

GC-B @ ~1 K?

by

K.R. Sreenivasan



Fifty Years of Research on Turbulence
and Acoustics: Colloquium honoring
Professor Geneviève Comte-Bellot

Ecole Centrale de Lyon

30 October 2009

**GC-B's Doctoral thesis on
Turbulent Channel Flow (1963),
translated into English as an ARC
Report by P. Bradshaw**

J. Fluid Mech. (1966), vol. 25, part 4, pp. 657-682
Printed in Great Britain

657

**The use of a contraction to improve the isotropy of
grid-generated turbulence**

By GENEVIÈVE COMTE-BELLOT† AND
STANLEY CORRISIN

Mechanics Department, The Johns Hopkins University

(Received 25 October 1965)

It is found that when the average kinetic energies of normal velocity components in decaying, grid-generated turbulence are equilibrated by a symmetric contraction of the wind tunnel, this equality can persist downstream. A second result is further confirmation of the fact that the best power-law fit to the inverse turbulent energy during the early part of decay is near $(x-x_1)^{1.25}$, for both rod grids and disk grids. The Kolmogorov decay law $\sim (t-t_1)^{1/2}$ is re-derived by a spectral method which is essentially equivalent to the original. Finally, a crude theoretical estimate of component energies in the straight duct after a weak contraction seems to support the experiments.

J. Fluid Mech. (1971), vol. 48, part 2, pp. 273-337
Printed in Great Britain

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**Simple Eulerian time correlation of full-
and narrow-band velocity signals in grid-generated,
'isotropic' turbulence**

By GENEVIÈVE COMTE-BELLOT

École Centrale de Lyon

AND STANLEY CORRISIN

The Johns Hopkins University

(Received 2 July 1970)

Space-time correlation measurements in the roughly isotropic turbulence behind a regular grid spanning a uniform airstream give the simplest Eulerian time correlation if we choose for the upstream probe signal a time delay which just 'cancels' the mean flow displacement. The correlation coefficient of turbulent velocities passed through matched narrow-band filters shows a strong dependence on nominal filter frequency (\sim wave-number at these small turbulence levels). With plausible scaling of the time separations, a scaling dependent on both wave-number and time, it is possible to effect a good collapse of the correlation functions corresponding to wave-numbers from 0.5 cm^{-1} , the location of the peak in the three-dimensional spectrum, to 10 cm^{-1} , about half the Kolmogorov wave-number. The spectrally local time-scaling factor is a 'parallel' combination of the times characterizing (i) gross strain distortion by larger eddies, (ii) wrinkling distortion by smaller eddies, (iii) convection by larger eddies and (iv) gross rotation by larger eddies.

**KRS, S. Tavoularis, R. Henry & S. Corrsin: Temperature fluctuations and scales
in grid-generated turbulence, *J. Fluid Mech.* **100**, 597-621 (1980)**

Helium at 4.2 K, $\nu = 2 \times 10^{-8}$ (air: 1.5×10^{-5})

J. Fluid Mech. (2002), vol. 452, pp. 189–197. © 2002 Cambridge University Press
DOI: 10.1017/S0022112001007194 Printed in the United Kingdom

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High-Reynolds-number turbulence in small apparatus: grid turbulence in cryogenic liquids

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C. M. White, A. N. Karpets and K. R. Sreenivasan

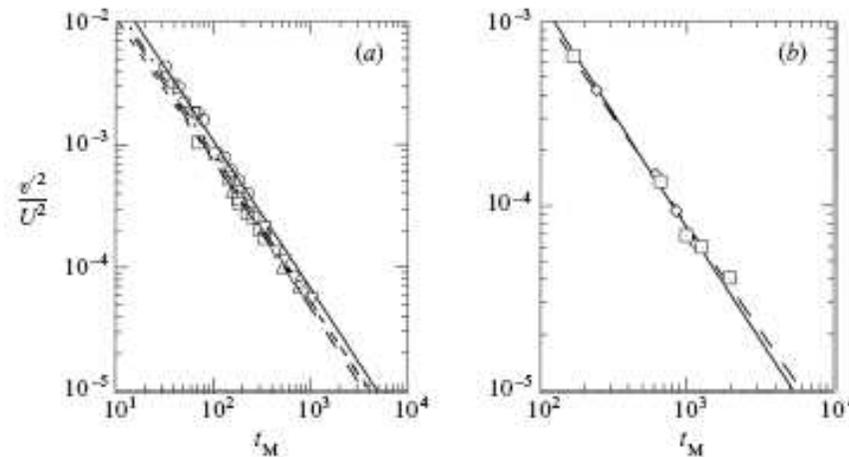
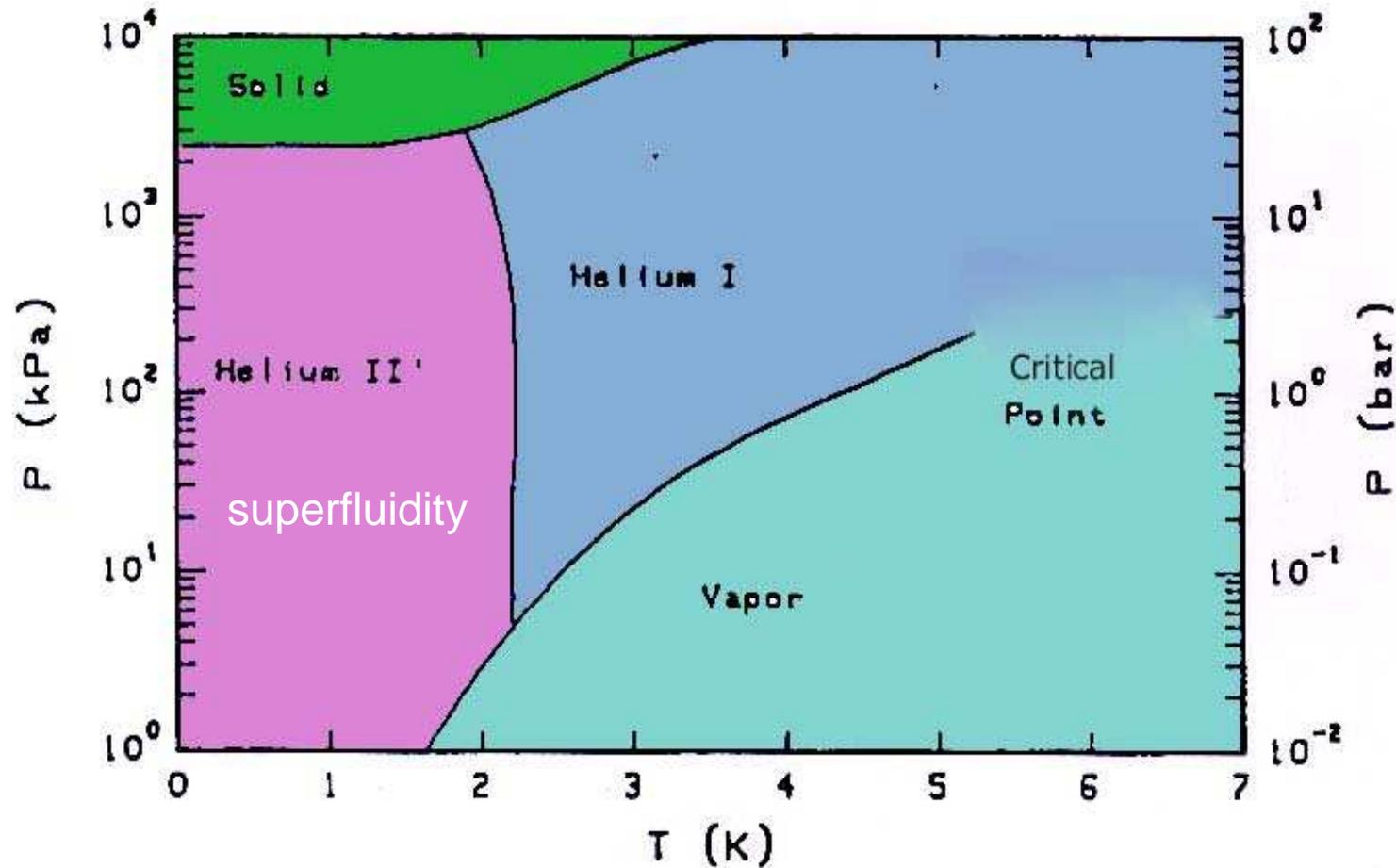


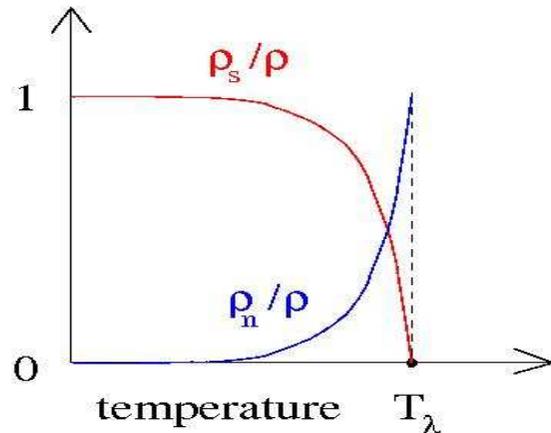
FIGURE 4. Temporal decay of the mean-squared-velocity fluctuation in the direction of motion of the grid at four Reynolds numbers for helium (*a*) and two mesh Reynolds numbers for nitrogen (*b*). The lines through the data are the least-square power-law fits. In (*a*) \circ , —, $R_M = 3.3 \times 10^4$; \square , —, $R_M = 6.6 \times 10^4$; \triangle , \cdots , $R_M = 1.32 \times 10^5$; ∇ , - · - ·, $R_M = 2.0 \times 10^5$. In (*b*) \diamond , —, $R_M = 9.1 \times 10^4$; \square , —, $R_M = 1.82 \times 10^4$. The Reynolds numbers remain large to the end of the range (so no ‘final period of decay’ is possible), and it is estimated that the boundary layer effects are negligible except perhaps for the last symbol for the nitrogen data.

Phase diagram of helium



The superfluid flow without friction (like a perfect fluid) and has spontaneously generated thin vortex structures (resembling the ideal vortex lines)

Phenomenological model for He II



Superfluid: density ρ_s , velocity v_s , no viscosity, no entropy, Euler fluid

Normal fluid: density ρ_n , velocity v_n , carries viscosity and entropy, Navier Stokes fluid

“coexisting but non-interacting and interpenetrating”

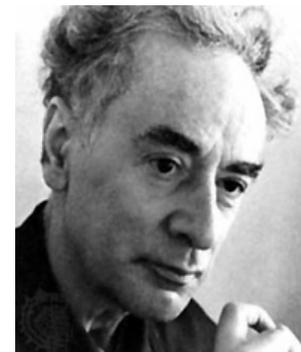
Landau: “We particularly emphasize that there is no division of the real particles of the liquid into “superfluid” and “normal” ones...which is a no more than a manner of expression...”



F. London 1900-1954



L. Tisza 1907-2009



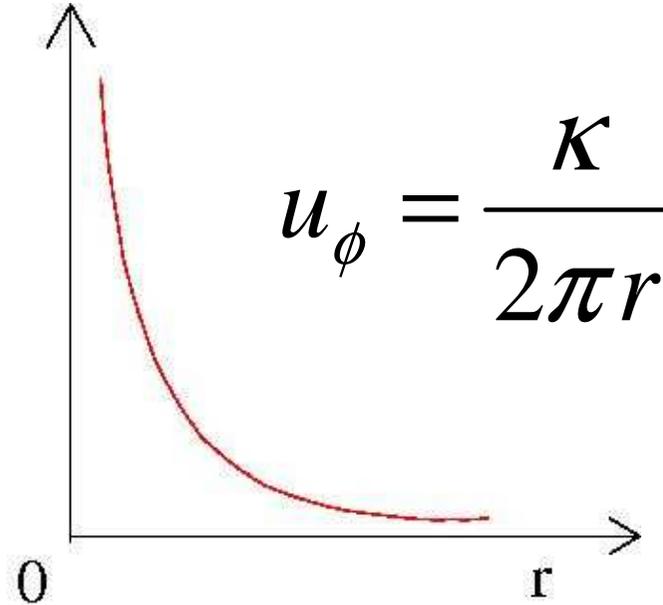
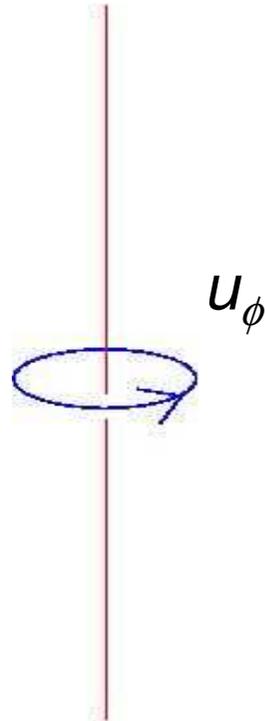
L.D. Landau 1908-1968

1962
Nobel
Prize

quantized vortices in helium II



Onsager
1903-1976



Wave function: $\psi = \psi_0 \exp(i\phi(r))$, $\psi_0 \rightarrow 0$ as $r \rightarrow 0$ and $\rightarrow 1$ as $r \rightarrow \infty$

Velocity is the gradient of $\phi(r)$. The increment of its gradient over any closed path must be a multiple of 2π , for the wave function to remain single valued.

“Thus, the well-known invariant called hydrodynamic circulation is quantized; the quantum of circulation is h/m .”

Onsager (1949)

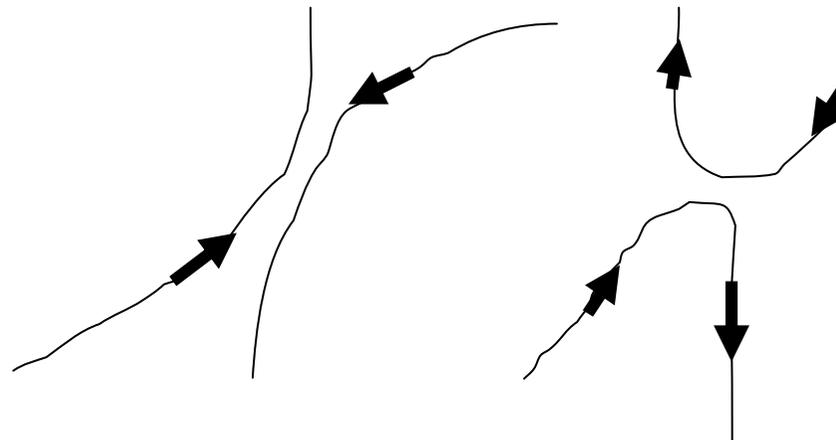
Except for a few angstroms from the center of the core, the laws obeyed are those of classical hydrodynamics [e.g., Biot-Savart].

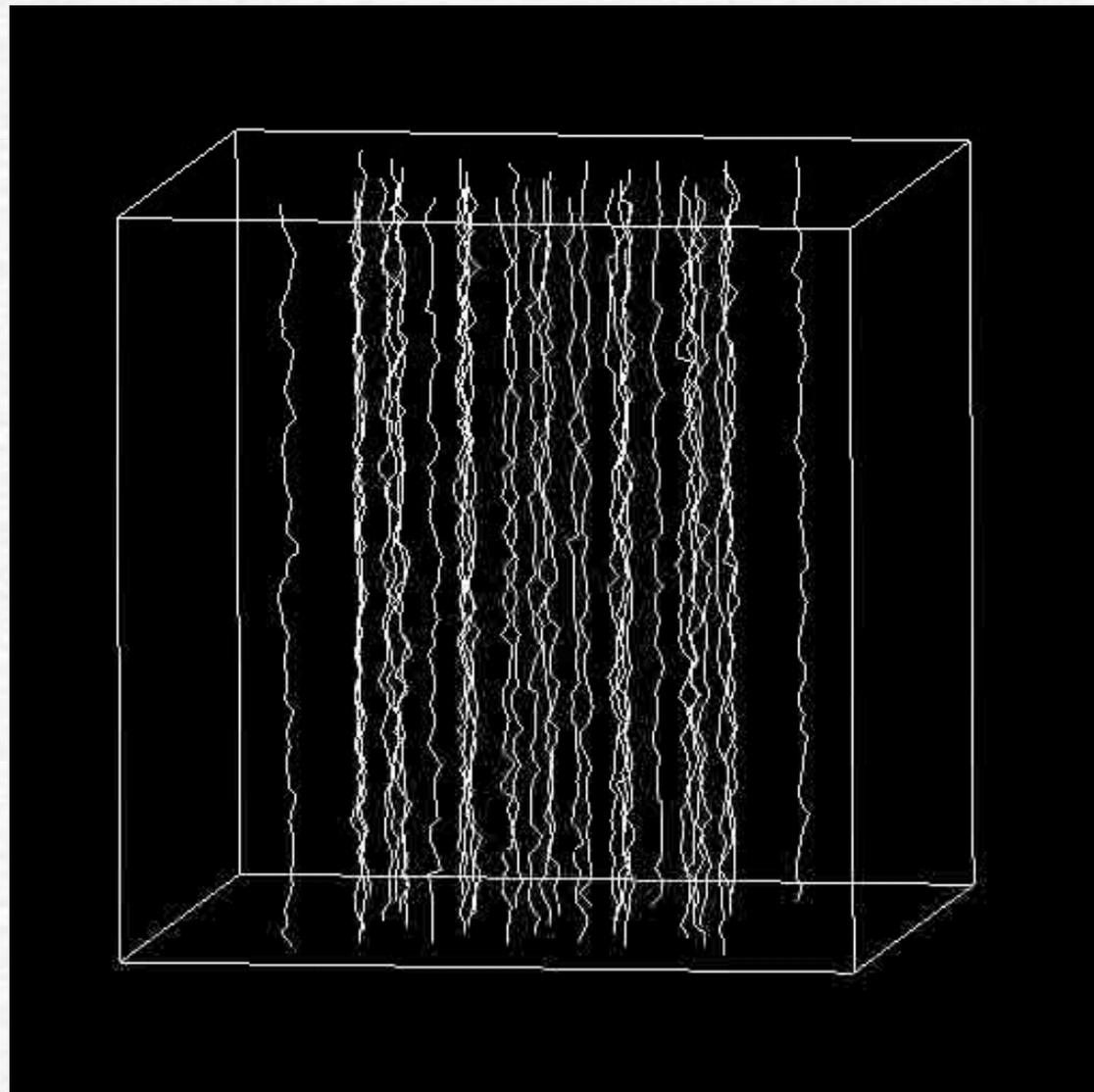


R.P. Feynman: 1918-1988

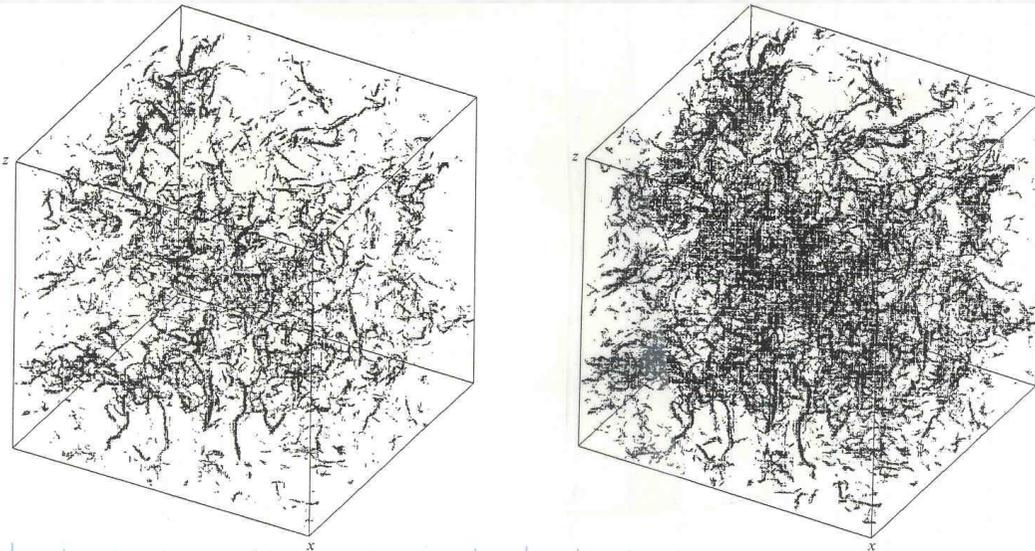
Prog. Low Temp. Phys. 1, 17 (1955)

If ... two oppositely directed sections of [vortex] line approach closely, ... the lines (which are under tension) may snap together and join connections a new way ...





Carlo Barenghi and colleagues



High-intensity vortex structures in homogeneous and isotropic turbulence (Vincenti & Meneguzzi 1991)

Vortex tangles (“**superfluid turbulence**”) by Tsubota, Araki & Nemirovskii 2000); pioneering simulations by K.W. Schwarz (1985)

Microscopic details of reconnection were explored by J. Koplik and H. Levine, *Phys. Rev. Lett.* **71**, 1375 (1993), by solving the nonlinear Schrödinger equation with quadratic nonlinearity — which is a good model for the wavefunction in BEC.

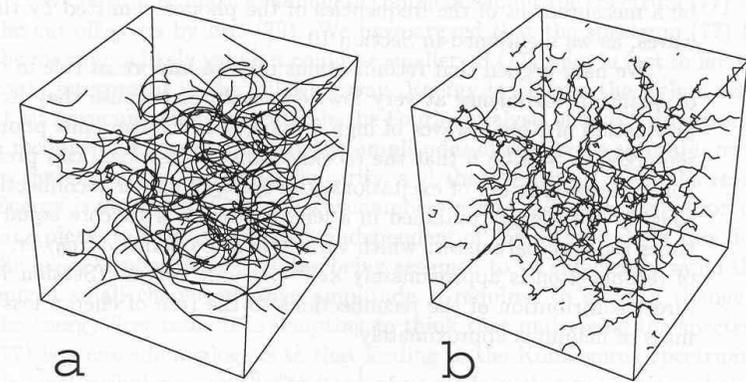
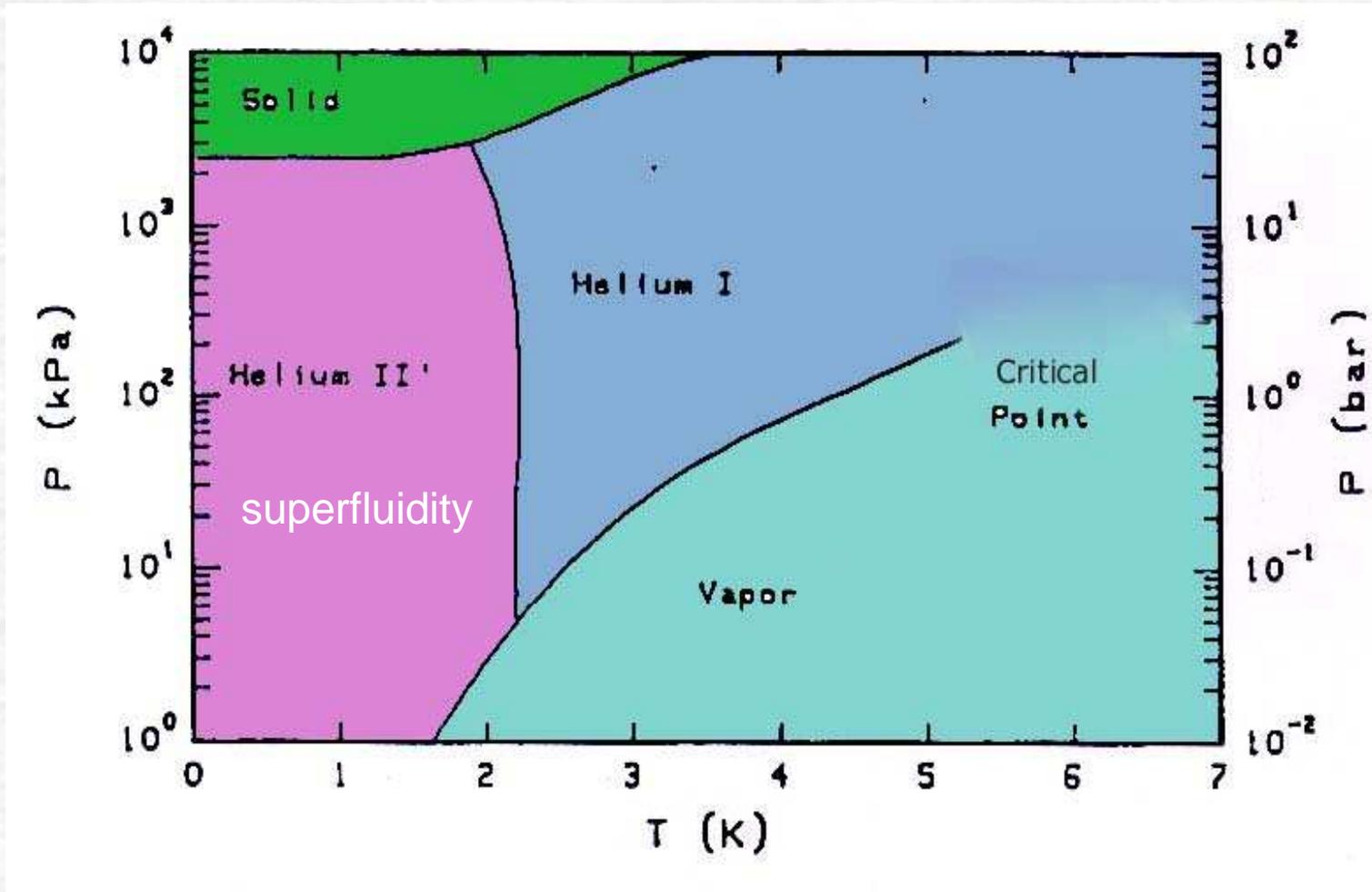


Fig. 10. Vortex tangles at (a) $T = 1.6\text{K}$ and (b) $T = 0\text{K}$. From Tsubota *et al.*⁵⁰.

Nature of superfluid turbulence, its connection to classical turbulence

Phase diagram of helium



GC-B @ ~ 1 K?

The energy decay in classical turbulence

1. Energy decay (definition)

$$d/dt \langle u^2 \rangle = -2\varepsilon$$

2. Dissipative anomaly (empirical)

$$\langle \varepsilon \rangle = C \langle u^2 \rangle^{3/2} / \Lambda$$

$$\begin{array}{c} \uparrow \\ O(1) \end{array}$$

Λ grows with time as a power law

Λ grows with time and could, at some point, be limited by the apparatus width, d . It is reasonable to assume that $\Lambda = \text{constant} = d$.

Consequences

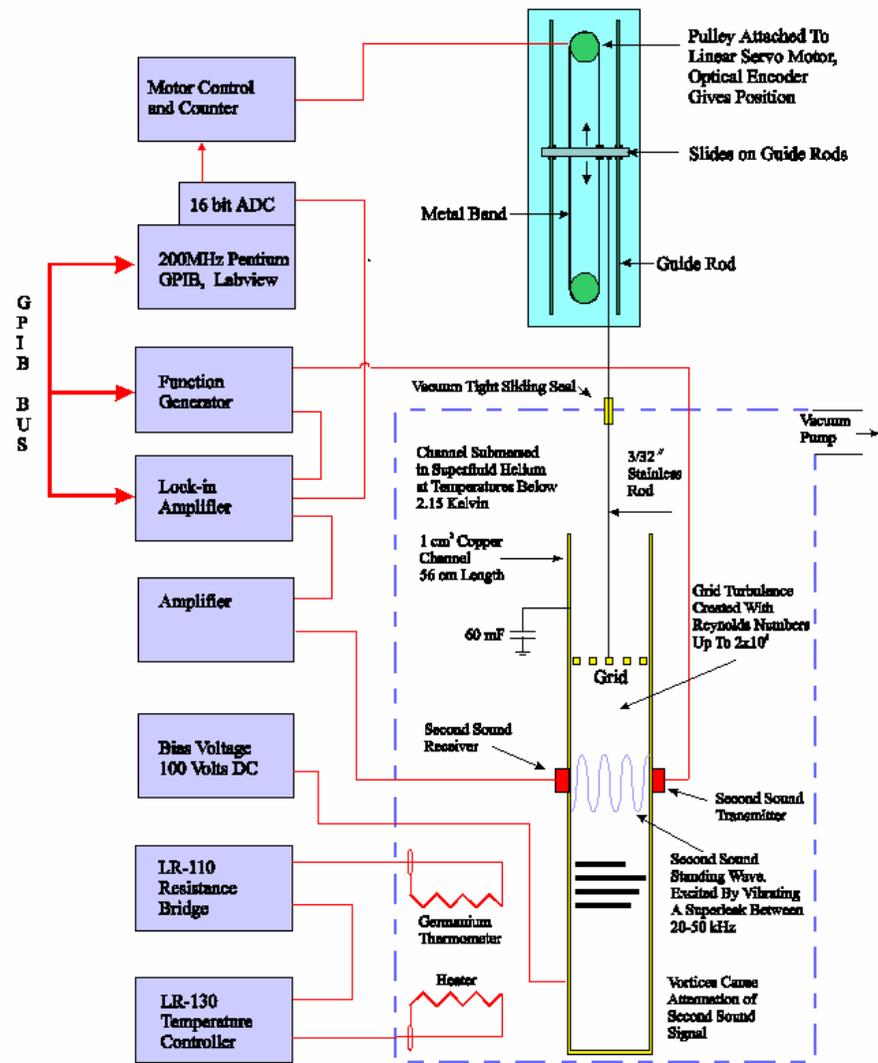
1. $\langle \varepsilon \rangle = d^2 t^{-3}$

2. Using the exact relation

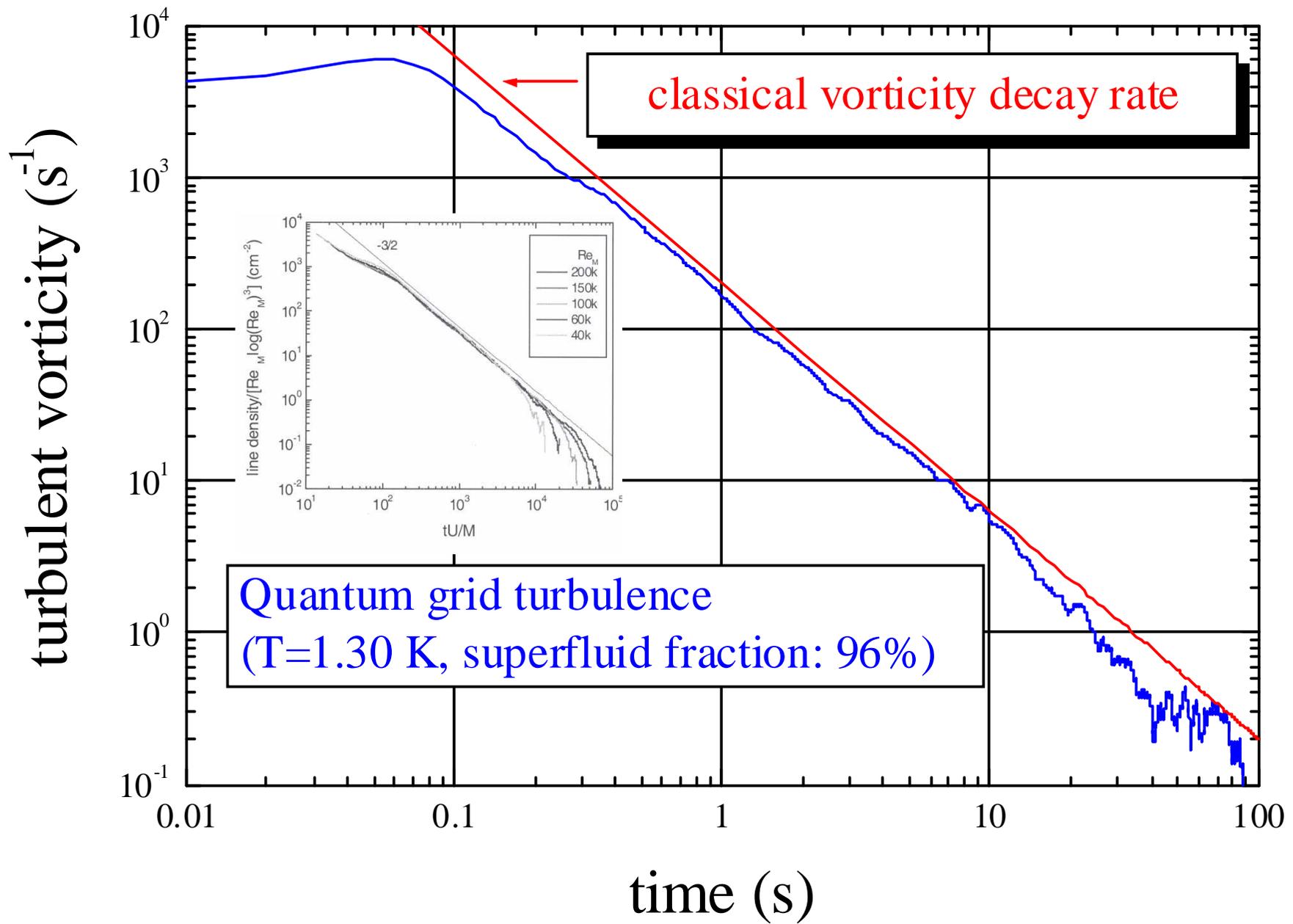
$$\langle \varepsilon \rangle = \nu \langle \omega^2 \rangle,$$

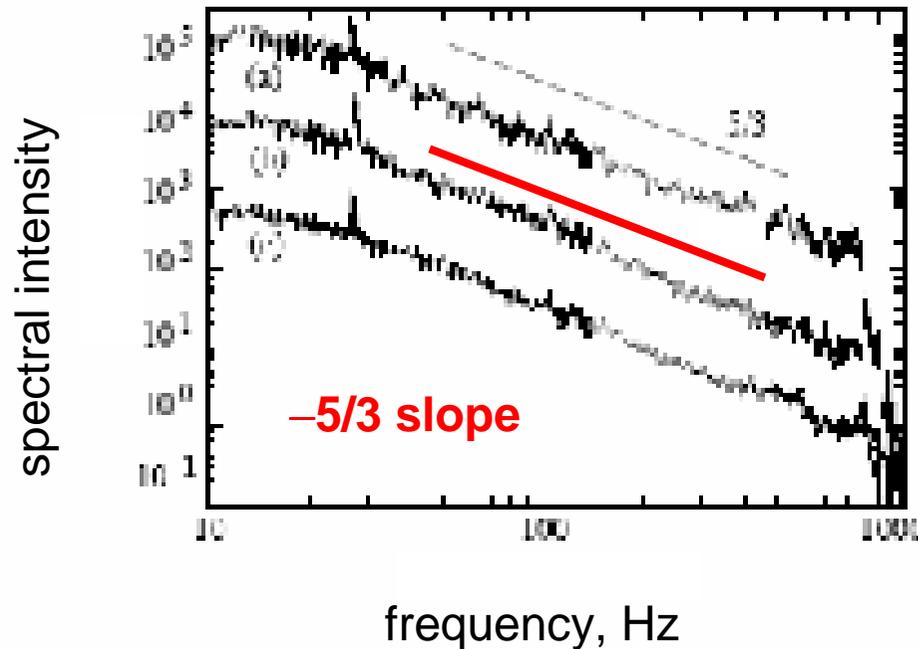
we obtain

$$\langle \omega^2 \rangle^{1/2} = (d/\nu^{1/2}) t^{-3/2}.$$



The apparatus for helium II grid turbulence
(R.J. Donnelly, S. Stalp, J.J. Niemela, W.F. Vinen, et al.)





In simulations:

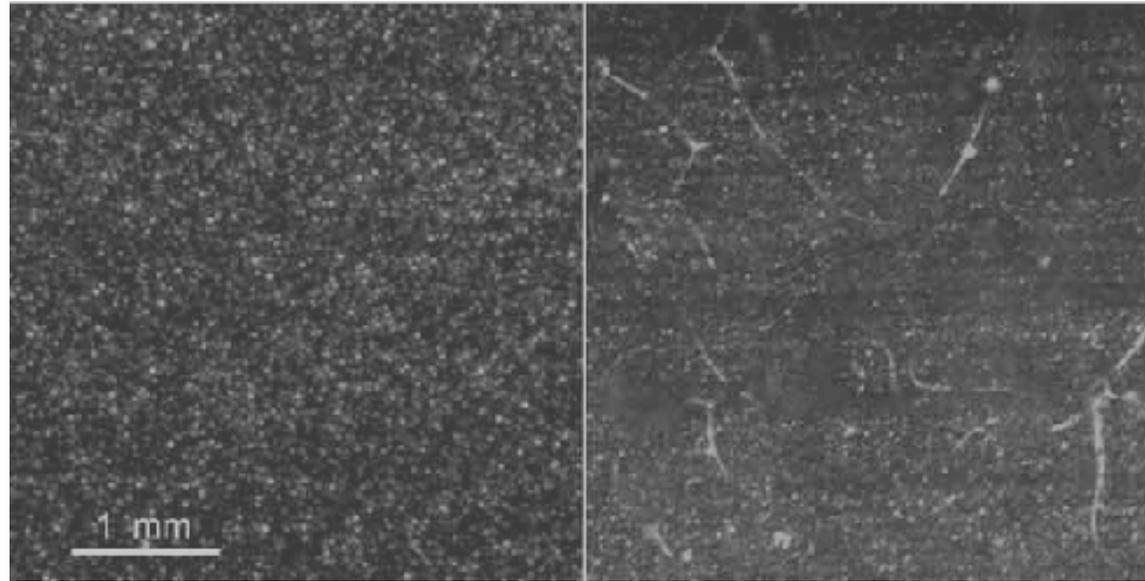
- C. Nore, M. Abid & M.E. Brachet, *Phys. Rev. Lett.* **78**, 3896 (1997)
- T. Araki, M. Tsubota & S.K. Nemirovskii, *Phys. Rev. Lett.* **89**, 145301 (2002)
- M. Kobayashi & M. Tsubota, *Phys. Rev. Lett.* **94**, 065302 (2005)
- P.E. Roche *et al.* *Europhys. Lett.* **77**, 66002 (2007)

Superfluid turbulence in Karman flow:

J. Maurer & P. Tabeling, *Europhys. Lett.* **43**, 29 (1998)

Obvious? Surprising?

50 years on...



~50 mK above T_λ

~50 mK below T_λ

The left panel shows a suspension of hydrogen particles just above the transition temperature. The right panel shows similar hydrogen particles after the fluid was cooled below the lambda point. Some particles have collected along filaments, while others are randomly distributed as before. Fewer free particles are apparent on the right only because the light intensity was reduced to highlight the brighter filaments in the image. Volume fraction $\cong 3 \times 10^{-5}$.

G.P. Bewley, D.P. Lathrop & KRS, Nature 441, 558 (2006)

What kind of particles?

Requirements on particle properties

- Must be small enough to follow the flow with fidelity (i.e., must respond to the smallest scales of the flow with fast response); in particular, must have the same density as the fluid (e.g., Maxey & Riley, *Phys. Fluids* **26**, 883 (1983))
- Must be large enough to be imaged with 'usable' illumination and detection equipment
- Must not cluster

In liquid helium

- Because of small apparatus and large Reynolds numbers, small scales are smaller than in water, demands on fidelity are higher; in particular, helium I has a density of 1/8 that of water
- Very small particles cannot be imaged
- Mutual attraction of particles and clustering cannot be suppressed by using surfactants as in water.

Particles that have worked

Nearly neutral particles of frozen mixtures of helium and hydrogen.

Bewley, Lathrop & KRS, *Nature* **441**, 558 (2006); *Experiments in Fluids* **44**, 887 (2008); Paoletti, Fiorito, KRS & Lathrop, *J. Phys. Soc. Jpn* **77**, 80702 (2008)



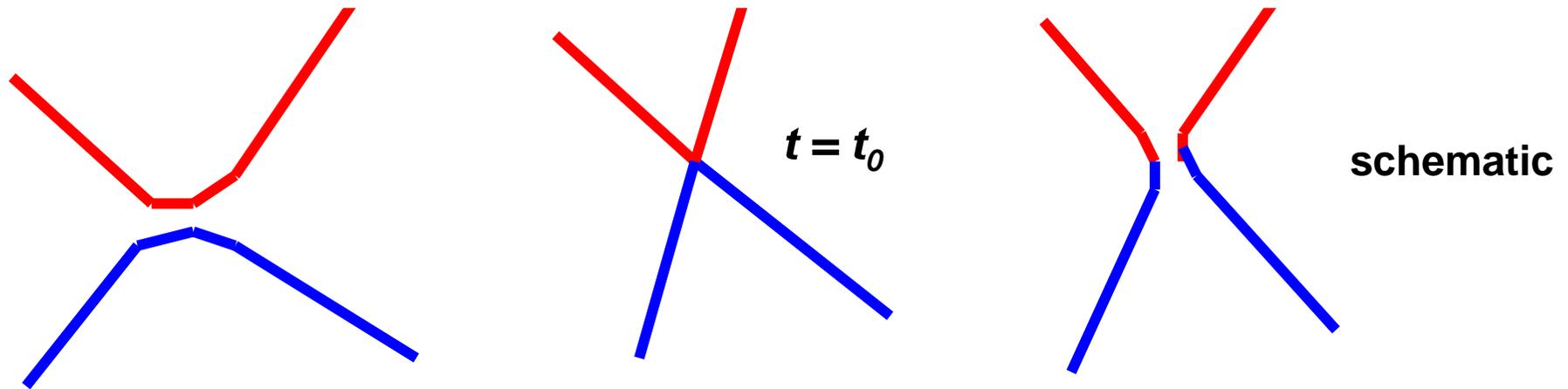
Greg Bewley



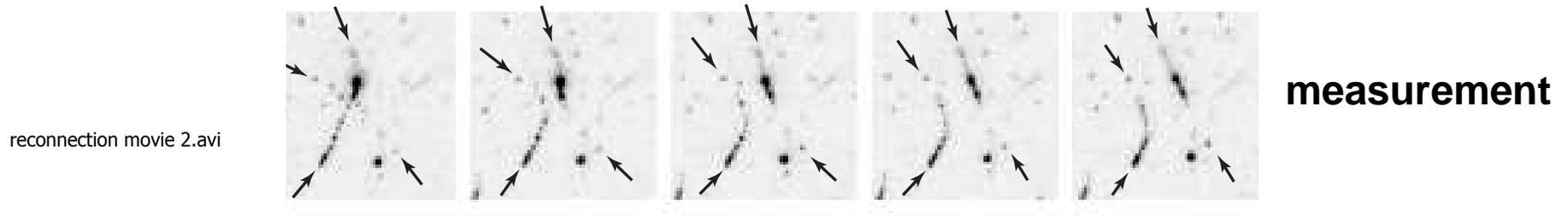
Matt Paoletti



Dan Lathrop



Schematic of cores of reconnecting vortices before and after reconnection at $t > t_0$.

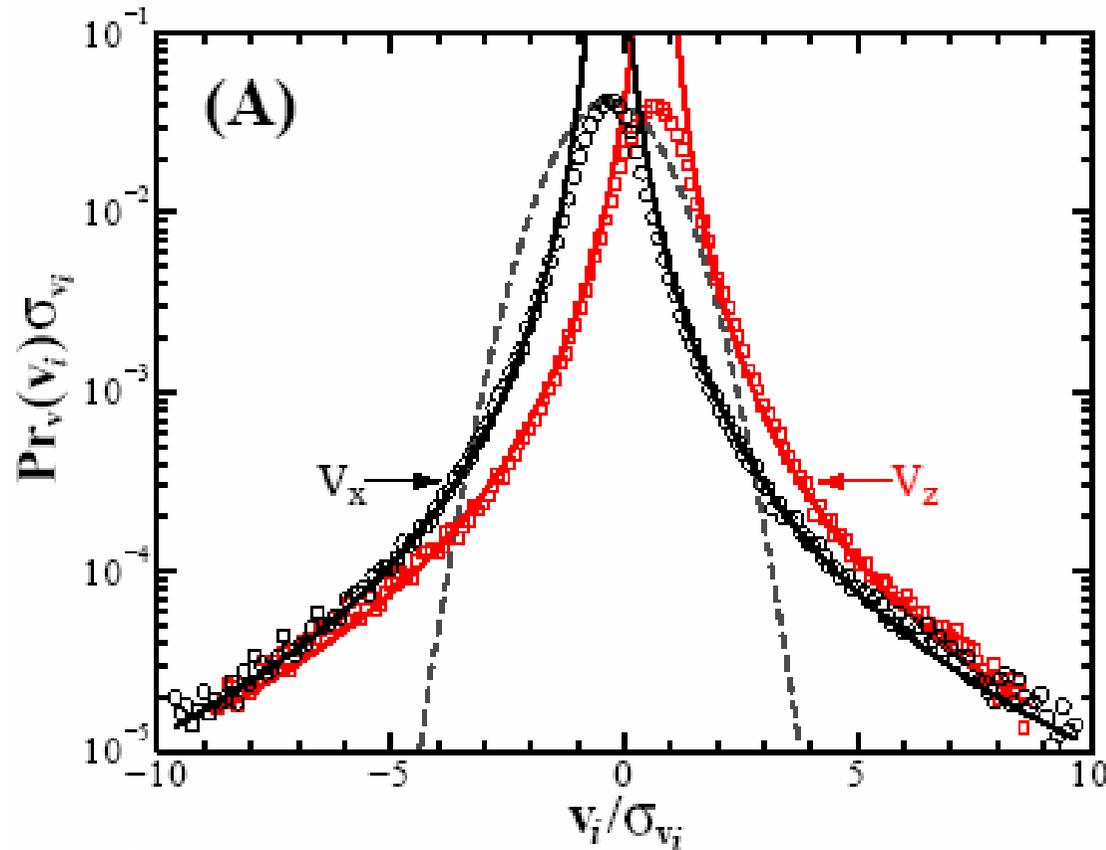


Images of hydrogen particles suspended in liquid helium, taken at 50 ms intervals, for $t > t_0$. Some particles are trapped on quantized vortex cores, while others are randomly distributed in the fluid. Before reconnection, particles drift collectively with the background flow. Subsequent frames show reconnection as the sudden motion of a group of particles.

Bewley, Poaletti, KRS & Lathrop, *PNAS* **105**, 13707 (2008)

Define delta

Nearly homogeneous turbulence following a counterflow



$$\text{Pr}(v) dv = \text{Pr}(t) dt$$
$$v = \kappa(t - t_0)^{-0.5}$$
$$\text{Pr}(v) \sim |v|^{-3}$$

No instances (away from solid boundary) where power-law tails exist for velocity distributions in classical turbulence.

Even by conditioning velocity PDFs on intense vorticity in classical turbulence, one finds no sign of anything other than a Gaussian.

Comparisons of classical and superfluid turbulence

Superfluid turbulence (helium II)

- Velocity distribution follows a power law
- Reconnections plays a crucial role
- Quantization of circulation imposes severe restrictions on the stretching of vortex line elements
- Dissipation mechanism is not well understood

Classical turbulence (3D)

- Velocity distribution is nearly normal
- The role of reconnections is not clear
- Vortex stretching plays a key role in scale-to-scale energy transfer
- Energy dissipation occurs because of fluid viscosity

Yet...

- $-5/3$ slope in the spectral form is common
- Decay law is the same as in classical turbulence
- The concept of eddy viscosity seems to apply in the decaying case

Only beginnings have been made to understand these aspects

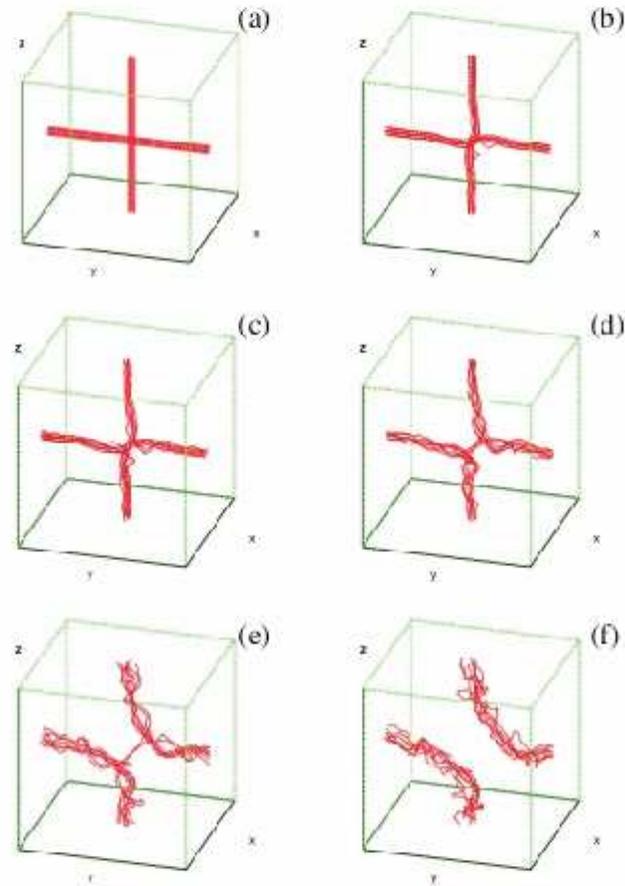
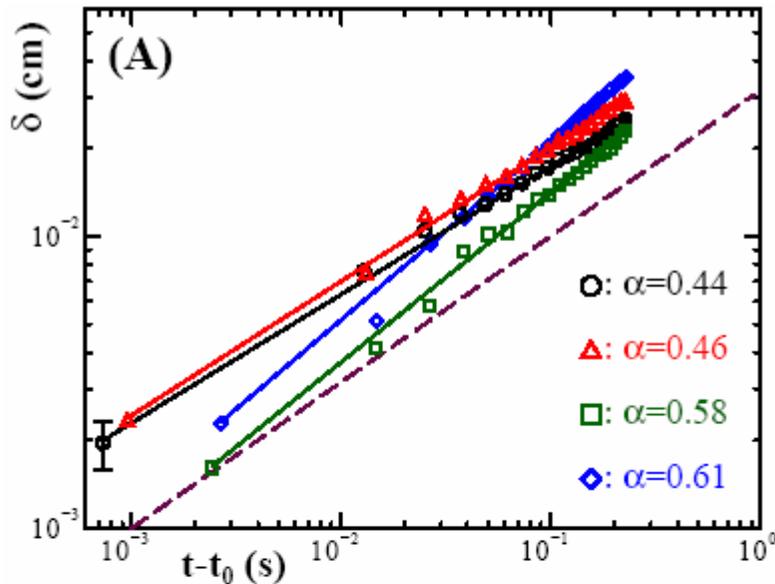


FIG. 1 (color online). Reconnection of two bundles of seven vortex strands each. (a) $t = 0$ s, (b) $t = 7.13$ s, (c) $t = 23.58$ s, (d) $t = 36.27$ s, (e) $t = 61.49$ s, (f) $t = 80.35$ s.

**Thank you for
your attention**

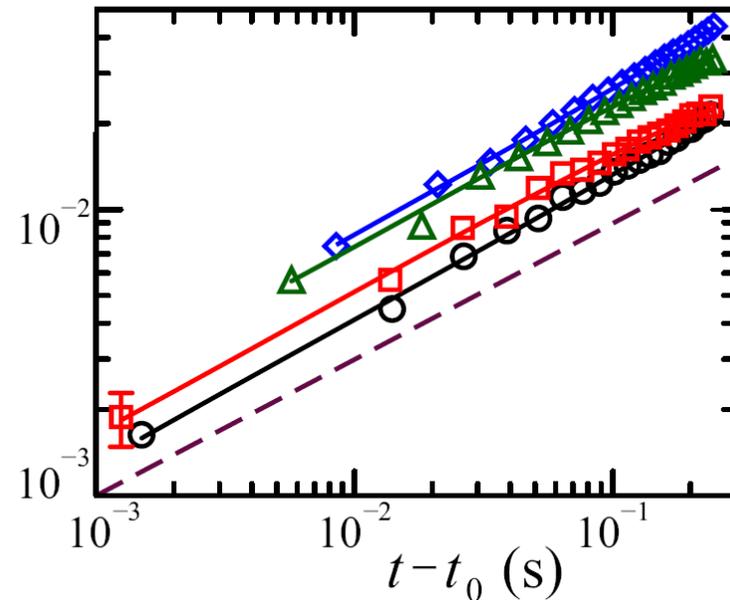
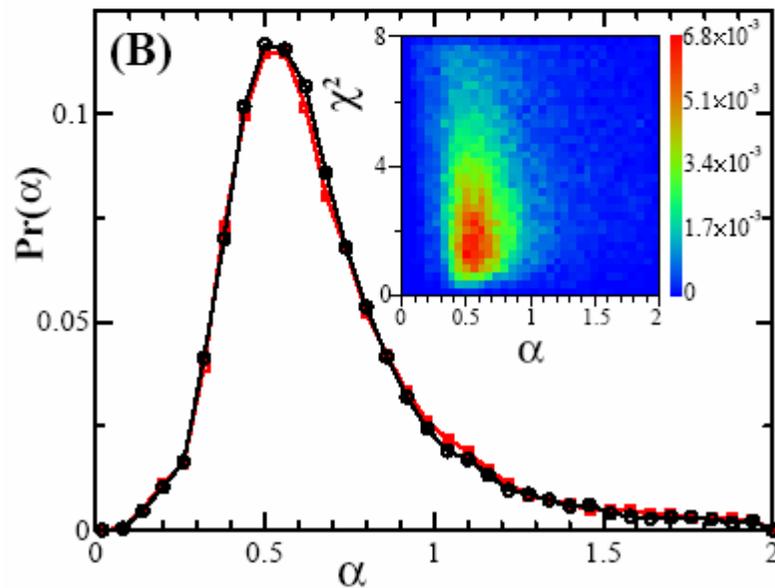


$$\delta(t) = A\kappa(t-t_0)^\alpha$$

dimensional analysis: $\alpha = 1/2$

Alternatively:

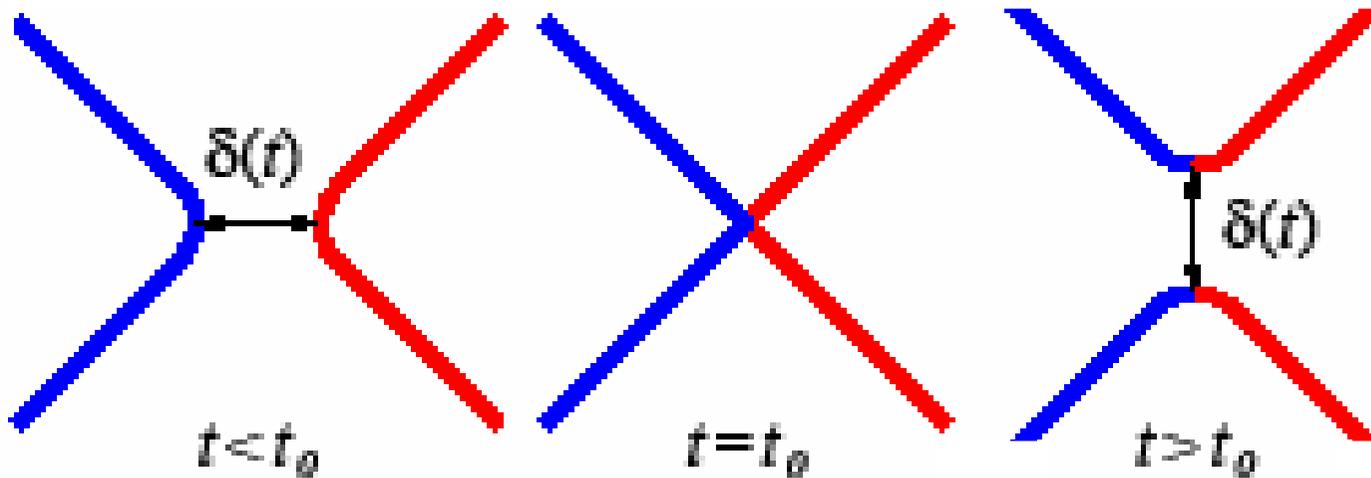
$$\delta(t) = A\kappa(t-t_0)^{1/2}[1+c(t-t_0)]$$



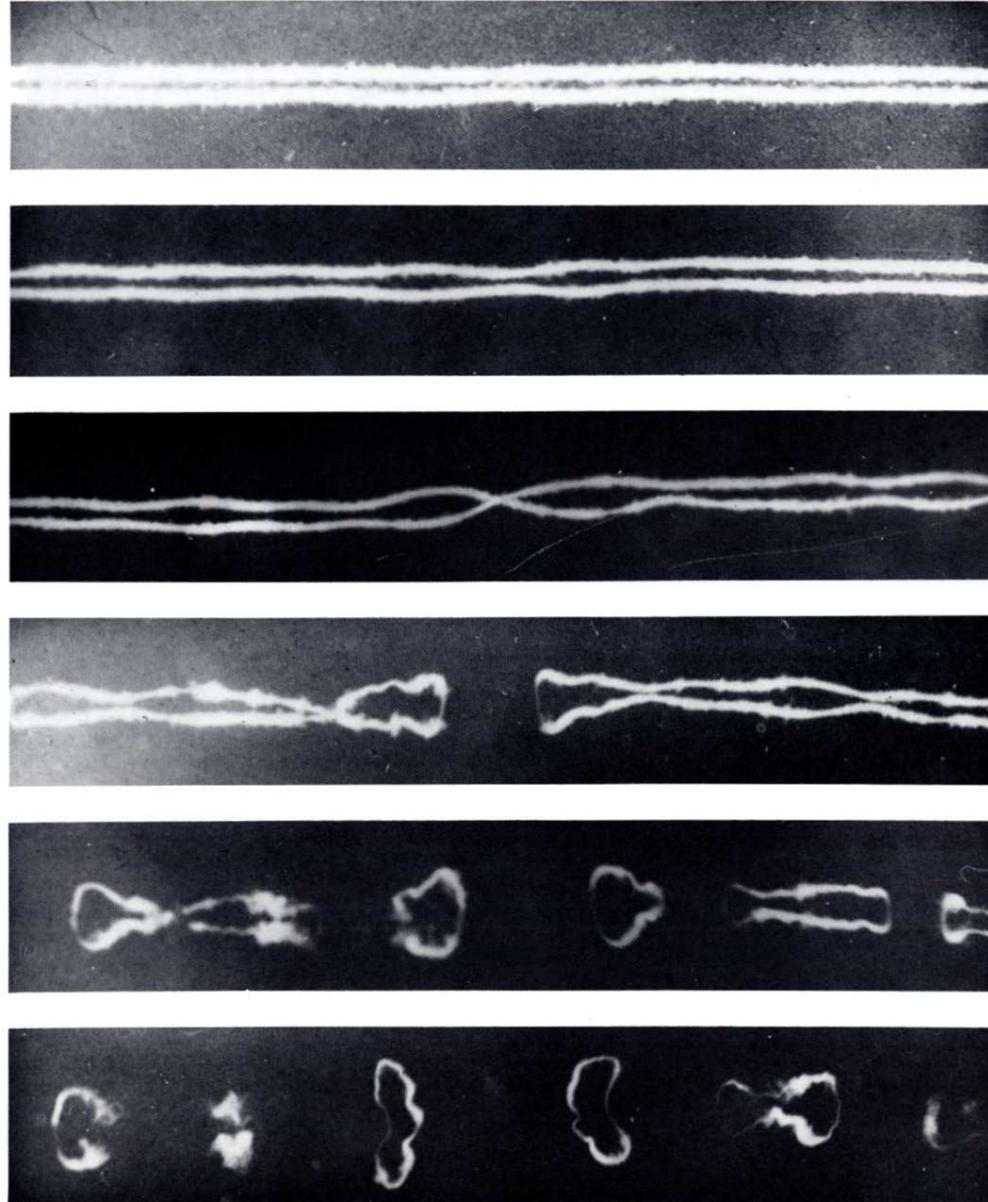
Paoletti, Fisher, KRS & Lathrop, *Phys. Rev. Lett.* (2008)

Reconnection statistics are time reversible?

Two vortices of opposite sign, which are attracted to each other, collide, splice parts of one to parts of the other, and move away from each other in a different direction.



Reconnecting vortex lines at the moment of reconnection, t_0 , and before and after t_0 .



116. **Instability of a pair of trailing vortices.** The vortex trail of a B-47 aircraft was photographed directly overhead at intervals of 15 s after its passage. The vortex cores are made visible by condensation of moisture. They slowly recede and draw together in a symmetrical nearly sinu-

soidal pattern until they connect to form a train of vortex rings. The wake then quickly disintegrates. This is commonly called Crow instability after the researcher who explained its early stages analytically. *Crow 1970, courtesy of Meteorology Research Inc.*

Vortices in classical fluids are macroscopic in scale (tens of centimeters thick in the previous example, perhaps mm-sized in laboratory flows). In both instances, vortex reconnection is strongly influenced by core dynamics and viscosity.

Helium II has superfluidity and vortices are \sim one Angstrom thick (atomic dimension), and the physics is less complex.

Comparisons of classical and superfluid turbulence

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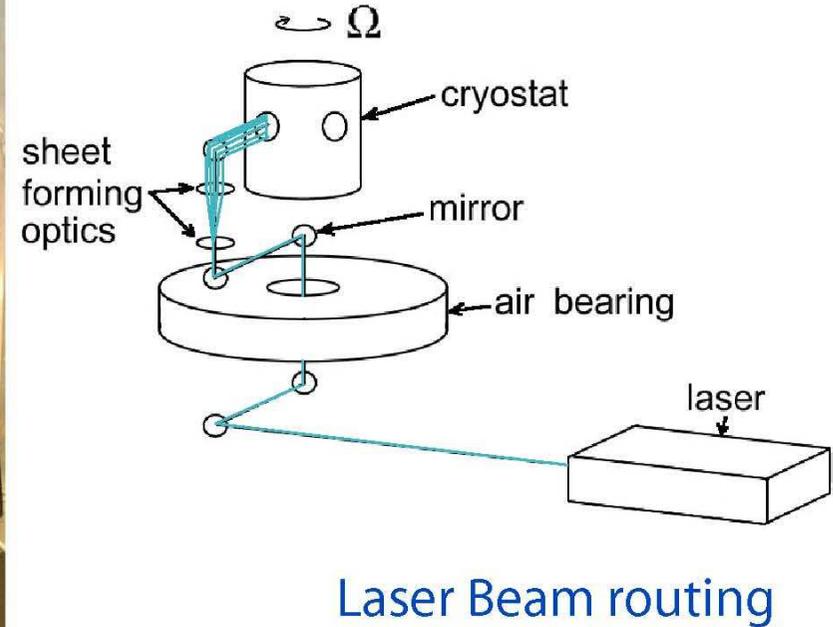
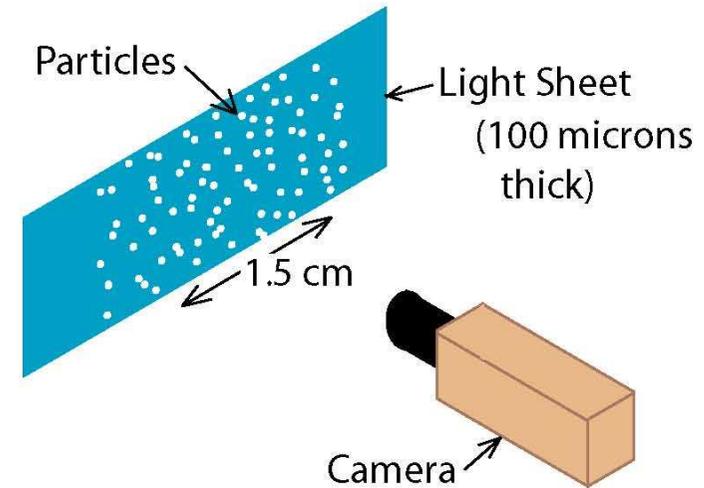
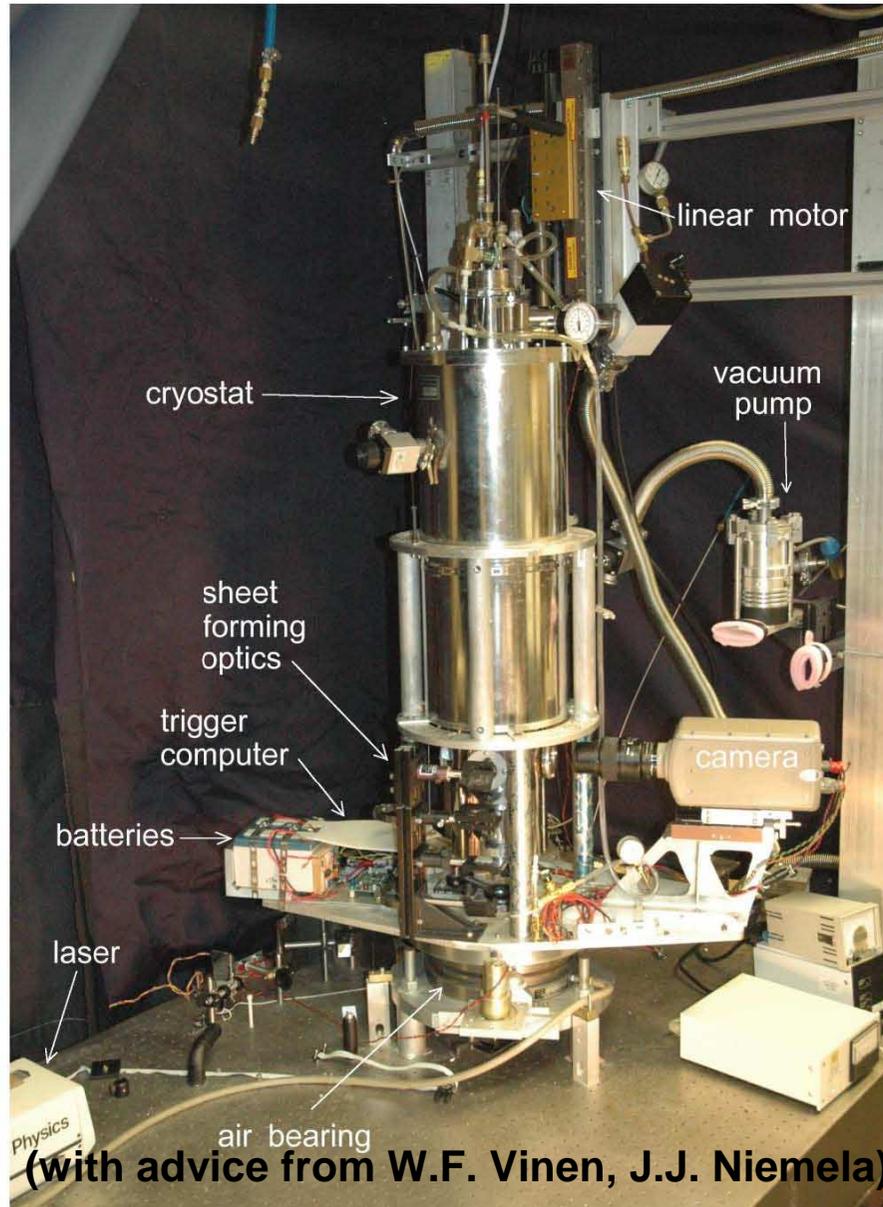
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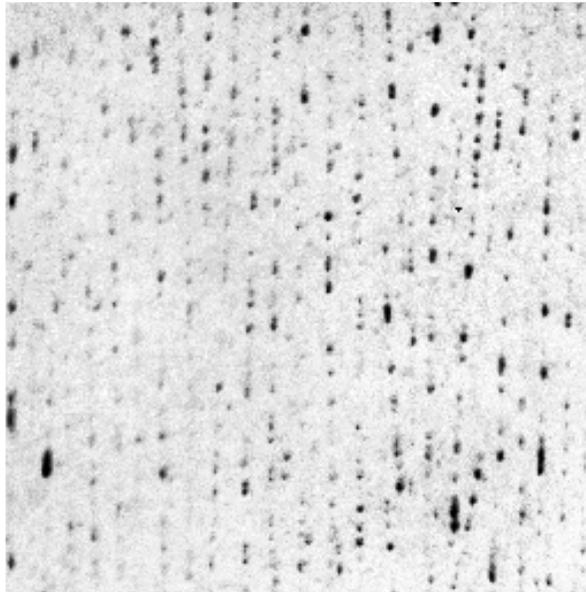
Brief chronological developments

1. 1950's: Hall and Vinen (indirect inference)
2. Late 1980's: Schwarz, Koplik, etc (vortex dynamics)
3. Mid-2000's: Visualization, measurements

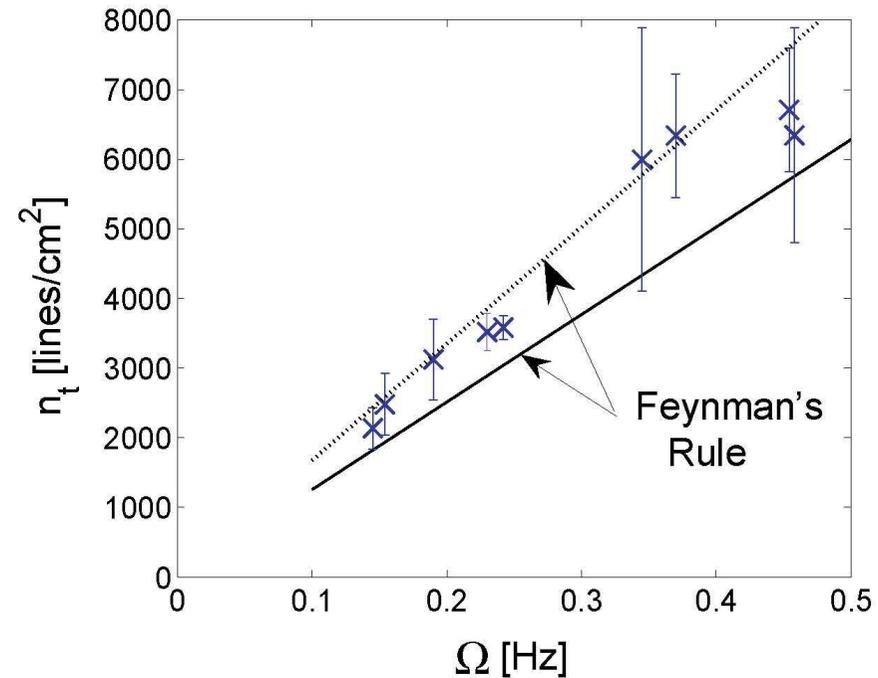
Apparatus



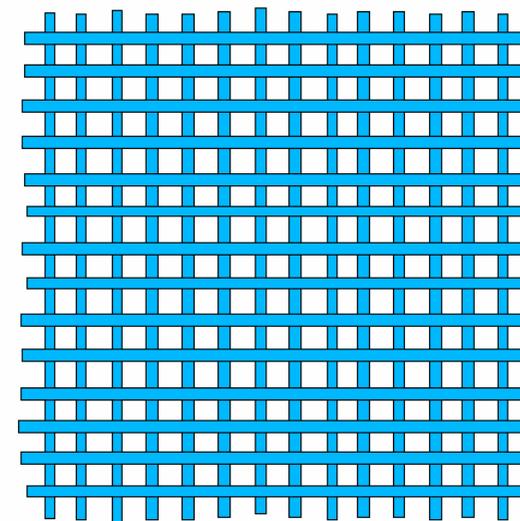
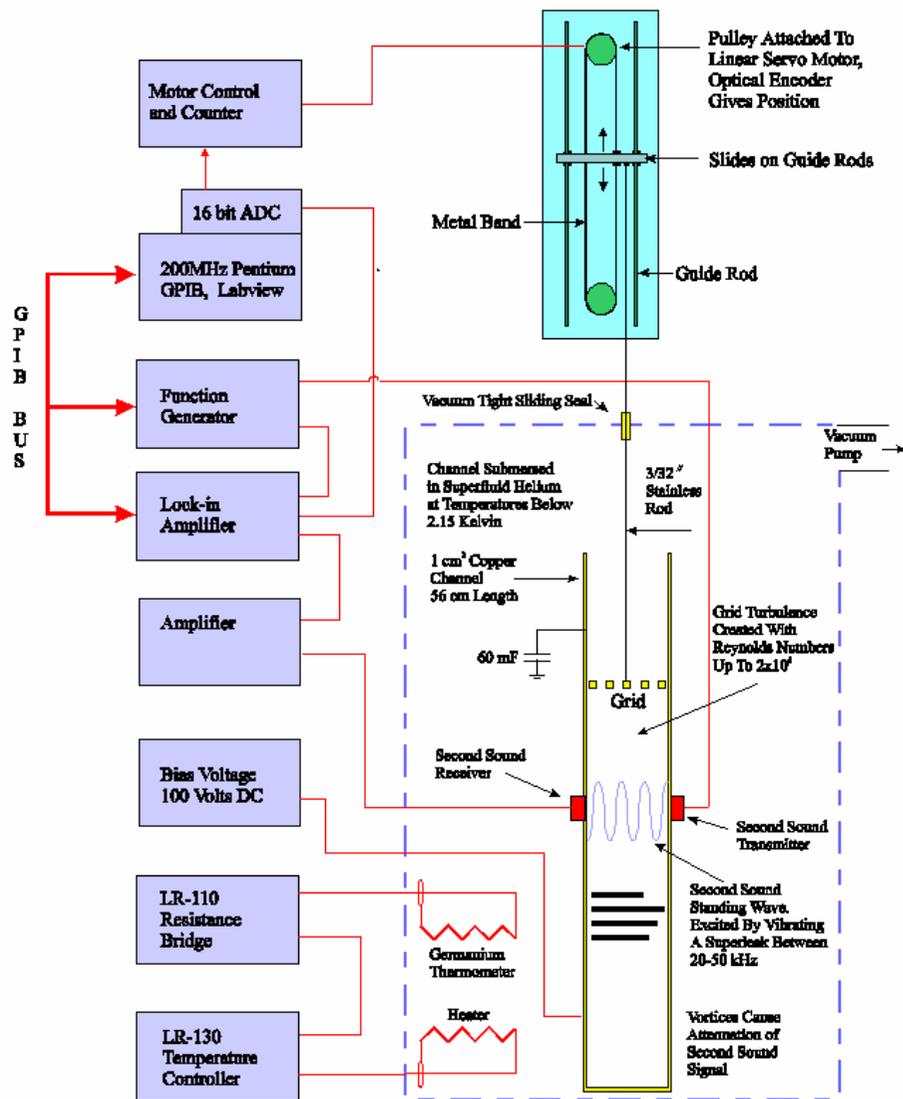
Number of vortices



c



The left panel shows an example of particles arranged along vertical lines when the system is rotating steadily about the vertical axis. The spacing of lines is remarkably uniform, although there are occasional distortions of the lattice and possible points of intersection. Their number follows Feynman's rule pretty well.



turbulence-generating grid
(as in Comte-Bellot & Corrsin)

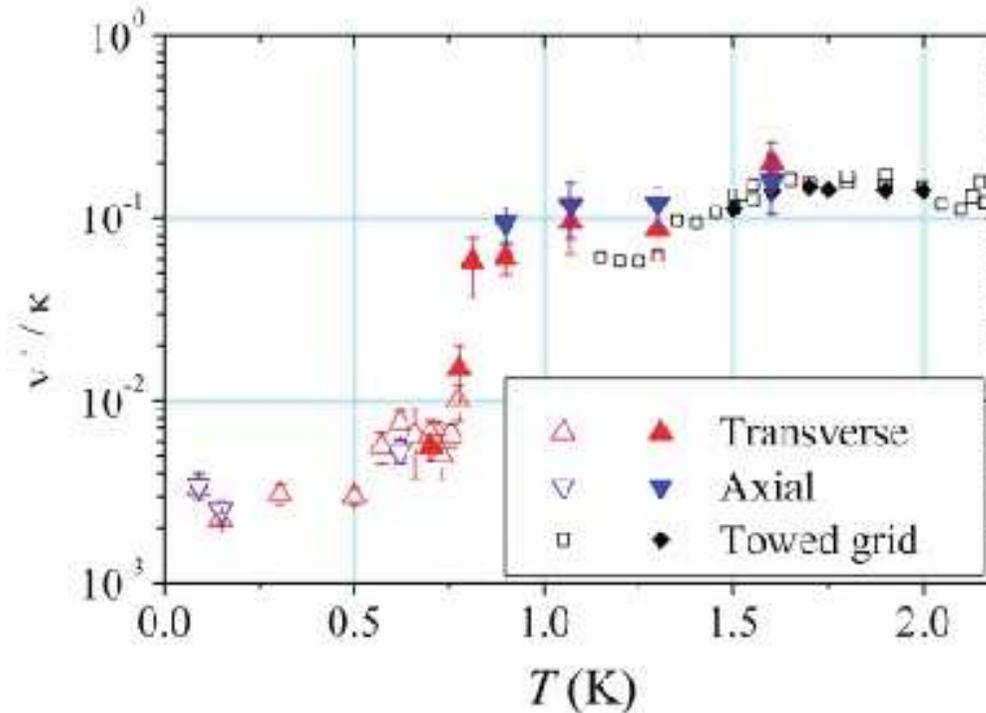
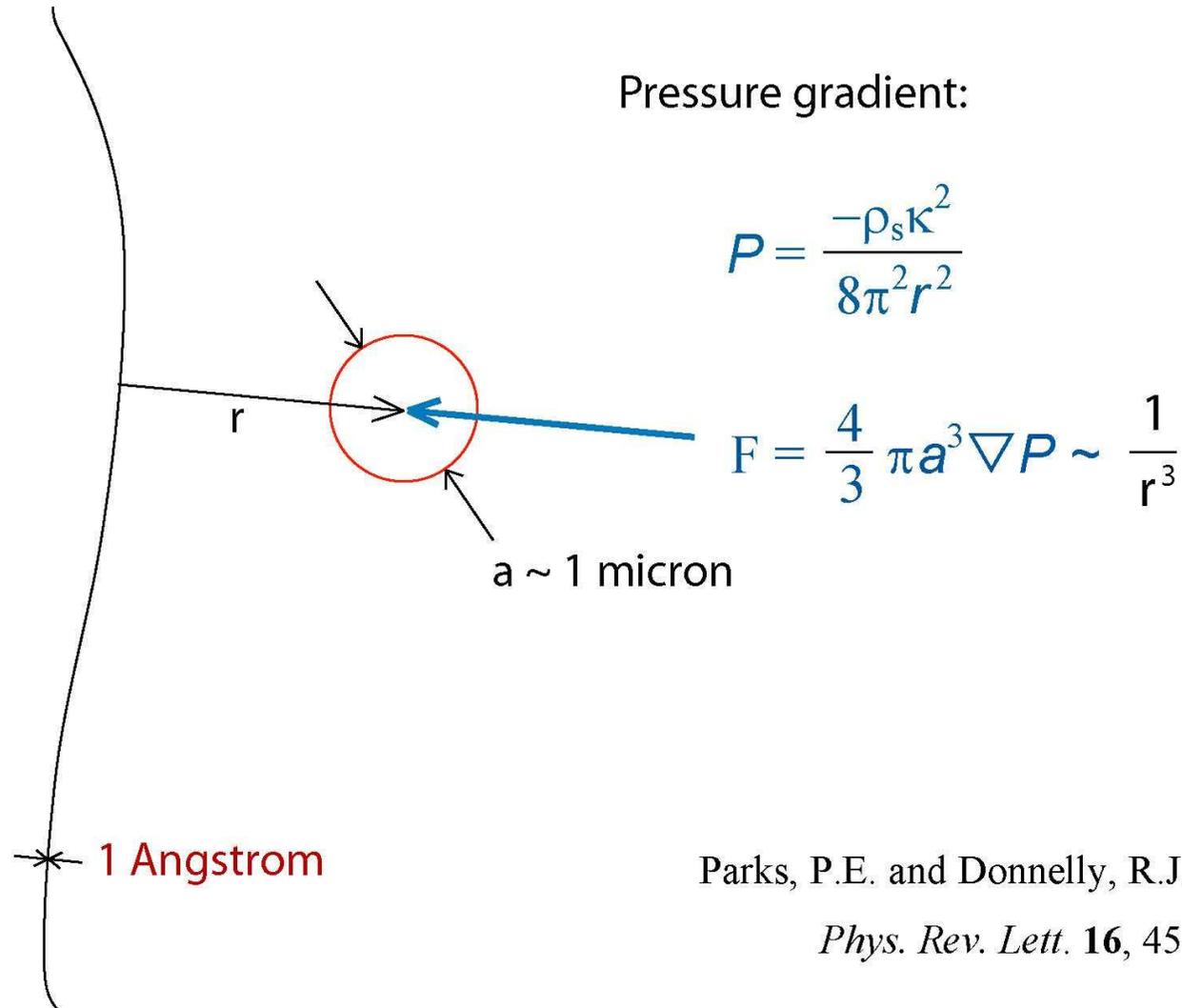


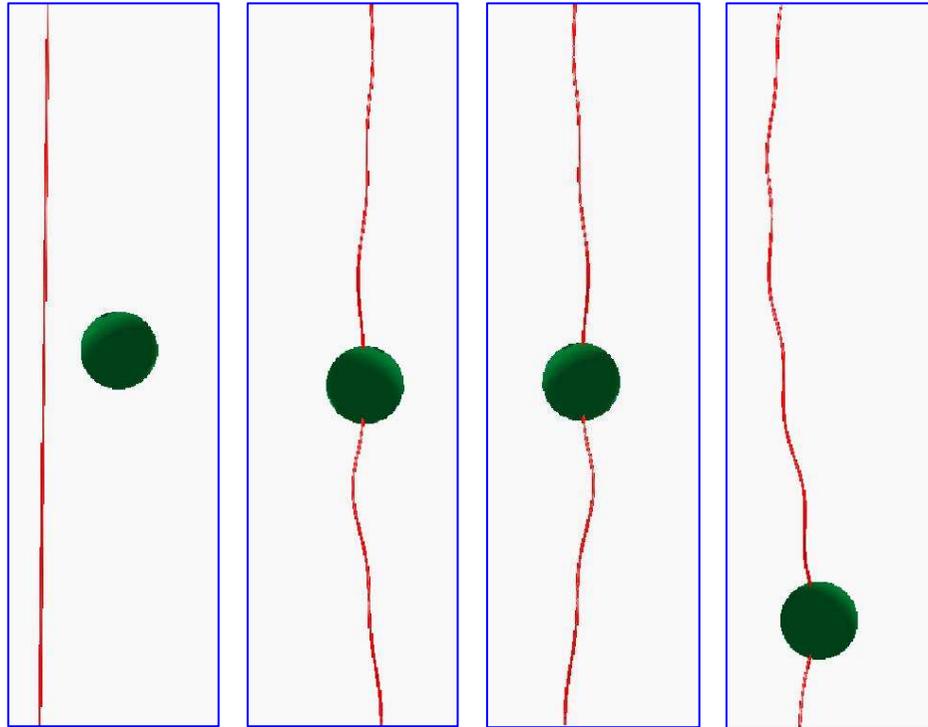
FIG. 5 (color online). The effective kinematic viscosity ν' after a spin down from $\Omega = 1.5$ rad/s measured in the transverse (Δ) and axial (∇) directions. Closed (open) triangles correspond to measurements with free ions (charged vortex rings). Error bars specify the uncertainty of fitting. Squares and diamonds: second sound measurements of grid turbulence [12,22].

Particle Trapping



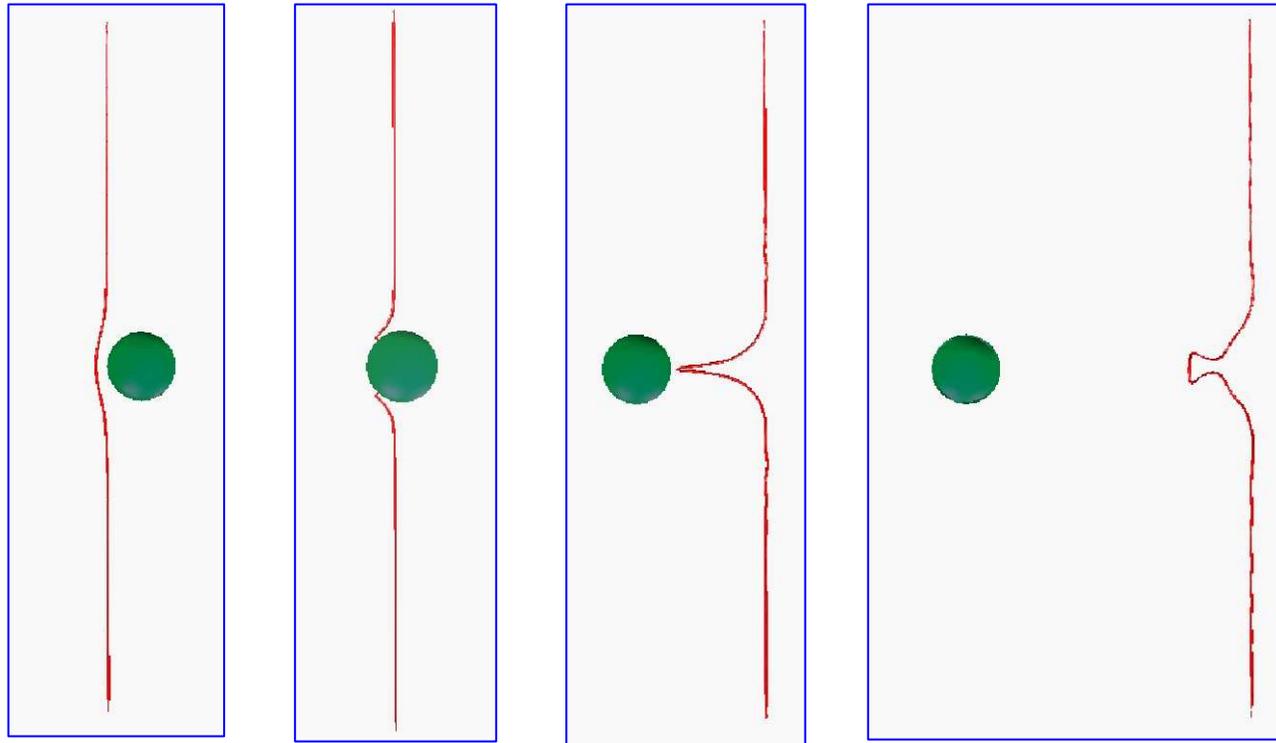
Parks, P.E. and Donnelly, R.J. (1966),
Phys. Rev. Lett. **16**, 45–48.

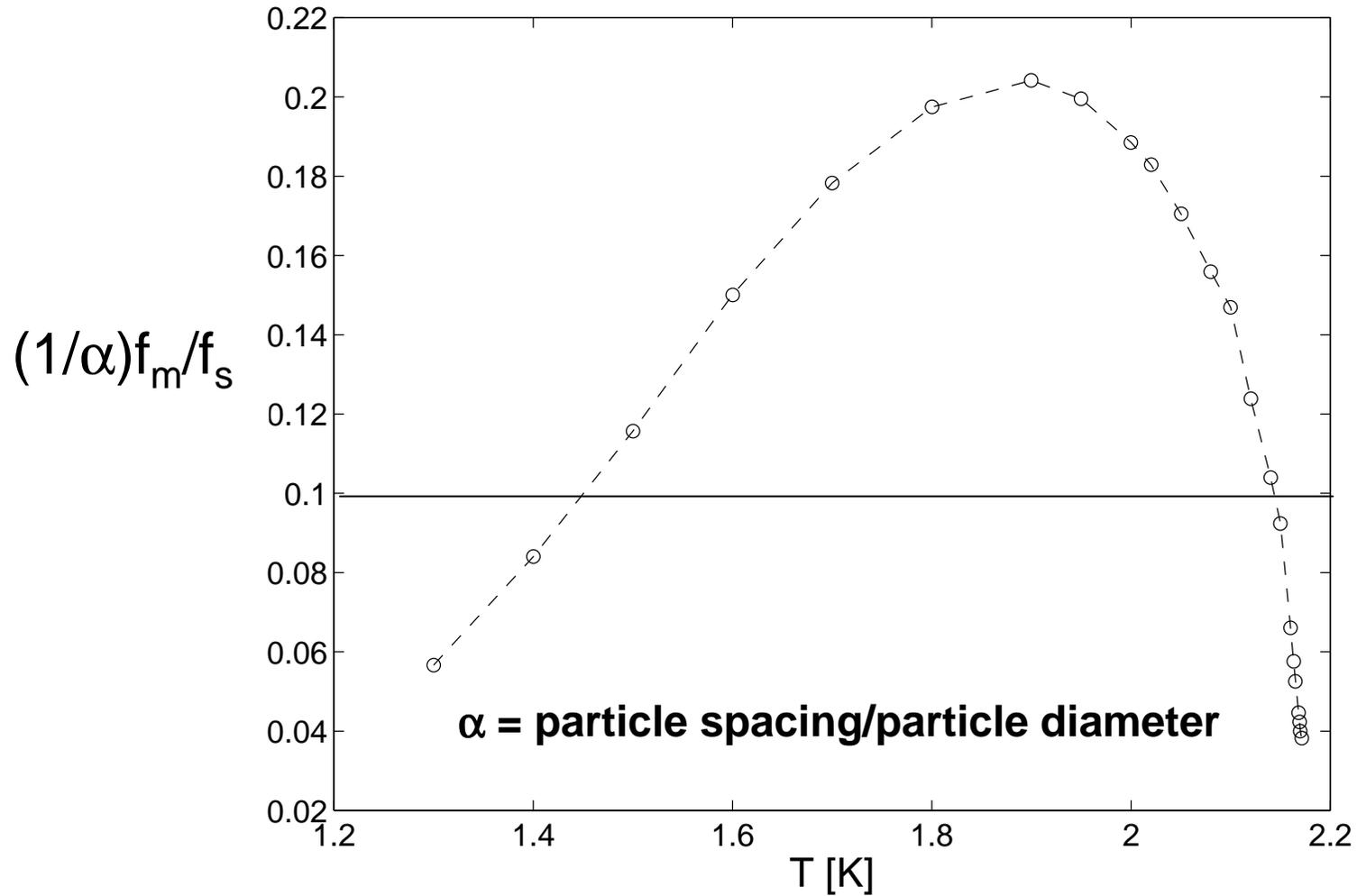
sphere is trapped by vortex



For a discussion of interaction between the fluid and particles in He II, see Sergeev, Barenghi & Kivotides, *Phys. Rev. B* **74**,184506 (2006); the simulations shown are by these authors.

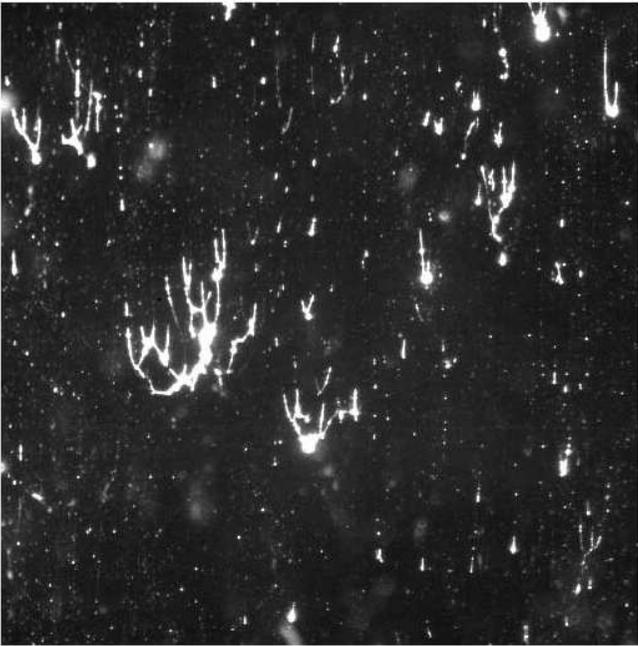
sphere escapes vortex



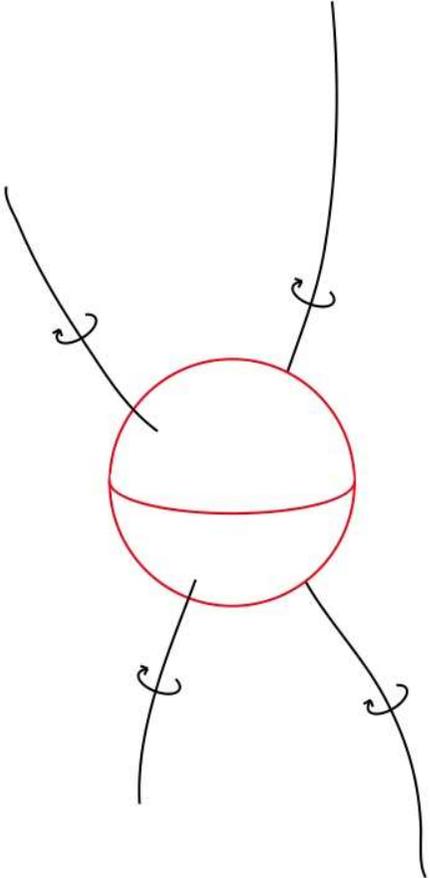
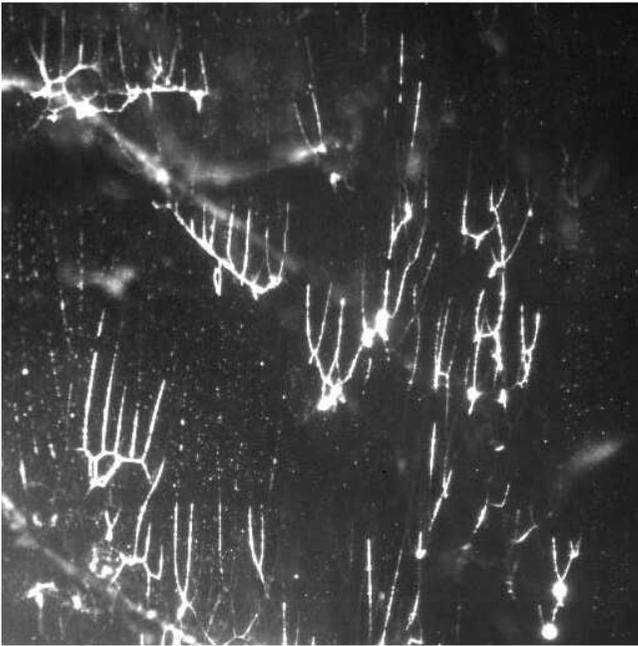


The ratio of the mutual friction force per unit length of a vortex to the drag on a particle trapped on the line. At about 2.17 K, the particle drag is equal to mutual friction if neighboring particles are about ten diameters apart.

Particles are not always passive tracers!

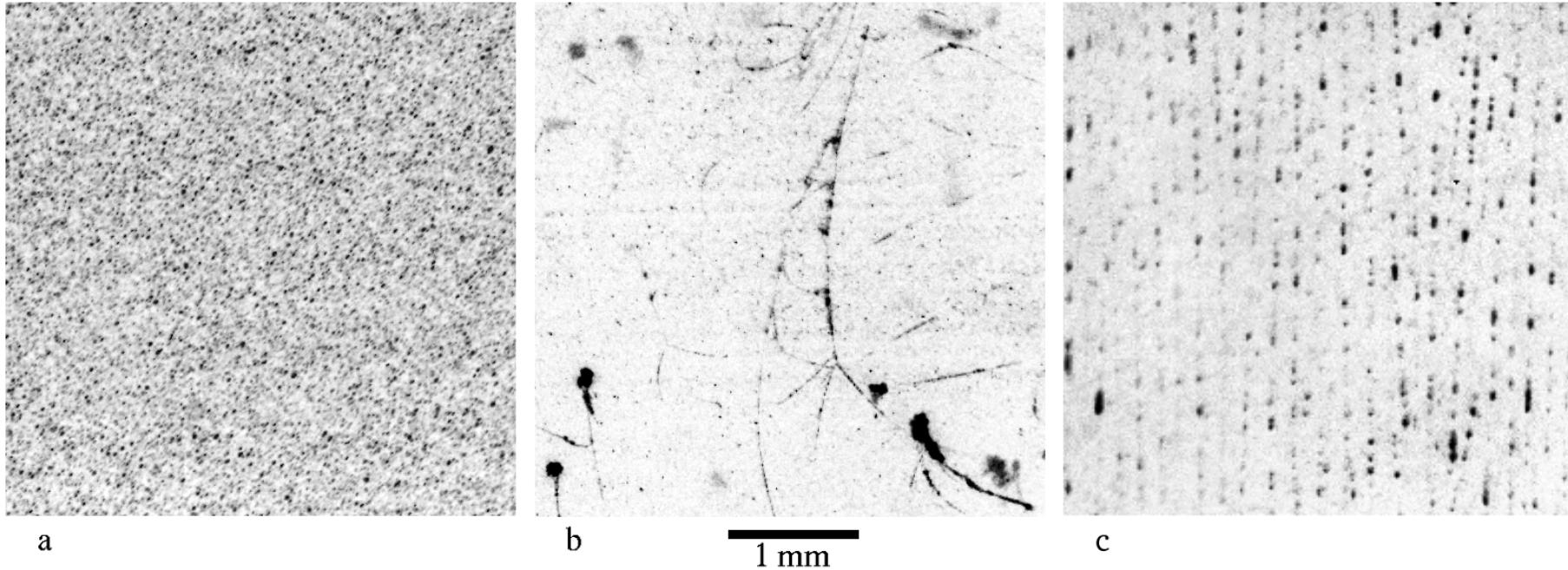


1 mm



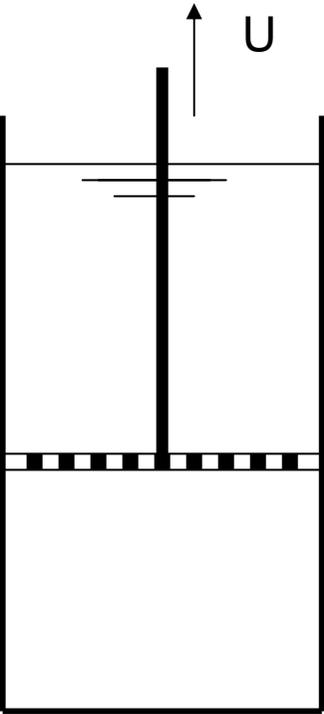
Summary

1. Using neutral particles of hydrogen-helium mixture, it has been possible to visualize superfluid vortex lines (and rings), and study their properties such as reconnection and decay.
2. These particles are not always passive so there is scope for further work. Interesting problems of particle-vortex interactions need to be studied further.
3. The superfluid turbulence appears to have the same spectral density in the inertial range as classical turbulence, posing interesting questions on the role of vortex stretching, dissipation mechanisms, etc.

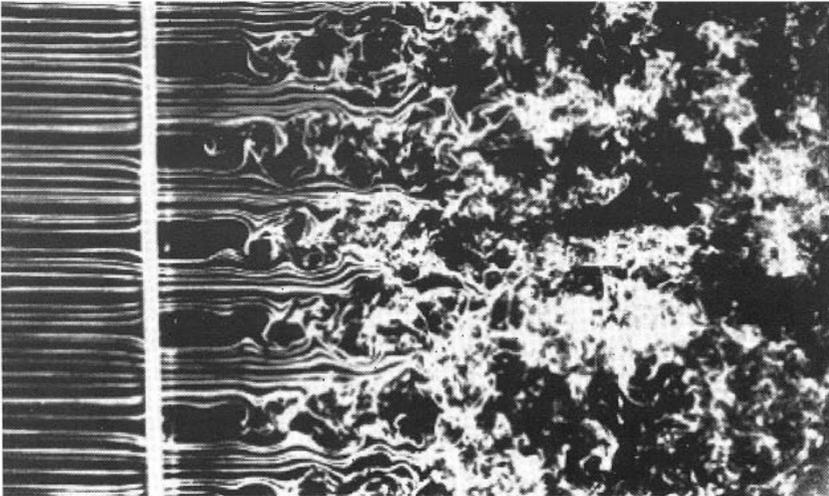


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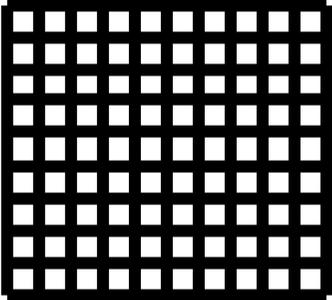
Turbulence behind grids



tank of water



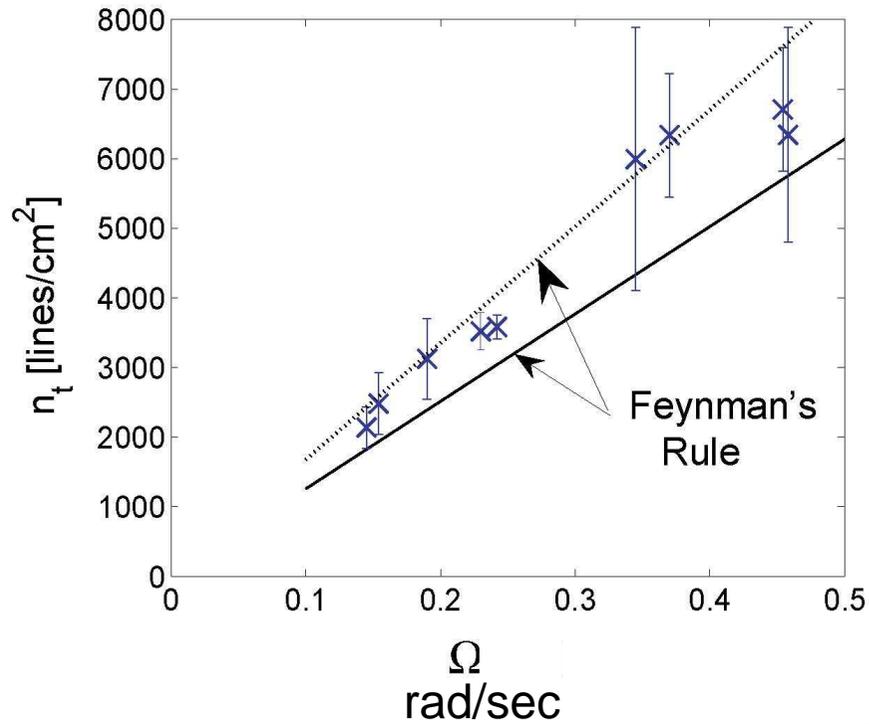
grid turbulence in air: Corke & Nagib



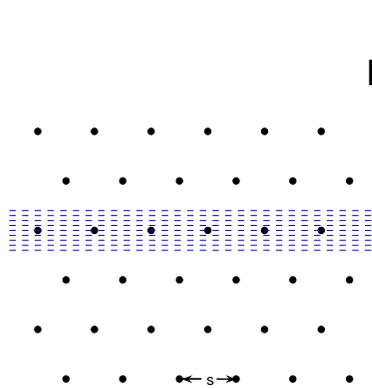
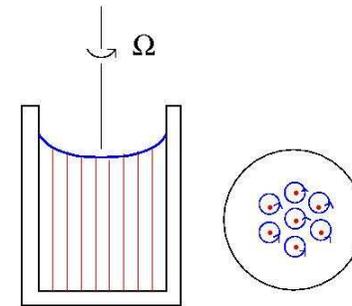
square grid of bars

nearly isotropic
turbulence is
generated.

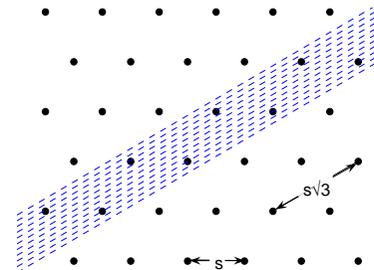
Lattice density

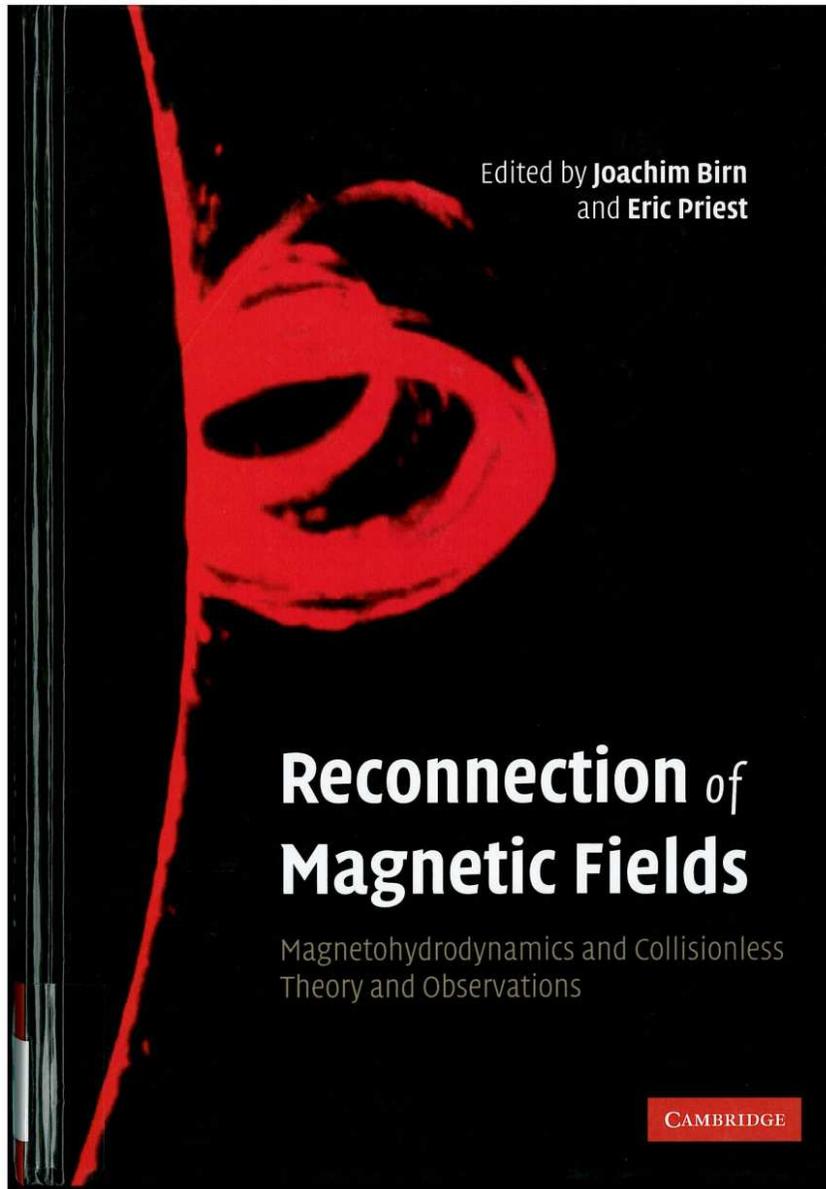


Feynman's rule
 $n_t \cong 2000\Omega$



$n_o \approx 2000\Omega$ lines/cm²





Some 850 references

Shelley, Meiron & Orszag *JFM* 246, 613-652 (1993)

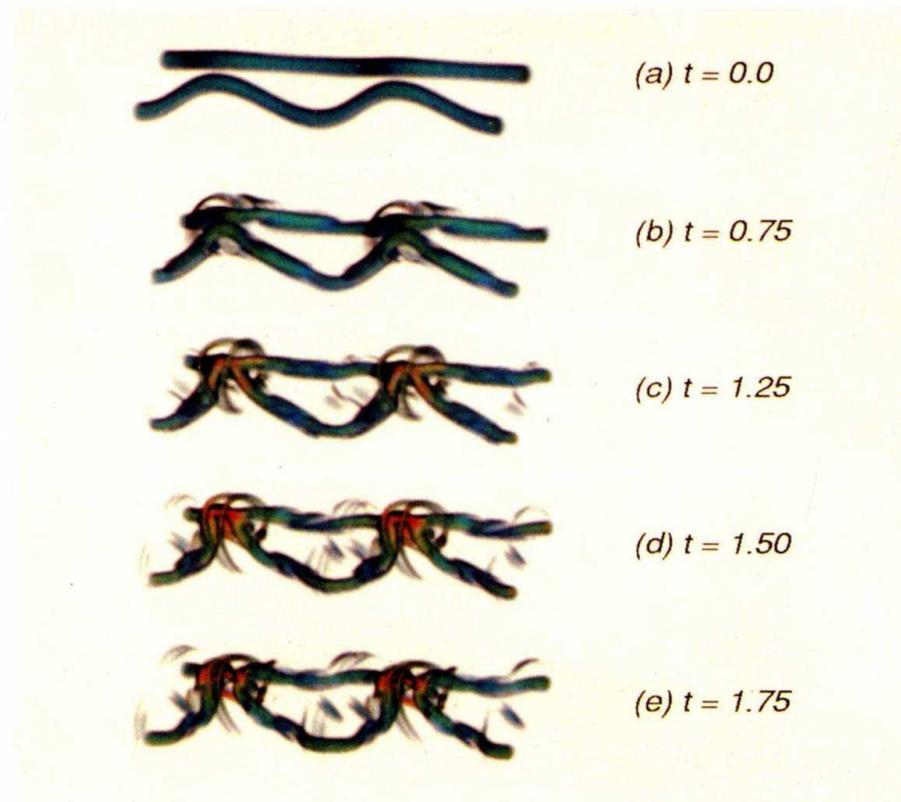


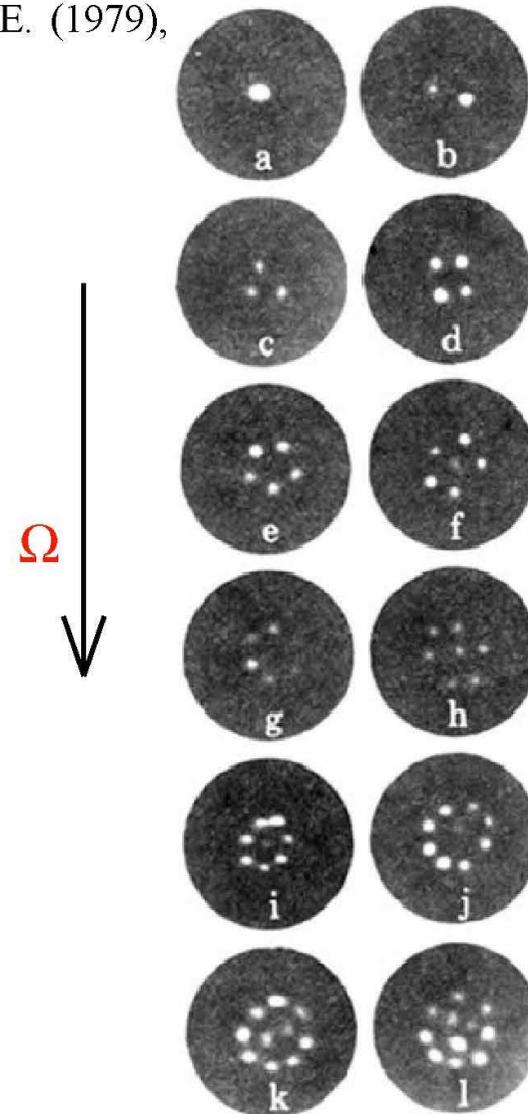
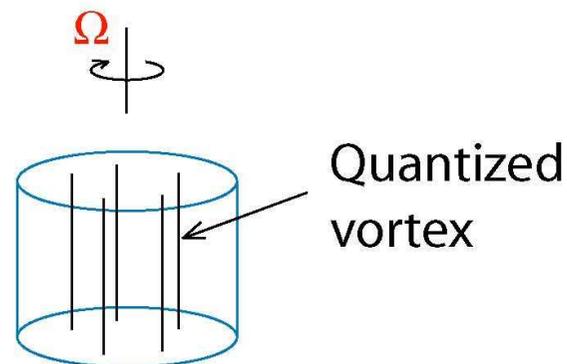
FIGURE 12. Volume rendering of vorticity magnitude for $Re=3500$ at (a) $t=0$, (b) 0.75, (c) 1.25, (d) 1.50, (e) 1.75. Two periods of the vortex tubes are shown here.

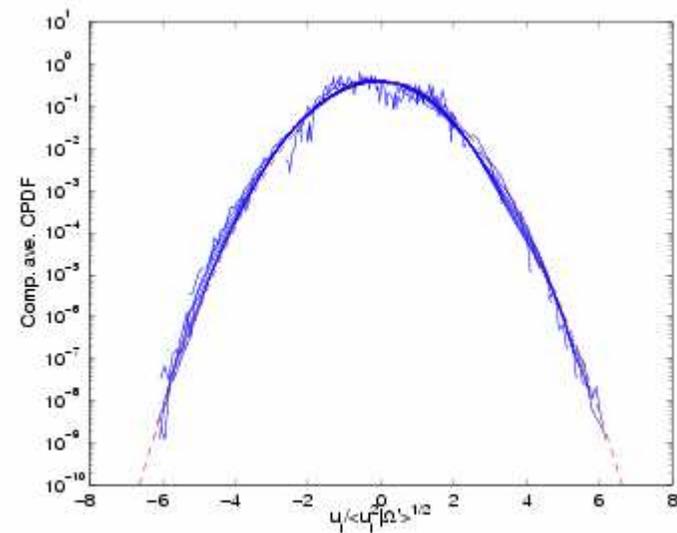
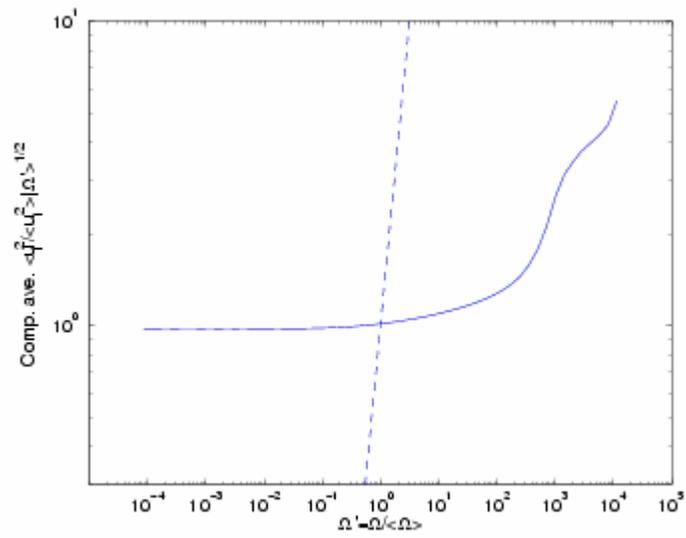
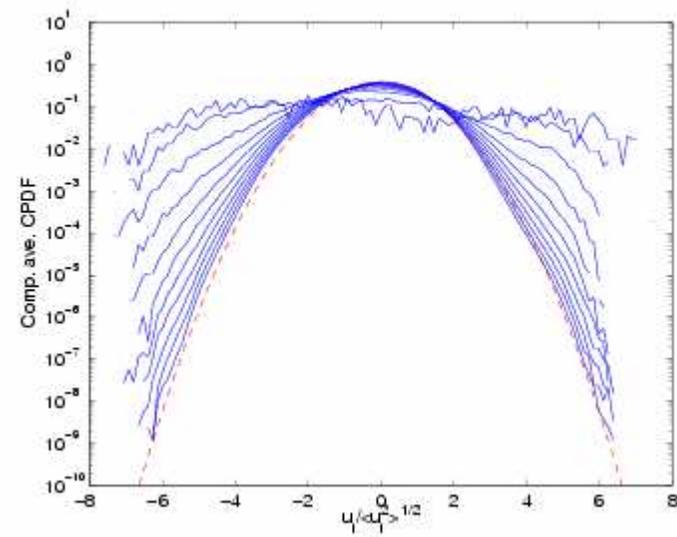
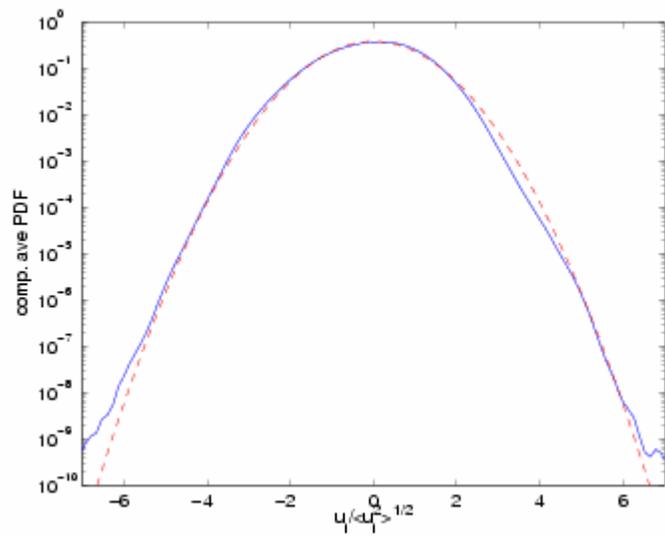
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Yarmchuk, E.J., Gordon, M.J.V. and Packard, R.E. (1979),
Phys. Rev. Lett. **43**, 214-217.

**technique not suitable for
visualizing tangled vortices**

indirectly inferred by Hall & Vinen, *Proc. Roy.
Soc.* **A238**, 204 (1956)

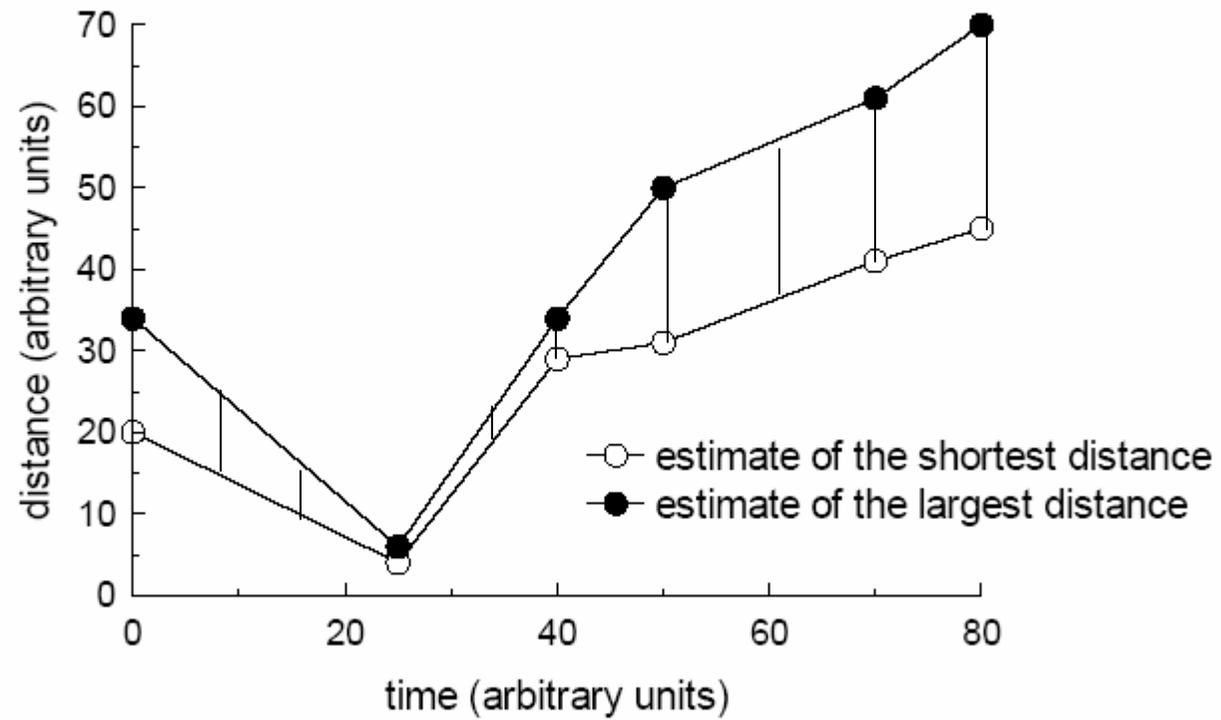


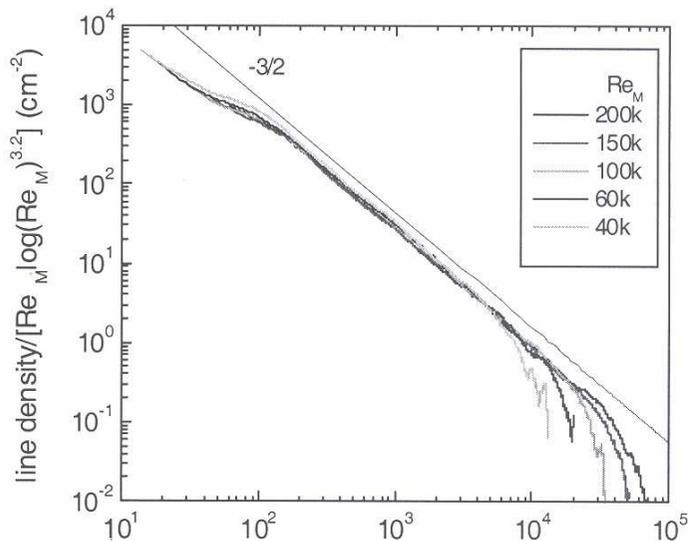
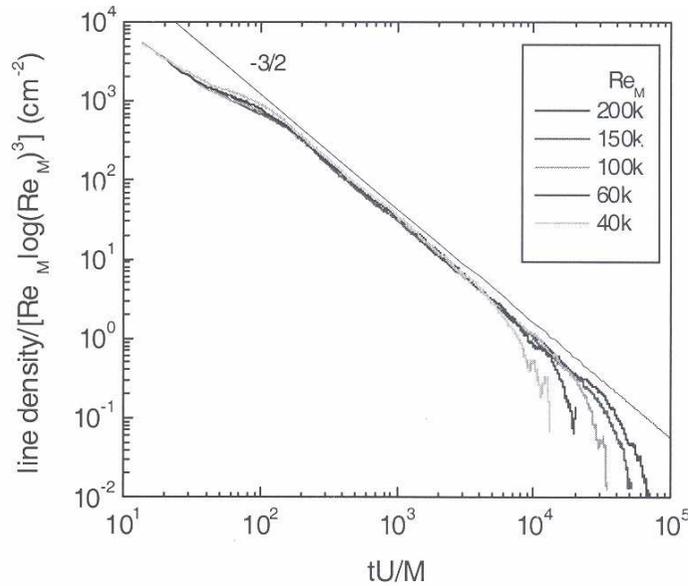


Velocity PDFs conditioned on strong vorticity

Vortex ring collision

(data from Koplik and Levine, PRL 76, 4745, 1996)





Because superfluid vorticity decays as $t^{-3/2}$, just as does classical vorticity, and the observed prefactors are as expected, the notion arises that the two turbulence fields are coupled in a range of scales. This is the hypothesis of coupled vorticity (Barenghi, Donnelly, Niemela, KRS, Vinen, Volovik, etc)

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