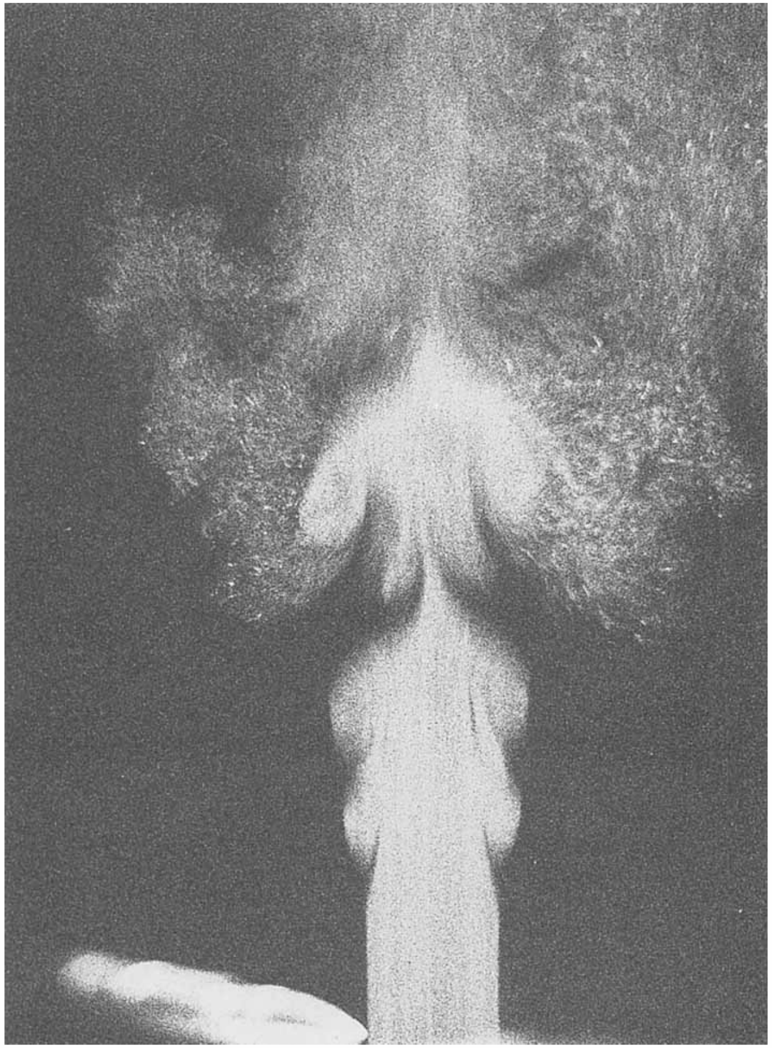
SELF-SUSTAINED OSCILLATIONS AND SOUND IN HOT JETS

Lutz LESSHAFFT, Patrick HUERRE
LadHyX
CNRS-École Polytechnique

Pierre SAGAUT
Institut d'Alembert
Université Pierre & Marie Curie

Colloquium in honor of Geneviève Comte-Bellot
Ecole Centrale de Lyon, October 29-30, 2009

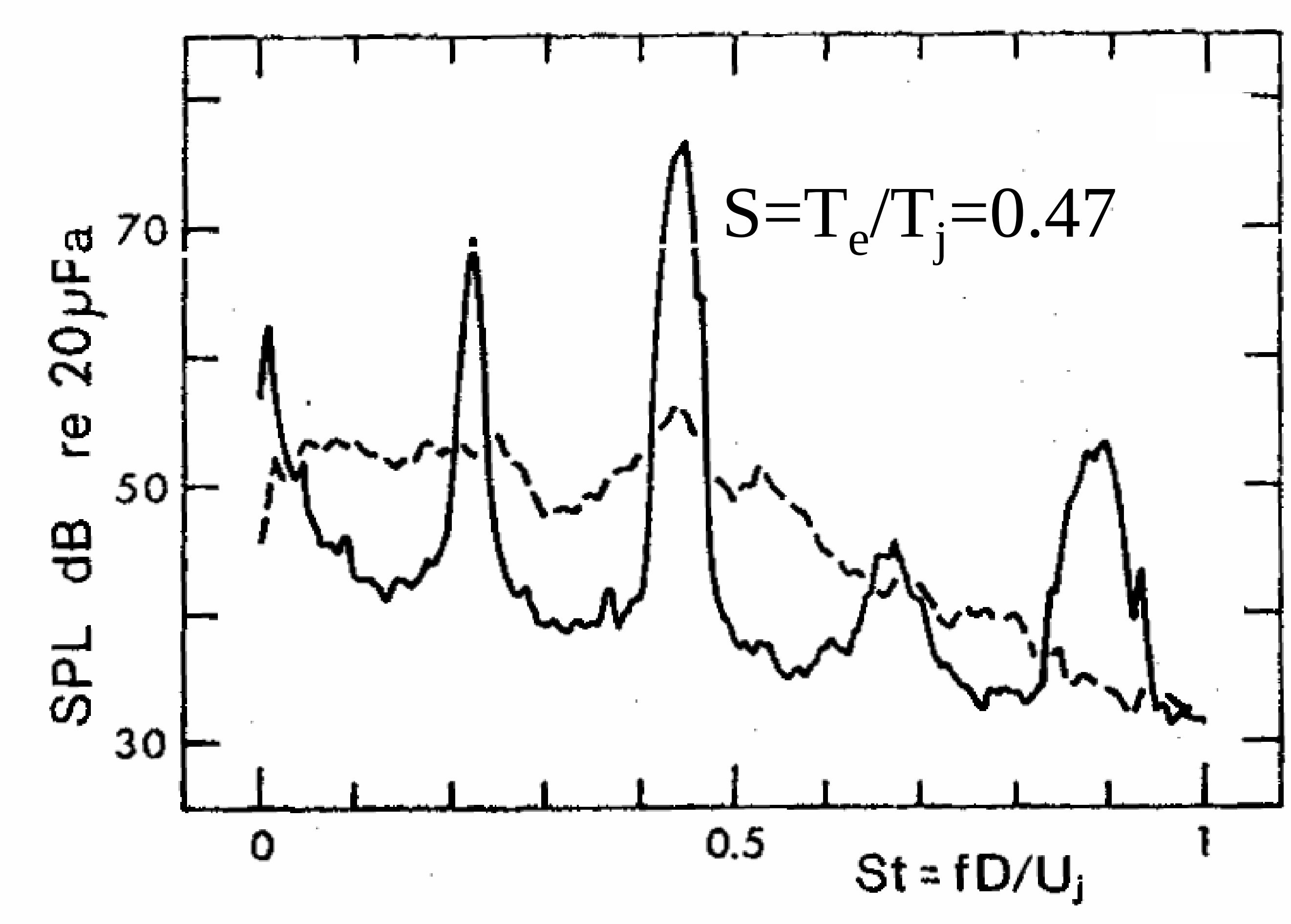
SELF-EXCITED OSCILLATIONS IN HOT JETS:
Mode II



Monkewitz & Sohn (1988) Monkewitz et al. (1990)

SELF-EXCITED OSCILLATIONS IN HOT JETS

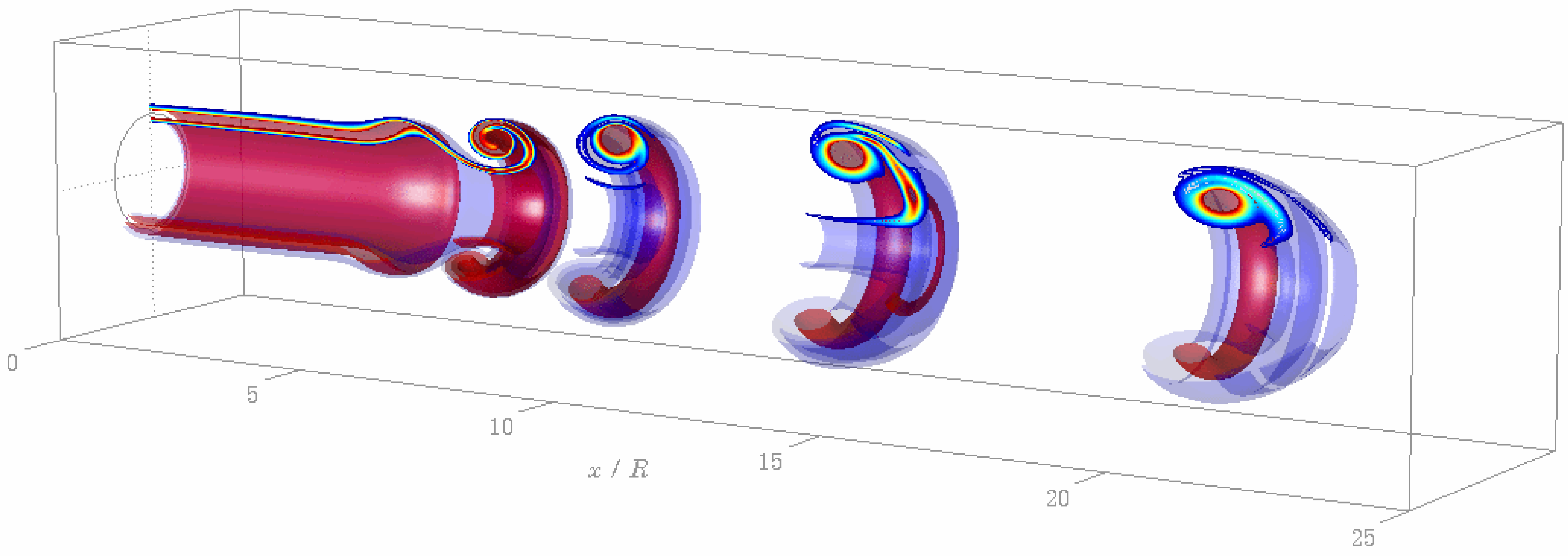
Onset of absolute instability: $S^{ca}=0.72$



Monkewitz & Sohn (1988) Monkewitz et al. (1990)

NONLINEAR GLOBAL MODE IN HOT JETS

$R/\theta = 20 \quad Re = 3750 \quad M = 0.1 \quad S = 0.60$



Lesshafft, H., Sagaut & Terracol (2006)

Outline

Local Parallel Flow Results

Nonlinear Global Modes in Hot Jets

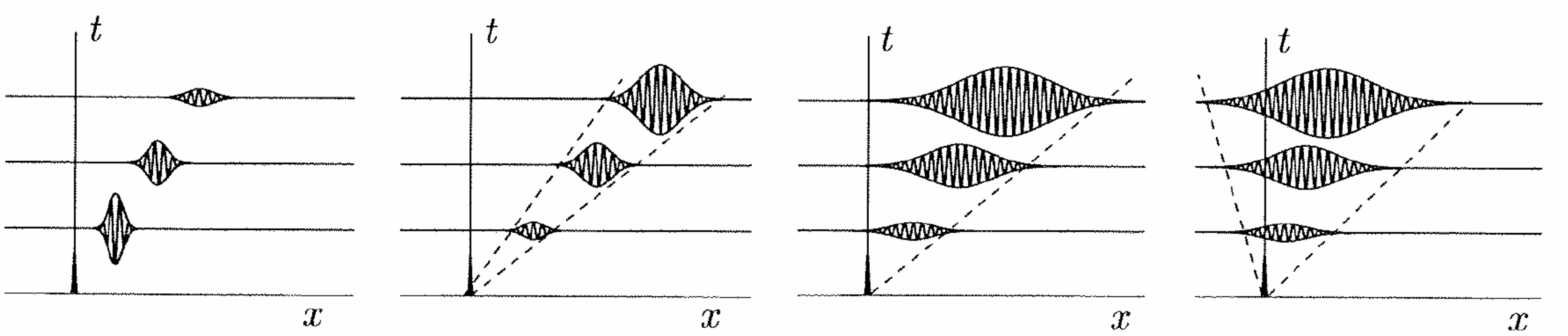
Sound Radiation by Global Modes in Hot Jets

LOCAL PARALLEL FLOW RESULTS

LINEAR IMPULSE RESPONSE: ABSOLUTE/CONVECTIVE INSTABILITY

1D model equation

Green's function or impulse response



Stable

Convectively unstable (CU)

Marginally CU/AU

Absolutely unstable (AU)

Briggs (1964) Bers (1983)

ABSOLUTE FREQUENCY

$$\boxed{\frac{\partial \Omega^l}{\partial k}(k_0) = 0}$$

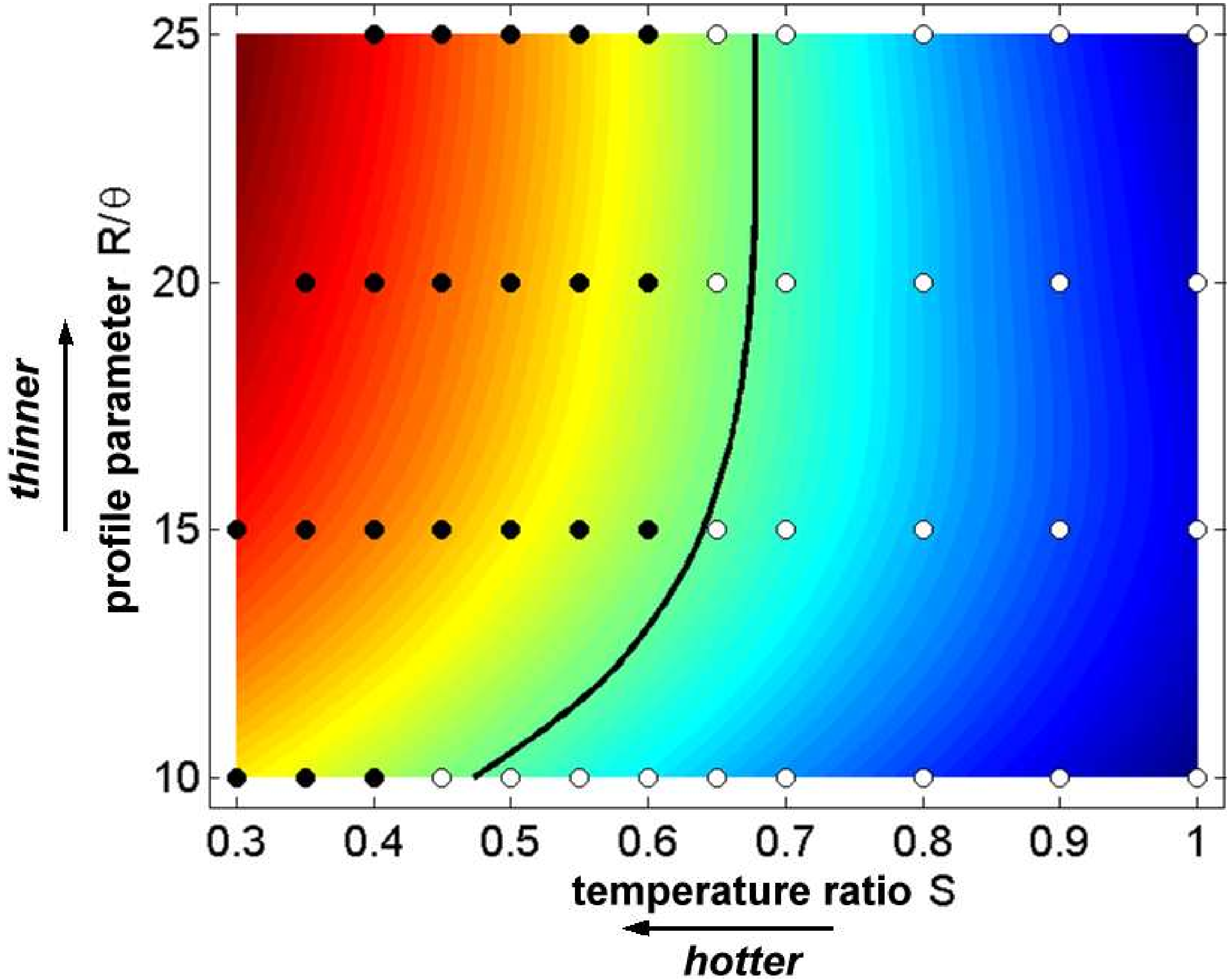
$$\boxed{\omega_0 = \Omega^l(k_0)}$$

Convective instability : $\omega_{0,i} < 0$

Absolute instability : $\omega_{0,i} > 0$

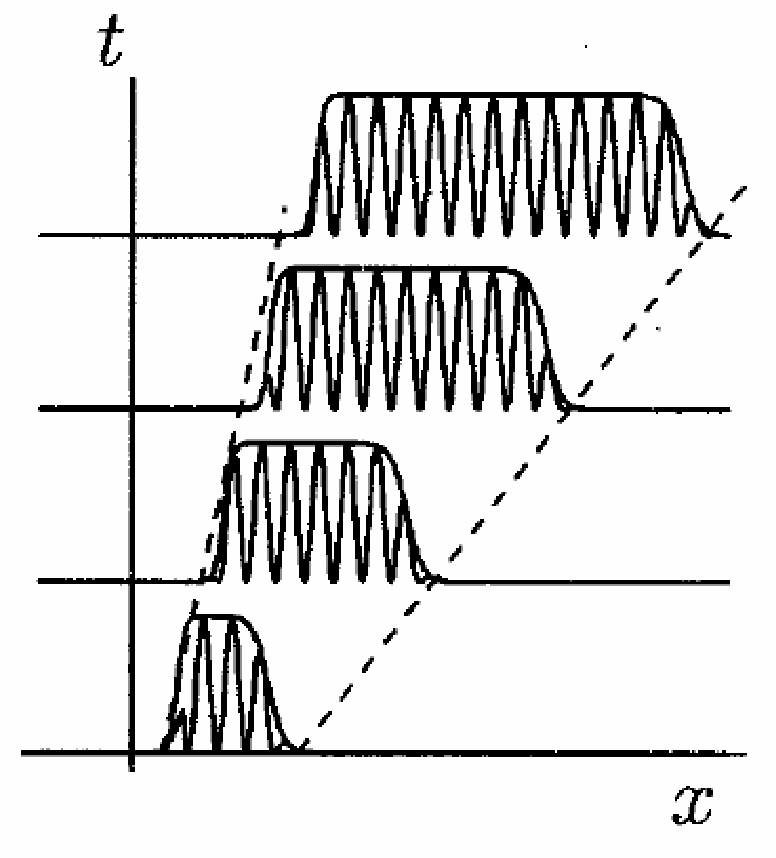
GLOBAL MODE ONSET & CU/AU ONSET

Re = 3750   M= 0.1

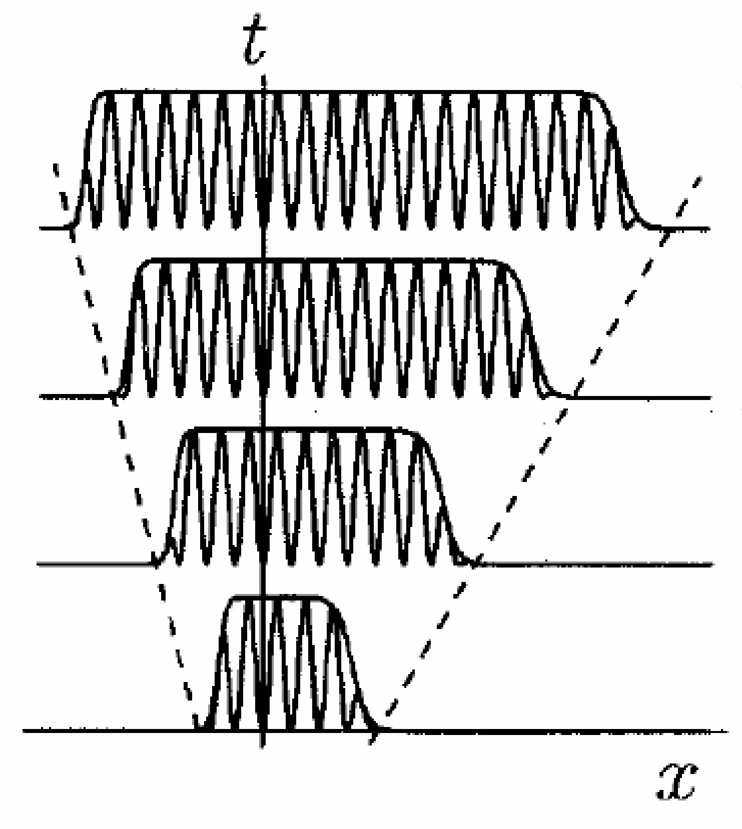


Lesshafft, H. & Sagaut (2006)

NONLINEAR CONVECTIVE VERSUS ABSOLUTE INSTABILITY



Convective instability

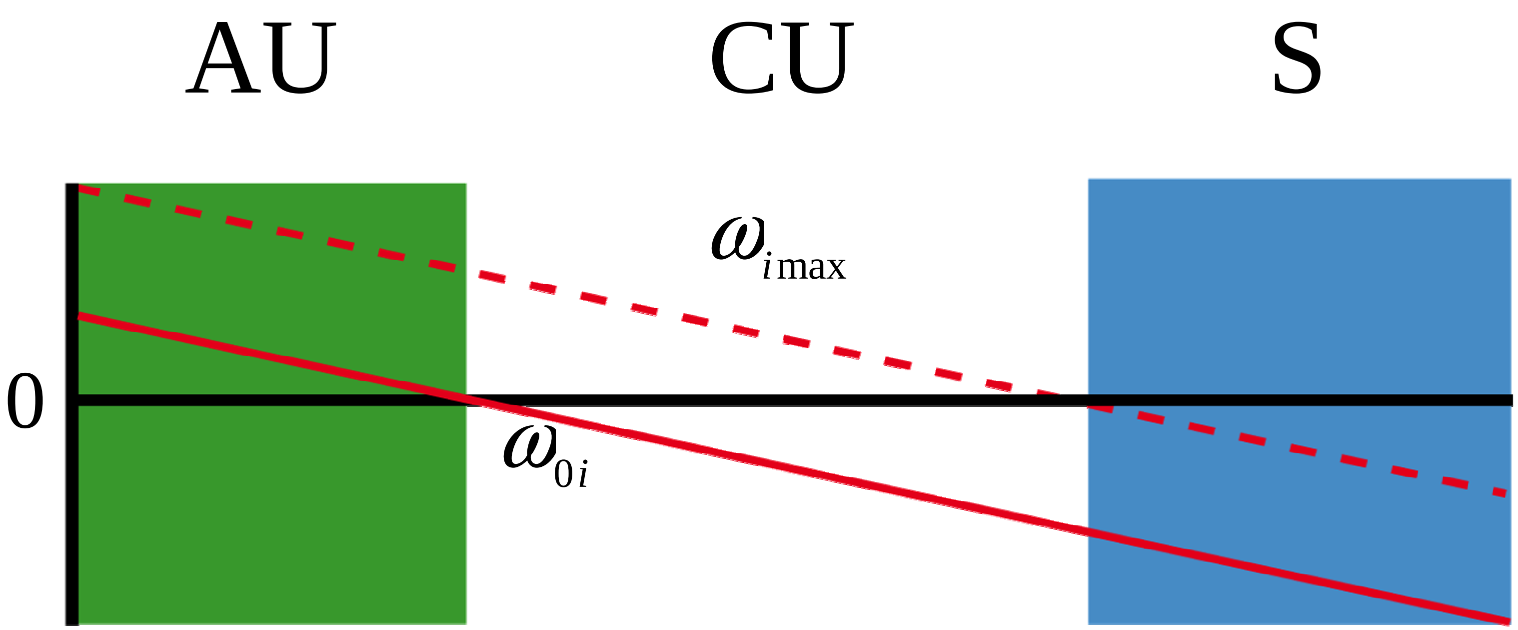


Absolute instability

Couairon & Chomaz (1997)

NONLINEAR GLOBAL MODES IN HOT JETS

Spatially developing flows in a semi-infinite domain

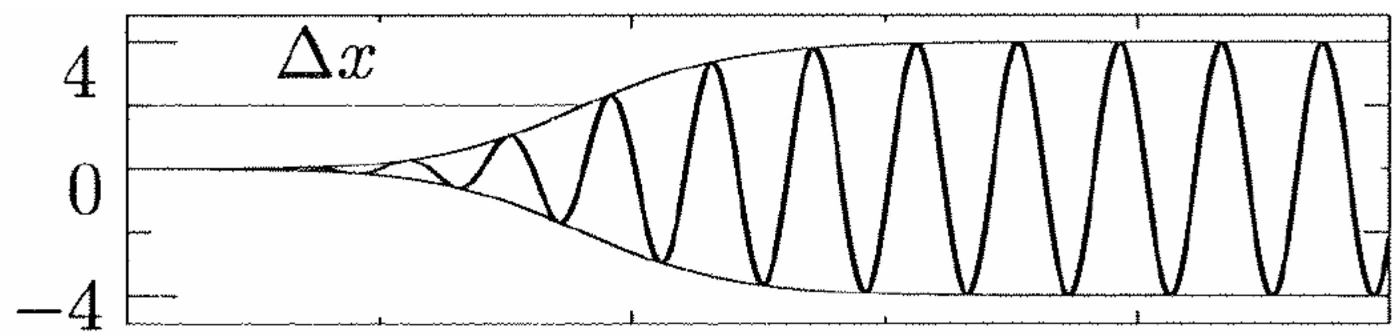


NONLINEAR GLOBAL FREQUENCY SELECTION CRITERION IN PARALLEL SEMI-INFINITE FLOWS

$$\omega = \Omega^{l}(k;X)$$

$$\frac{\partial \Omega^{l}}{\partial k}(k_0;X=0)=0$$

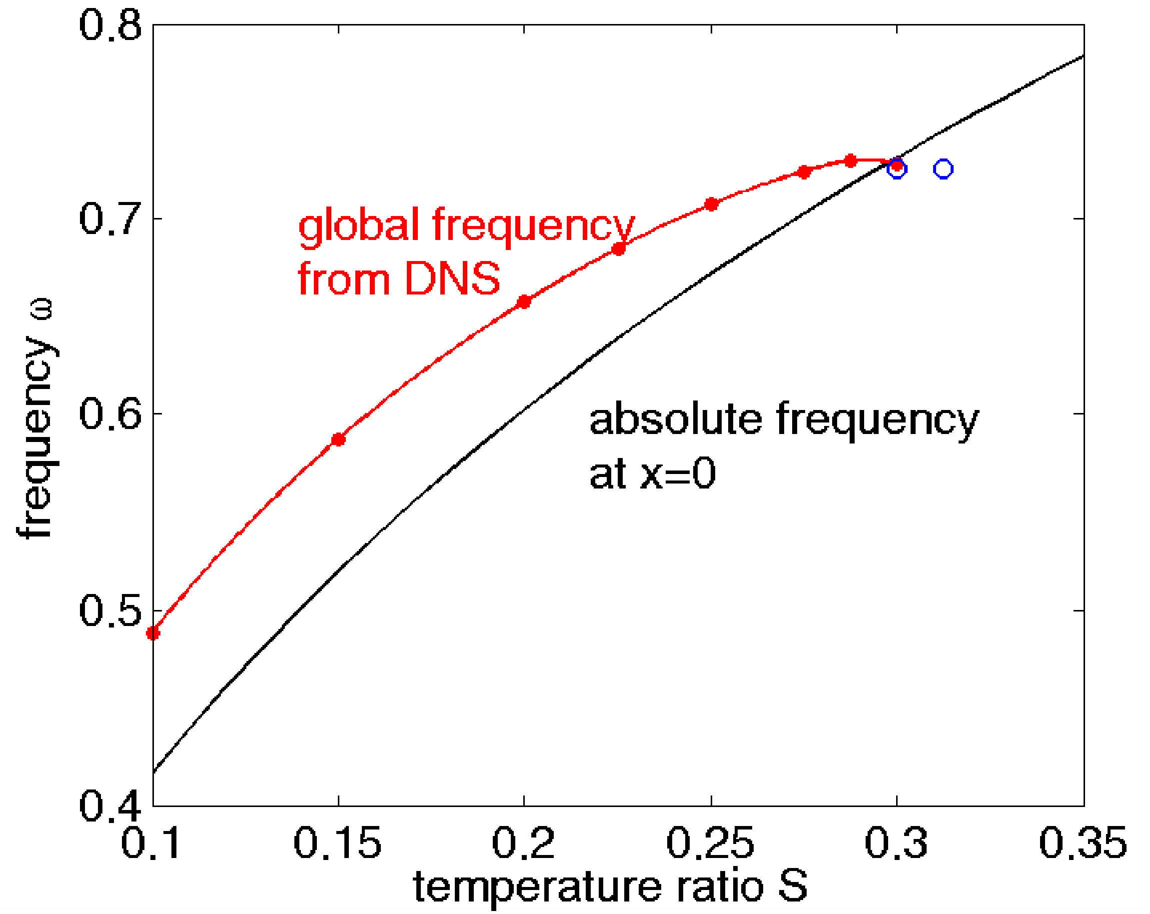
$$\omega_0 = \Omega^{l}(k_0;X=0)$$



Couairon & Chomaz (1999)

FREQUENCY SELECTION CRITERION

$R/\theta = 10 \quad Re = 1000 \quad M = 0.1$



Lesshafft, H., Sagaut & Terracol (2006)

SOUND RADIATION BY GLOBAL MODES IN HOT JETS

LIGHTHILL EQUATION

$$\frac{1}{c_\infty^2}\frac{\partial^2 p'}{\partial t^2} - \frac{\partial^2 p'}{\partial x_i^2} = \underbrace{\frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}}_{\text{Reynolds stresses and viscosity}} - \underbrace{\frac{\partial^2 \rho_e}{\partial t^2}}_{\text{"excess density" fluctuations}}$$

$$T_{ij} = (\rho u_i)_b u'_j + (\rho u_i)' u_{bj} + (\rho u_i)' u'_j - \tau'_{ij}$$

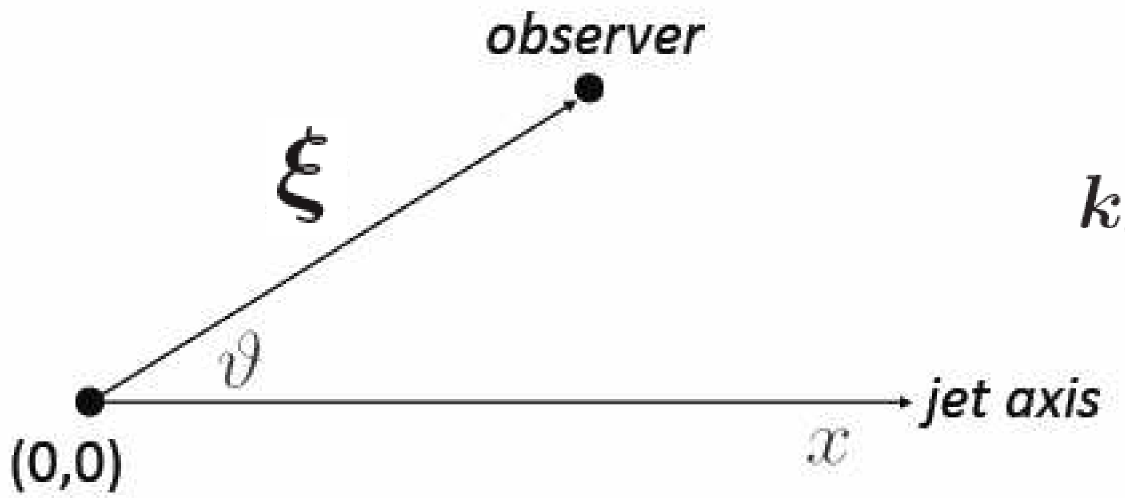
$$\rho_e = \rho' - p'/c_\infty^2$$

Source terms are known from DNS.

➛ individual contributions may be evaluated by solving the Lighthill equation.

LIGHTHILL EQUATION

Possible to construct a formal solution for far field pressure, based on free space Green's function:



$$\boldsymbol{k}_a = \omega / c_\infty \, \mathbf{e}_\xi$$

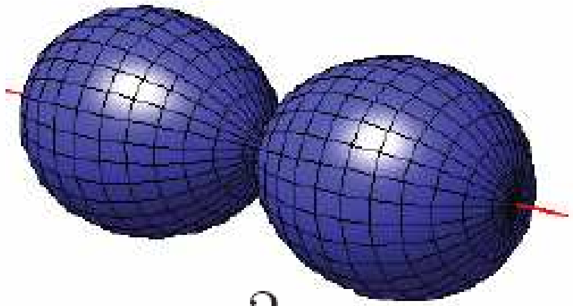
$$\hat{p}(\xi, \vartheta, \omega) = -\frac{k_a^2}{2\xi} \, e^{ik_a\xi} \left(I^x_{xx} + I^x_{rx} + I^x_{rr} + I^x_{\varphi\varphi} - c^2_\infty I^x_e \right)$$

SOLUTION TO LIGHTHILL EQUATION

Radially compact sources

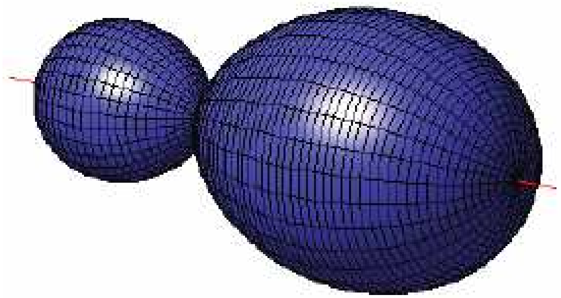
$$\hat{p}(\xi, \vartheta, \omega) = -\frac{k_a^2}{2\xi}\, e^{ik_a \xi} \left(I^x_{xx} + I^x_{rx} + I^x_{rr} + I^x_{\varphi\varphi} - c_\infty^2 I^x_e \right)$$

- Reynolds stresses and viscosity: quadrupoles



$\cos^2 \vartheta$

$$I^x_{xx} = \underbrace{J_0(\alpha)\, \tilde{T}^x_{xx}(k_a}_{\text{antenna factor}} \cos\vartheta, \omega)\, \underbrace{\cos^2 \vartheta}_{\text{quadrupole}}$$



$$\alpha = -k_a \sin\vartheta$$

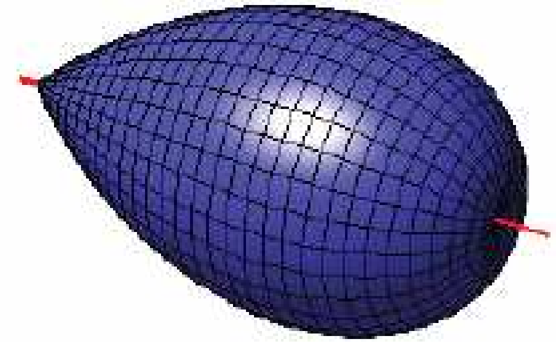
SOLUTION TO LIGHTHILL EQUATION

Radially compact sources

$$\hat{p}(\xi,\vartheta,\omega) = -\frac{k_a^2}{2\xi}\, e^{ik_a\xi}\left(I^x_{xx} + I^x_{rx} + I^x_{rr} + I^x_{\varphi\varphi} - c_\infty^2 I^x_e\right)$$

- excess density: arbitrary shape (formally monopole)

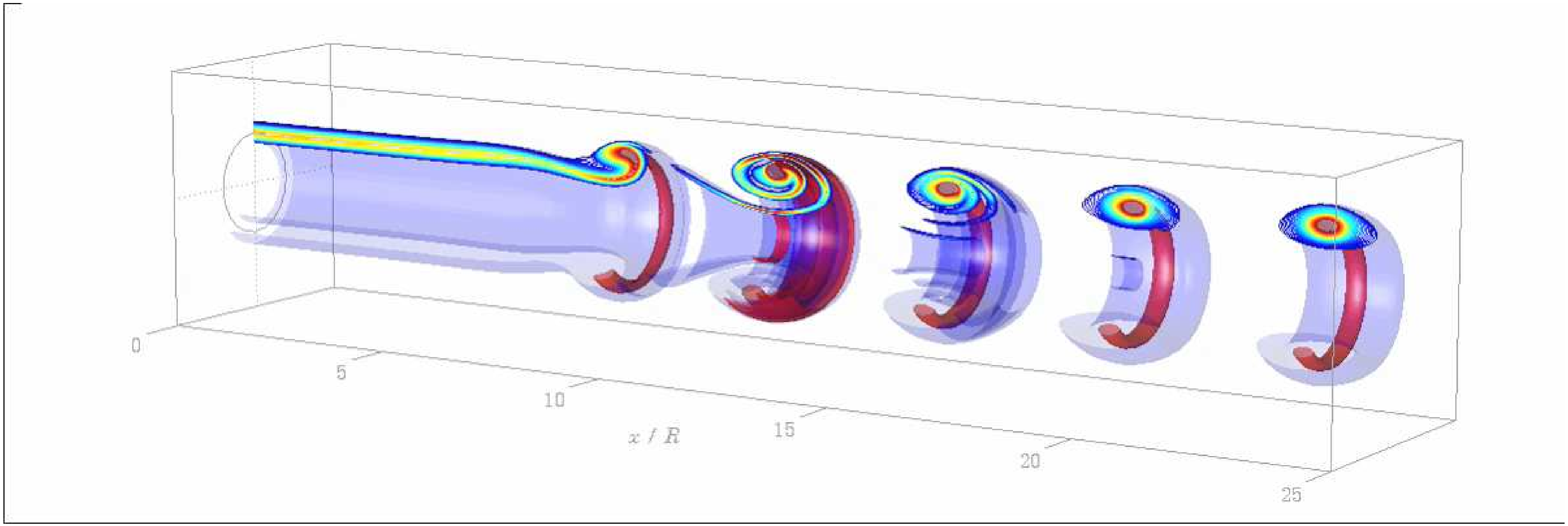
$$I^x_e = J_0(\alpha)\, \underbrace{\tilde{\rho}^x_e(k_a\cos\vartheta,\omega)}_{\text{antenna factor}}$$



SOUND FIELD OF GLOBAL MODE IN HOT JETS

Choose a case without vortex pairing (thick shear layer):

$R/\theta = 10 \quad Re = 2000 \quad M = 0.1 \quad S = 0.40$

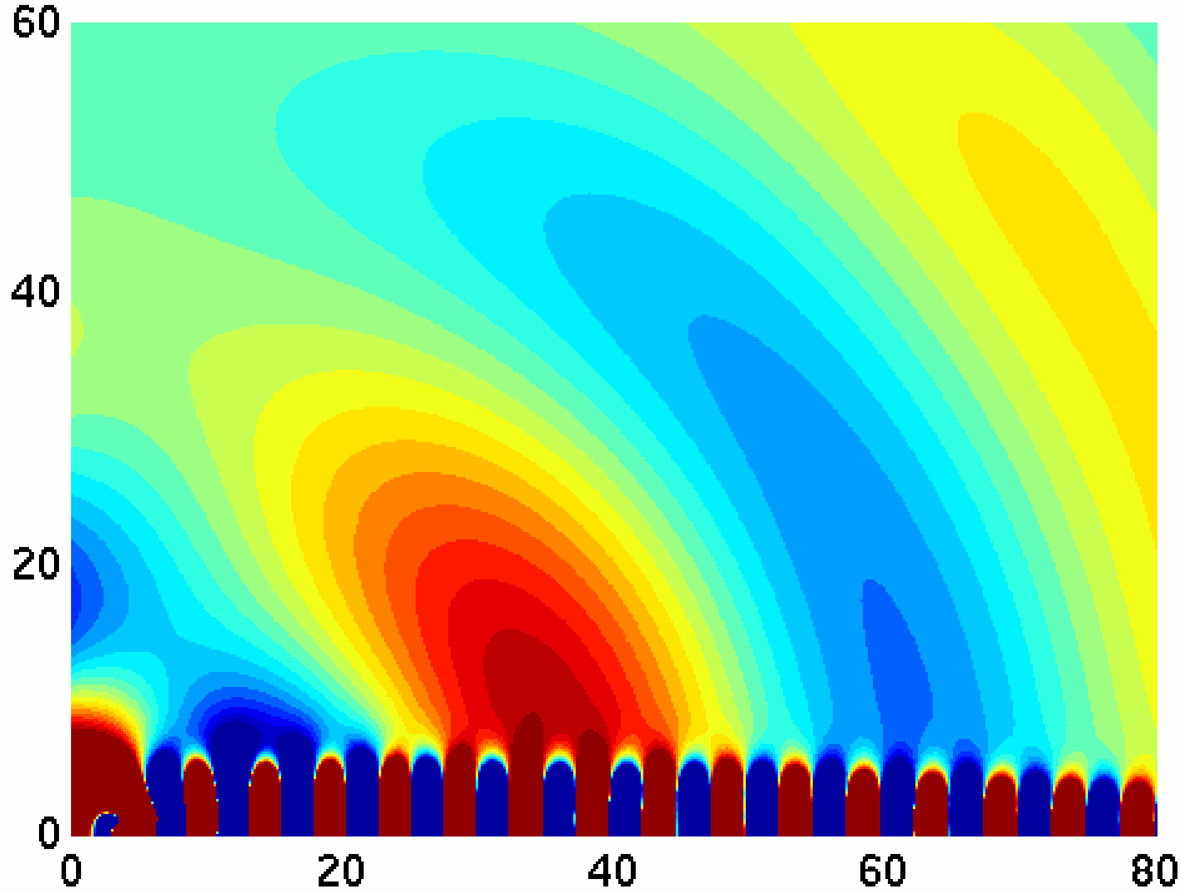


Lesshafft, H. & Sagaut (2009)

SOUND FIELD OF GLOBAL MODE IN HOT JETS

$R/\theta = 10$ $Re = 2000$ $M= 0.1$ $S = 0.40$

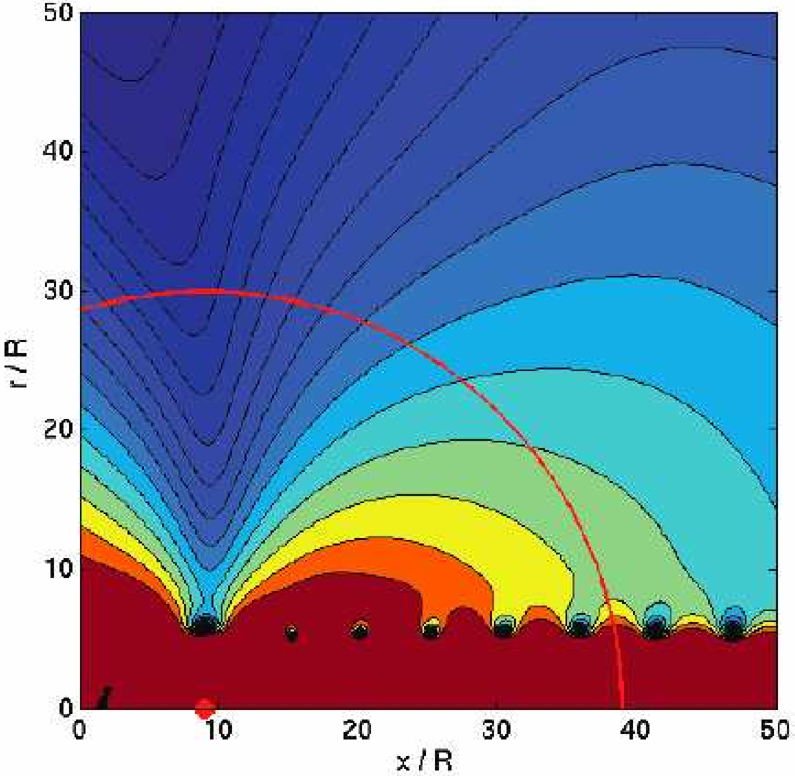
Lesshafft, H.
& Sagaut (2009)

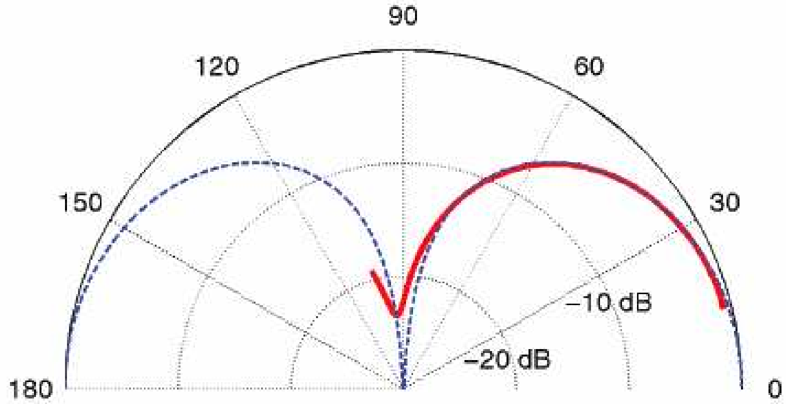


DNS: pressure oscillations, near and far field

SOUND FIELD OF GLOBAL MODE IN HOT JETS

DNS : far field pressure amplitude





Acoustic directivity

$|p(\theta)| \propto |\cos\theta|$

Dipole sound directivity -

What is the acoustic source mechanism?

Lesshafft, H.& Sagaut (2009)

DIRECTIVITY PATTERN OF RADIATED SOUND

Dissection of the "excess density" term (Lilley 1974):

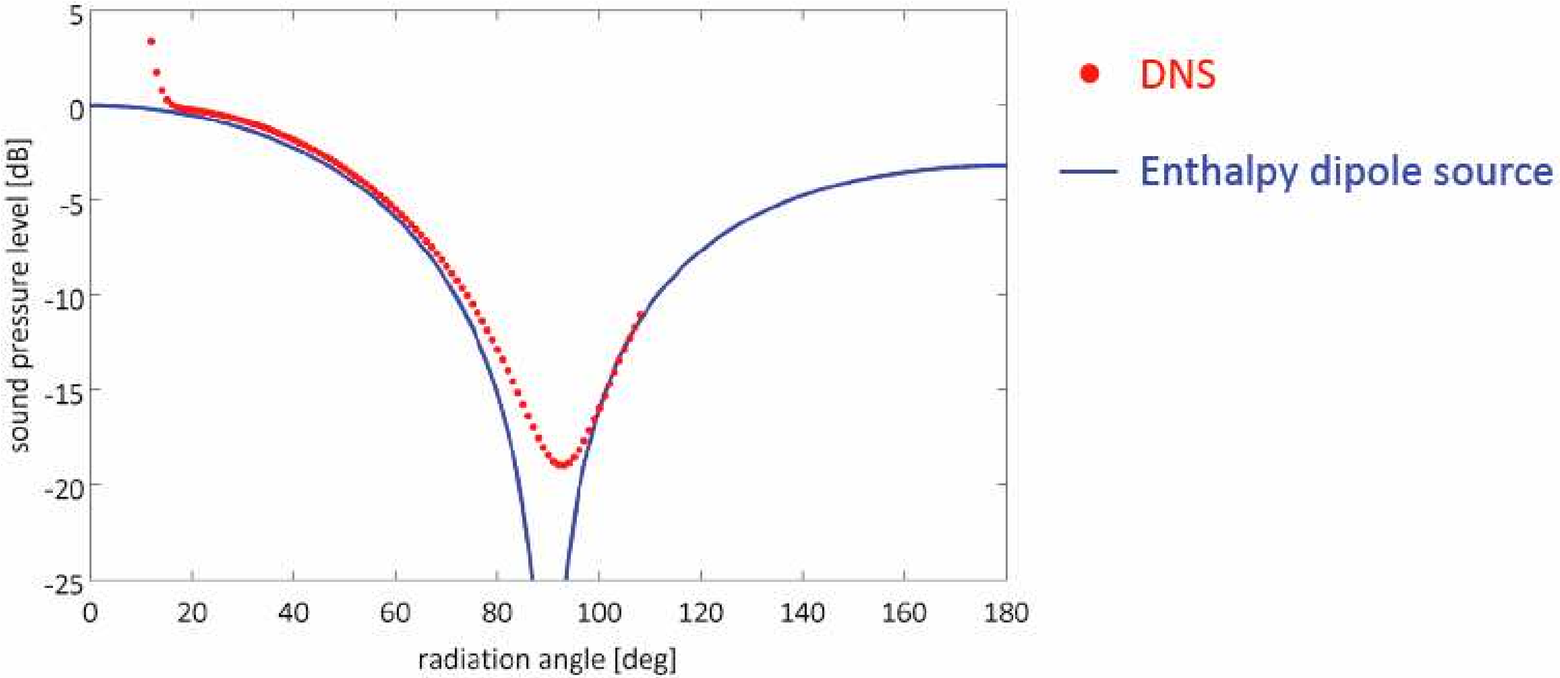
kinetic energy:	$\rho\,\lvert\vec{u}\rvert^2$	monopole
enthalpy:	$\rho\,\vec{u}\,(h_s - h_\infty)$	dipole
dissipation:	$\nabla T\,, \quad \underline{\underline{\tau}} \cdot \vec{u}$	dipole

➛ plug in source term distributions from DNS

Lesshafft, H. & Sagaut (2009)

DIRECTIVITY PATTERN OF RADIATED SOUND

Compare enthalpy dipole source (Lilley 1974) and DNS:



Excellent agreement:

Acoustic field of this self-excited hot jet is dominated by enthalpy-related dipole source term.

Lesshafft, H. & Sagaut (2009)

CONCLUSIONS

Self-excited synchronised states in hot jets may be interpreted as nonlinear global modes which « live » on an underlying steady basic flow displaying a region of absolute instability .

The wavemaker responsible for the onset of the oscillations is located at the AU upstream boundary (criterion for flows of semi-infinite streamwise extent).

Global modes in hot jets give rise to a dipole-like sound field due to the streamwise acceleration of enthalpy fluctuations in the axial direction.

Superdirectivity features are found to be mild since global mode envelope is non-Gaussian and Mach number is low.

DIRECTIVITY PATTERN OF RADIATED SOUND
Dissection of density term (continued)

$$\frac{\partial^2 \rho_e}{\partial t^2} = \frac{1}{c_\infty^2}\frac{\partial^2}{\partial t^2}K(\boldsymbol{x},t) - \frac{1}{c_\infty}\frac{\partial^2}{\partial t \partial x_i}H_i(\boldsymbol{x},t) - \frac{1}{c_\infty}\frac{\partial^2}{\partial t \partial x_i}D_i(\boldsymbol{x},t)$$

Monopole
$$K(\boldsymbol{x},t) = \frac{\gamma - 1}{2}\rho|\boldsymbol{u}|^2$$

Dipole
$$H_i(\boldsymbol{x},t) = \frac{\gamma - 1}{c_\infty}\rho u_i(h_\infty - h_s)$$

Dipole
$$D_i(\boldsymbol{x},t) = \frac{\gamma - 1}{c_\infty}(\tau_{ij}u_j - q_i)$$

Lilley ((1974, 1996)