The Importance of Hot-Wire Measurements in Aeroacoustics

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Outline

- A little history
- Noise source modeling
- Hot-wire measurements and analysis
- Role in high speed jet noise
- Summary

Early References

- Comte-Bellot and Mathieu (1958) “Sur la détermination expérimentale des coefficients de sensibilité aux fluctuations de vitesse et de temperature des anémomètres à fil chaud,” C. R. Acad. Sci., 246(23): 3219-3222.

- Comte-Bellot & Corrsin (1966) “The use of a contraction to improve the isotropy of grid-generated turbulence,” *JFM*, **25**(4): 657-682.

- Comte-Bellot & Corrsin (1971) “Simple Eulerian time correlation of full- and narrow-band velocity signals in grid-generated ‘isotropic’ turbulence,” *JFM*, **48**(2): 237-337.

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Hot-Wire Sensitivity

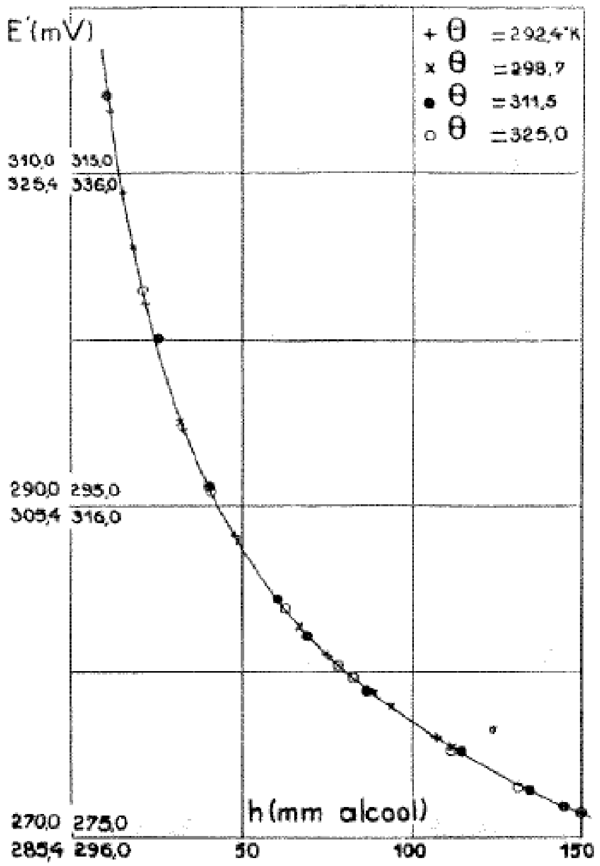

Fig. 1. — Pour l'échelle des E' prendre les valeurs :
en haut à gauche pour Θ = 292,4° K ;
en haut à droite pour Θ = 298,7° K ;
en bas à gauche pour Θ = 311,5° K ;
en bas à droite pour Θ = 325,0° K.

Comte-Bellot & Mathieu (1958)

Hot-Wire Measurements (1960's)

- Issues:
  - Making the wires
  - Electronic multiplication
  - Space-time correlations
  - Minimizing wake interference effects
  - Spectral analysis

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Instrumentation

- Hot-wires
  - Platinum-rhodium (3.5μm x 0.4mm) soldered to the tips of jewelers' broaches
- Electronic multiplication by "quarter-square" principle

$$\overline{u_i u_j} = \frac{1}{4}\left[\overline{\left(u_i + u_j\right)^2} - \overline{\left(u_i - u_j\right)^2}\right]$$

- Time delay for cross-correlation
  - Moving head on Sangamo 284RB tape recorder
- Spectral analysis
  - Hewlett-Packard 302A (constant bandwidth) wave analyzer

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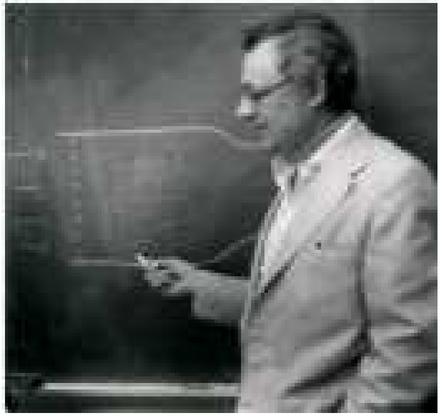
Hewlett-Packard Wave Analyzer

“Corrsin” Wind Tunnel



Johns Hopkins University, Maryland

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"Source Terms"

- Lighthill (1952)

$$\frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}; \quad T_{ij} = \rho u_i u_j + (p_{ij} - a_o{}^2 \rho \delta_{ij})$$

- Lilley (1974)

$$\frac{D}{Dt}\frac{\partial^2 \left(u_i u_j\right)}{\partial x_i \partial x_j} - 2\frac{\partial U}{\partial x_i}\frac{\partial^2 \left(u_i u_j\right)}{\partial x_i \partial x_j}$$

- Goldstein (2003)

$$T'_{ij} = -\left(\rho v''_i v''_j - \bar{\rho} \tilde{v}''_i v''_j\right)$$

Cross Correlations

- Second Order

$$R_{ij}(\mathbf{x},\boldsymbol{\eta},\tau)=\overline{u_i(\mathbf{x},t)u_j(\mathbf{x}+\boldsymbol{\eta},t+\tau)}$$

- Fourth-Order (I)

$$R_{ijkl}(\mathbf{x},\boldsymbol{\eta},\tau)=\overline{u_iu_j(\mathbf{x},t)u_ku_l(\mathbf{x}+\boldsymbol{\eta},t+\tau)}$$

$$r_{ijkl}(\mathbf{x},\boldsymbol{\eta},\tau)=\frac{R_{ijkl}(\mathbf{x},\boldsymbol{\eta},\tau)}{\sqrt{\overline{\left(\overline{u_iu_j}\right)^2(\mathbf{x})\left(\overline{u_ku_l}\right)^2(\mathbf{x}+\boldsymbol{\eta})}}}$$

- Fourth-Order (II)

$$C_{ijkl}(\mathbf{x},\boldsymbol{\eta},\tau)=\overline{\left(u_iu_j-\overline{u_iu_j}\right)(\mathbf{x},t)\left(u_ku_l-\overline{u_ku_l}\right)(\mathbf{x}+\boldsymbol{\eta},t+\tau)}$$

$$c_{ijkl}(\mathbf{x},\boldsymbol{\eta},\tau)=\frac{C_{ijkl}(\mathbf{x},\boldsymbol{\eta},\tau)}{\sqrt{\overline{\left(u_iu_j-\overline{u_iu_j}\right)^2}(\mathbf{x})\overline{\left(u_ku_l-\overline{u_ku_l}\right)^2}(\mathbf{x}+\boldsymbol{\eta})}}$$

2nd and 4th Order Statistics

$$i = j = k = l = \alpha$$

$$r_{\alpha\alpha\alpha\alpha}(\mathbf{x},\mathbf{0},0) = \overline{u_\alpha^{\ 4}}(\mathbf{x}) / \left[\overline{u_\alpha^{\ 2}}(\mathbf{x})\right]^2 = T_\alpha(\mathbf{x})$$: Flatness factor

$$c_{\alpha\alpha\alpha\alpha}(\mathbf{x},\boldsymbol{\eta},\tau) = \frac{r_{\alpha\alpha\alpha\alpha}(\mathbf{x},\boldsymbol{\eta},\tau) - 1}{\sqrt{\left[T_\alpha(\mathbf{x}) - 1\right]\left[T_\alpha(\mathbf{x}+\boldsymbol{\eta}) - 1\right]}}$$

Quasi-normal approximation

$$R_{ijkl} = R_{ik}R_{jl} + R_{il}R_{jk} + R_{ij}R_{kl}$$

Lighthill (1993)

$$c_{\alpha\alpha\alpha\alpha}(\mathbf{x},\boldsymbol{\eta},\tau) = r_{\alpha\alpha}^{\ 2}(\mathbf{x},\boldsymbol{\eta},\tau)$$ Eqn. (11)

$$r_{\alpha\alpha\alpha\alpha} = 1 + \left(T_\alpha(\mathbf{x}) - 1\right) r_{\alpha\alpha}^{\ 2}$$

Cross Spectral Density and Cross Coherence

Cross spectral density

$$S_{ijkl}\left(\mathbf{x},\boldsymbol{\eta},\omega\right)=\frac{1}{2\pi}\int_{-\infty}^{\infty} C_{ijkl}\left(\mathbf{x},\boldsymbol{\eta},\tau\right)e^{i\omega\tau}d\tau$$

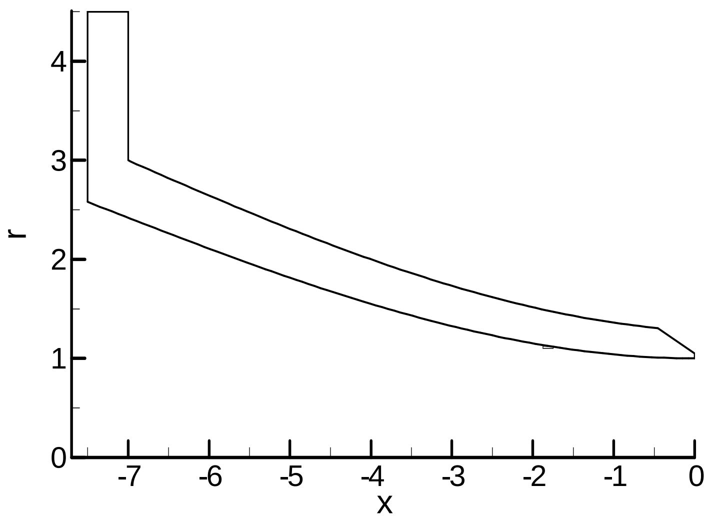
$$C_{ijkl}\left(\mathbf{x},\boldsymbol{\eta},\tau\right)=\int_{-\infty}^{\infty} S_{ijkl}\left(\mathbf{x},\boldsymbol{\eta},\omega\right)e^{-i\omega\tau}d\tau$$

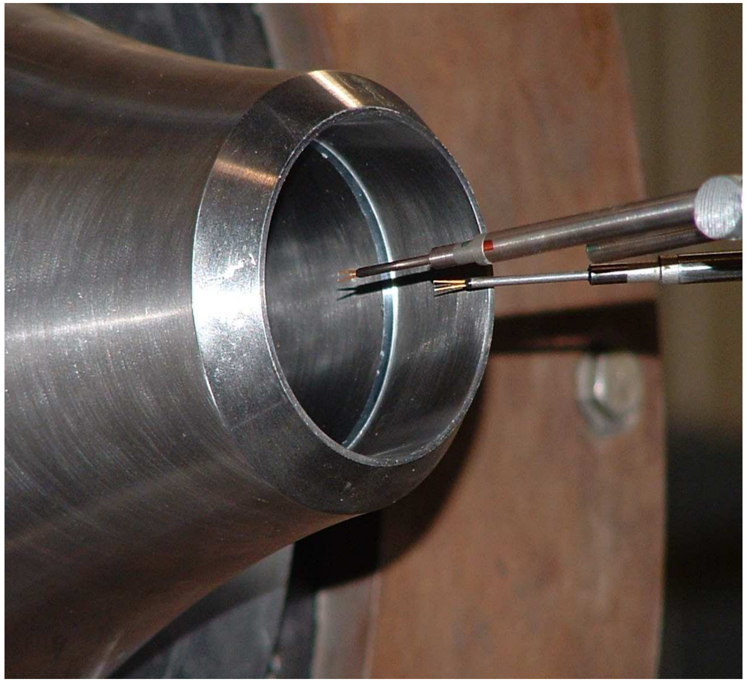
Complex cross coherence

$$s_{ijkl}\left(\mathbf{x},\boldsymbol{\eta},\omega\right)=\frac{S_{ijkl}\left(\mathbf{x},\boldsymbol{\eta},\omega\right)}{\sqrt{S_{ijkl}{}^{2}\left(\mathbf{x},\omega\right)S_{ijkl}{}^{2}\left(\mathbf{x}+\boldsymbol{\eta},\omega\right)}}$$

Experimental Facility

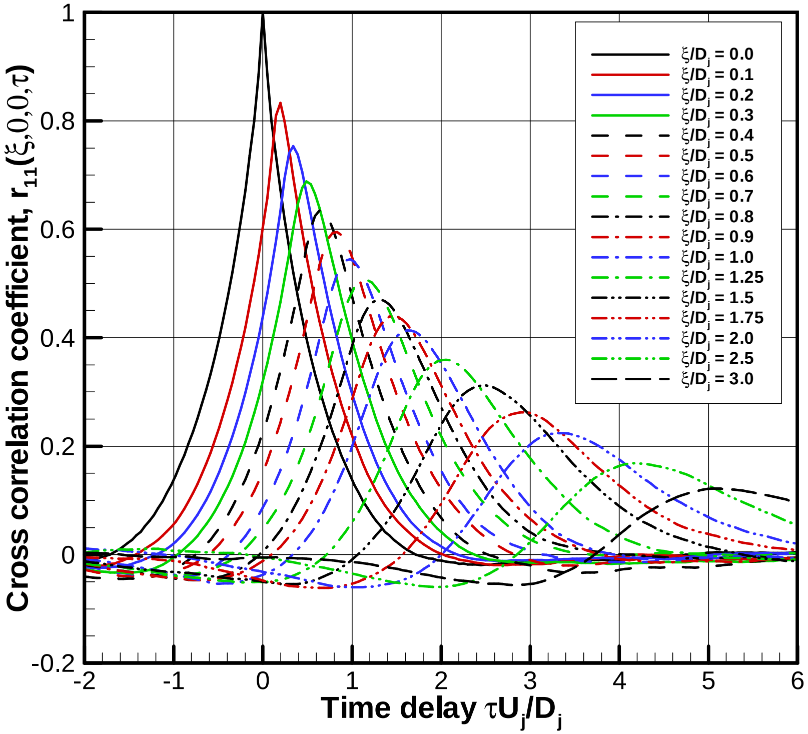
- Nominal exit velocity: 285 ft/sec
- Jet exit diameter: 2 inches

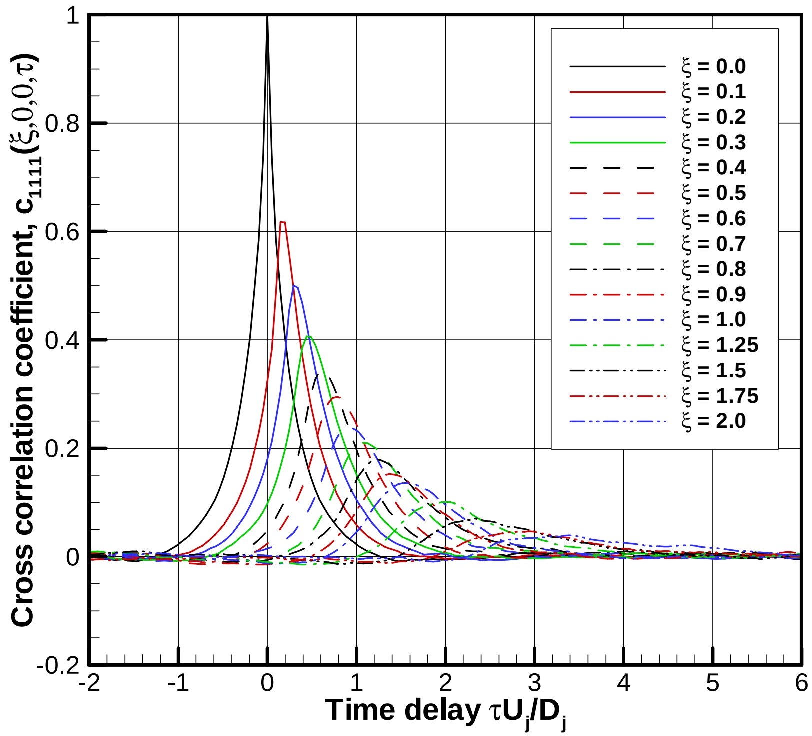




Cross Correlation Coefficients

$x / D_j = 5, \; r / D_j = 0.5$





$r_{11}\left(\mathbf{x}, \boldsymbol{\eta}, \tau\right)$

$c_{1111}\left(\mathbf{x}, \boldsymbol{\eta}, \tau\right)$

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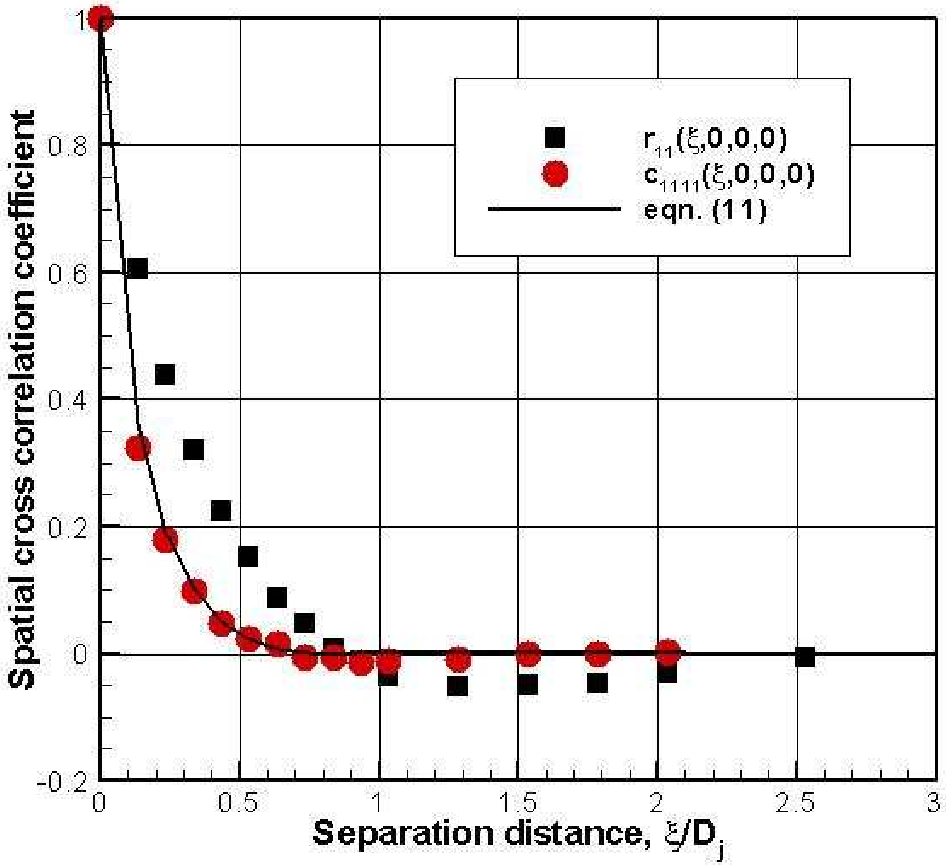
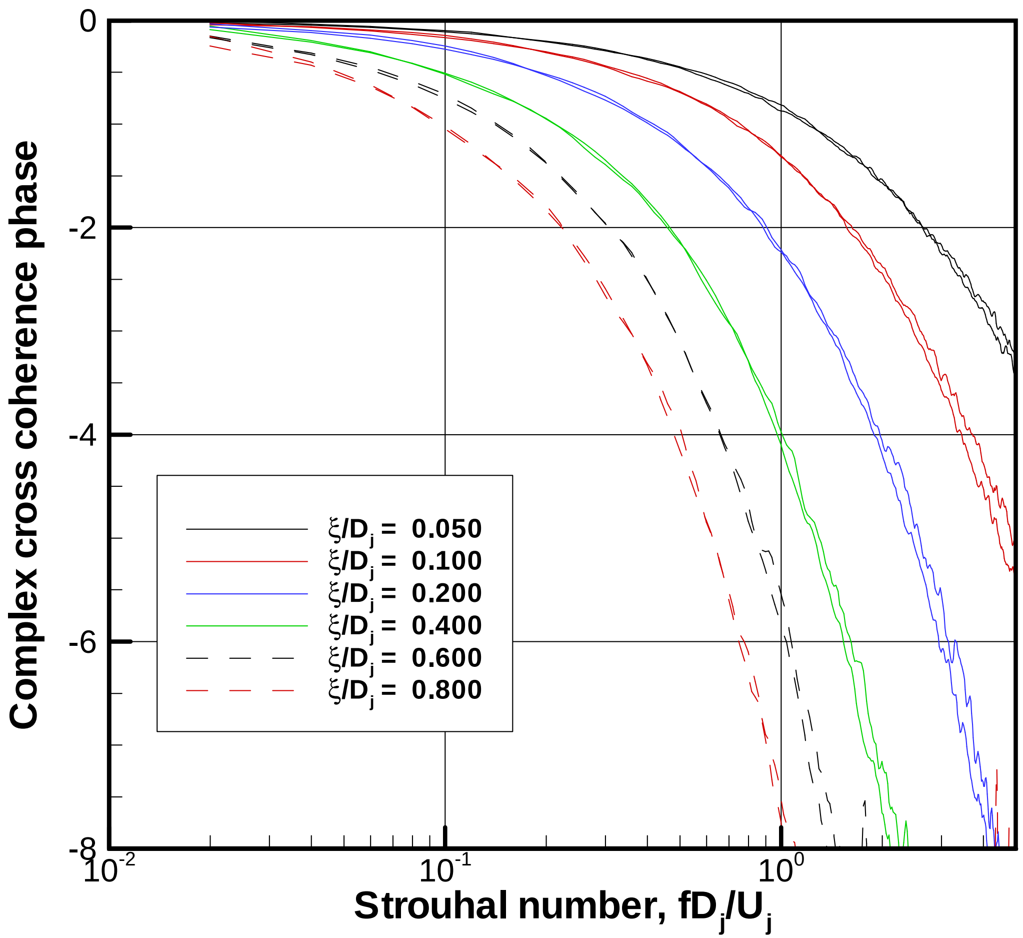
Spatial Cross Correlation

$x / D_j = 5, \; r / D_j = 0.5$



Complex Cross Coherence: Phase

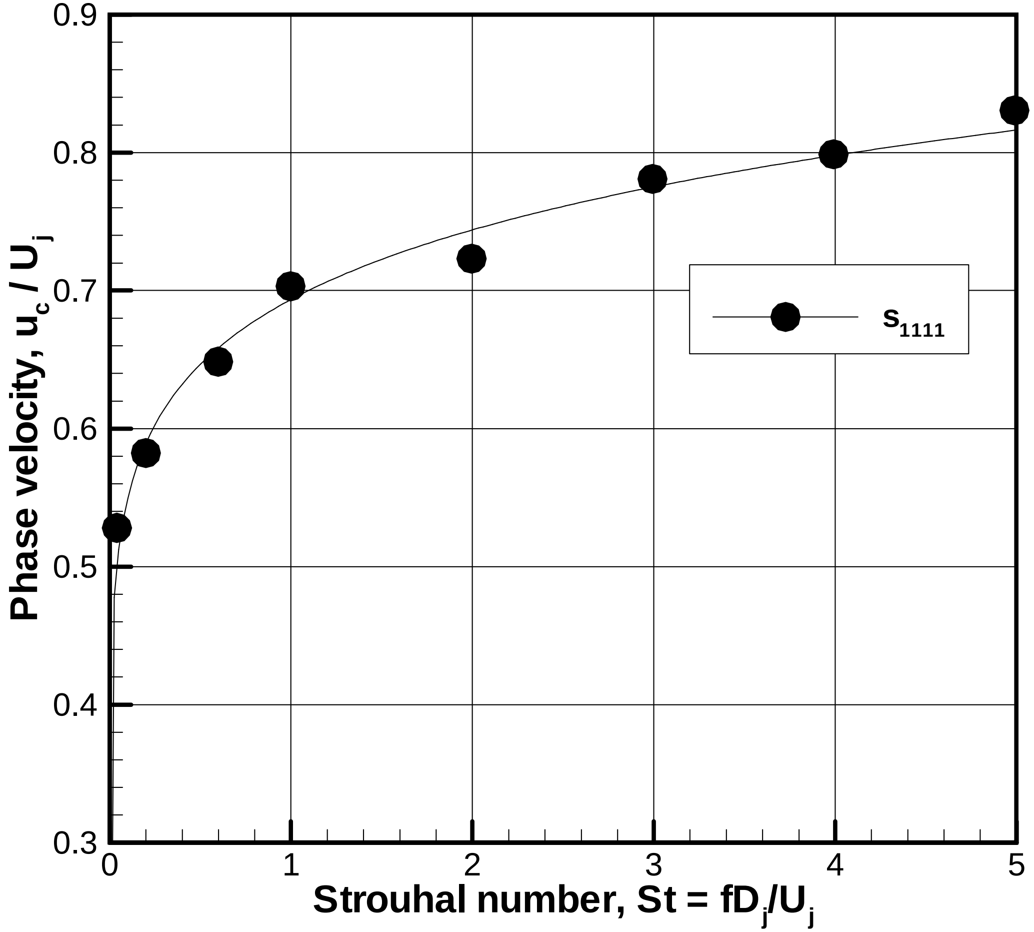
$x / D_j = 5,\ r / D_j = 0.5$



$$\frac{u_c(St)}{U_j} = \frac{2\pi St}{\left| d\phi / d\xi \right|}$$

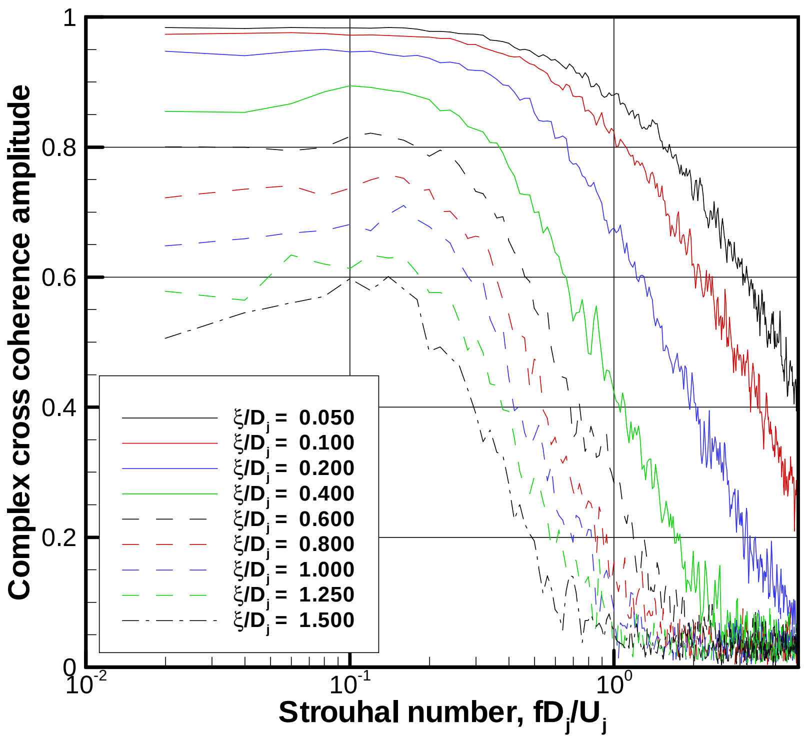
Phase Velocity: Frequency Dependence

$$x / D_j = 5,\ r / D_j = 0.5$$

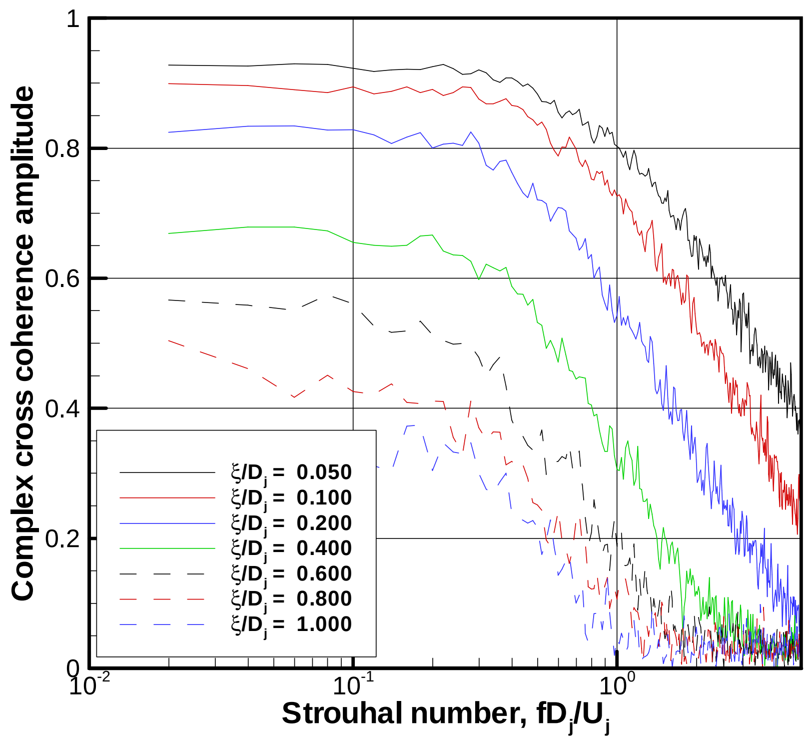


Complex Cross Coherence: Amplitude

$x/D_j = 5,\ r/D_j = 0.5$



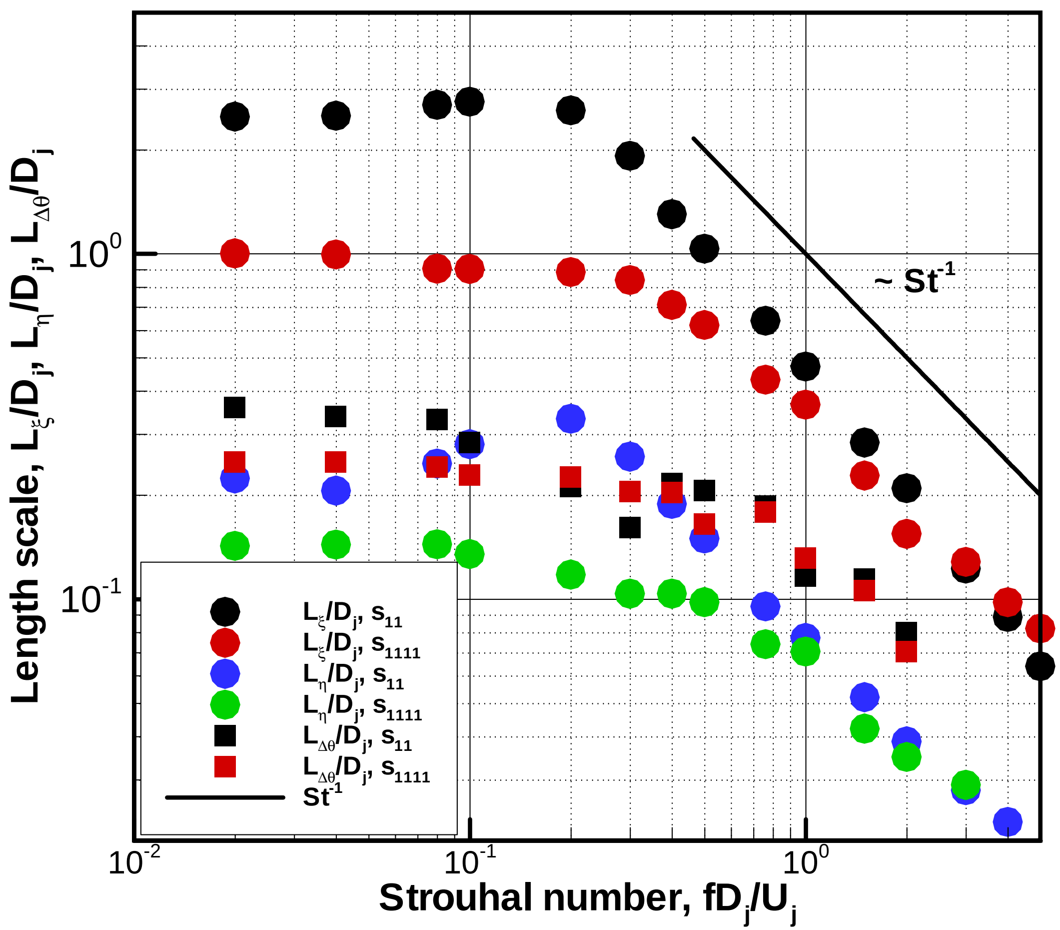
$$\left|s_{11}\left(\mathbf{x},\boldsymbol{\eta},\omega\right)\right|$$



$$\left|s_{1111}\left(\mathbf{x},\boldsymbol{\eta},\omega\right)\right|$$

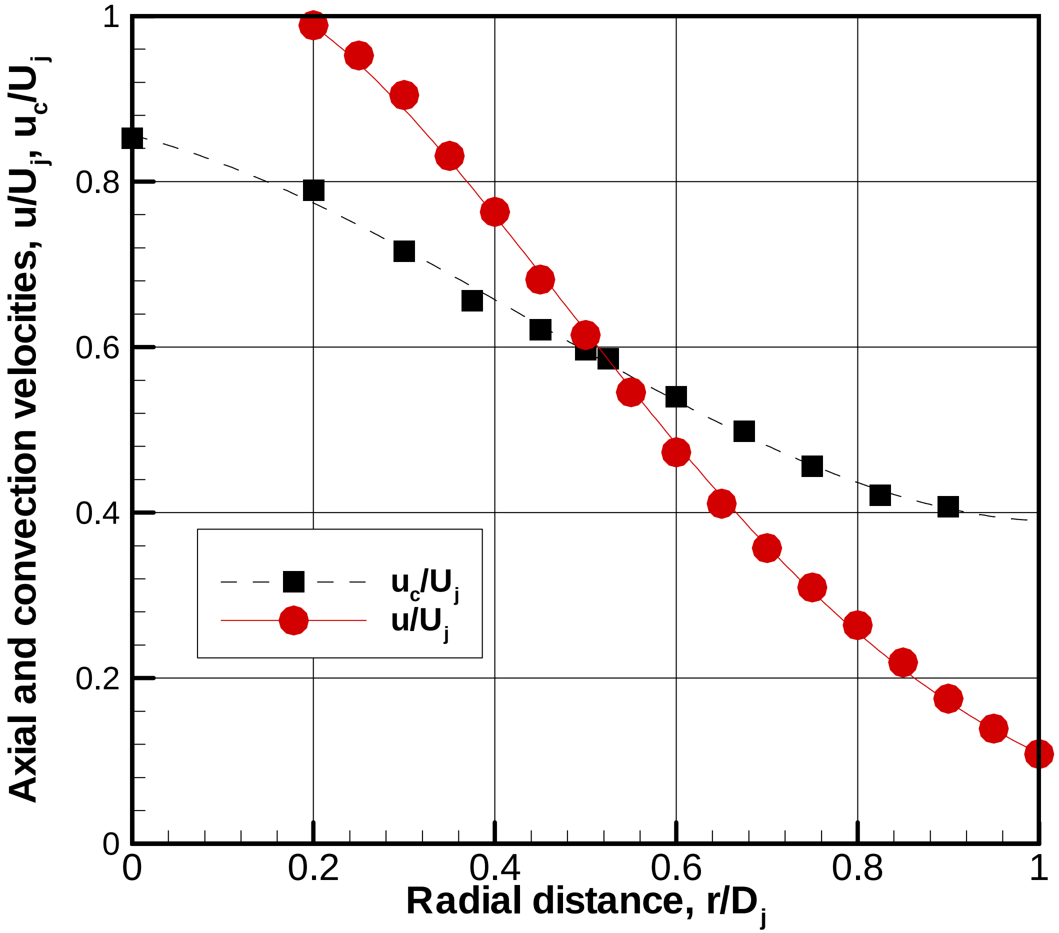
Length Scales

$x / D_j = 5, \ r / D_j = 0.5$



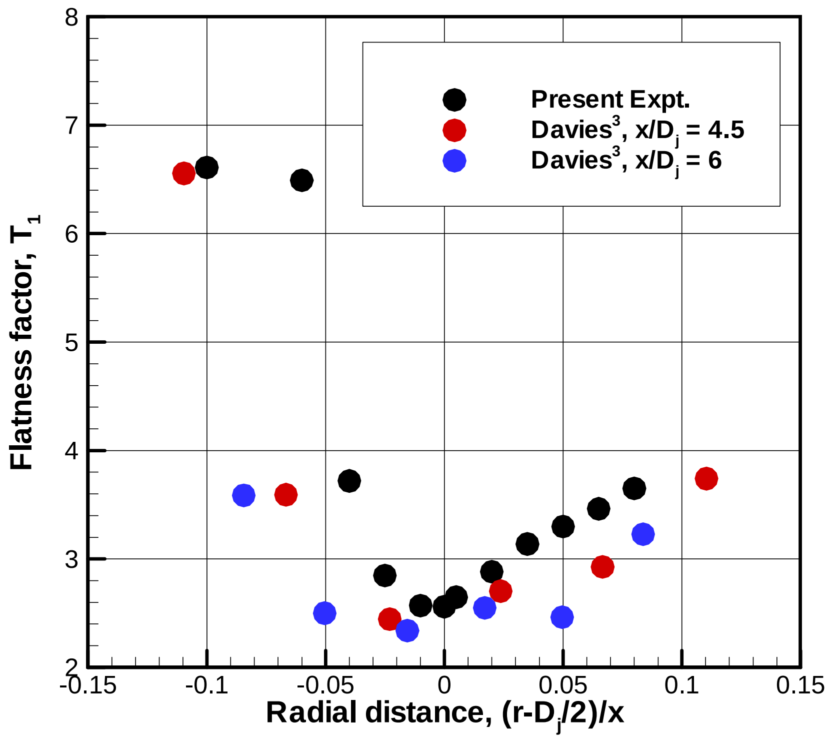
Radial Variation: Convection Velocity

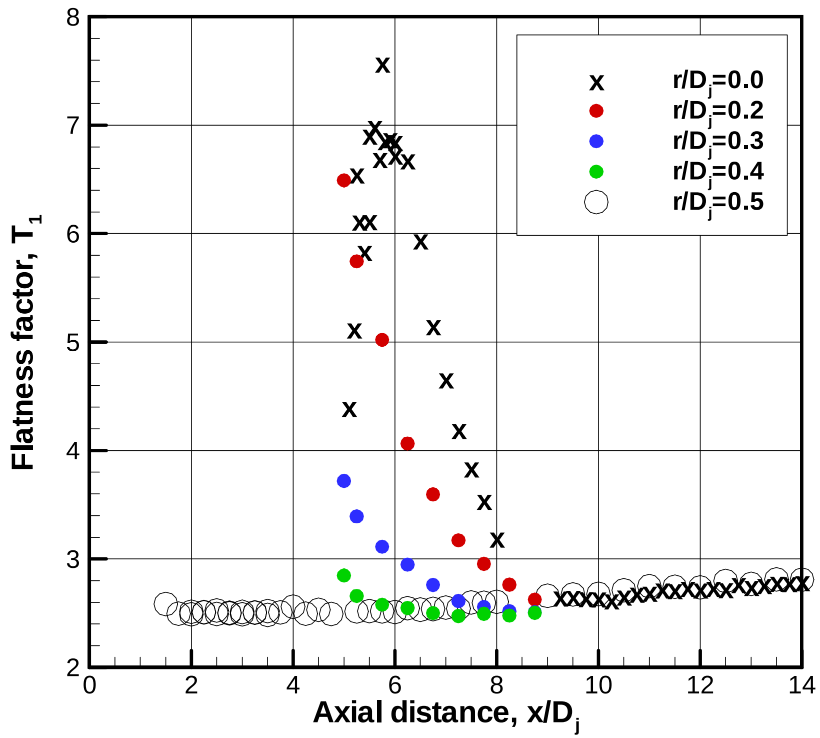
$x / D_j = 5$



Flatness Factor

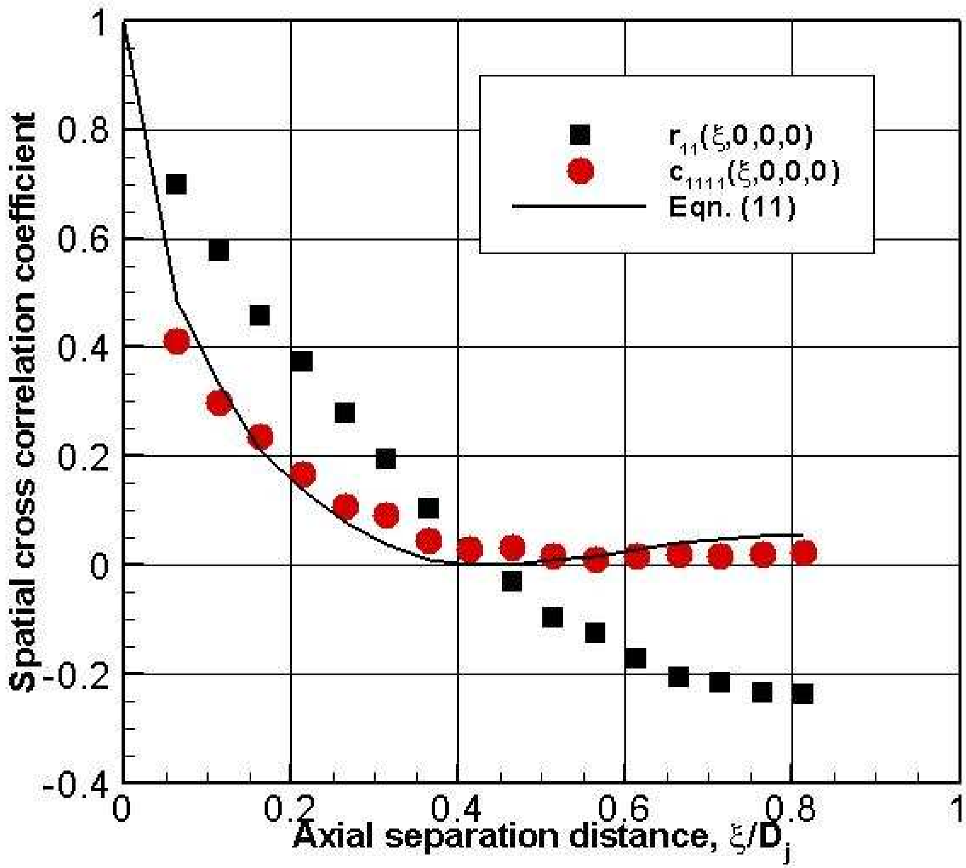
$$x / D_j = 5$$





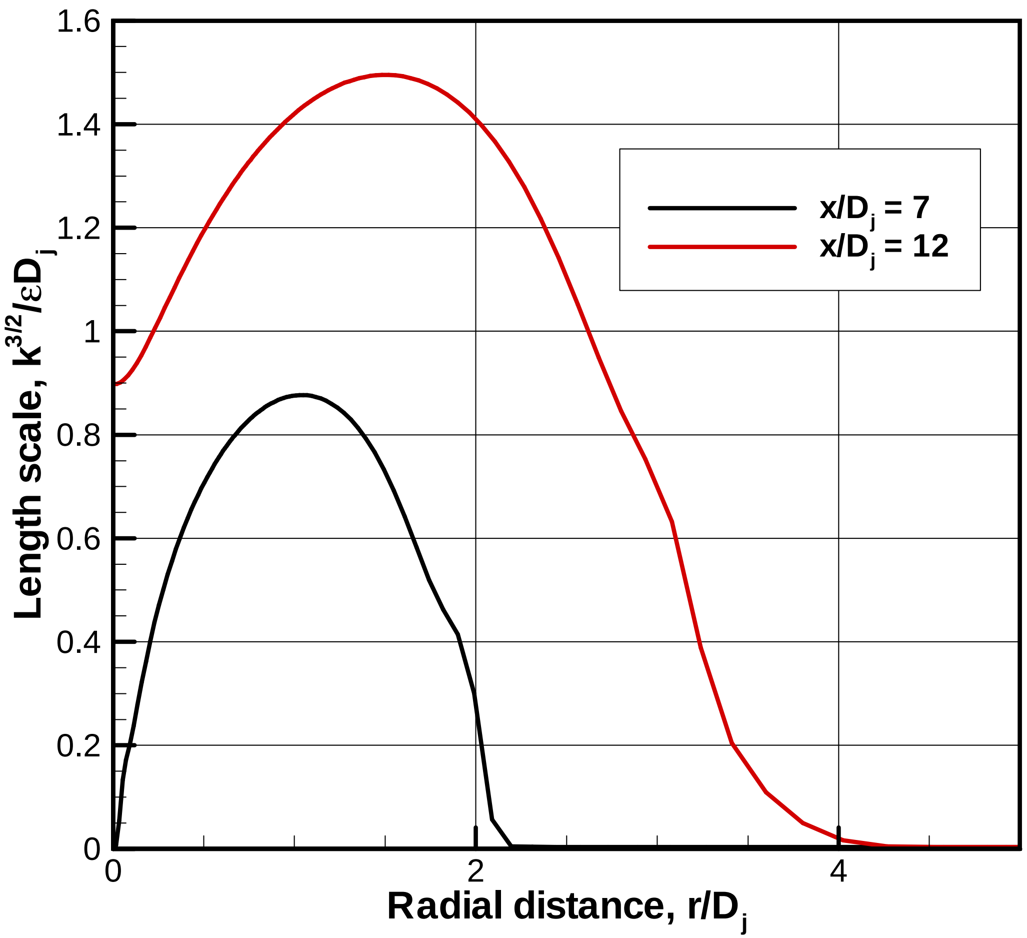
Spatial Cross Correlations

$x / D_j = 5,\ r / D_j = 0.0$



Length Scales

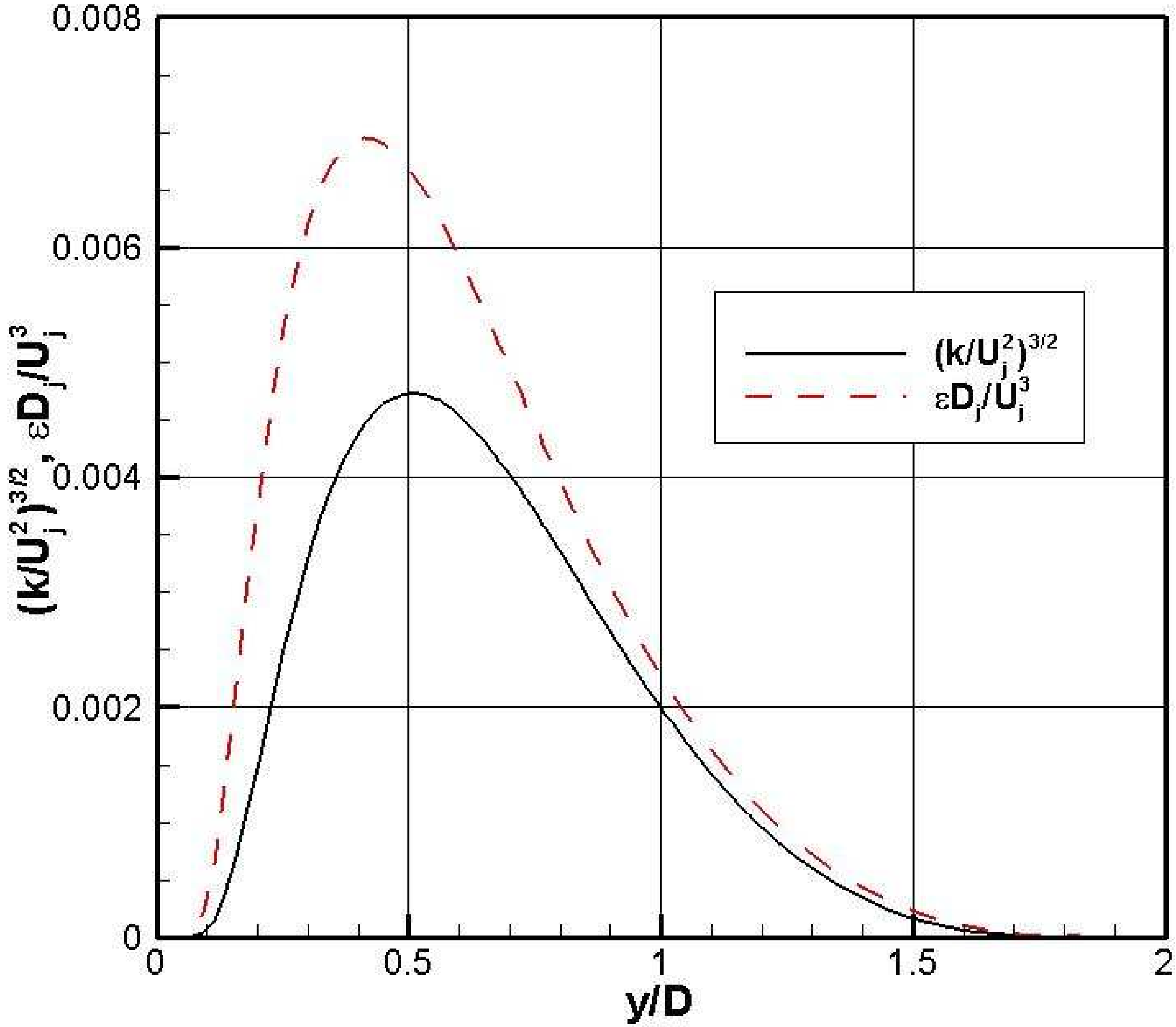
$$L = k^{3/2} / \varepsilon$$



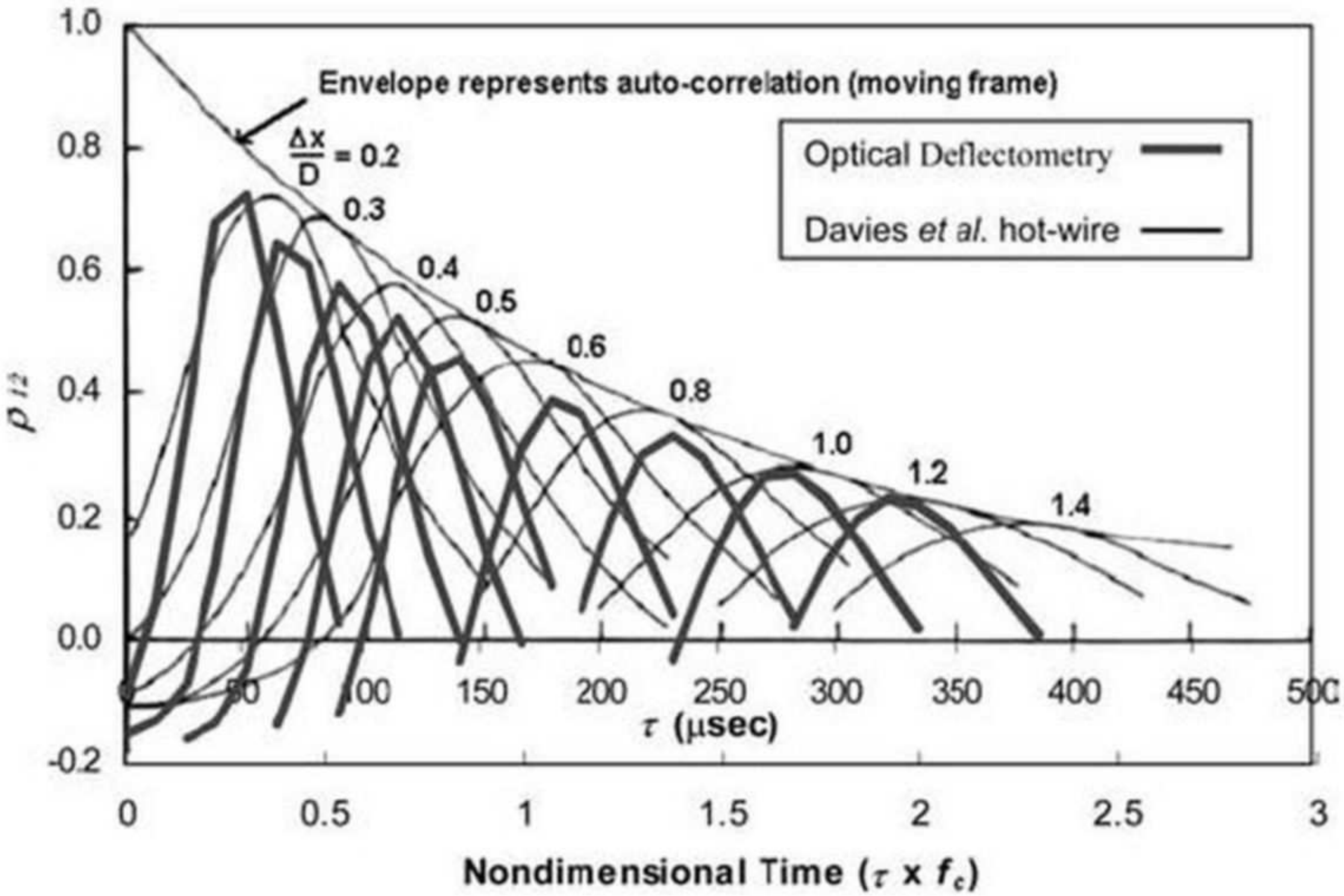
Length scale on lip line:

- Larger than overall length scale
- Less than length scale for second order fluctuations at low frequencies
- Equal to length scale for fourth order fluctuations at low frequencies

RANS Calculations



High Speed Jet Noise Models



Doty & McLaughlin (2005)

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Broadband Shock-Associated Noise

Problem: Predict the component of noise due to presence of shocks and expansions contained in supersonic jets operating off-design

Only know:

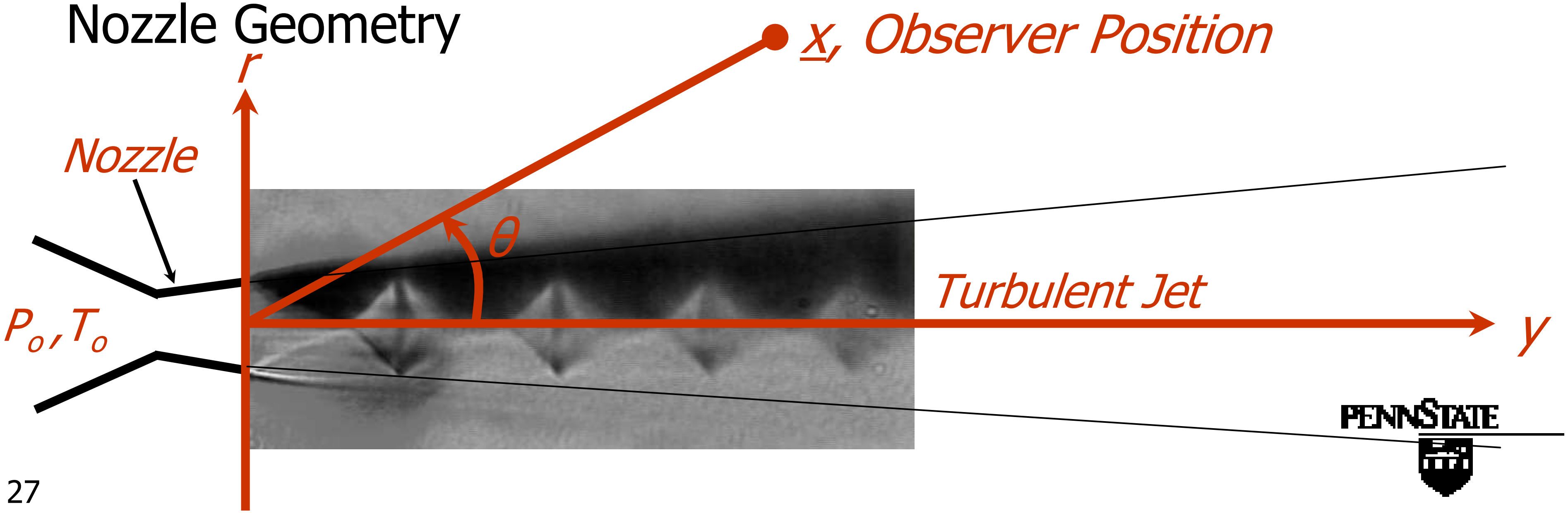
$NPR = p_o / p_\infty$

$TTR = T_o / T_\infty$

Nozzle Geometry

Need to know:

Far-field spectral density of fluctuating pressure



Present Approach

- Equations are separated into a linear operator (the linearized Euler equations: LEE) and interaction terms (sources)
- Solution for the far field pressure written in terms of the vector Green's function for the LEE
- Pressure autocorrelation and spectral density are obtained
- Model introduced for turbulence statistics
- Shock cell represented in terms of its wavenumber content
- Expression obtained for spectral density in terms of quantities provided by a RANS CFD solution with a two-equation turbulence model

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Final Prediction Formula

Model for turbulent velocity cross correlation,

$$R^{v}(\mathbf{y},\boldsymbol{\eta},\tau)=K\exp\left[-|\tau|/\tau_s\right]\exp\left[-(\xi-\bar{u}_c\tau)^2/l^2\right]\exp\left[-\left(\eta^2+\zeta^2\right)^2/l_\perp^2\right]$$

where, $\boldsymbol{\eta}=(\xi,\eta,\zeta)$

Final formula for spectral density,

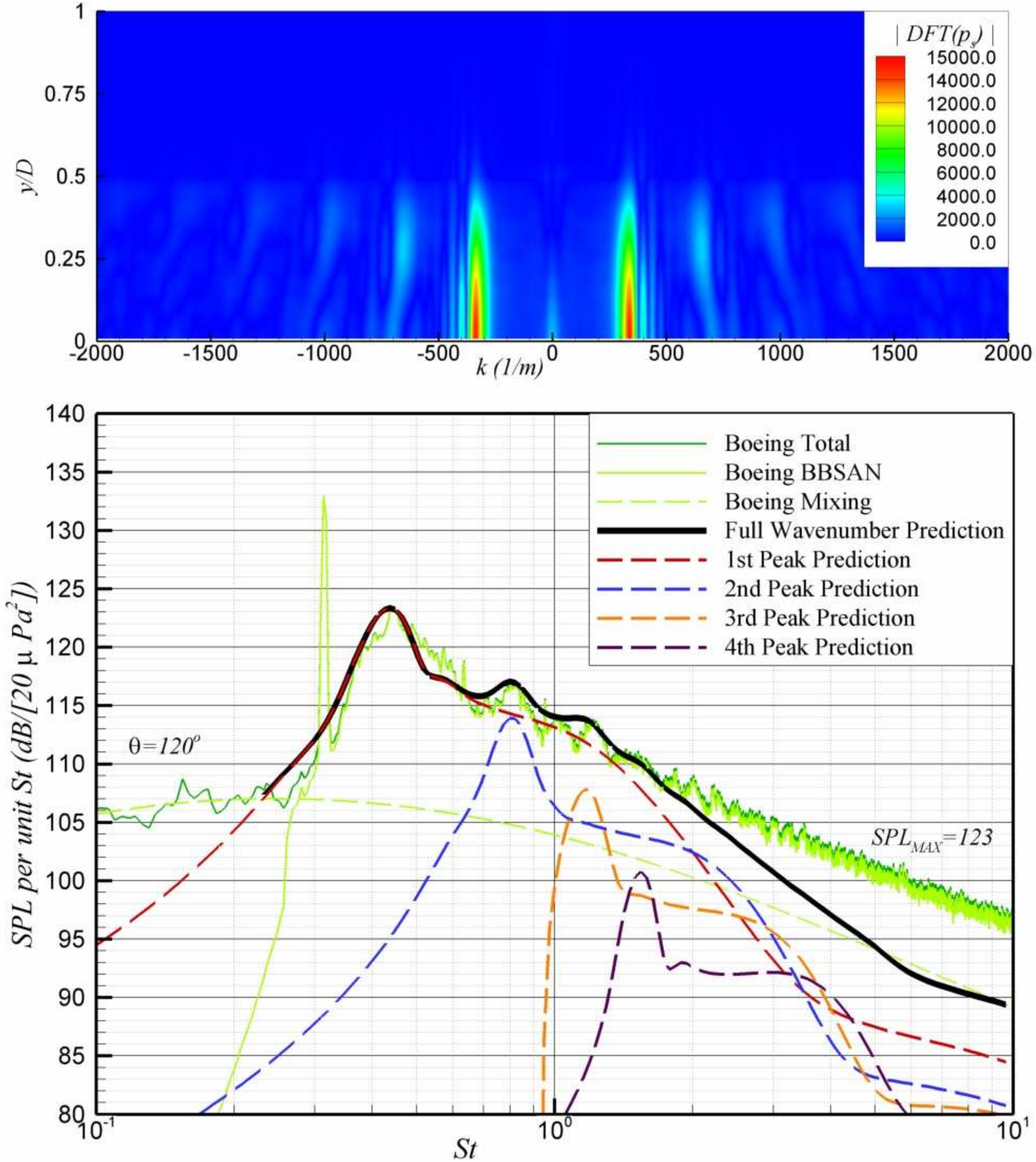
$$S(\mathbf{x},\omega)=\frac{1}{\pi\sqrt{\pi}a_\infty^4x^2}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\left\{\frac{Kl_\perp^2}{l\tau_s}p_s(\mathbf{y})\tilde{p}_s(k_1,y_2,y_3)\exp(ik_1y_1)\right.$$
$$\left.\times\frac{\omega^2\tau_s^2\left\{\exp\left[-l^2(k_1-\omega\cos\theta/a_\infty)^2/4-\omega^2l_\perp^2\sin^2\theta/4a_\infty^2\right]\right\}}{\left[1+(1-M_c\cos\theta+\bar{u}_ck_1/\omega)^2\omega^2\tau_s^2\right]}\right\}dk_1d\mathbf{y}$$

Wavenumber Region

$M_d = 1.0$

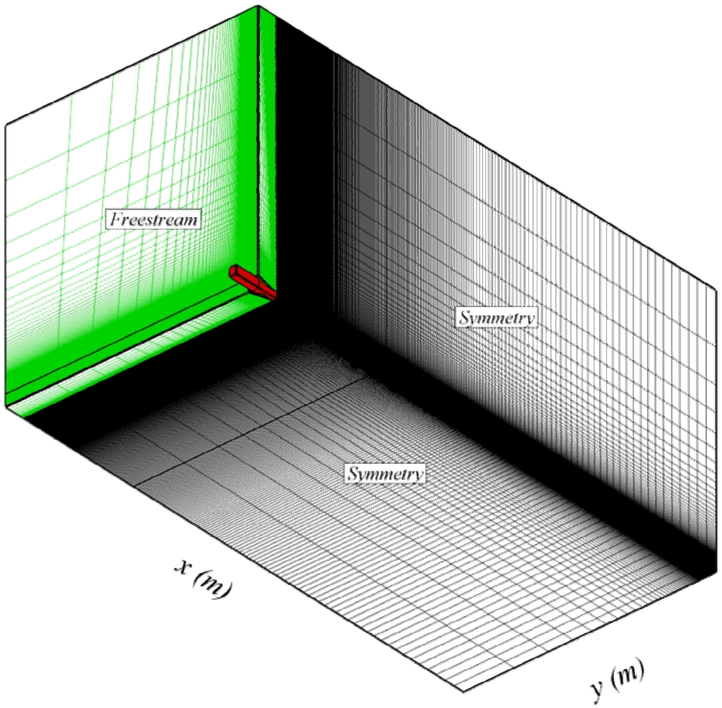
$M_j = 1.5$

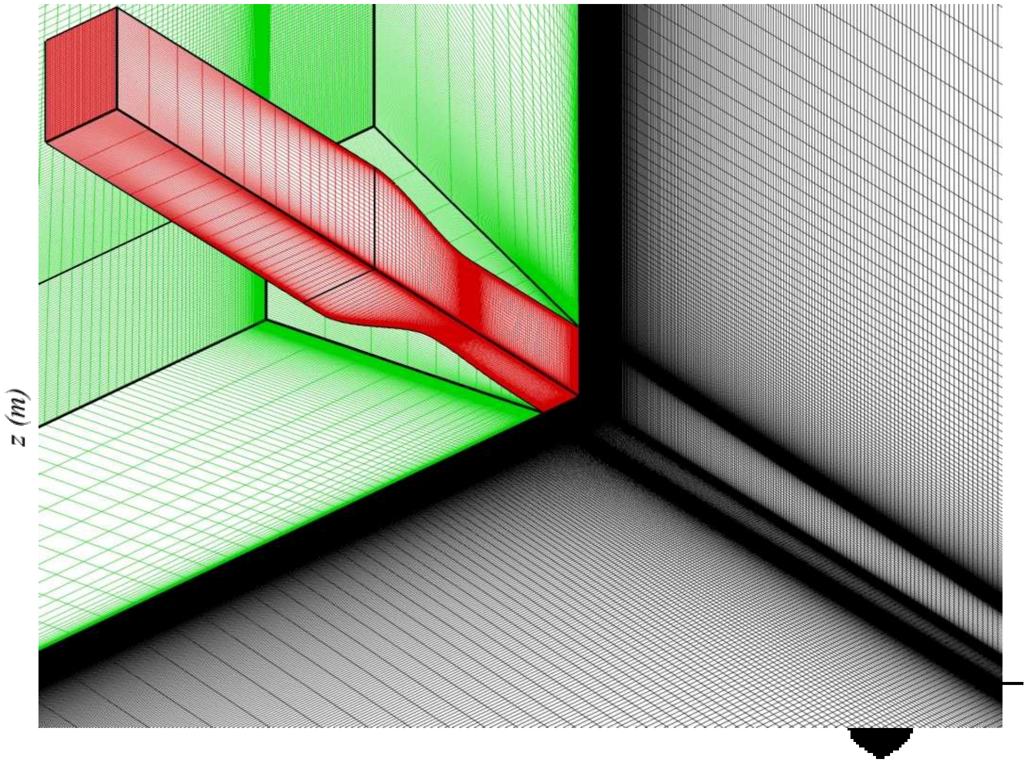
$TTR = 1.0$



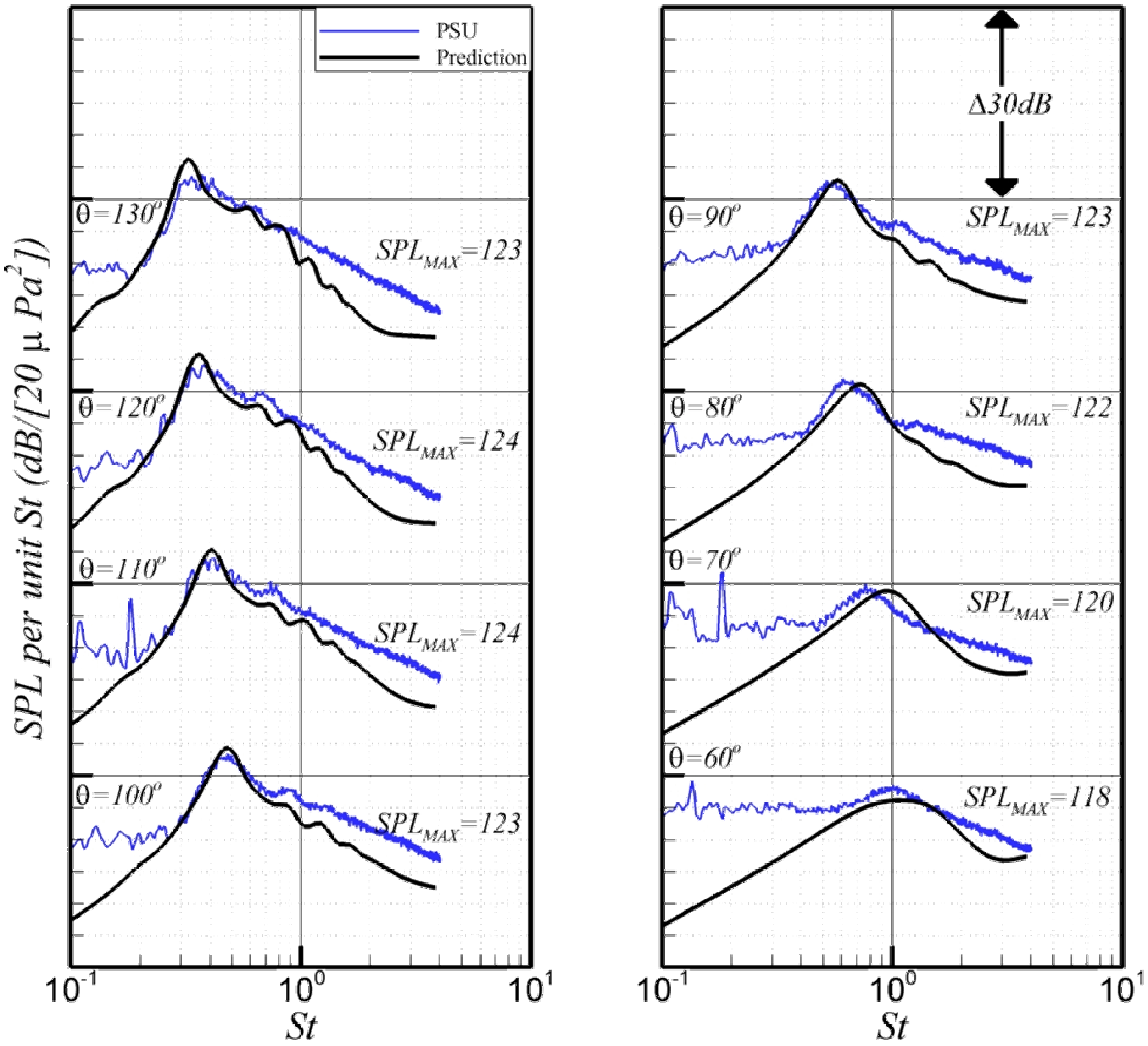
Rectangular Jet Operating Conditions

M_d	M_j	NPR	TTR	D_e (m)	D_{ej} (m)	f_c	β	T_j (K)	u_j (m/s)
1.50	1.30	2.77	1.00	0.01778	0.01693	22888	0.748	219.1	385.7
1.50	1.70	4.94	1.00	0.01778	0.01896	24499	0.800	185.8	464.5
1.50	1.30	2.77	2.20	0.01778	0.01693	33792	0.748	482.0	572.1
1.50	1.70	4.94	2.20	0.01778	0.01896	36334	0.800	408.7	688.9





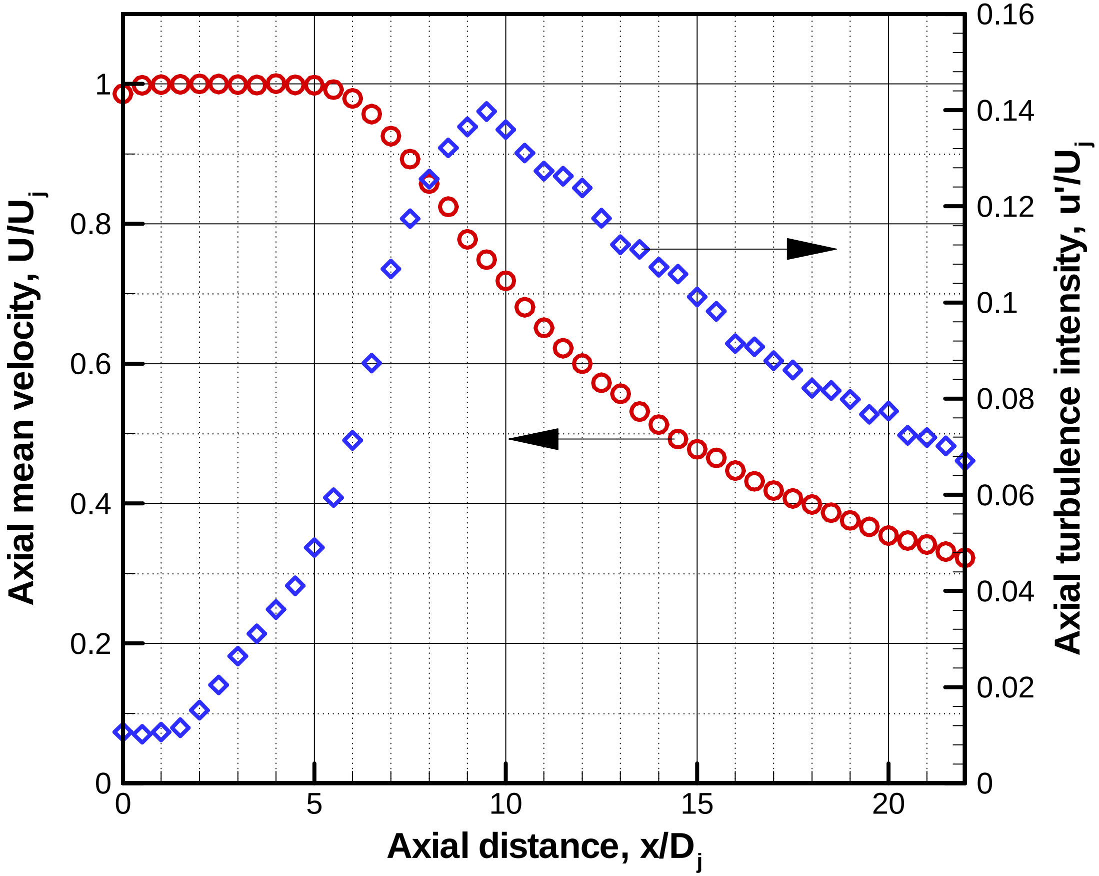
Rectangular Jet
$M_d = 1.5$ $M_j = 1.7$ $TTR = 1.0$



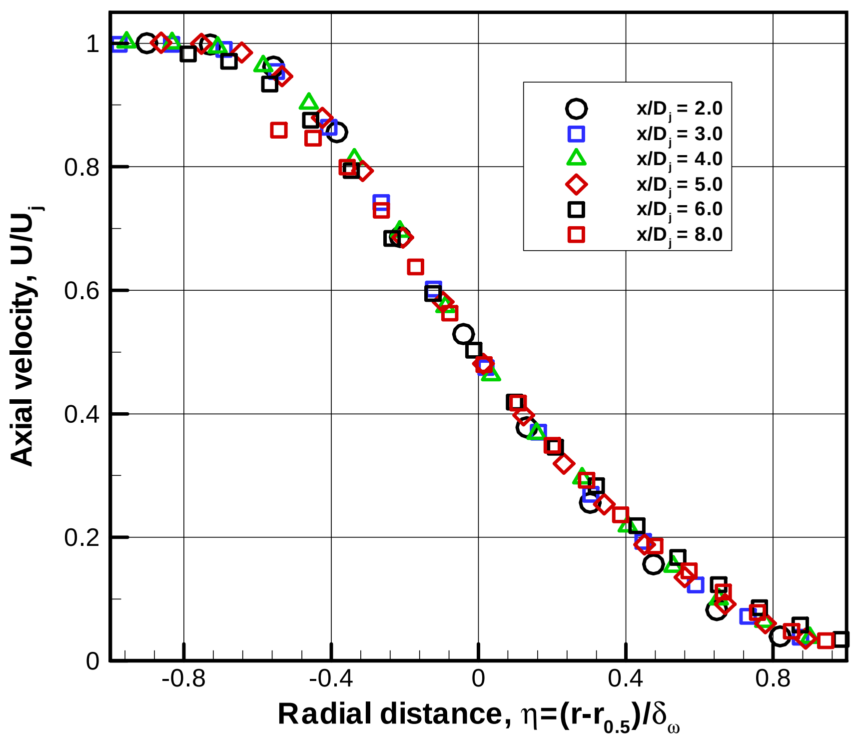
Summary

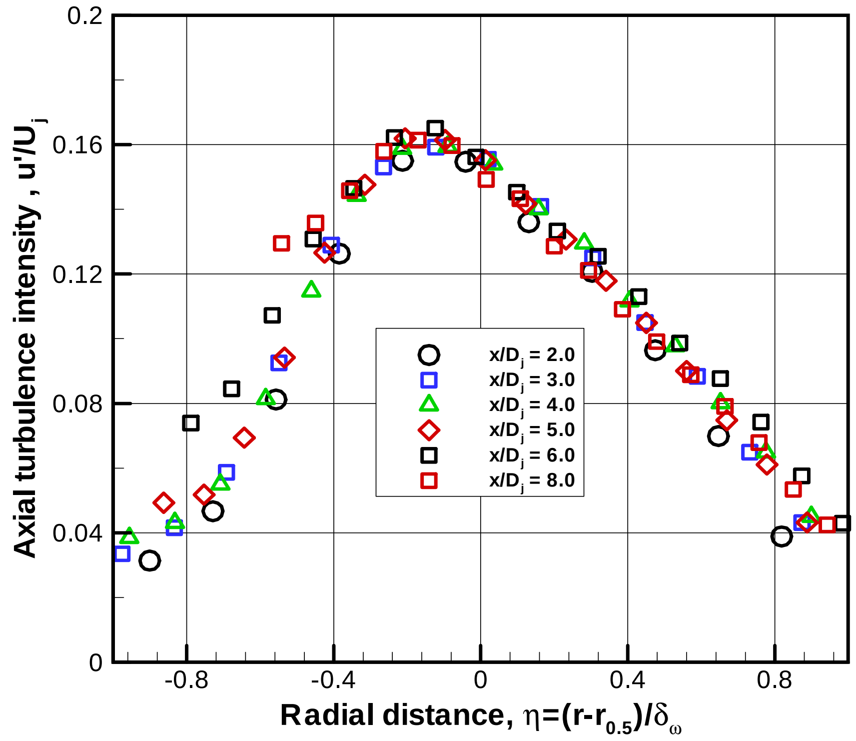
- Hot-wire measurements have played an important role in noise source modeling
- Other instrumentation clearly has a role
- Use of RANS to determine scales is problematic
- BBSAN can be predicted using an *acoustic analogy* and RANS

Centerline Traverse



Mean Flow Similarity





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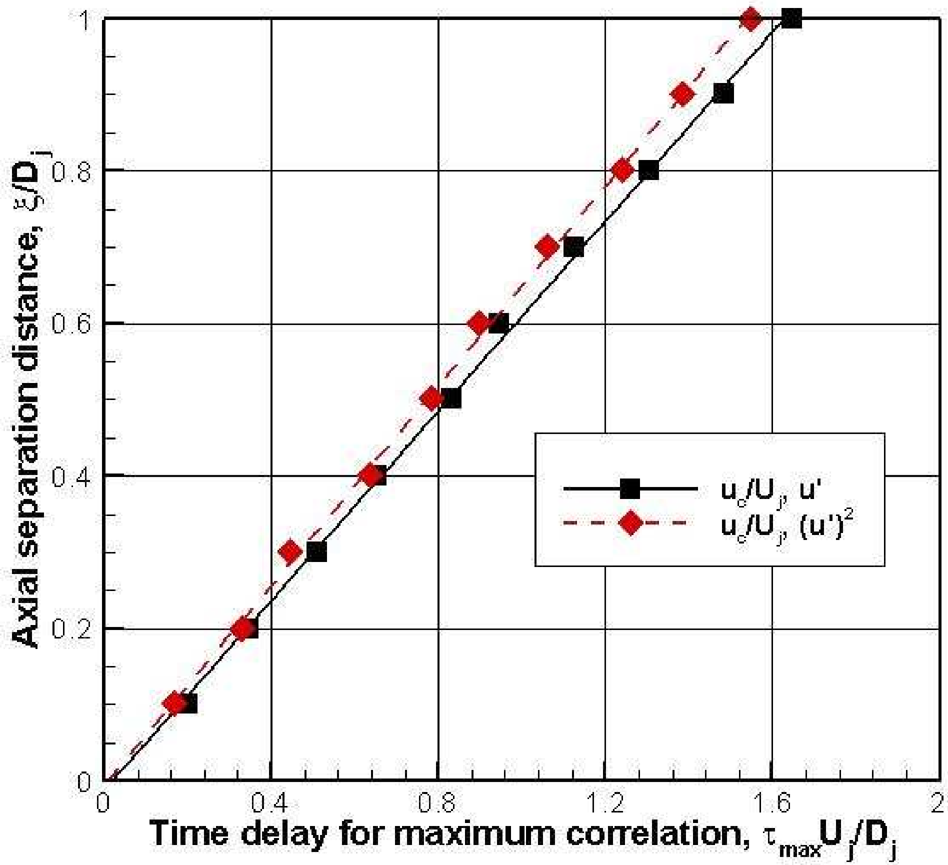
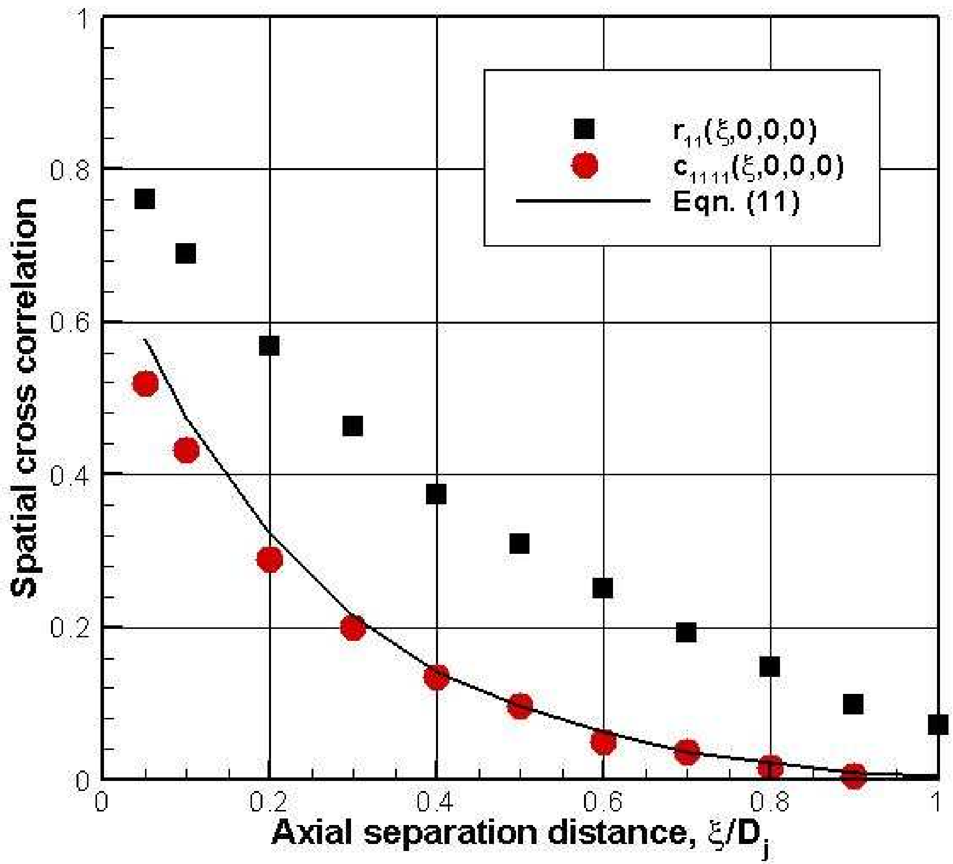
Convection Velocity

$x / D_j = 5,\ r / D_i = 0.5$



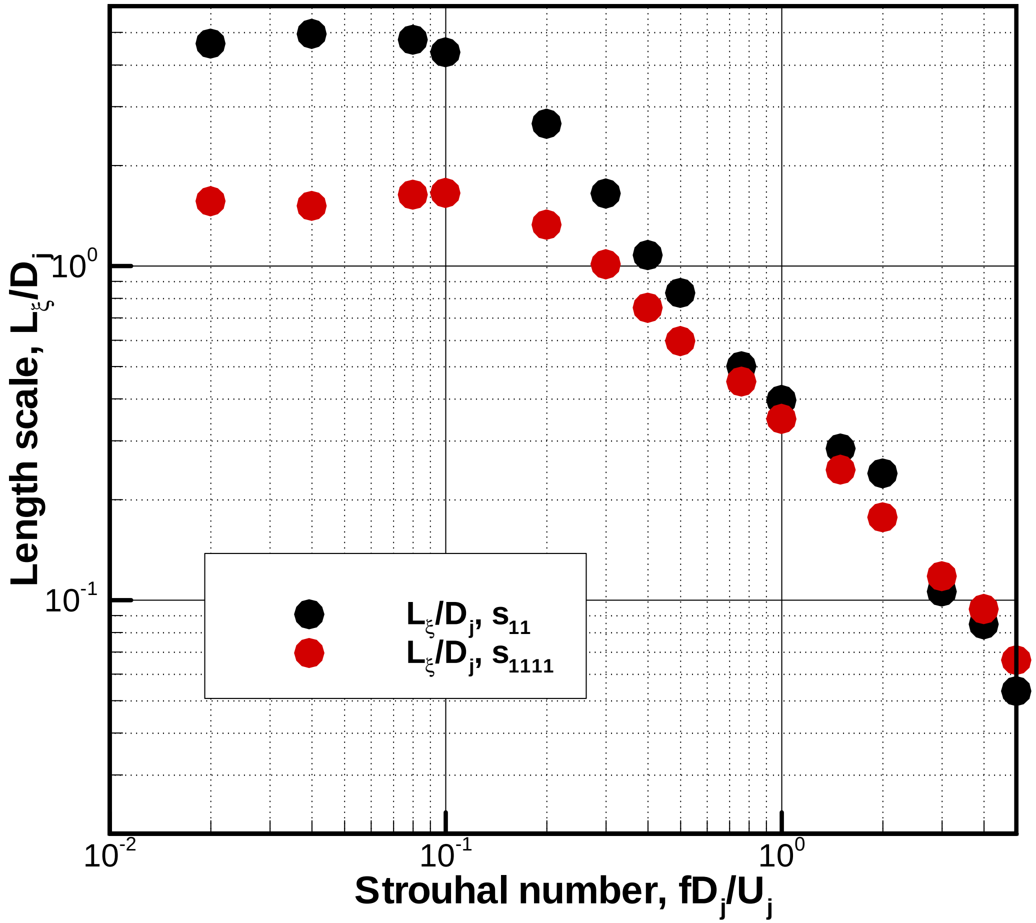
Spatial Cross Correlations

$x / D_j = 9,\ r / D_j = 0.5$



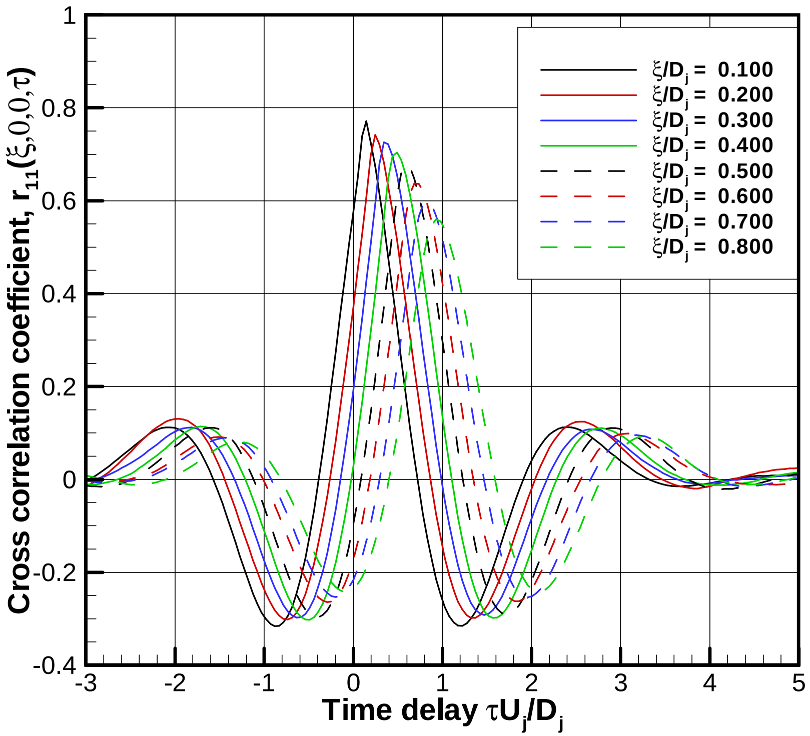
Length Scales

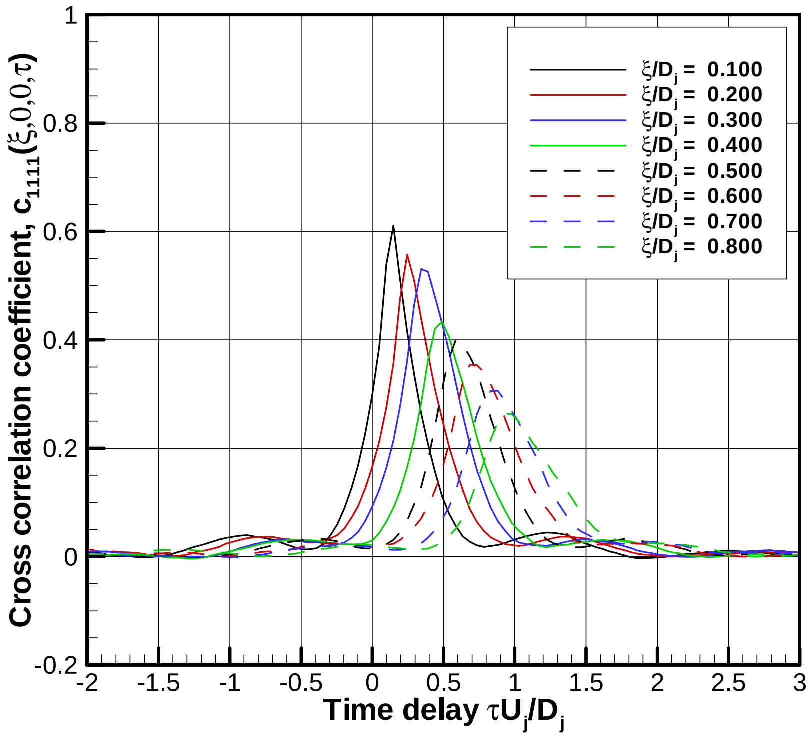
$x/D_j = 9,\ r/D_j = 0.5$



Cross Correlation Coefficients

$x/D_j = 5,\ r/D_j = 0.0$





$r_{11}(\mathbf{x}, \boldsymbol{\eta}, \tau)$

$c_{1111}(\mathbf{x}, \boldsymbol{\eta}, \tau)$

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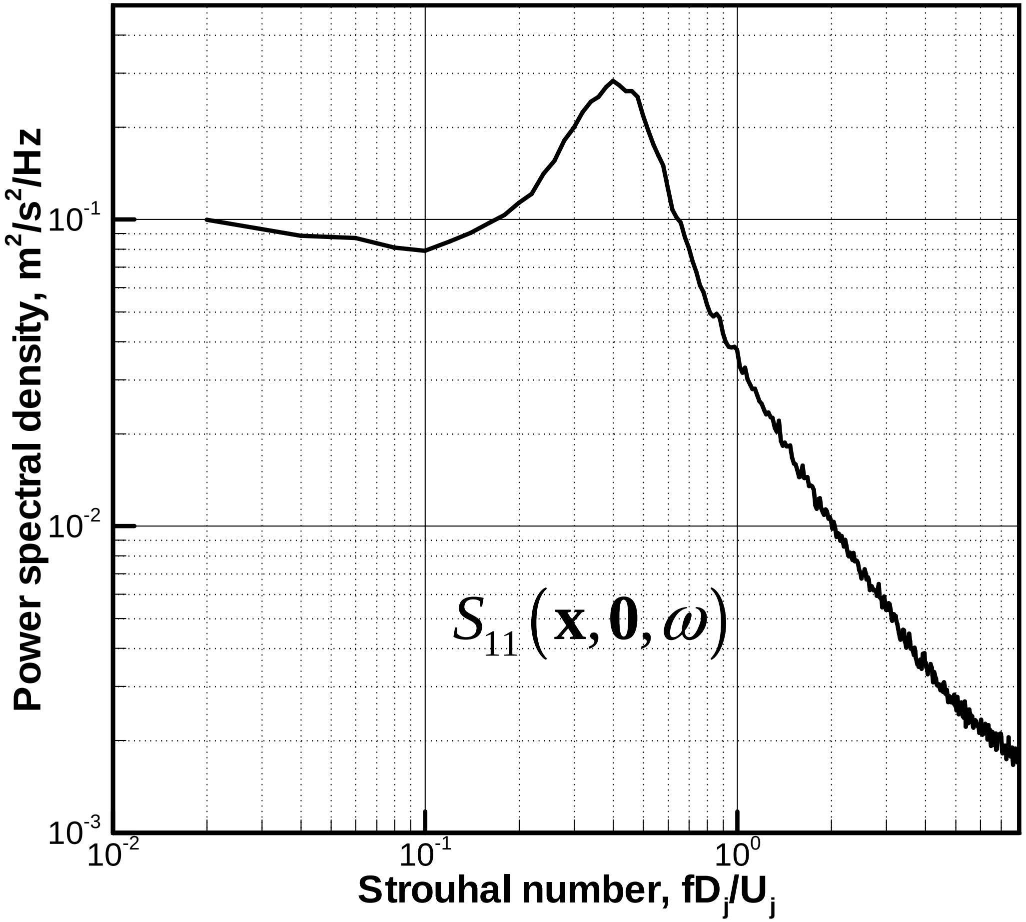
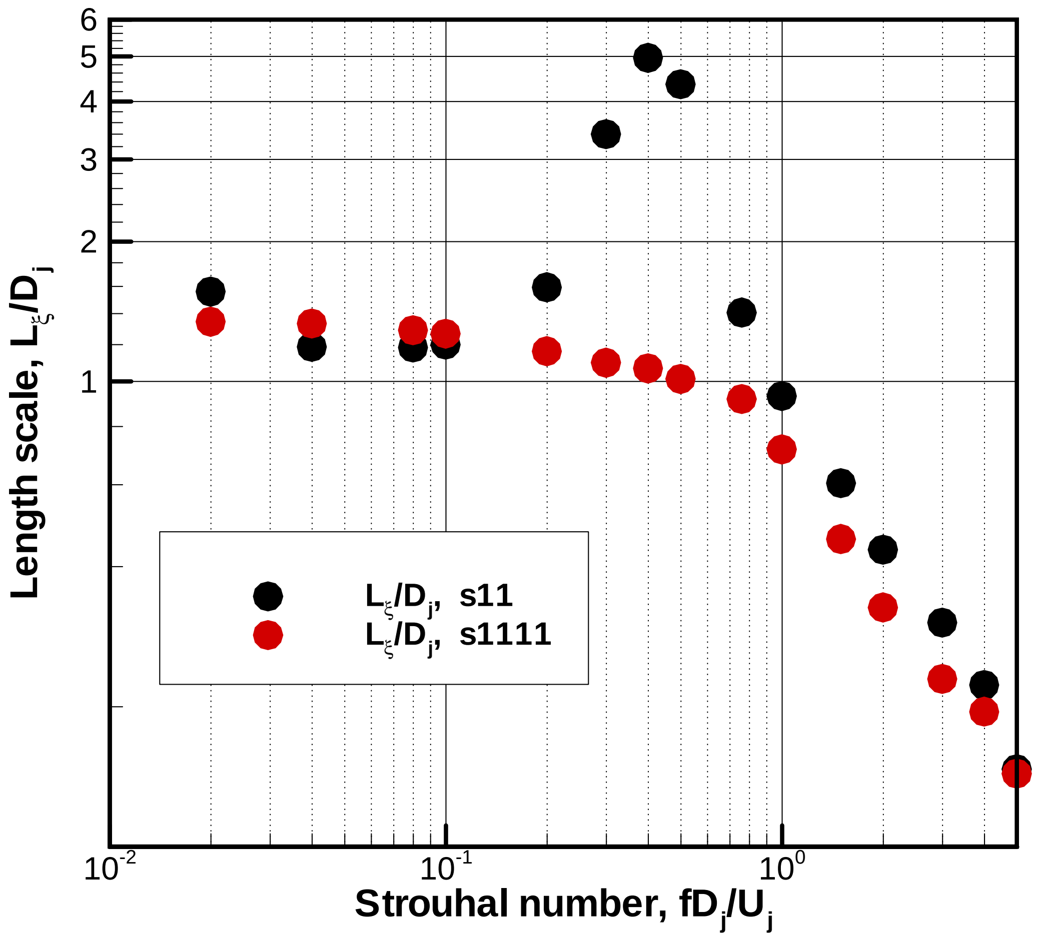
Power Spectral Density

$x/D_j = 5, \; r/D_j = 0.0$



Length Scales

$x / D_j = 5,\ r / D_j = 0.0$



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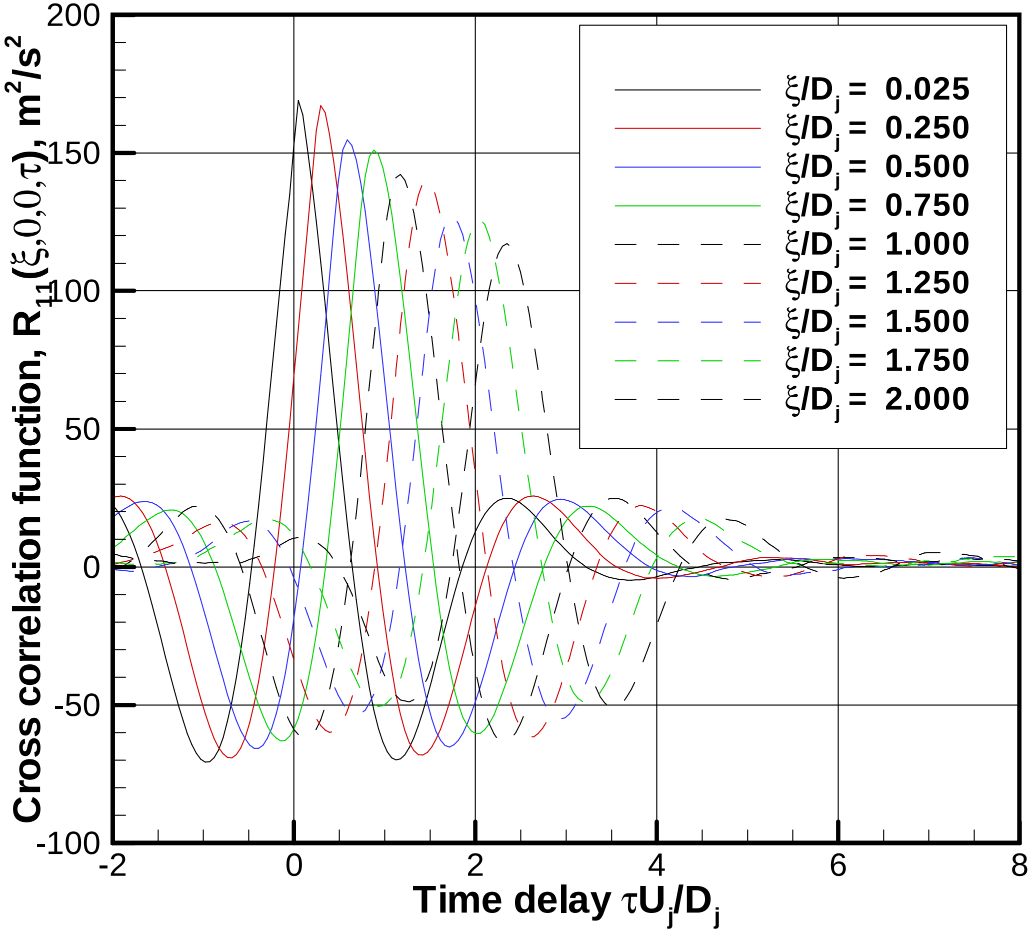
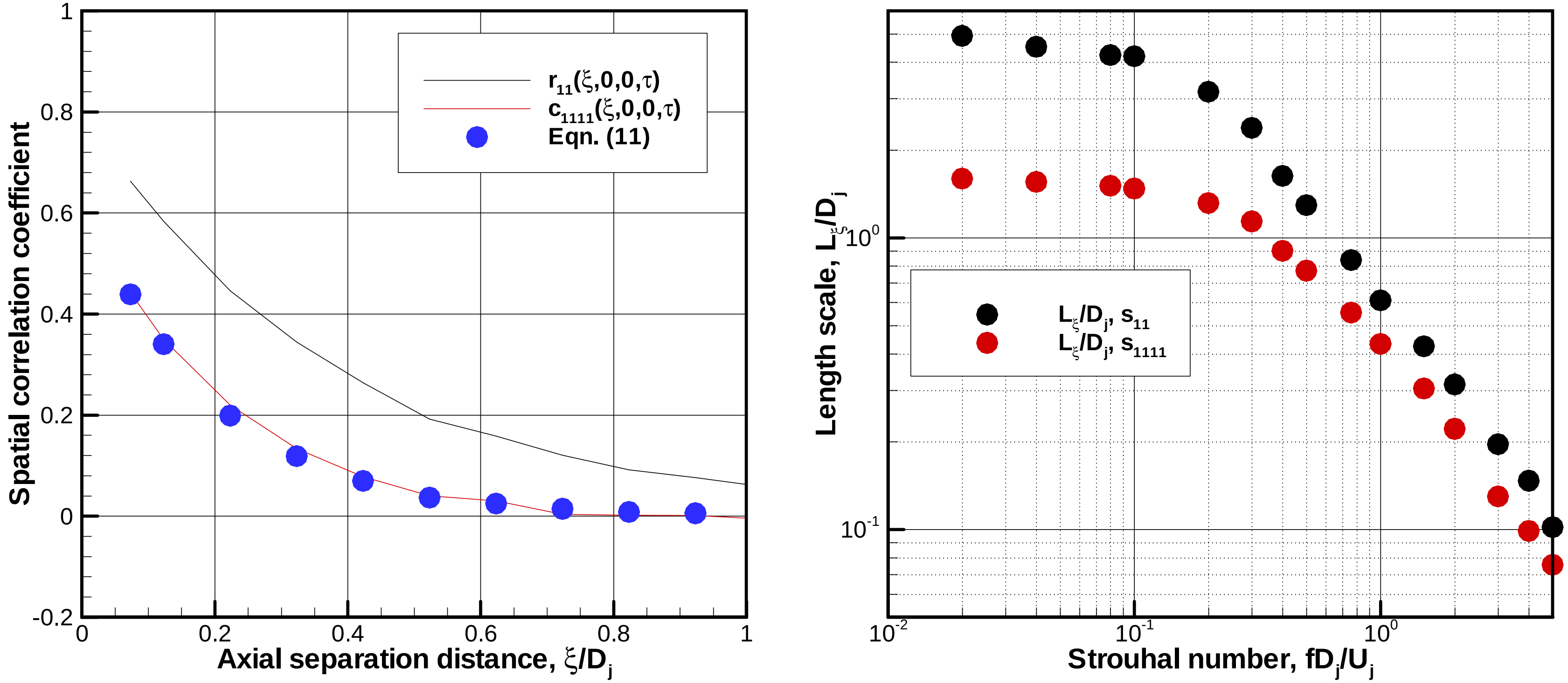
Dimensional Cross Correlation

$x/D_j = 5,\ r/D_j = 0.0$

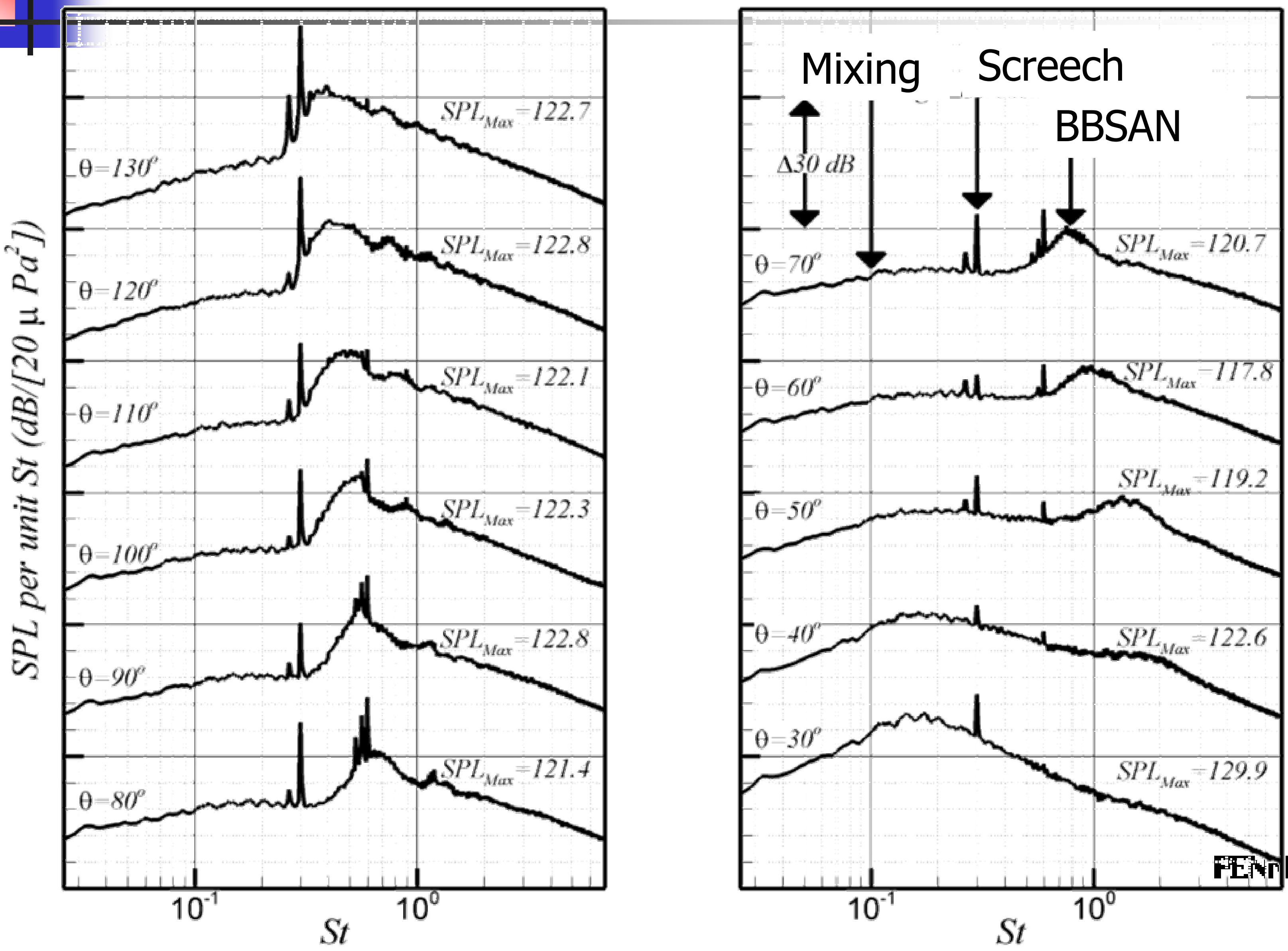


Downstream Centerline Measurements

$x / D_j = 9, \; r / D_j = 0.0$



Supersonic Jet Noise Sources



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Present Approach

- Based on the Euler equations in terms of the logarithm of pressure and velocity
- Variables separated into four components*
  - Long time average
  - Shock cell structure
  - Coherent turbulent fluctuations
  - Fluctuations associated with the interaction of the turbulent fluctuations with the shock cells – this is the Broadband Shock-Associated Noise (BBSAN)

**This follows Tam's general formulation*

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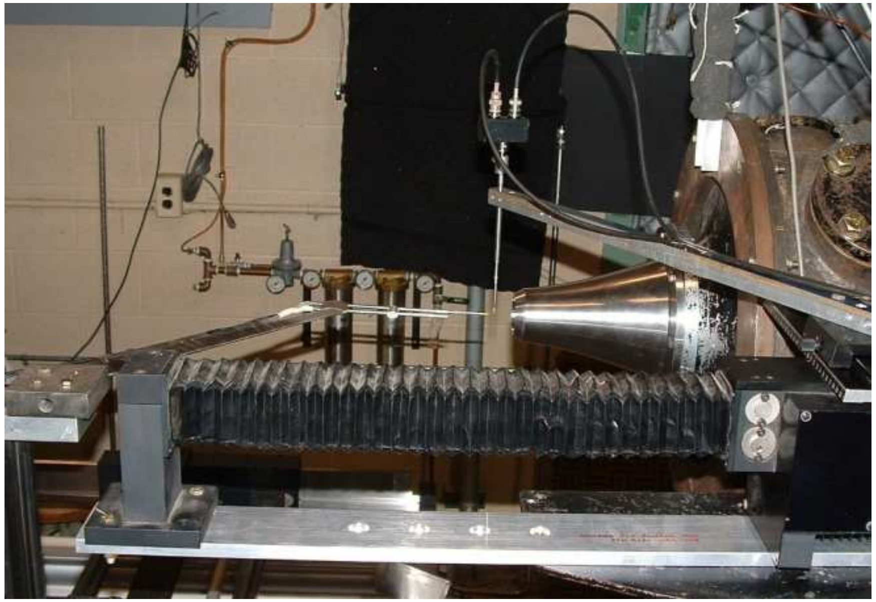
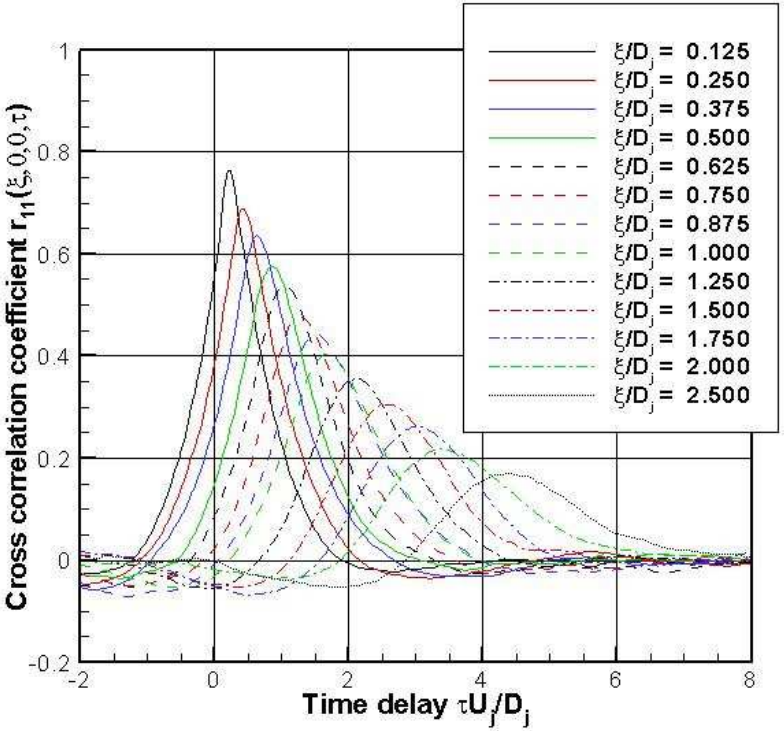
Two Cross-Wires

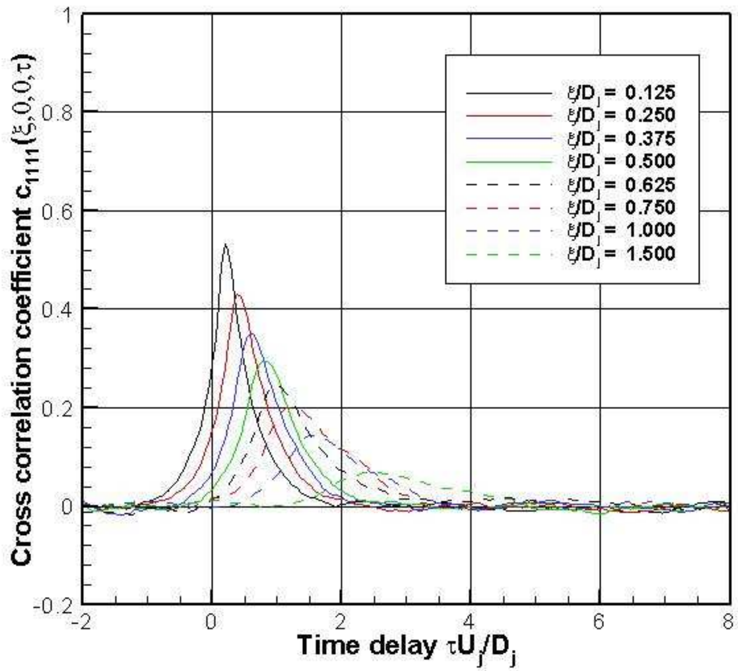


Cross Correlation Coefficients

$x / D_j = 5,\ r / D_j = 0.5$



$r_{11}(\mathbf{x}, \boldsymbol{\eta}, \tau)$



$c_{1111}(\mathbf{x}, \boldsymbol{\eta}, \tau)$

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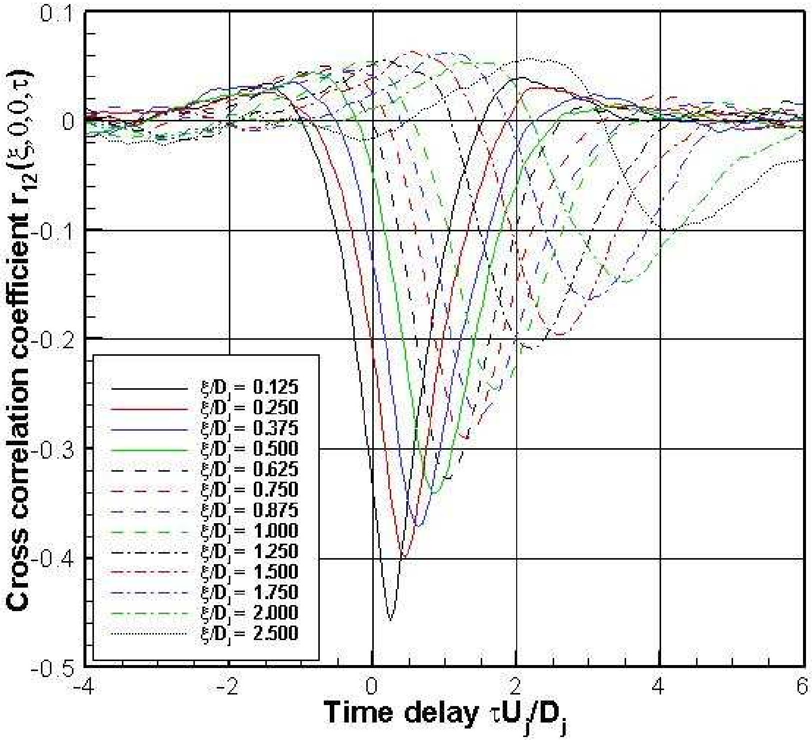
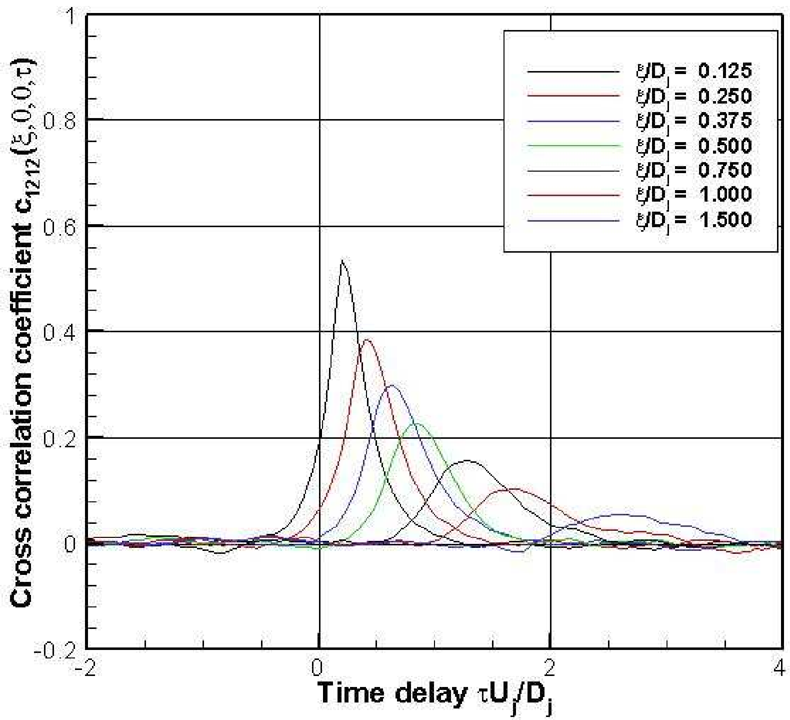
Cross Correlation Coefficients

$x/D_j = 5, \ r/D_j = 0.5$



$r_{12}(\xi,0,0,\tau)$



$c_{1212}(\xi,0,0,\tau)$