use in a theory leading to a possible structure of the Universe. Our results are compared with current available observations and measurements.
- Our study uses as its base the speculations of Thornhill on the properties of Dark Matter. Thornhill, who before his retirement was a widely respected fluid dynamicist, has over the past 25 years had his work on ethereal cosmology largely ignored, mainly because he had the temerity to cast doubts on the postulate introduced by Einstein in 1905.
- In this study we introduce our approach to cosmology based on fluid dynamics alone commencing with the beginning of the Universe expansion. We introduce the speed of propagation in Dark Matter as equal to the speed of light, which is now a function of the properties of the medium, and there is no limit to the speed of the Dark Matter particulate here treated as a continuous fluid.

Vacuum Universe

- + Einstein's postulate that all intergalactic space was a void
- → The rejection of an ether
- The theory of relativity

 \rightarrow

- The space-time properties of the universe
- The analogy between the propagation of light and that of sound
- Covariance of Wave Motion

Dark Matter

- > The mass of the universe
- > The mystery of the missing mass
- > The present model of dark matter explains why its properties have escaped measurement arising from its near vacuum partial pressure 10⁻¹⁴ (Pa). Current measurements have a resolution of near 10⁻⁸ (Pa).
- The present model of dark matter shows the universe is accelerating outwards. Hubble measurements show the age of the universe is about 13x10⁹ (years). Its main long-term linear expansion commenced about 10⁴ (years) following the hot big-bang.

Covariance of Wave Motion

The characteristic wave hyperconoid

$$(dx)^2 + (dy)^2 + (dz)^2 = c^2 (dt)^2$$

must be invariant under transformation in all frames of reference moving with the local velocity. This involves a <u>Galilean</u> transformation.

The <u>Lorentz</u> transformation,

$$(dx)^2 + (dy)^2 + (dz)^2 - c^2 (dt)^2$$

is <u>invariant</u> under transformation but wave surfaces are not invariant under this transformation.

Thus in a Lorentzian vacuum universe <u>wave motion</u> is not possible. Particle motion exists as in Newton's <u>corpuscular</u> <u>motion</u>, and in Einstein's discovery of the <u>photo-electric effect</u>.

WAVE MOTION – MOVING SOURCE

MEDIUM at **REST**

$$c(t - \tau) = |x - y|$$

$$R = |x - y|$$

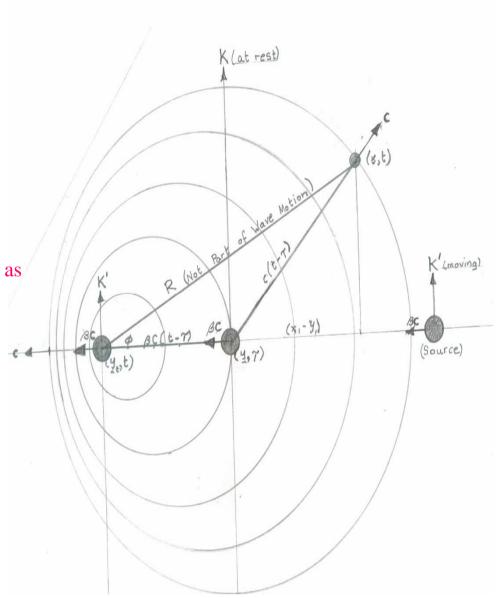
$$t$$

$$\mathcal{E} = V/c$$

$$c(t - \tau) (1 - \mathcal{E}^2)^{1/2} + \mathcal{E} R\cos^{\frac{\alpha}{2}} / (1 - \mathcal{E}^2)^{1/2}$$

$$= R(1 - \mathcal{E}^2 \sin^2 ^{\frac{\alpha}{2}})^{1/2} / (1 - \mathcal{E}^2)^{1/2}$$

This result equals $c(t - \tau)$ as a function of |x - y| as used in a Lorentz transformation,



THE EFFECT OF GRAVITY

- ➤ Thornhill argued that to satisfy <u>Mach's Principle</u>, inertia must be solely determined by the surrounding masses. The equations of motion are in a framework at rest with all fluid motion initially at rest. In such a frame there can exist no gravitational force.
- In the moving frame of the expanding universe the outwards acceleration is balanced by a centripetal force directed to the origin.
- The theory of <u>General Relativity</u> requires the Universe to be decelerating but it is based on the local theory of space warping. (Relativity strictly applies only to a <u>Vacuum Universe</u>)
- In the presence of <u>Dark Matter</u> the force directed towards the origin is what we call the Gravitational Force as in the <u>Newtonian theory</u>, involving the total integrated mass of the Universe.

Michelson-Morley Experiment

- Maxwell developed the equations for electromagnetic propagation in <u>free space</u>.
- Maxwell claimed wave motion required a
- + <u>mass-less ether</u> to support wave motion in space.
- Michelson and Morley devised an experiment to discover the properties of such an ether.
- Michelson and Morley were unaware that the earth in its motion around the sun is covered by a <u>boundary layer</u>, and so even if an ether had existed, it would have been impossible to measure its relative motion with earth.

Expanding Universe

- Hubble measurement suggests the Universe was created by a hot big-bang some 13 billion years ago.
- Following the fall in temperature, we assume dark matter was created and was responsible for the generation of <u>electricity and magnetism</u>.
- The early Universe, after thousands of years, had a temperature of the order of 10⁶ (K) and was composed of Hydrogen/Helium in the ratio 75%/25%,

Our model of the linear Universe expansion is the result of the pressure generated in the hot gas causing an impulse, such as a spherical piston like motion, so that the outer boundary is set in motion with an initial acceleration. Its value at $t = t_0$ is determined by the overpressure. We have discussed previously that, following the hot big-bang, the embryonic Universe would grow rapidly, but that short-time growth is reduced before the long-term linear expansion commences. The asymptotic solution for the Dark Matter Universe is independent of the initial assumptions.

The Solution of Euler's Equation

The radial speed of the spherical boundary is

$$U(t) = 2 \sqrt{3} c_0 (1 - R_0/R)^{1/2}$$

The exact relation between R and t is

$$(t - t_0) = (R_0 / 2 \sqrt{3} c_0) (\sqrt{(R/R_0)} \sqrt{(R/R_0 - 1)} + \ln(\sqrt{(R_0/R_0)} + \sqrt{(R/R_0 - 1)})$$

Early Universe

- Following the hot big-bang a quantum universe was created with an infinite array of particles forming an extremely hot soup. The speed of these particles would have been of order the speed of light, and there would have existed multiple collisions and shock waves.
- Ya Zel'dovich and colleagues have suggested that under gravitational interference the cosmic soup would have been formed into flat pancake like structures with the heavier particles forming the boundaries and the lighter particles controlling the inner space.
- Ultimately the boundary particles engaged in forming the galaxies containing the bulk of the subatomic and atomic matter out of the sea of the lightest medium, we are referring to as dark matter.

The role of the pressure

- It is assumed that, on the scale of the Universe, Dark Matter may be treated as a continuum.
- In most studies on the structure of the universe the pressure exerted by the medium in intergalactic space is either ignored, as in studies based on a vacuum universe, or, if included, it is treated as a function only of time.
- The linear expansion of the universe must be dominated by a radial motion. This is exerted by the spherically symmetric force exerted by the pressure created by this enormous mass of dark matter.
- The <u>negative pressure gradient</u> is responsible for a <u>positive outwards radial</u> <u>acceleration</u> of the outer boundary of the universe enclosing the 10¹¹ galaxies, and is opposed by an equal radial force, which we may infer is a physical measure of gravity.

The Expansion of the Universe

- The assumption is the early universe was very small in comparison with its size at the commencement of the expansion and in the present epoch.
- + From the Hubble measurements, $T/T_0 = R_0 / R = t_0 / t$

with
$$T_a/T_0 \ll 1$$
 and $R_0/R_a \ll 1$

where suffix 'a' refers to the present epoch and '0' refers to the commencement of the expansion. Thus

 $R_0/R_a = 2.7 \text{ (K)}/10^6 \text{ (K)}$, an order of <u>one millionth</u>, from the measurements of **Penzias** and **Wilson**, and **Hubble**.

We assume that the growth of the size of the Universe was initially exponential but the rate of growth then decreased until it approached a linear expansion. However we find using Thornhill's theory that the present assumed spherical structure of the Universe is quite independent of its earlier history.

CONCLUSIONS 111

- Einstein's work on relativity and the introduction of the Vacuum Universe revolutionized work on COSMOLOGY and mathematical physics. In effect it linearised all work on electromagnetism.
- > The claim that the equations of fluid dynamics are not laws of Nature is shown to be invalid.
- > The duality of light transmission through corpuscular motion or by wave propagation is not in doubt except in the case of wave motion in a void.
- All work on COSMOLOGY recognizes that the mass of the Universe exceeds that of the measured visible matter by a large factor. The missing mass is referred to as Dark Matter.
- Our suggestion, based on the work of Thornhill, is that Dark Matter occupies all intergalactic space. The model used for Dark Matter satisfies the laws of physics and is assumed to have the

CONCLUSIONS 1V

- > Our suggestion, based on the work of Thornhill, is that Dark Matter occupies all intergalactic space. The model used for Dark Matter satisfies the laws of physics and is assumed to have the properties of a perfect but exotic gas since it has, so far, not been measured. Current searches are world-wide.
- > Following Thornhill, our model of Dark Matter, leads to an exact solution of the Euler Equations for the asymptotic properties of a spherically expanding Universe dominated by the flow of Dark Matter.
- > The solution is independent of conditions existing at the commencement of the expansion. Its boundary in the present epoch maintains its positive acceleration.