Advances in Large-eddy simulations to explore turbulence

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- **I- Formalism of incompressible large-eddy simulations (LES).**
- **II- Isotropic turbulence decay.**
- **III- Channel (with spanwise rotation).**
- **IV- Mixing in incompressible coaxial jets.**
- **V- Subsonic and supersonic jets.**

M. Lesieur, O, Métais & P. Comte, 2005, “Large-Eddy Simulations of Turbulence", Cambridge University Press.

M. Lesieur, 2008, “Turbulence in Fluids", Springer (4 th edition).

LIMITS OF DIRECT NUMERICAL SIMULATIONS (DNS)

• One considers motion equations for a monophasic Newtonian fluid (Navier-Stokes and energy equations). We assume existence and uniqueness of solutions.

• Dissipative scale $l_D = k_D^{-1}$, such that smaller wavelenghts are damped by molecular viscosity (Kolmogorov scale away from walls).

• Direct-numerical simulation : it is a *deterministic* solution of Navier-Stokes and related equations. Thanks to an apropriate projection on a spatio-temporal grid of the various partial-differential operators, one advances with time starting from a given initial state, with prescribed spatial boundary conditions. The typical grid mesh Δx must be smaller than l_D.

• Numerical schemes should be precise enough (high order if possible). Depends on the domain complexity.

• In developed turbulence, and if L characterizes large scales, the number of spatial grid points necessary for a well-resolved DNS is $\approx (L/l_D)^3$. One finds $\approx 10^{15}$ points for a commercial-plane wing (DNS possible in $30 \approx 50$ years), 10^{18} points for the atmospheric boundary layer, more for a fast-breader reactor core.

• LES allow to reduce drastically the number of collocation points.

INCOMPRESSIBLE LES : PHYSICAL SPACE

- $\rho = \rho_0$, Δx fixed length characterizing the spatial grid mesh ($l_D < \Delta x < L$). $G_{\Delta x}(\overrightarrow{x})$ is a low-pass spatial filter of width Δx, chosen in order to eliminate *subgrid scales* of wave length $< \Delta x$. We pose

$$\bar{f}(\overrightarrow{x}, t) = f * G_{\Delta x} = \int f(\overrightarrow{y}, t) G_{\Delta x}(\overrightarrow{x} - \overrightarrow{y}) d\overrightarrow{y} \ .$$

The filter commutes with spatial and temporal partial derivatives (if the grid is uniform).

- Navier-Stokes equations (NS) :

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j}(u_i u_j) = -\frac{1}{\rho_0} \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}(2\nu S_{ij}) \quad \text{with} \quad S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

- filtered NS :

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j}(\bar{u}_i \bar{u}_j) = -\frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}(2\nu \bar{S}_{ij} + \bar{u}_i \bar{u}_j - \overline{u_i u_j})$$

- $T_{ij} = \bar{u}_i \bar{u}_j - \overline{u_i u_j}$, *subgrid-scale tensor.*
- Eddy-viscosity assumption : $T_{ij} = 2\nu_t(\overrightarrow{x}, t)\ \bar{S}_{ij} + (1/3) T_{ll}\ \delta_{ij}$.

- NS/LES equations

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j}(\bar{u}_i \bar{u}_j) = -\frac{1}{\rho_0}\,\frac{\partial \bar{P}}{\partial x_i} + \frac{\partial}{\partial x_j}[2(\nu + \nu_t)\bar{S}_{ij}]\ ,\ \ \nu_t = (\Delta x) V_{\Delta x}\ .$$

- Modified pressure ("macro-pressure") : $\bar{P} = \bar{p} - (1/3)\rho_0 T_{ll}$.
- Continuity : $\partial \bar{u}_j / \partial x_j = 0$.
- Mixing of a scalar satisfying a Lagrangian heat Fourier equation

$$\frac{\partial T}{dt} + \frac{\partial}{\partial x_j}(T\ u_j) = \frac{\partial}{\partial x_j}\left\{\kappa \frac{\partial T}{\partial x_j}\right\}$$

(molecular diffusivity κ). Problem crucial in combustion modelling and in geophysical turbulence. Eddy-diffusivity assumption κ_t , determined thanks to a turbulent Prandtl (resp. Schmidt) number ν_t/κ_t .

$$\frac{\partial \bar{T}}{dt} + \frac{\partial}{\partial x_j}(\bar{T}\ \bar{u}_j) = \frac{\partial}{\partial x_j}\left\{(\kappa + \kappa_t)\frac{\partial \bar{T}}{\partial x_j}\right\}\ .$$

- *Smagorinsky's model* (*Mon. Weath. Rev.*, 1963) : $V_{\Delta x} \sim \Delta x \sqrt{\bar{S}_{ij}\bar{S}_{ij}}$. Improvements by Germano, Piomelli, Moin & Cabot, (*Phys. Fluids*, 1991), with a local dynamic evaluation of the constant by double filtering.

INCOMPRESSIBLE LES : FOURIER SPACE

• We assume turbulence in an infinite domain, statistically homogeneous. Spatial Fourier transform (FT)

$$\hat{f}(\overrightarrow{k},t) = \left(\frac{1}{2\pi}\right)^3 \int e^{-i\overrightarrow{k}.\overrightarrow{x}} \quad f(\overrightarrow{x},t)\, d\overrightarrow{x} \quad .$$

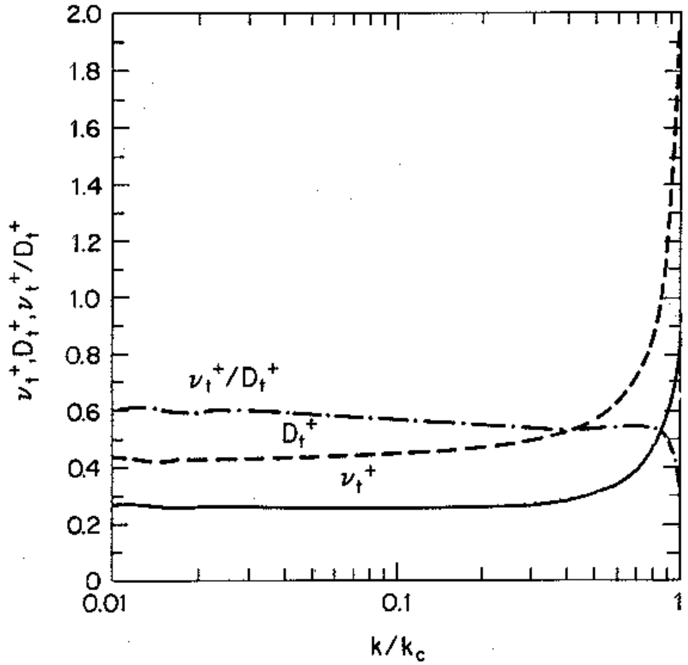
• The chosen filter is a sharp cutoff low-pass filter :

$$\overline{\hat{f}} = \hat{f} \;\text{ for }\; k = |\overrightarrow{k}| < k_C = \frac{\pi}{\Delta x} \;;\; \overline{\hat{f}} = 0 \;\text{ for }\; k > k_C \;.$$

• Kinetic-energy spectrum $E(k,t)$ (isotropic turbulence) : $E(k,t)\delta k$ is the mean kinetic energy per unit mass in a spatial frequency band $[k, k+\delta k]$ (*Mean*, in the sense of a statistical average on an ensemble of realizations $<>$).

• NS in Fourier space. Pressure is eliminated by projection in the incompressibility plane (plane perpendicular to $\overrightarrow{k}$) of the advection term, that is ik_j FT $\{u_i u_j\}$. We have FT $\{$dissipative term$\}$ $= -\nu k^2 \hat{u}_i(\overrightarrow{k},t)$.

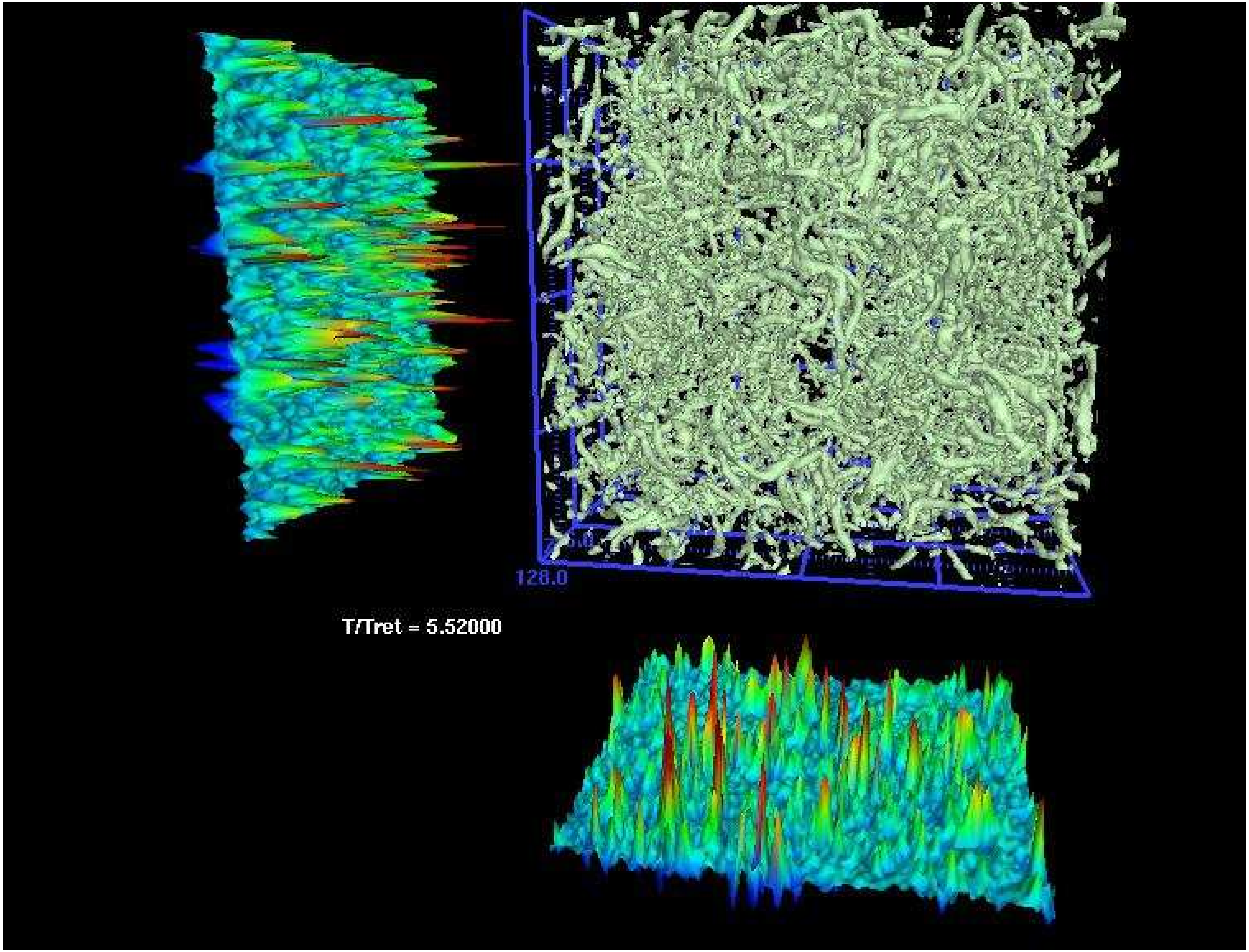
• Non-linear interactions involve *triads* such that $\overrightarrow{k} = \overrightarrow{p} + \overrightarrow{q}$. Sub-grid modelling turns out to evaluate transfers such that $k < k_C$, p or $q > k_C$.



• NS/LES in Fourier space : a spectral eddy viscosity $\nu_t(k|k_C)$ is added to ν. It is evaluated thanks to kinetic-energy transfers across k_C given by a turbulence two-point statistical closure (EDQNM, not to be confused with RANS one-point closure models) :

$$\nu_t(k|k_C) = 0.441\ {C_K}^{-3/2} \left[\frac{E(k_C)}{k_C}\right]^{1/2} X\left(\frac{k}{k_C}\right) \quad ,$$

assuming k_C belongs to an inertial Kolmogorov range $E(k) = C_K \epsilon^{2/3} k^{-5/3}$. *Plateau-peak model* of Chollet & Lesieur (*J. Atmos. Sci.*, 1981). $X(k/k_C) \approx 1$ for $k/k_C < 1/3$.


128.0
T/Tret = 5.52000

• *Spectral dynamic model* (Lamballais, Métais & Lesieur, *Theor. Comp. Fluid. Dyn.*, 1998) : accounts for a k^{-m} spectrum at the cutoff.

• Application to the decay of isotropic turbulence at zero molecular viscosity in a periodic box. Gaussian initial velocity field. Pseudo-spectral numerical methods, initial peak at $k_i = 4$. Formation and evolution of spaghetti-type vortices, visualized by iso-surfaces at a positive threshold of

$$Q = (1/2)(\Omega_{ij}\Omega_{ij} - S_{ij}S_{ij}) = \nabla^2 p/2\rho$$

(Q *criterion* of Hunt, Wray & Moin, 1988).

• It is possible that the existence of these vortices is responsible for turbulence multifractal character at small scales (Frisch, *Cambridge Univ. Press*, 1995).

KINETIC-ENERGY DECAY

"Academic" problem, interesting for isotropic turbulence theory (Batchelor,*Cambridge Univ. Press*, 1953), grid-turbulence experiments (Comte-Bellot & Corrsin, *J. Fluid Mech.*, 1966 ; Warhaft & Lumley, *J. Fluid Mech.*, 1978), validation of CFD codes, physics of liquid helium (Stalp, Skrbeck & Donnelly, *Phys. Rev. Let., 1999*), cosmology (decay of early universe ?).

$$\frac{1}{2} < \overrightarrow{u}^2 >= v^2 = \text{Cons } t^{-\alpha_E}.$$

• Batchelor predicts theoretically $\alpha_E = 1$.

• Lesieur & Schertzer's relation (*Journal de Mécanique*, 1978)

$$E(k,0) \propto k^s \quad , \quad T(k,t) \propto k^4, \quad E(k,t) = \text{Cons } t^{\gamma(s)} k^s \ \text{ for } \ k \to 0.$$

Assuming a self-similar decaying kinetic-energy spectrum

$$E(k,t) = v^2 l \ F(kl), \ l = \frac{v^3}{\epsilon}, \ \epsilon = -\frac{1}{2}\frac{dv^2}{dt}$$

where F is a dimensionless function, one finds

$$\alpha_E = 2\frac{s+1-\gamma(s)}{s+3} \quad .$$

$\gamma(s) = 0$ for $s < 4$. For $s = 4$, an E.D.Q.N.M. calculation gives $\gamma = 0.16$ and $\alpha_E = 1.38$.

• Comte-Bellot & Corrsin (1966) perform an experiment of grid turbulence and measure $\alpha_E = 1.26$ (Warhaft-Lumley, $\alpha_E = 1.34$). Theoretically, Geneviève assumes a time-invariant k^4 infrared spectrum, a $k^{-5/3}$ decaying ultraviolet spectrum and finds $\alpha_E = 10/7 = 1.43$.

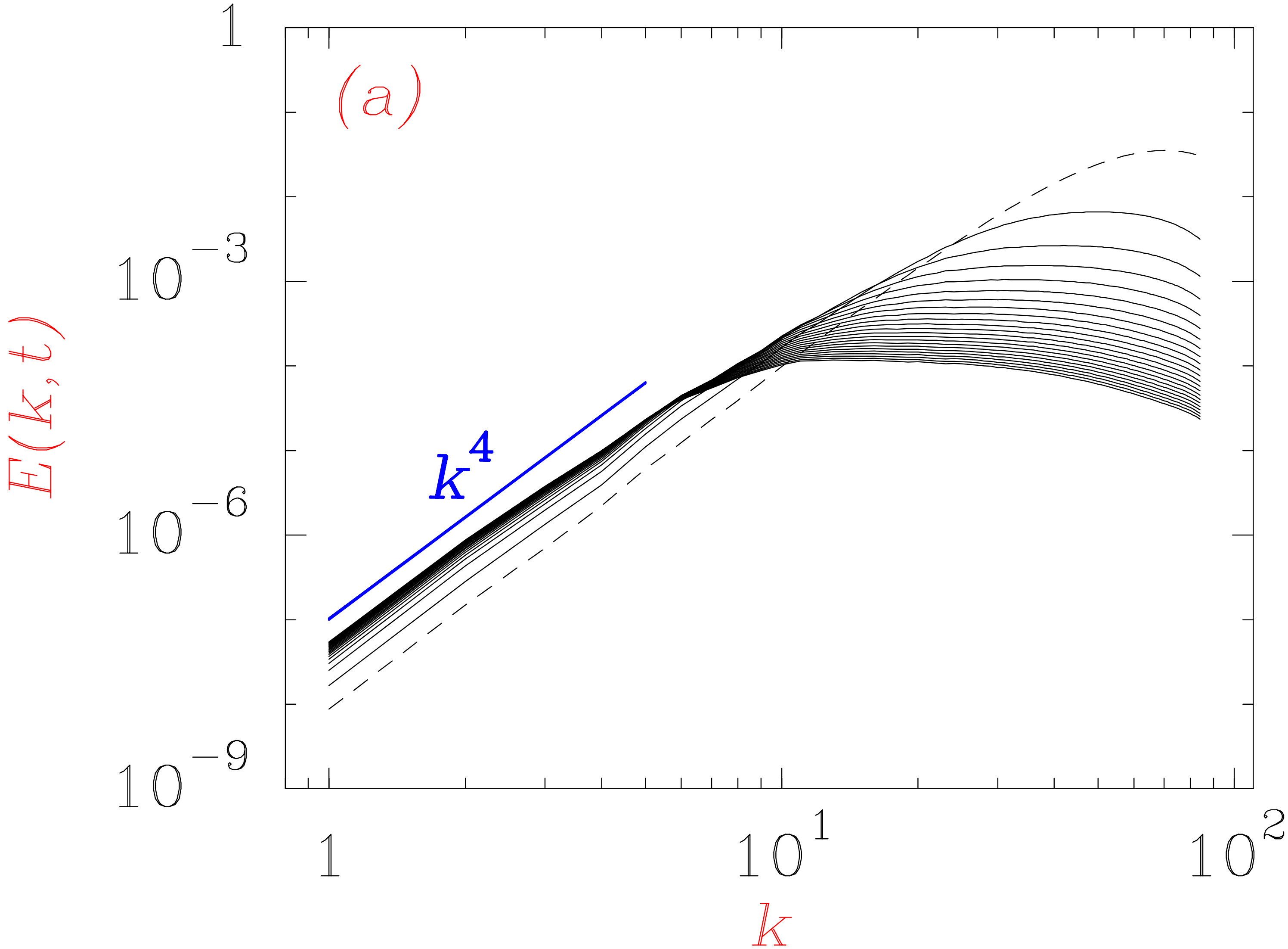
• Same result as Kolmogorov (1941-b) assuming the time invariance of Loitzianskii integral $I(t) = \int r^2 U_{ii}(r,t) d\overrightarrow{r}$, whith $U_{ij}(r,t) =< u_i(\overrightarrow{x},t) u_j(\overrightarrow{x}+\overrightarrow{r},t) >$, which is proportional to $t^{\gamma(4)}$.

• Landau & Lifchitz (*Fluid Mechanics*) show this invariance assuming angular-momentum conservation in the flow. Problems due to viscous-dissipation and boundary-conditions effects (they assume the flow in a compact domain with zero velocity at the wall).

• LES of Ossia & Lesieur (*Journal of Turbulence*, 2000) confirms the permanence of large eddies with $s = 2$, yielding $\alpha_E \approx 1.2 = 6/5$ (Saffman, 1967). With $s = 4$ an inviscid LES with 256^3 collocation points using pseudo-spectral methods gives asymptotically $\alpha_E = 1.40$. Loitzianskii integral still increases slightly at the end of the computation.

• Very high resolution DNS (1024^3 points, same pseudo-spectral methods) by Ishida, Davidson & Kaneda (*J. Fluid Mech.*, 2006) with a k^4 infrared spectrum shows eventually the saturation of $I(t)$ to a constant, with $\alpha_E = -10/7$. May be too influenced by high molecular viscosity.

• Eyink & Thompson (*Phys. Fluids*, 2000) consider non-integer values of s, following Burgers equation study of Gurbatov, Frisch et al. *J. Fluid Mech.*, 1997).



(a)
1
10^-3
10^-6
10^-9
E(k,t)
k^4
1
10^1
10^2
k

Weakly-compressible non-rotating channel with 2 spanwise grooves

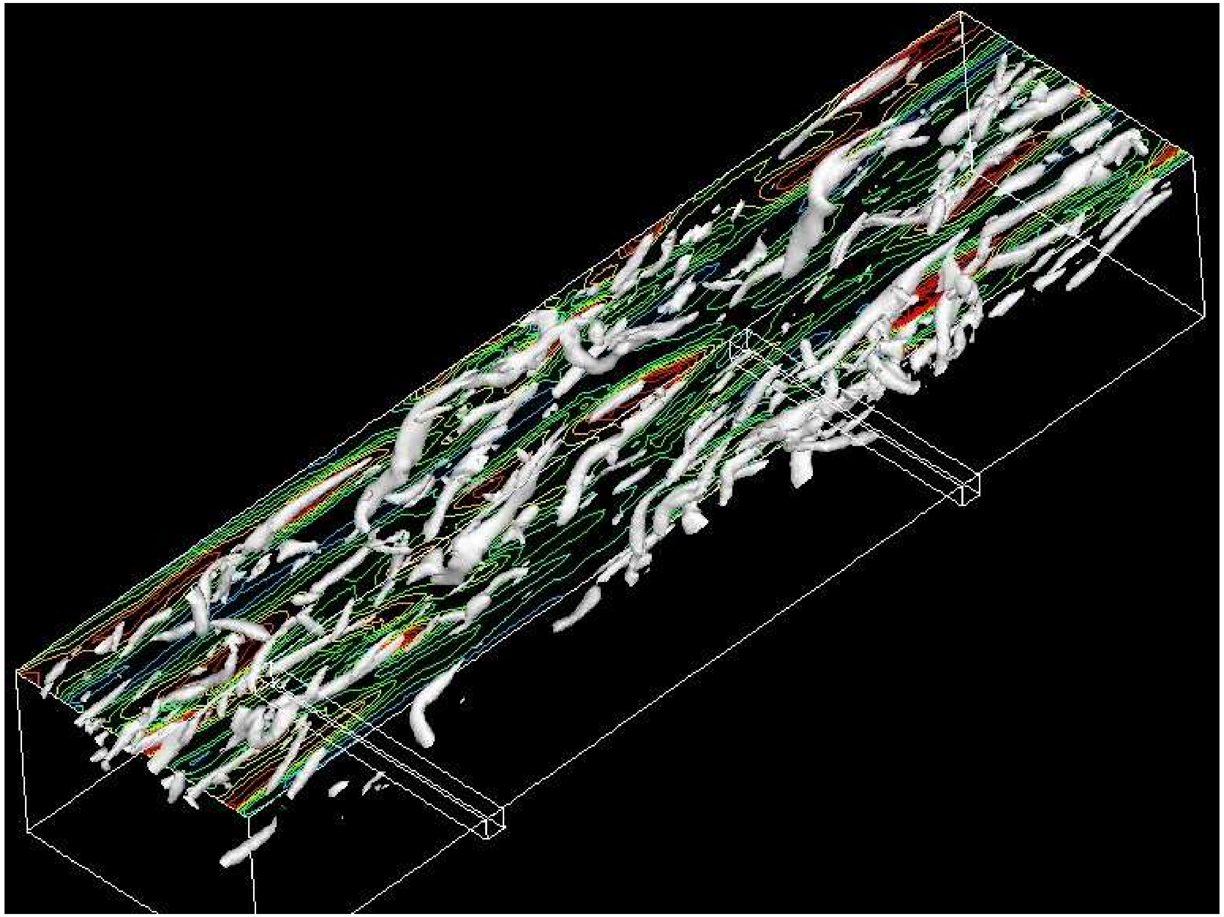
• Periodic channel with 2 small spanwise square cavities on one wall (Dubief & Delcayre *J. Turbulence*, 2000). Reynolds number $h^+ = h/\delta_v = 160$ in wall units, with $\delta_v = \nu/v_*$. One recalls

$$\rho v_*^2 = \mu \; d\bar{u}/dy$$

at the wall.

• Weakly-compressible flow (Mach 0.3, based on bulk velocity and sound speed at the wall). Numerical code of Grenoble MOST team COMPRESS (4th order MacCormack method, Normand & Lesieur, *Theor. Comp. Fluid. Dyn.*, 1992). Résolution $200 \times 128 \times 64$. Propagation of longitudinal hairpin vortices and associated low-and high-speed streaks (see movie). LES is 3 times faster than DNS.

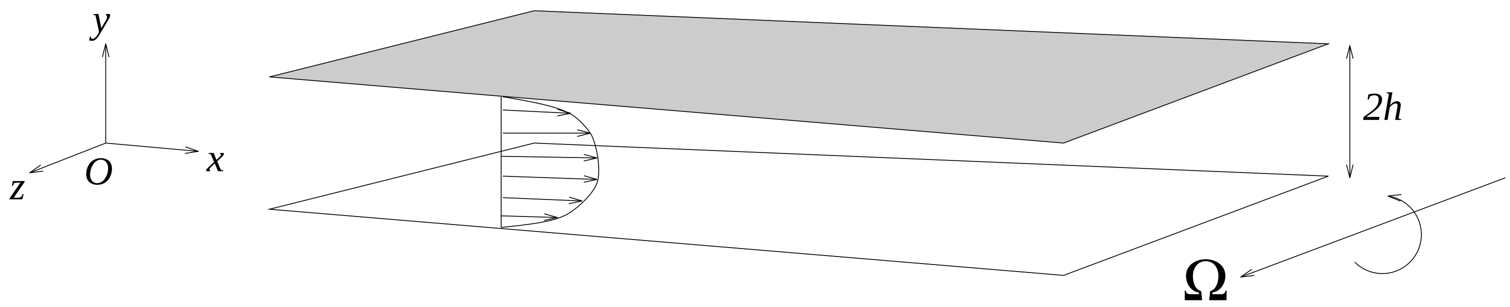
• Pioneering experimental work on the incompressible channel carried out by Geneviève in Grenoble during the sixties. Measured the longitudinal velocity fluctuation u' skewness factor, with a positive value at the wall. Associated to a predominance of high-speed streaks upon the low-speed ones. Geneviève measured also Favre space-time correlations in the channel,



Non-rotating incompressible plane channel

Lamballais' LES (1998) at $h^+ = 395$ agree very well statistically (first and second-order velocity moments) with Kim's DNS (1992). First grid point at $y^+ = 1$ with a stretched grid. LES is 70 times faster than DNS.

Plane channel rotating about a spanwise axis



- With rotation (Coriolis acceleration in Navier-Stokes, centrifugal accélération in the pressure gradient), problem important in industry (turbomachinery in hydraulics and aerospace engineering) and environment (meteorology, oceanography, internal geophysics).

$$\text{Local Rossby number} \quad R_o(y,t) = -\frac{1}{2\Omega}\frac{d\bar{u}(y,t)}{dy} \quad , \ \text{initial Rossby } R_o(0,0) = -\frac{3}{R_{ot}}$$

- The flow evolves to create a range of local Rossby equal to -1 in the anticyclo-

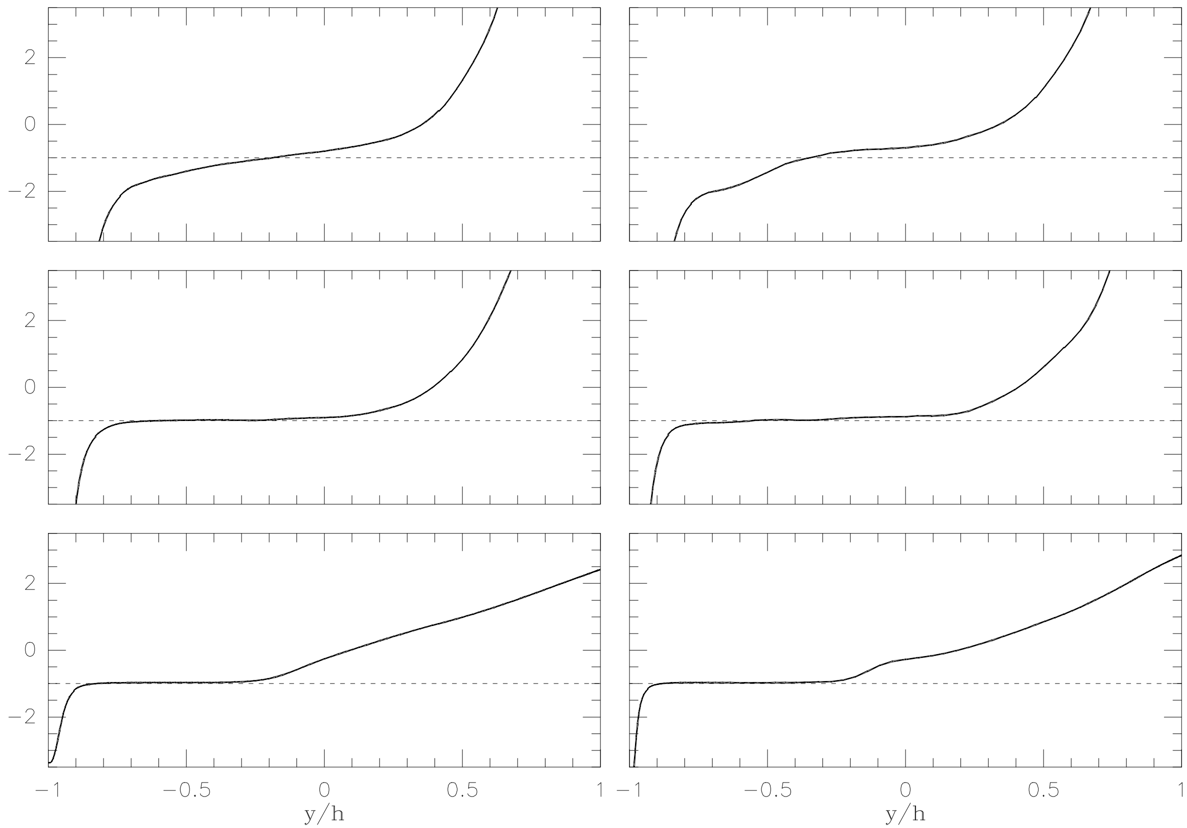


Fig. 1 – Local Rossby number distribution in DNS (left) and LES (right). From top to bottom, minimal Rossby : -18, -6 and -2.

nic range (pressure side), which replaces and extends the logarithmic range. Found experimentally by Johnston et al. (*J. Fluid Mech.*, 1972).

• Corresponds to a longitudinal alignment of the *absolute vorticity* $\vec{\omega} + 2\vec{\Omega}$.

• Holds also for anticyclonic mixing layers. Here, primary KH vortices are replaced by strong longitudinal vortices (analogy with centrifugal instabilities). See 3D instability studies and numerical simulations (DNS and LES) of Flores, Métais, Riley & Yanase (*Phys. Fluids*, 1993, *J. Fluid Mech.*, 1995).

LES MODELS IN R^3

• For industrial applications in more complex geometries, spectral LES have been adapted approximately in physical space : eddy viscosity computed thanks to a local kinetic-energy spectrum, determined with the aid of the second-order velocity structure function (*structure-function model*, Métais & Lesieur, *J. Fluid Mech.*, 1992).

$$V_{\Delta x} = 0.105\ C_K^{-3/2}\ \langle[\overline{\vec{u}}(\vec{x},t) - \overline{\vec{u}}(\vec{x}+\vec{r},t)]^2\rangle_{\|\vec{r}\|=\Delta x}^{1/2}$$

local statistical average between $\vec{x}$ and 6 (or 4) closest points. Extension to irregular grids by extrapolation (Kolmogorov 2/3 law). But bad behaviour at the walls as Smagorinsky...

• For shear flows, two excellent models which eliminate the eddy-viscous damping

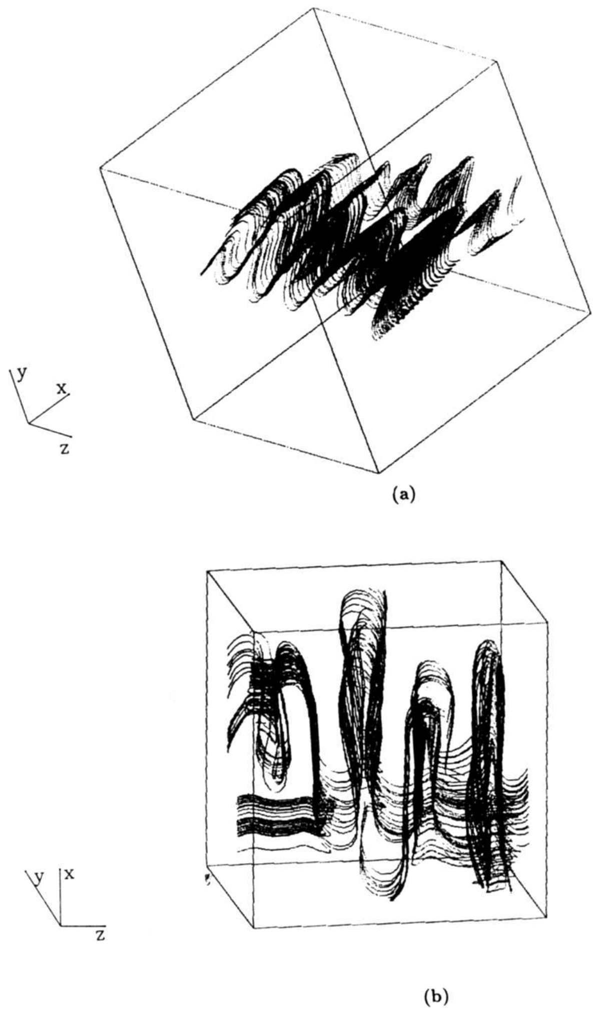


FIG. 2 – Absolute vortex filaments in DNS of a rotating mixing layer ($R_o^{(i)} = -5$), courtesy O. Métais).

due to large-scale velocity gradients : *selective structure-function model* and *filtered structure-function model* (cf Lesieur & Métais, *Ann. Rev. Fluid Mech.*, 1996). Models incorporated in CEA and ALSTOM codes for the development of nuclear reactors, with velocity-temperature coupling. They are also in FLUENT.

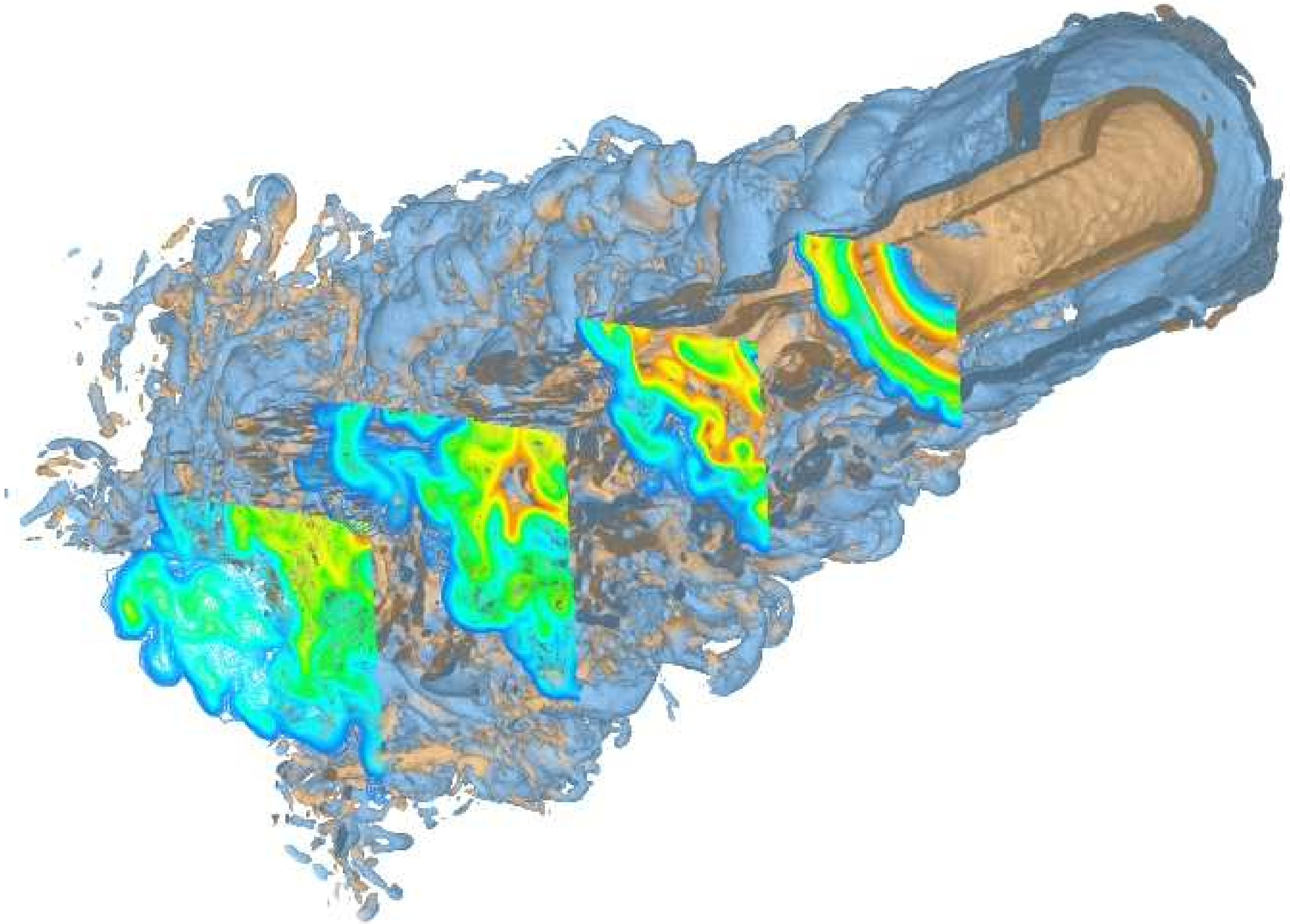
MIXING IN INCOMPRESSIBLE COAXIAL JETS

Flows very important for aircraft turbojets to reduce the emitted noise (with low outside velocity). Exist in rocket engines (with high outside velocity), under two phases. LES study of the latter case in monophasic incompressible conditions with passive-scalar transport (Balarac, 2006). Basic upstream velocity (Michalke & Herman *J. Fluid Mech.*, 1982)

$$\bar{u}(r) = \frac{U_1 + U_2}{2} + \frac{U_2 - U_1}{2} \tanh\left(\frac{r - R_1}{2\Theta_{01}}\right) \quad \text{for} \quad \mathrm{r} \leq \mathrm{R_m}$$

$$\bar{u}(r) = \frac{U_1 + U_2}{2} + \frac{U_2 - U_1}{2} \tanh\left(\frac{r - R_2}{2\Theta_{02}}\right) \quad \text{for} \quad \mathrm{r} \geq \mathrm{R_m} \quad .$$

perturbed by a weak white noise. LES (filtered structure-function model) at Reynolds number 30000 and 50 million collocation points. Vorticity norm coloured by tangential vorticity and four cross sections of passive-scalar distribution.



LES OF COMPRESSIBLE ROUND JET (IDEAL GAS)

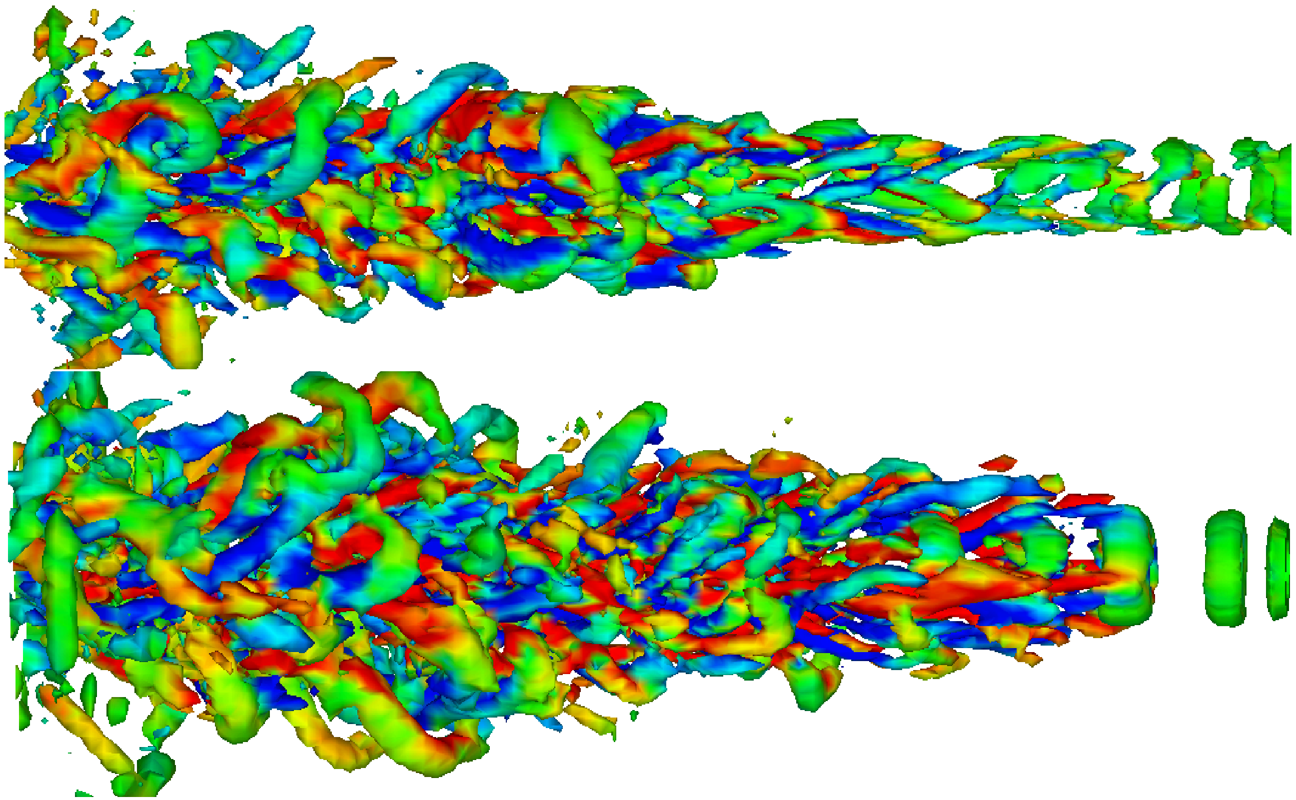
• NS equations under a flux form (mass, linear momentum, total energy) are still filtered with the "bar-filter". The problem is much simplified by introducing Favre filters weighted by density, and analogous to Favre averaging in turbulence (Favre, *C.R.Acad.Sci.*, 1958)

$$\overline{\rho f} = \bar{\rho} \tilde{f} \quad .$$

We introduce also a "macro-température", which turns out to be related to the macro-pressure through the ideal-gases law. Eddy coefficients are not changed.

• "Free" round jet (Reynolds 36000, Mach 0.7 and 1.4). Filtered structure-function model. Numerical code COMPRESS (with characteristics methods+ sponge zone). Q coloré par ω_x. The supersonic jet is much more focused than the subsonic (Maidi & Lesieur, *J. Turbulence*, 2005), with an increased potential-core length (book by Gatski & Bonnet, *Elsevier*, 2009).

• Forced jet (Maidi et al. *J. Turbulence*, 2006).



CONCLUSIONS

• LES have to be assessed by comparison with good laboratory experiments (such as those done by Geneviève and collaborators) and real DNS. They are exceptional tools to study vortex dynamics and statistics. They have revealed part of turbulence misteries.

• Compared with DNS, LES are faster by a factor of 3 (low Reynolds) to 70 (high Reynolds).

• LES models parameters do not need any adjustment to forcings or external actions such as separation, ensemble rotation, heating, compressibility.

• LES give deterministic informations on kinematic and thermal fluctuations, essential for systems safety.

• The coupling of LES and unstationary RANS methods (which are in fact weakly-resolved LES) gives rise to decisive advances for the simulation of more complex industial systems.

• The huge increase of computer ressources might eventually suppress the need for RANS methods.