New issues in LES of turbulent flows: *multiphysics and uncertainty modelling*

Pierre Sagaut

Institut Jean Le Rond d'Alembert

Université Pierre et Marie Curie- Paris 6, France

http://www.lmm.jussieu.fr/~sagaut

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An example of complex flow

Motivation

CAD definition
&
mesh generation

Simplified/incomplete
boundary conditions

Physical modelling
errors

**Complex system simulation
with uncertainties**

Unknown/varying
flow parameters

Discretization errors

Outline

1. Response surface via Polynomial Chaos

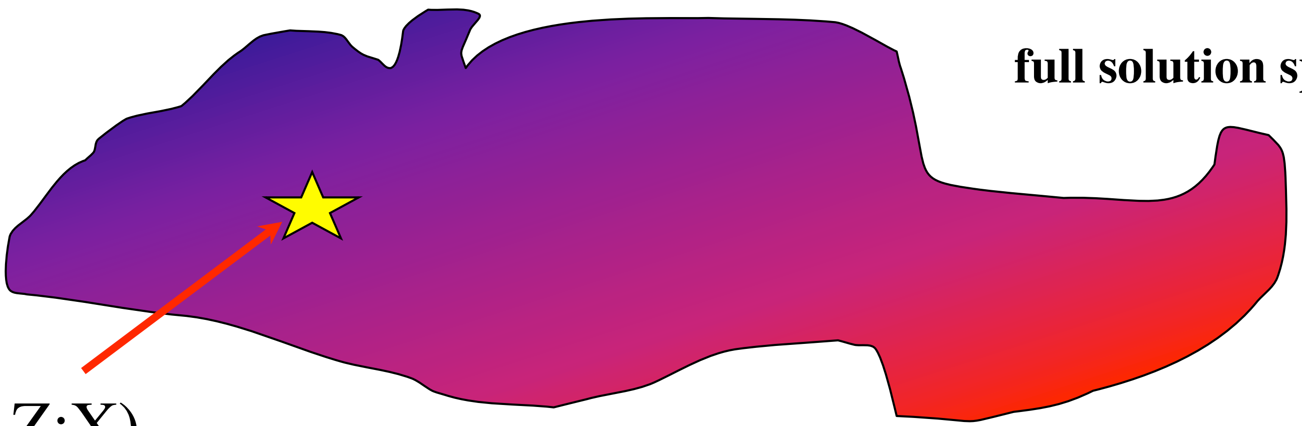
2. Uncertainty in subgrid model calibration

3. Putting error/uncertainty bars on LES results

Uncertain system description

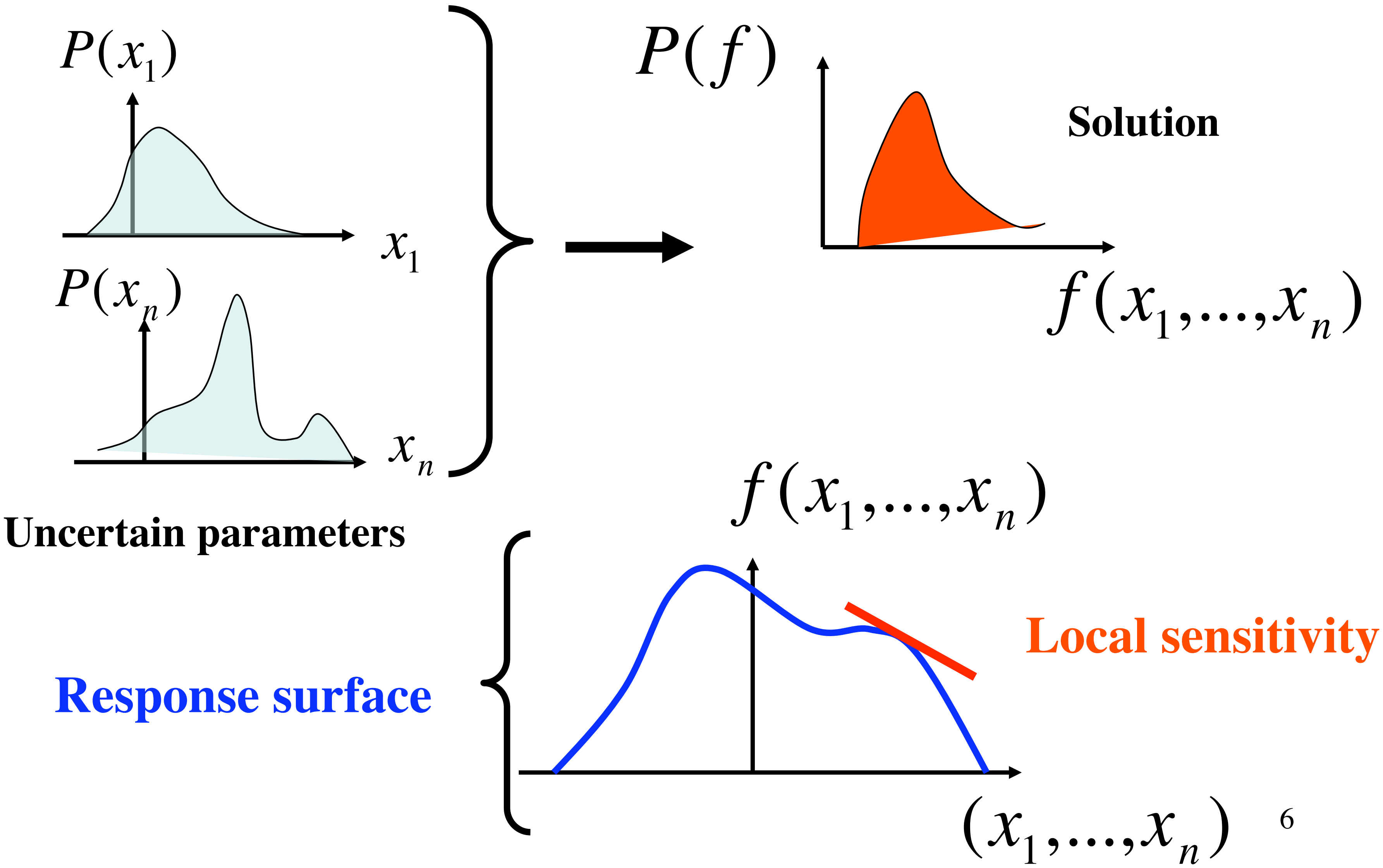
What is the space of solutions spanned by uncertain parameters ?

$$f = f(t, Z; x_1, x_2, \ldots, x_n) \qquad \frac{df}{dt} = G(t, Z; x_1, \ldots, x_n)$$



Uncertain system description (cont'd)

Probability Density Function



Generalized Polynomial Chaos

- Wiener (1938) : Homogeneous Chaos Theory
- Solution with uncertain random parameter

$$u(x,t;\xi) = \sum_{i=1,N} u_i(x,t)\Psi_i(\xi)$$

Uncertain parameter

Orthogonal polynomial basis functions

⇨ Pseudo-spectral Galerkin projection of spanned solutions

Generalized Polynomial Chaos (cont'd)

Distribution	Optimal polynomial basis
Gaussian	Hermite
Gamma	Laguerre
Beta/uniform	Legendre
Binomial	Krawtchouk

gPC post-processing

$$\begin{cases} \langle u(x,t) \rangle = u_0(x,t) \\ Var\left(u(x,t)\right) = \sum\limits_{i=1,N} \left[u_i^2(x,t) \langle \psi_i^2 \rangle \right] \end{cases}$$

Uncertain subgrid model calibration

(Lucor, Meyers & Sagaut, J. Fluid Mech, 2007)

- **Classical Smagorinsky-Lilly model**

$$\nu_t = C_\infty^2 \Delta^2 \left\langle 2\bar{S}_{ij}\bar{S}_{ij} \right\rangle^{1/2} \quad C_\infty = \frac{1}{\pi}\left(\frac{2}{3K_0}\right)^{3/4} \sim 0.17 - 0.18$$

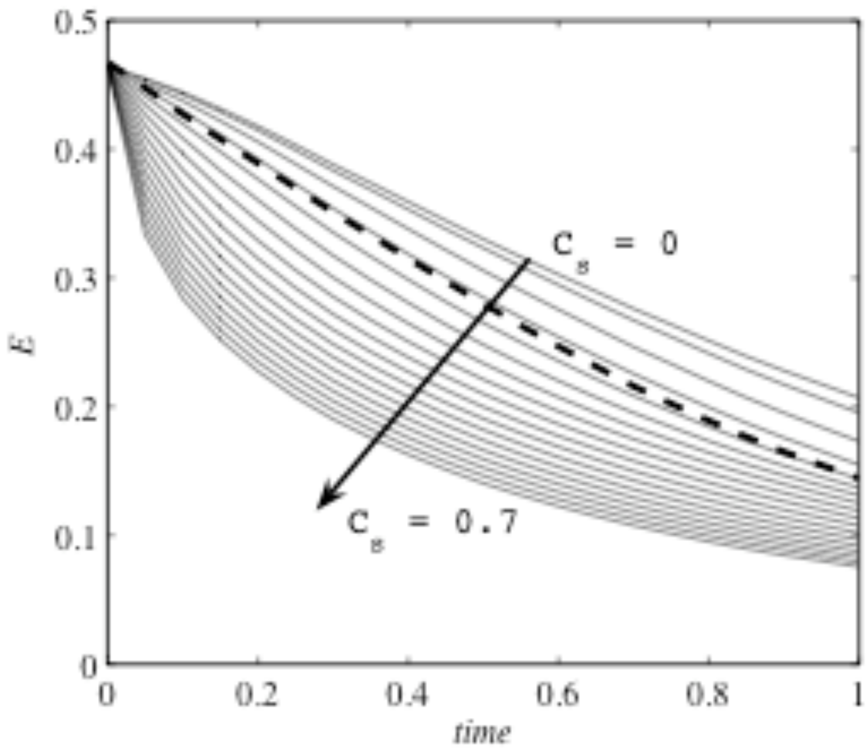
- **Exact Smagorinsky constant expression** *(Meyers & Sagaut, J. Fluid Mech, 2007)*

$$C(L/\Delta, Re_L) = \frac{C_\infty}{\gamma}\Phi^{-3/4}\sqrt{1 - \left(\frac{\gamma\eta}{C_\infty\Delta}\right)^{4/3}\Phi}$$

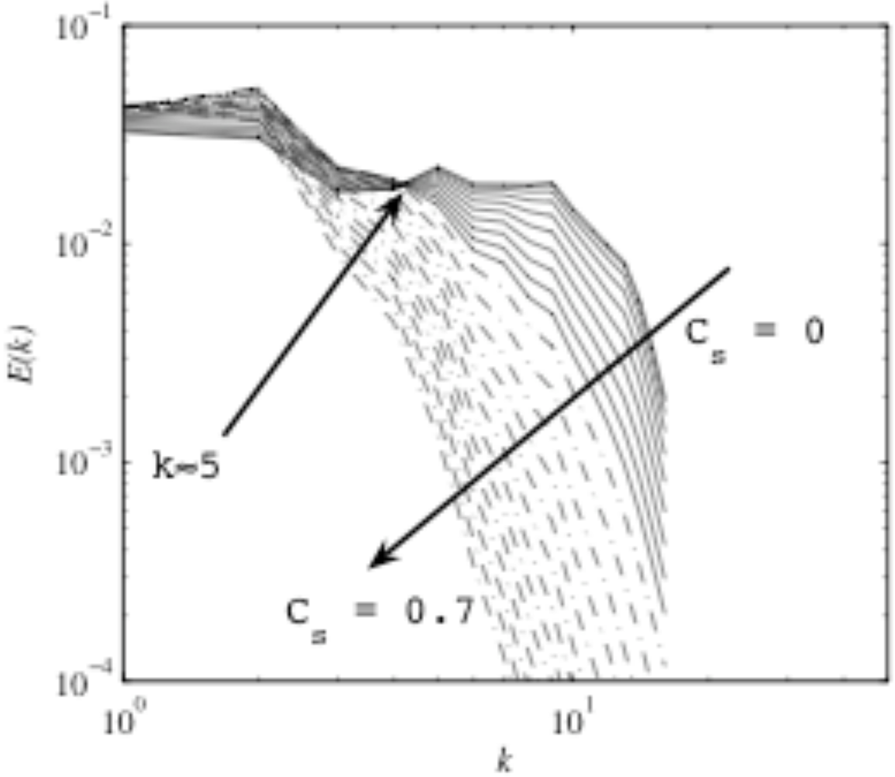
Case-dependent parameters

$$\begin{cases} \Phi(L/\Delta, Re_L) = \dfrac{4}{3}\dfrac{1}{(\gamma\pi L/\Delta)^{4/3}}\displaystyle\int_0^\infty x^{1/3} G^2(x/L) f_L(x) f_\eta(x Re_L^{-3/4}) dx \\ \gamma = \dfrac{\Delta}{\pi}\left(\dfrac{4}{3}\displaystyle\int_0^\infty x^{1/3} G^2(x) dx\right)^{3/4} \end{cases}$$

Decaying HIT with uncertain Cs

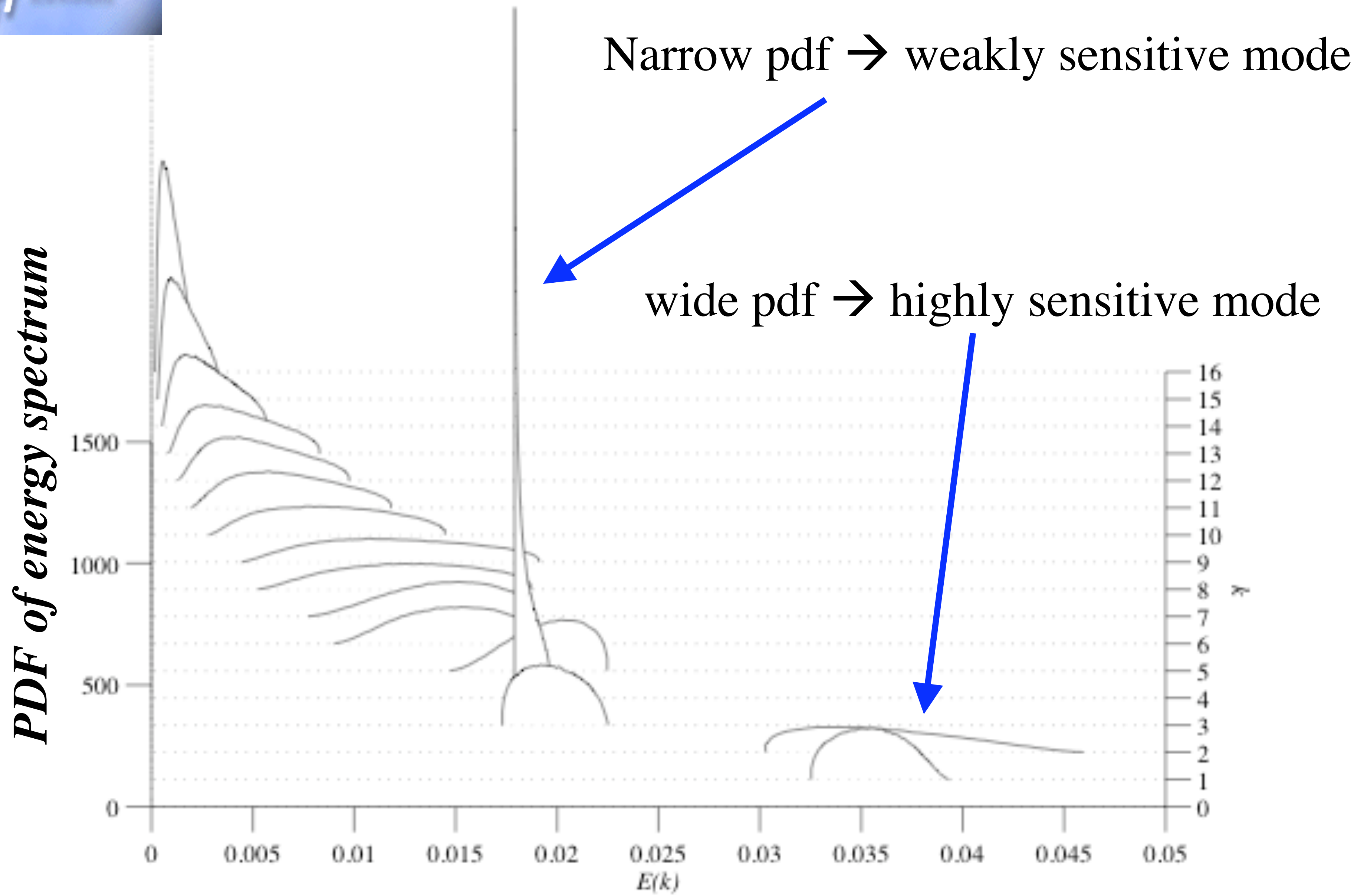


TKE decay



Final TKE spectrum

Decaying HIT with uncertain Cs (cont'd)



Cs as a stochastic variable

Meyers-Meneveau spectrum shape

$$E(k) = C_K \varepsilon^{2/3} k^{-5/3} (kl)^{-\beta} f_L(kL) f_\eta(k\eta)$$

$$f_\eta(k\eta) = \exp(-\alpha_1 k\eta)\left(1 + \frac{\alpha_2 (k\eta/\alpha_4)^{\alpha_3}}{1 + (k\eta/\alpha_4)^{\alpha_3}}\right)$$

$$f_L(kL) = \left(\frac{kL}{[(kL)^p + \alpha_5]^{1/p}}\right)^{5/3+\beta+2}$$

Uncertain parameters!

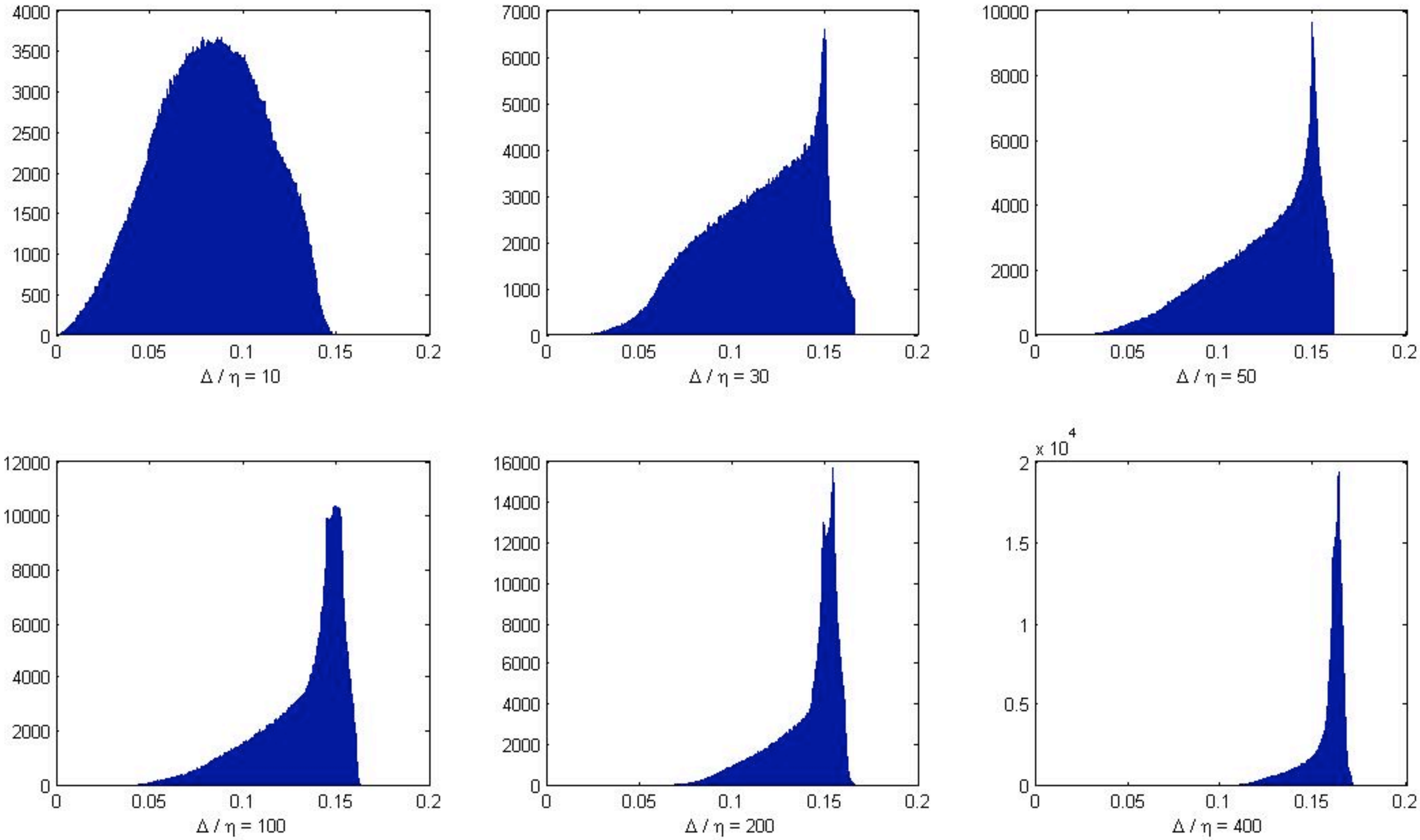
Cs as a stochastic variable

Constrained problem with

$$
\begin{aligned}
\int_0^{+\infty} x^{-5/3-\beta} C_K f_L(k\eta) f_\eta(k\eta Re^{-3/4}) d(k\eta) &= 1 \\
\int_0^{+\infty} x^{1/3-\beta} Re^{-3\beta/4} C_K f_L(k\eta Re^{3/4}) f_\eta(k\eta) d(k\eta) &= 1/2 \\
\int_0^{+\infty} x^{7/3-\beta} Re^{-3\beta/4} C_K f_L(k\eta Re^{3/4}) f_\eta(k\eta) d(k\eta) &= -\frac{7S_3}{12\sqrt{15}}
\end{aligned}
$$

Pdf of Cs

$$Re_\lambda = 732$$

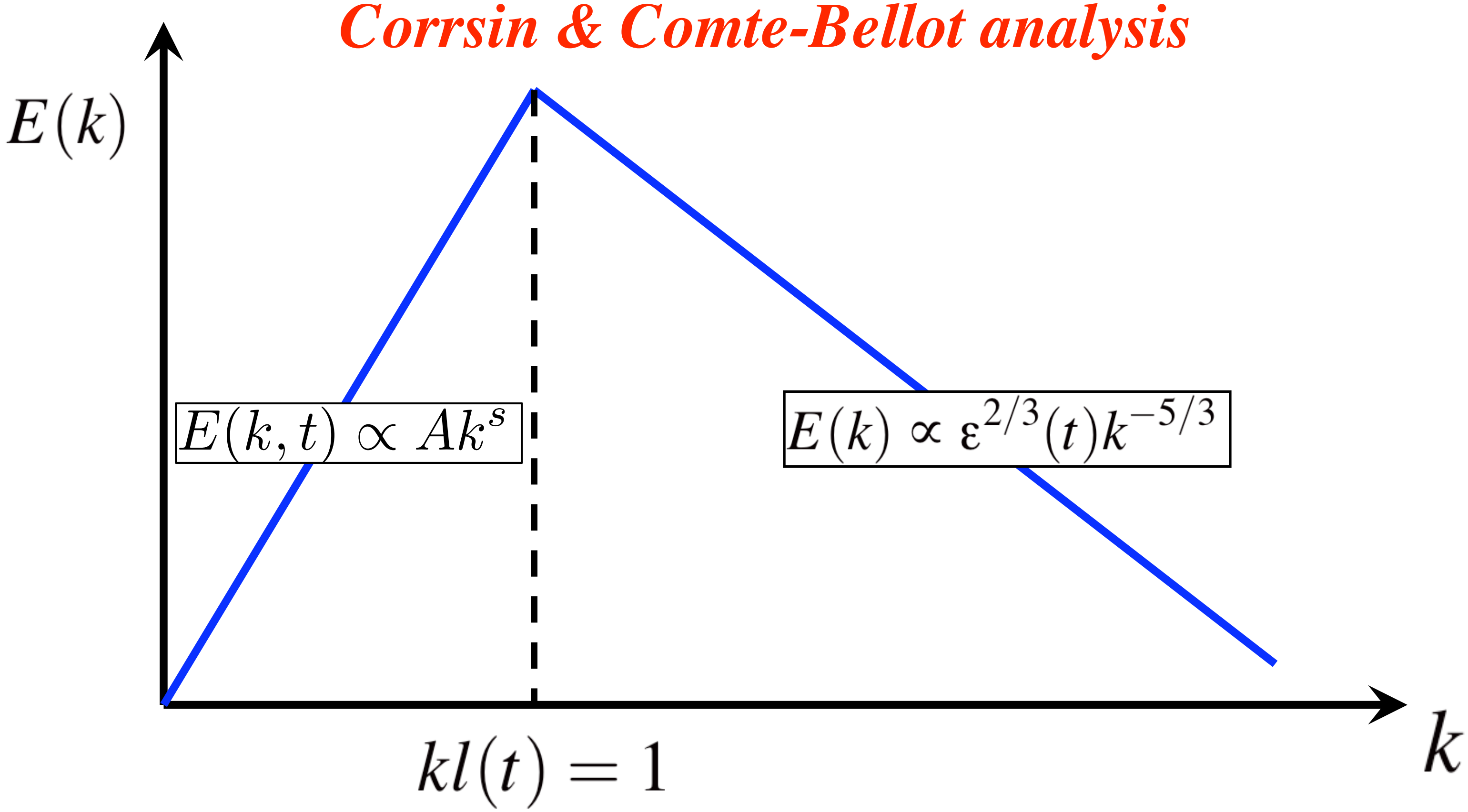


Stochastic Cs analysis

Cs statistical moments	$\Delta/\eta = 10$	$\Delta/\eta = 30$	$\Delta/\eta = 50$	$\Delta/\eta = 100$	$\Delta/\eta = 200$	$\Delta/\eta = 400$
mean value	0.083158	0.12272	0.13368	0.13905	0.14495	0.15761
variance	$5.21e^{-4}$	$3.33e^{-4}$	$3.77e^{-4}$	$2.79e^{-4}$	$1.88e^{-4}$	$1e^{-4}$
partial variances						
α_2	$5.19e^{-4}$	$1.06e^{-4}$	$2.64e^{-5}$	$8.93e^{-6}$	$4.55e^{-6}$	$1.42e^{-6}$
α_3	$1.39e^{-6}$	$1.95e^{-4}$	$2.94e^{-4}$	$2.26e^{-4}$	$1.56e^{-4}$	$8.69e^{-5}$
α_4	$8.79e^{-34}$	$1.76e^{-33}$	$2.19e^{-33}$	$2.51e^{-33}$	$2.84e^{-33}$	$2.77e^{-33}$
$\alpha_2\alpha_3$	$4.71e^{-7}$	$3.21e^{-5}$	$5.71e^{-5}$	$4.41e^{-5}$	$2.74e^{-5}$	$1.2e^{-5}$
$\alpha_2\alpha_4$	$3.96e^{-35}$	$1.14e^{-35}$	$4.19e^{-36}$	$6.32e^{-37}$	$5.35e^{-37}$	$3.3e^{-36}$
$\alpha_3\alpha_4$	$5.85e^{-37}$	$1.27e^{-35}$	$4.11e^{-35}$	$5.55e^{-36}$	$2.38e^{-35}$	$1.56e^{-35}$
deterministic mean value	0.07857	0.1252	0.14042	0.1475	0.15173	0.16211

Break: uncertain grid turbulence decay

Corrsin & Comte-Bellot analysis



$$
\begin{aligned}
A &\propto 1 \\
L(t) &\propto t^{2/(3+s)} \\
K(t) &\propto t^{-2(1+s)/(3+s)} \\
Re(t) &\propto t^{(1-s)/(3+s)}
\end{aligned}
$$

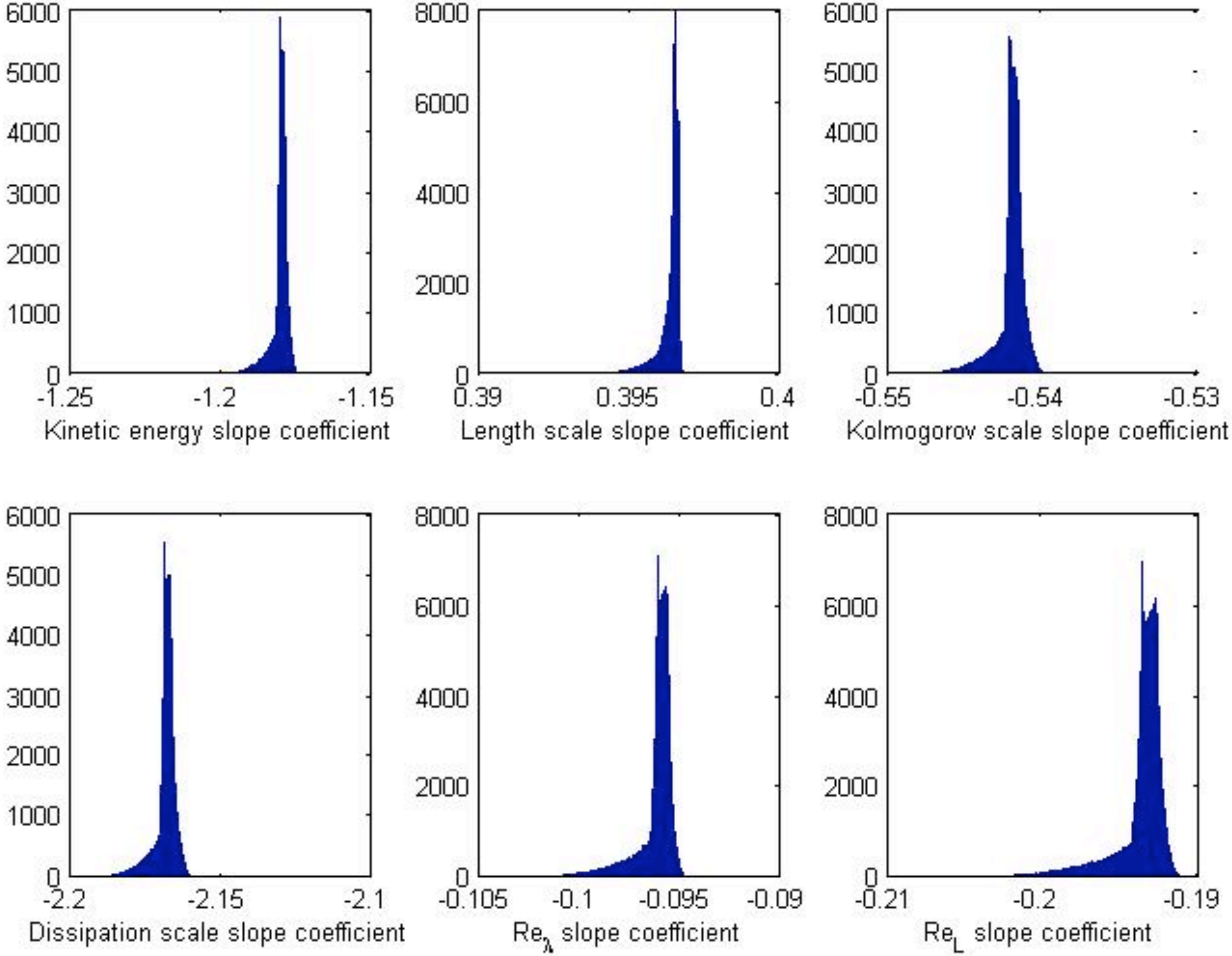
Break: uncertain grid turbulence decay

	$s=1$	$s=2$	$s=3$	$s=4$	$s=+\infty$
$\mathcal{K}(t)$	$\propto t^{-1}$	$\propto t^{-6/5}$	$\propto t^{-4/3}$	$\propto t^{-10/7}$	$\propto t^{-2}$
$\varepsilon(t)$	$\propto t^{-2}$	$\propto t^{-11/5}$	$\propto t^{-7/3}$	$\propto t^{-17/7}$	$\propto t^{-3}$
$L(t)$	$\propto t^{1/2}$	$\propto t^{2/5}$	$\propto t^{1/3}$	$\propto t^{2/7}$	Cste
$Re_L(t)$	Cste	$\propto t^{-1/5}$	$\propto t^{-1/3}$	$\propto t^{-3/7}$	$\propto t^{-1}$

Saturation effect (bounded physical domain):

$$\begin{aligned} A &\propto 1 \\ L(t) &\propto 1 \\ \mathcal{K}(t) &\propto t^{-2} \\ Re(t) &\propto t^{-1} \end{aligned}$$

EDQNM/gPC analysis (Saffman spectrum)



Break: uncertain grid turbulence decay

Isotropic turbulence decay exponent:

$$n = \begin{cases} -\dfrac{(s+1)}{2} & Re \ll 1 \\ -2\dfrac{(s+1)}{(s+3)} & Re \gg 1 \end{cases}, \quad (1 \leq s \leq 4)$$

Corresponding parameter tuning (k-ε model):

$$C_{\varepsilon_2} = \begin{cases} 1 + \dfrac{2}{(s+1)} & Re \ll 1 \\ 1 + \dfrac{(s+3)}{2(s+1)} & Re \gg 1 \end{cases}, \quad (1 \leq s \leq 4)$$

Putting error bars on LES data

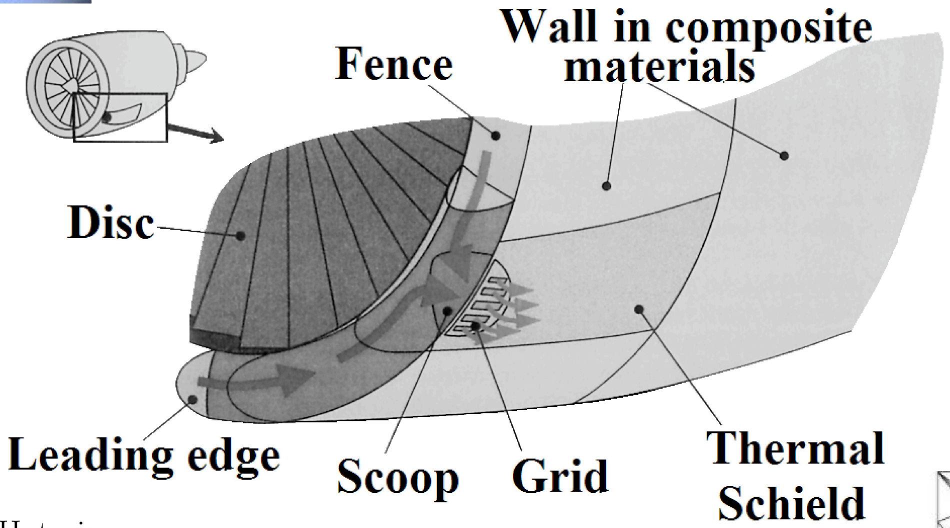
- In complex configurations:
  - optimal values of subgrid models are not known
  - best tuning of artificial viscosity parameter not known

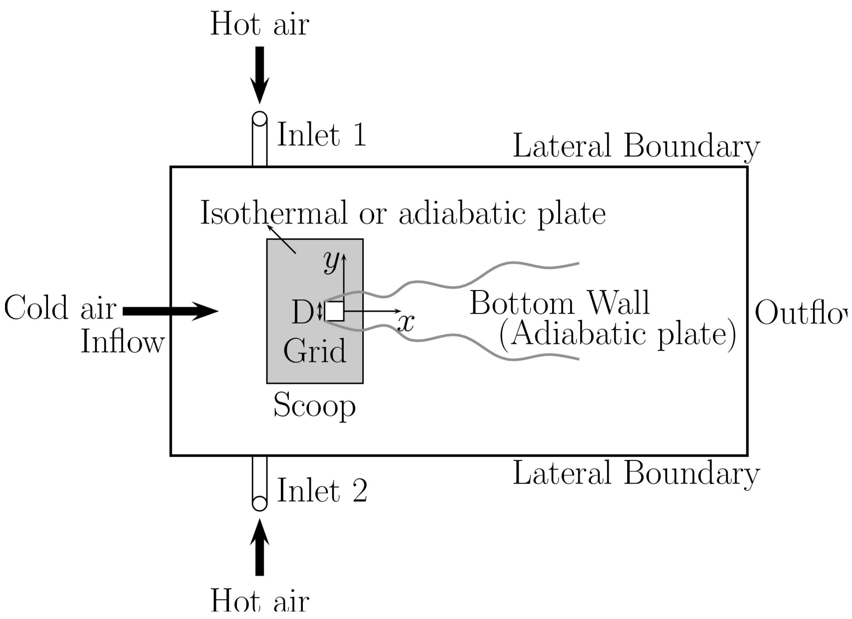
➪ these two parameters are considered as uncertain parameters

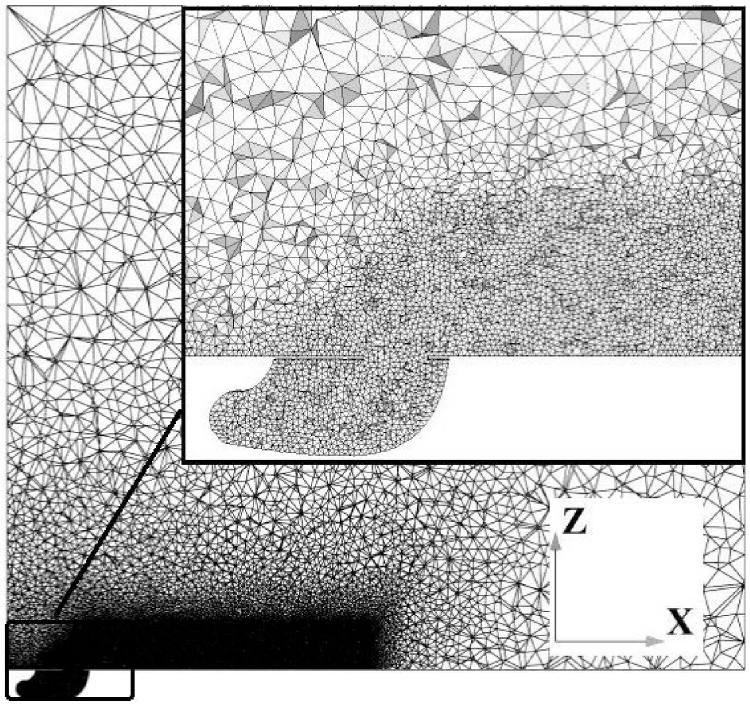
➪ comparison with experimental data should account for possible numerical result variability

Case study

(Jouhaud & Sagaut, J. Fluid Engng, 2008)







Response surface via Kriging

- Optimal linear unbiaised statistical predictor
- Based on sampling points (1 sample = 1 usual simulation)
- Several variants have been developed (cokriging, …)
- Kriging methods also provide an estimation of the interpolation error
- Sampling points can be generated dynamically to minimize the interpolation error (adaptive refinement)

Basic Kriging Method

Estimator at position *x*

Estimated function at position x_s

$$\hat{f}(x) = c^T(x, x_s)C^{-1}(x, x_s)f(x_s)$$

Covariance vector

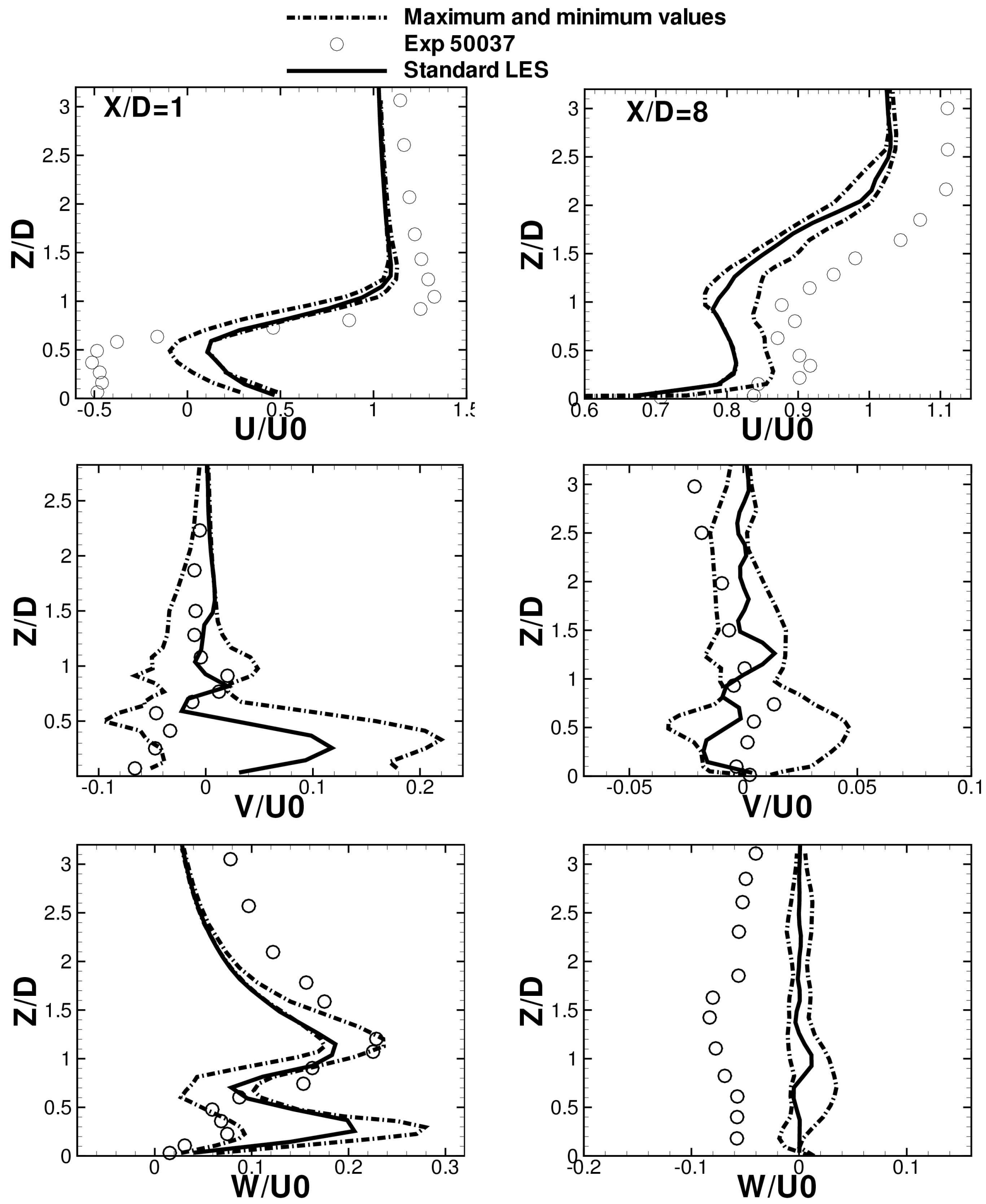
Covariance matrix

$$c(x, x_s) = \begin{pmatrix} Cov(x, x_1) \\ \dots \\ Cov(x, x_n) \end{pmatrix}$$

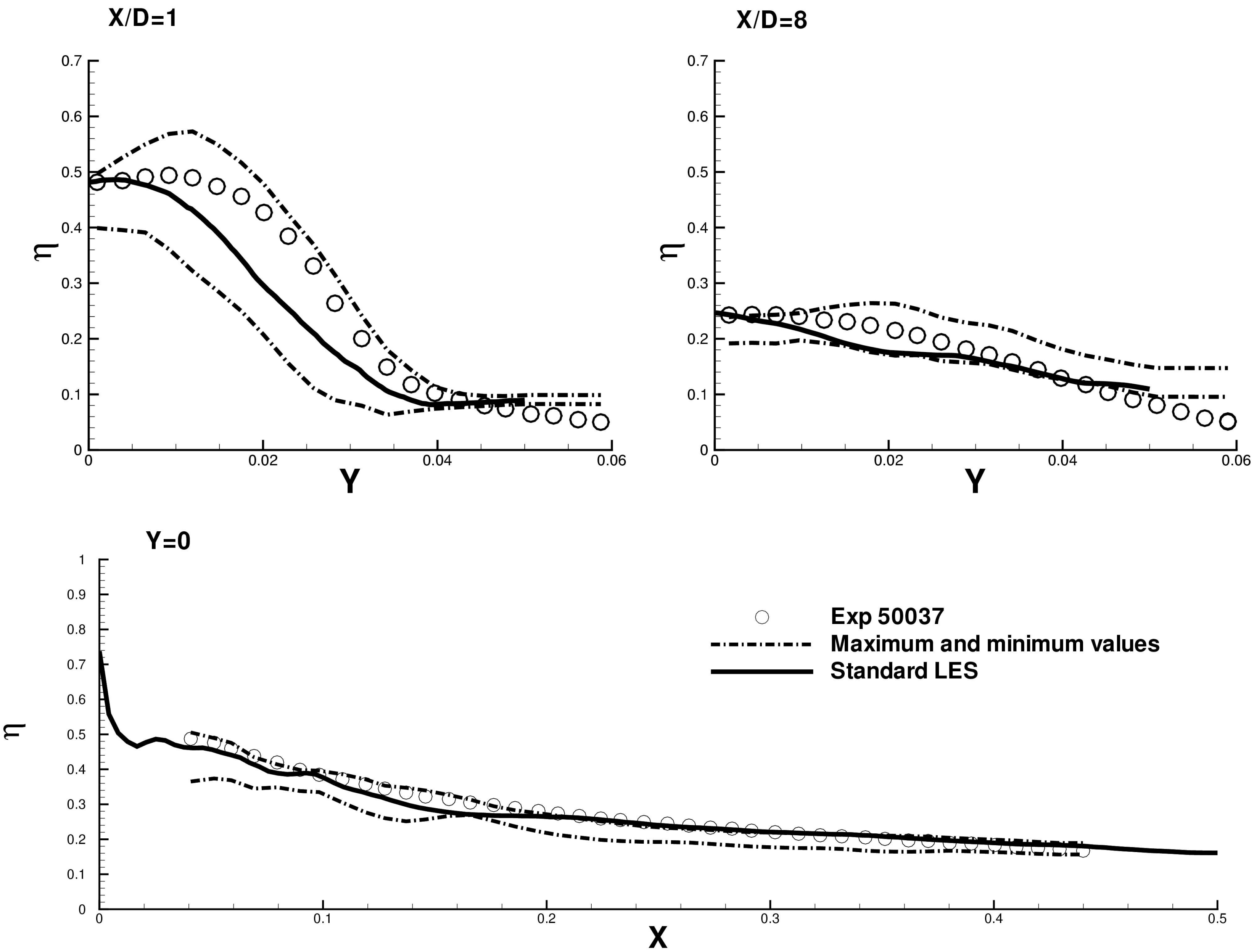
$$C(x_s, x_s) = \begin{pmatrix} \sigma^2 & Cov(x_1, x_2) \dots Cov(x_1, x_n) \\ Cov(x_2, x_1) & \sigma^2 \quad \dots Cov(x_2, x_n) \\ \dots & \dots \quad \dots \quad \dots \\ Cov(x_n, x_1) Cov(x_n, x_2) \dots & \sigma^2 \end{pmatrix}$$

a priori covariogram function: $Cov(y, z) = \sigma^2 \exp(-|y - z|)$

Mean flow predicted by LES



Mean temperature field



Defining the best LES solution

Which solution is the best LES solution ?

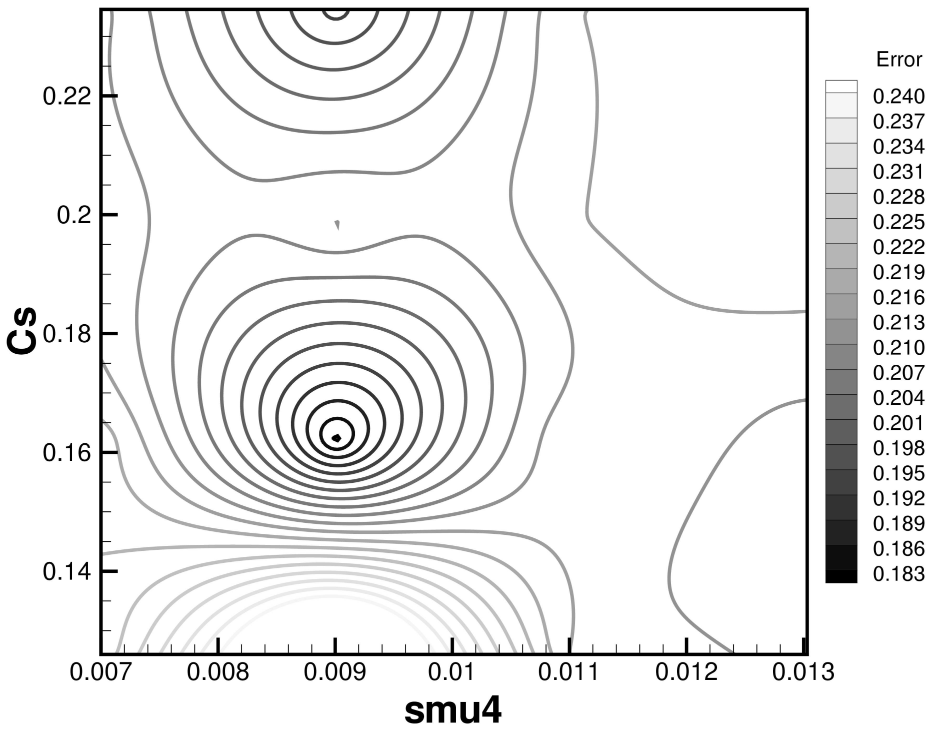
- If some experimental data are available:
  - some error functions can be defined
  - solutions with the lowest error norm can be identified

  ➪ « clean » definition of the best LES solution(s)

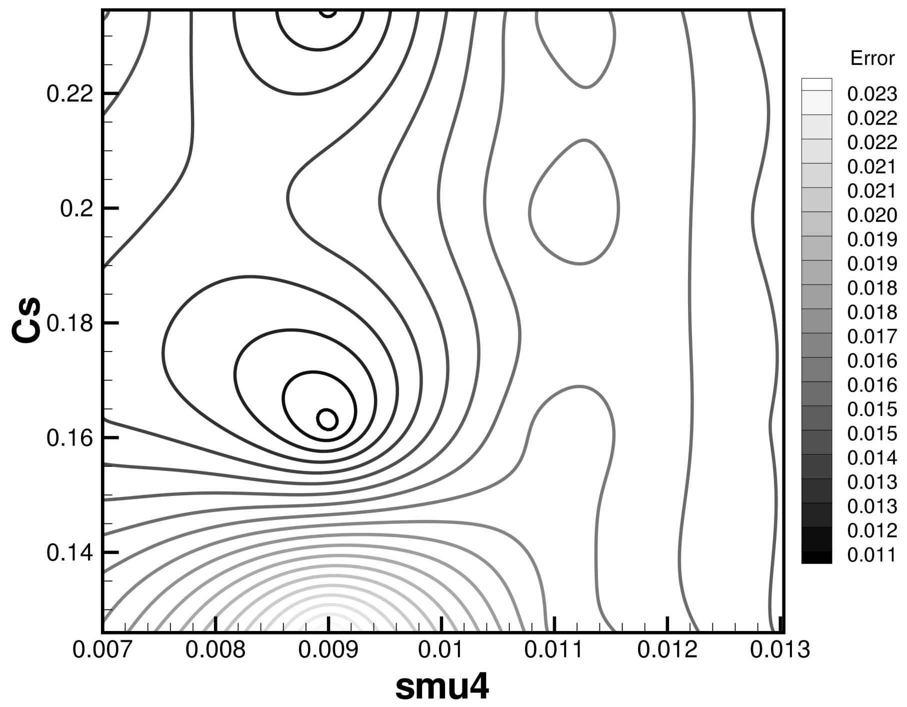
- « best » LES a priori depends on the error measure

Error map

Kriging-based response surface of error at X/D=8

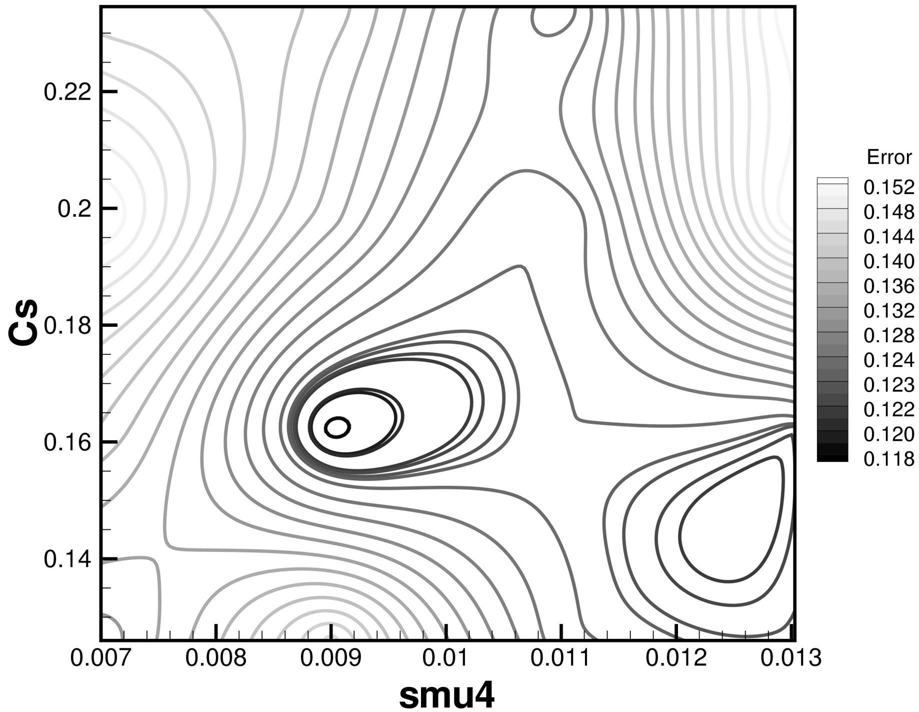


L_1 norm

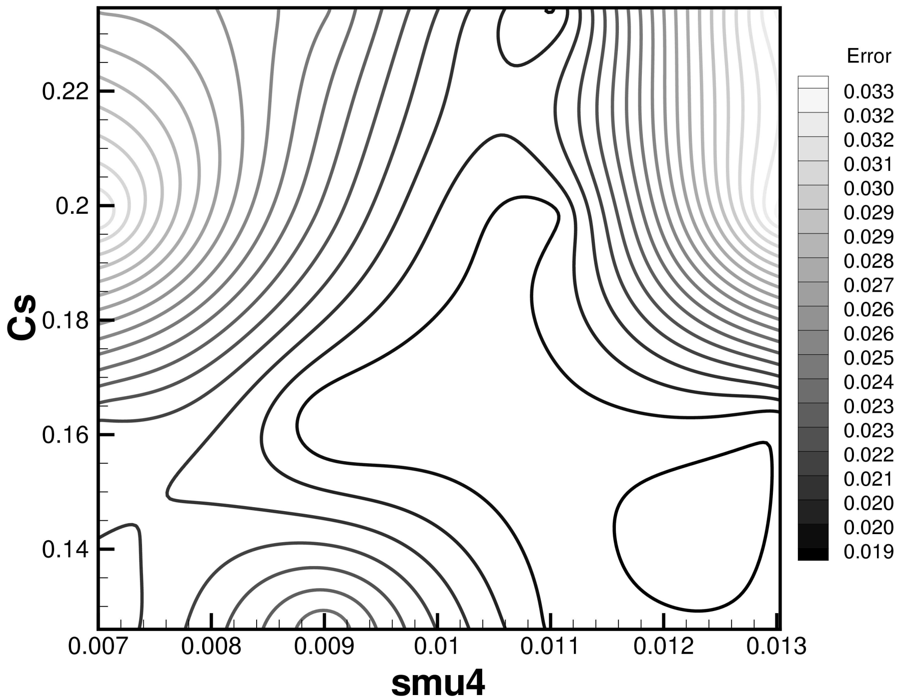


L_2 norm

Kriging-based response surface of global error at both locations X/D=8 and X/D=1

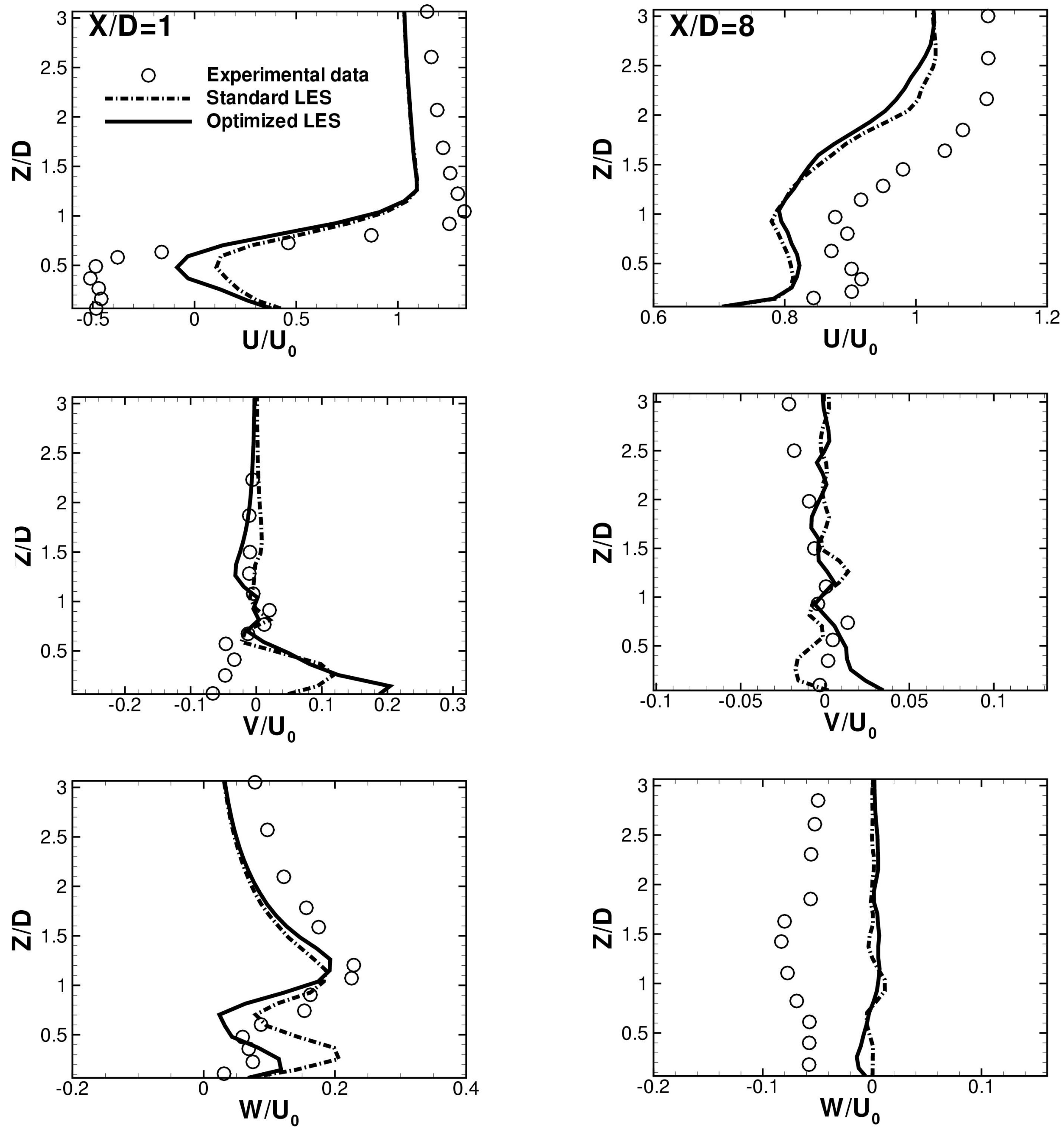


L_1 norm

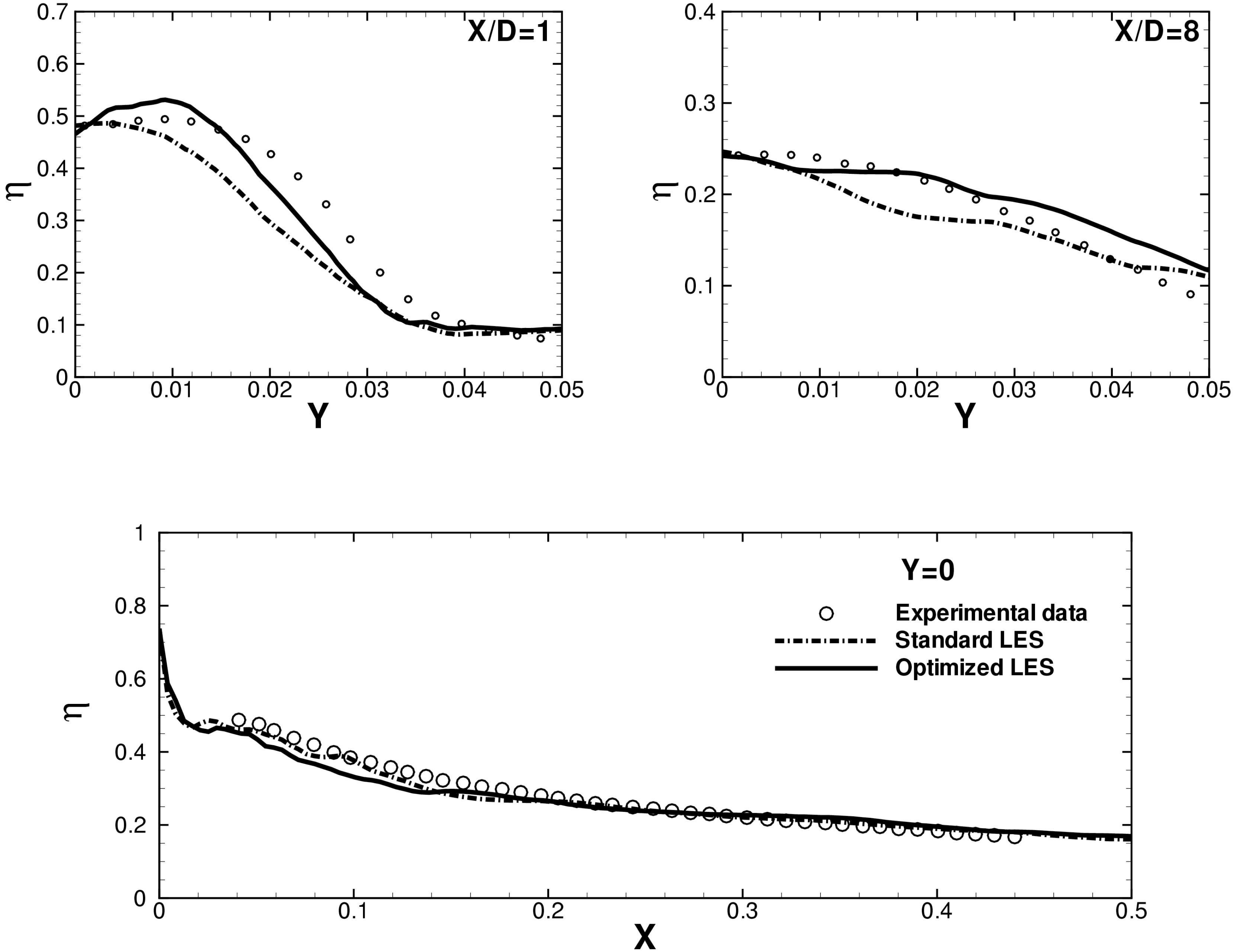


L_2 norm

Best LES solution



Best LES solution



Conclusions

- Validation/certification not trivial !
- Uncertainties are ubiquitious in almost all application fields
- Mathematical tools do exist !
- Computational ressources now available

➩Next step in modelling !

Thank you!