SciDAC
Scientific Discovery through Advanced Computing

AFOSR
AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

Numerical Experiments with Shock Turbulence Interaction

Sanjiva K. Lele
Johan Larsson, Soshi Kawai
Stanford University

Celebrating 50 Years of Turbulence and Acoustics Research
Symposium Honoring Prof. G. Comte Bellot, ECL, Lyon, France

October 28-29, 2009

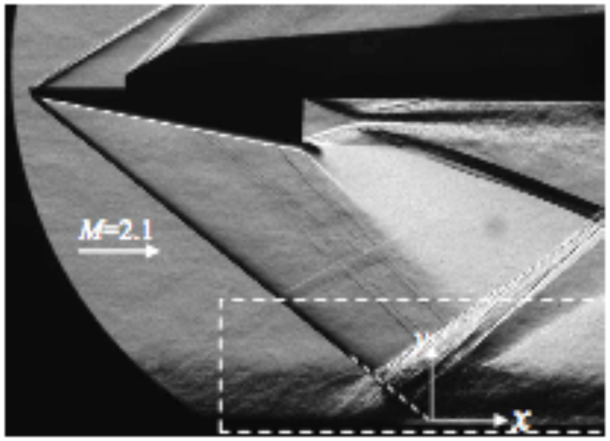
Supported by the Department of Energy under the SciDAC -II program, and AFOSR-MURI program
Computations performed at the National Energy Research Scientific Computing Center under the ERCAP program, and at Argonne National Laboratory under the INCITE program.

Motivation (1)

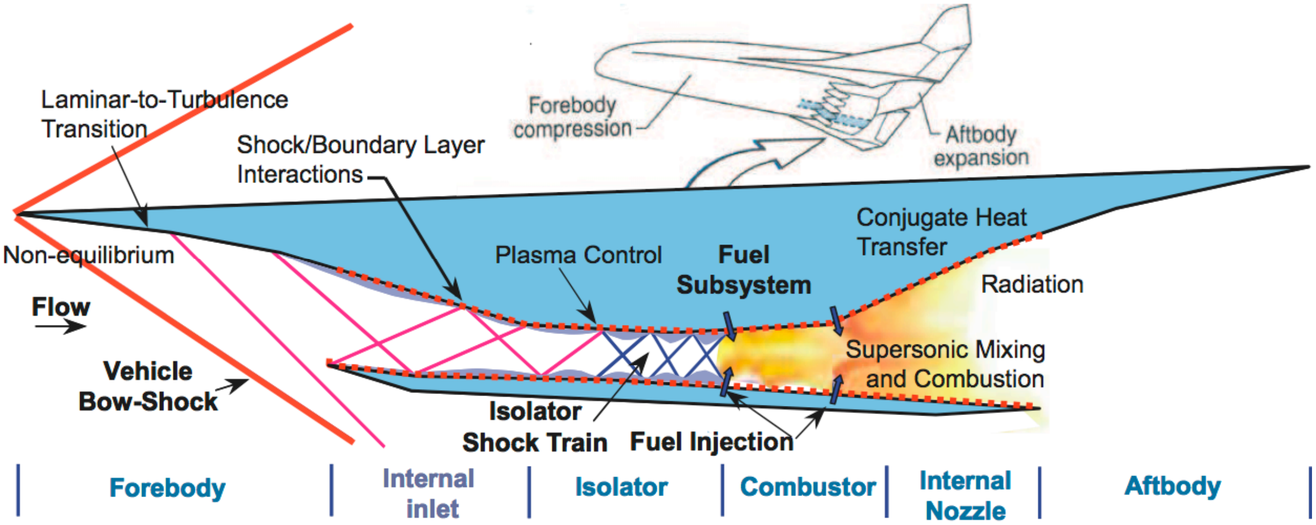
Shock-turbulence interaction is central to high-speed propulsion systems yet its current models are quite inaccurate

Examples: scramjet engine; supersonic inlets

- Control of shockwave-turbulent boundary interactions in supersonic aircraft inlets (bleed system) causes a 15-20% performance penalty;
- Stability of the shock-train system limits the operability of a scramjet engine



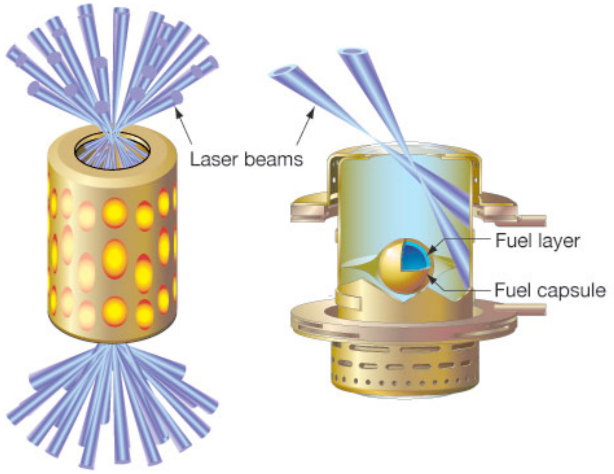
Shock-Boundary layer interaction



Scramjet engine

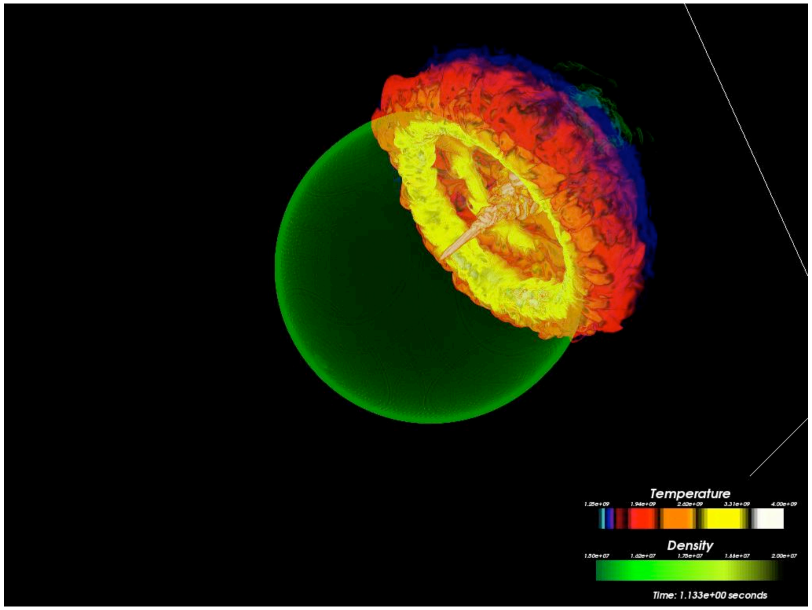
Motivation (2)

- Interactions between strong shocks and interfaces lead to flow instabilities, which drive turbulent mixing
- Accurate prediction of shock-induced turbulent mixing remains an open problem
  - Examples: Rayleigh-Taylor mixing, Richtmyer-Meshkov mixing
  - Applications: inertial confinement fusion, supernova explosions



Source: lasers.llnl.gov/programs/nic/icf/

Inertial confinement fusion by shock-induced implosion of a deuterium/tritium capsule



Source: flash.uchicago.edu

Supernova explosion

The key challenge:

- Simulation of broadband turbulence requires numerical methods with minimum numerical dissipation
    - Capture the widest range of resolvable scales on a given grid
    - Spectral methods; high-order methods
- Simulation of flows with shocks requires numerical dissipation for shock capturing
- The key challenge:

How to design a numerical method that handles these conflicting requirements?

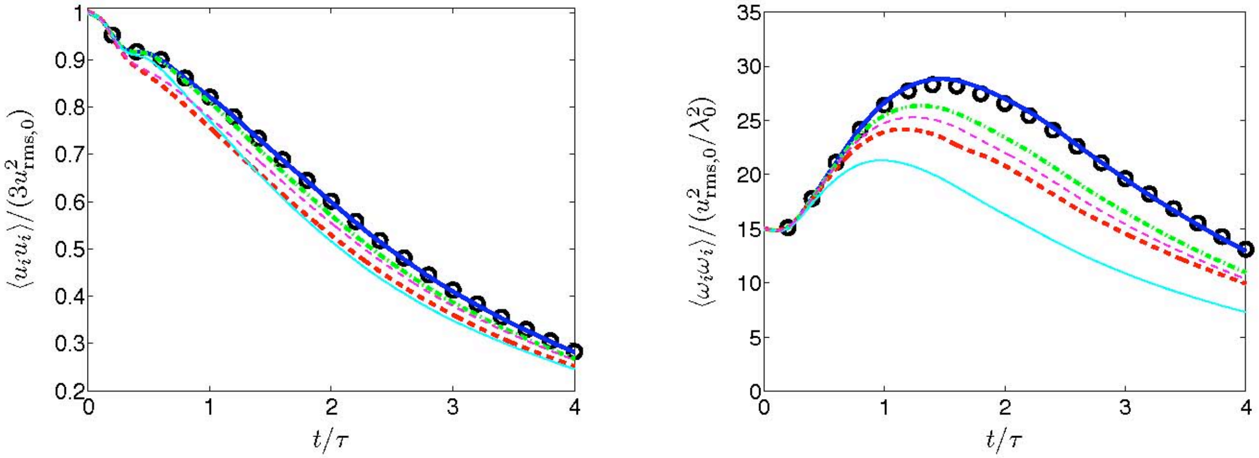
Assessment of Numerical Algorithms:
(for shock-turbulence interaction simulations)

- Broad range of high-resolution algorithms
  - High-order WENO (7th order)
  - High-order compact differences with selective artificial viscosity (STAN; STAN-I)
    Cook & Cabot, 2007, and modified Kawai & Lele, 2008;
    Mani, Larsson & Moin, 2009, Bhagatwala & Lele, 2009
  - Hybrid (8th order central differening + WENO) Larsson & Gustavsson, 2008
  - High order central difference + Wavelet sensor based artificial dissipation
    ADPDIS3D (Yee & Sjogreen, 2007)

- Broad range of benchmark test cases
  - Noh problem (strong implosion)
  - Shu-Osher problem
  - Interaction of vorticity/Entropy wave with normal shock
  - Taylor-Green problem
  - Compressible isotropic turbulence

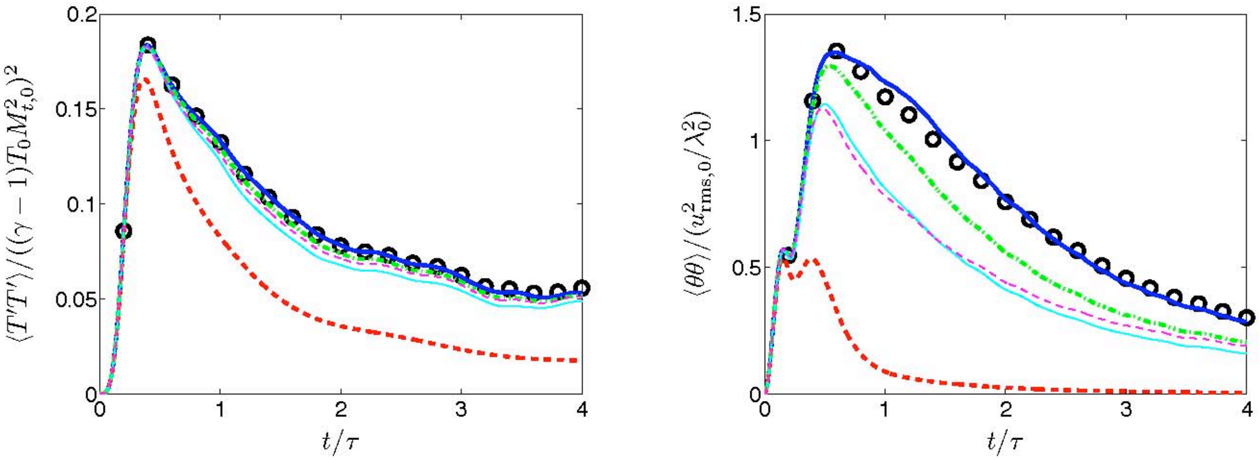
Details in paper by Johnsen et al. to appear in J. Comput. Phys, 2009

Numerical Algorithm Assessment: Compressible Isotropic Turbulence (with eddy shocklets)



(a) Kinetic energy.

(b) Enstrophy.



(c) Temperature.

(d) Dilatation, $\theta = \partial_j u_j$.

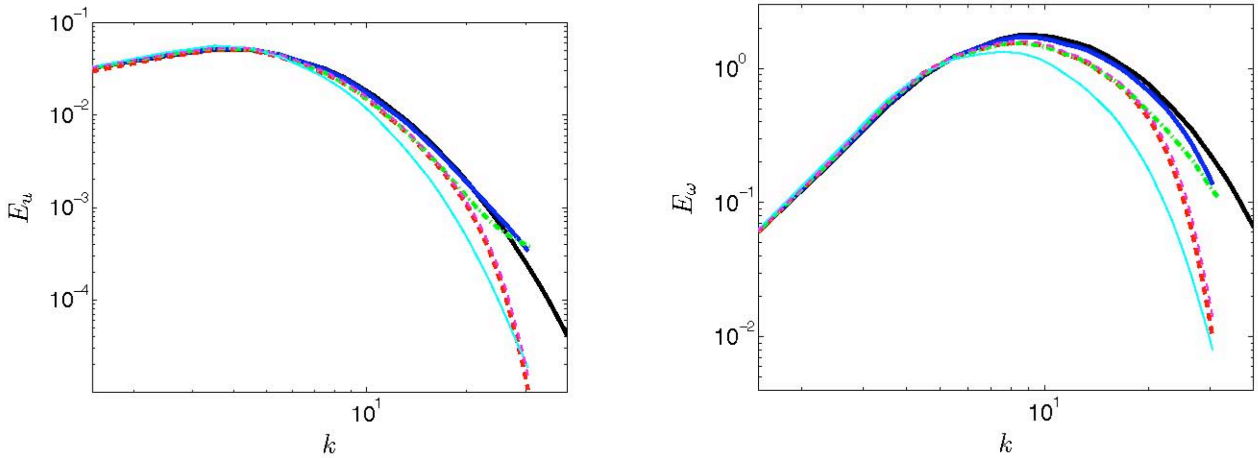
Code	Color	Line style
Reference	black	varying
Stan	red	dashed
Stan-I	magenta	dashed (thin)
Hybrid	blue	solid
WENO	cyan	solid (thin)
ADPDIS3D	green	dashed-dotted
Shock Fit	black	dotted

Table 2: Color and line legend for the plots.

Figure 11: Temporal evolution of the variance of different quantities for the isotropic turbulence problem on 64^3 grid. The reference is the solution on a 256^3 grid spectrally filtered to a 64^3 grid (circles).

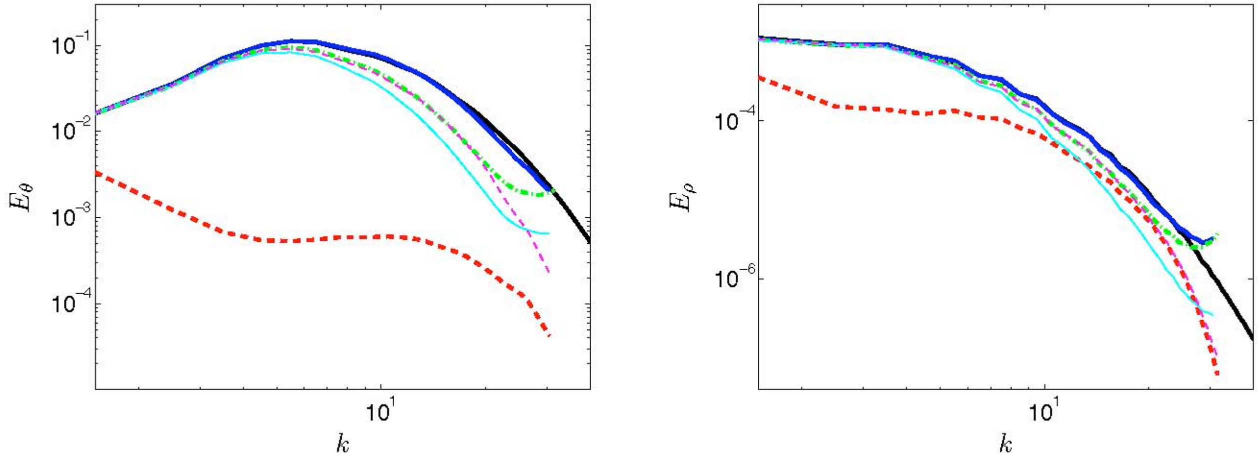
Numerical Algorithm Assessment:
Compressible Isotropic Turbulence (with eddy shocklets)

Algorithm comparison



(a) Velocity.

(b) Vorticity.



(c) Dilatation.

(d) Density.

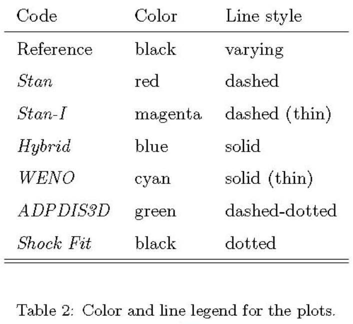
Code	Color	Line style
Reference	black	varying
Stan	red	dashed
Stan-I	magenta	dashed (thin)
Hybrid	blue	solid
WENO	cyan	solid (thin)
ADPDIS3D	green	dashed-dotted
Shock Fit	black	dotted

Table 2: Color and line legend for the plots.

Figure 13: Spectra at $t/\tau = 4$ for the isotropic turbulence problem on a 64^3 grid. The reference is the solution on a 256^3 grid.

Numerical Algorithm Assessment:
Effective Bandwidth

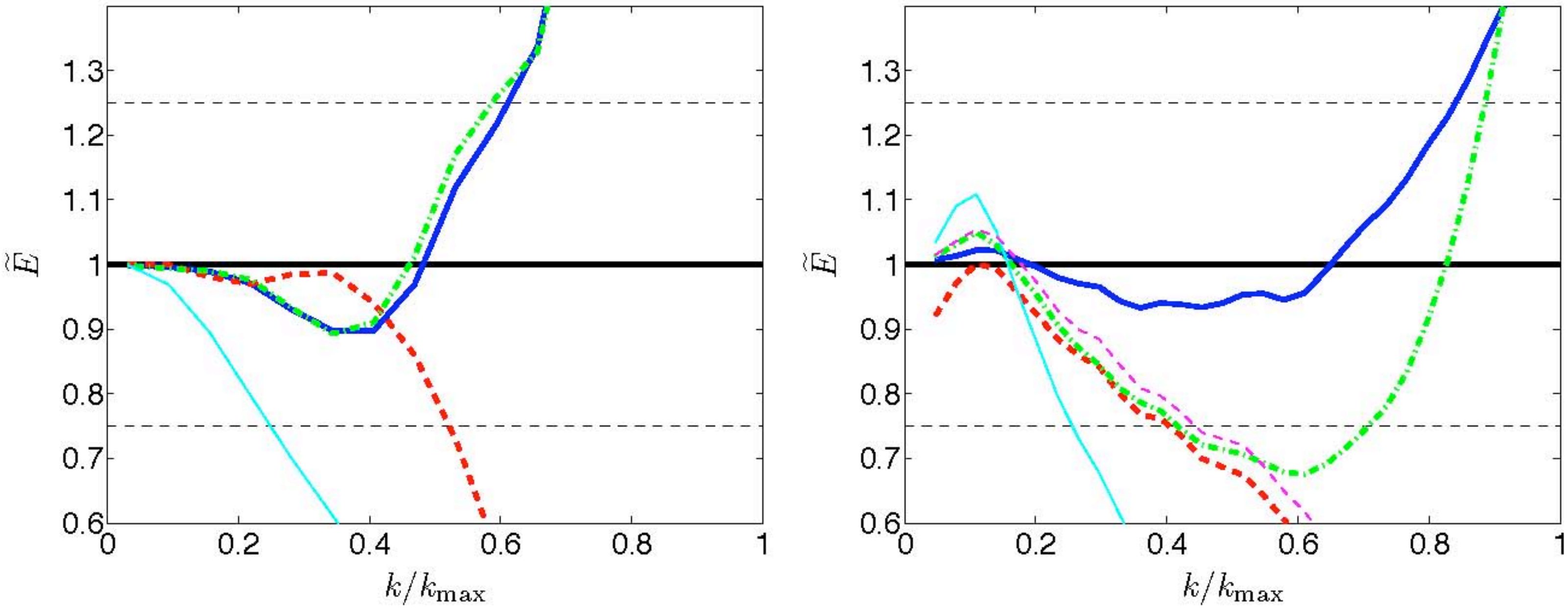
- Use compensated spectra E(k)/E_{exact} (k) to define effective bandwidth



Code	Color	Line style
Reference	black	varying
Stan	red	dashed
Stan-I	magenta	dashed (thin)
Hybrid	blue	solid
WENO	cyan	solid (thin)
ADPDIS3D	green	dashed-dotted
Shock Fit	black	dotted

Table 2: Color and line legend for the plots.

Algorithm comparison



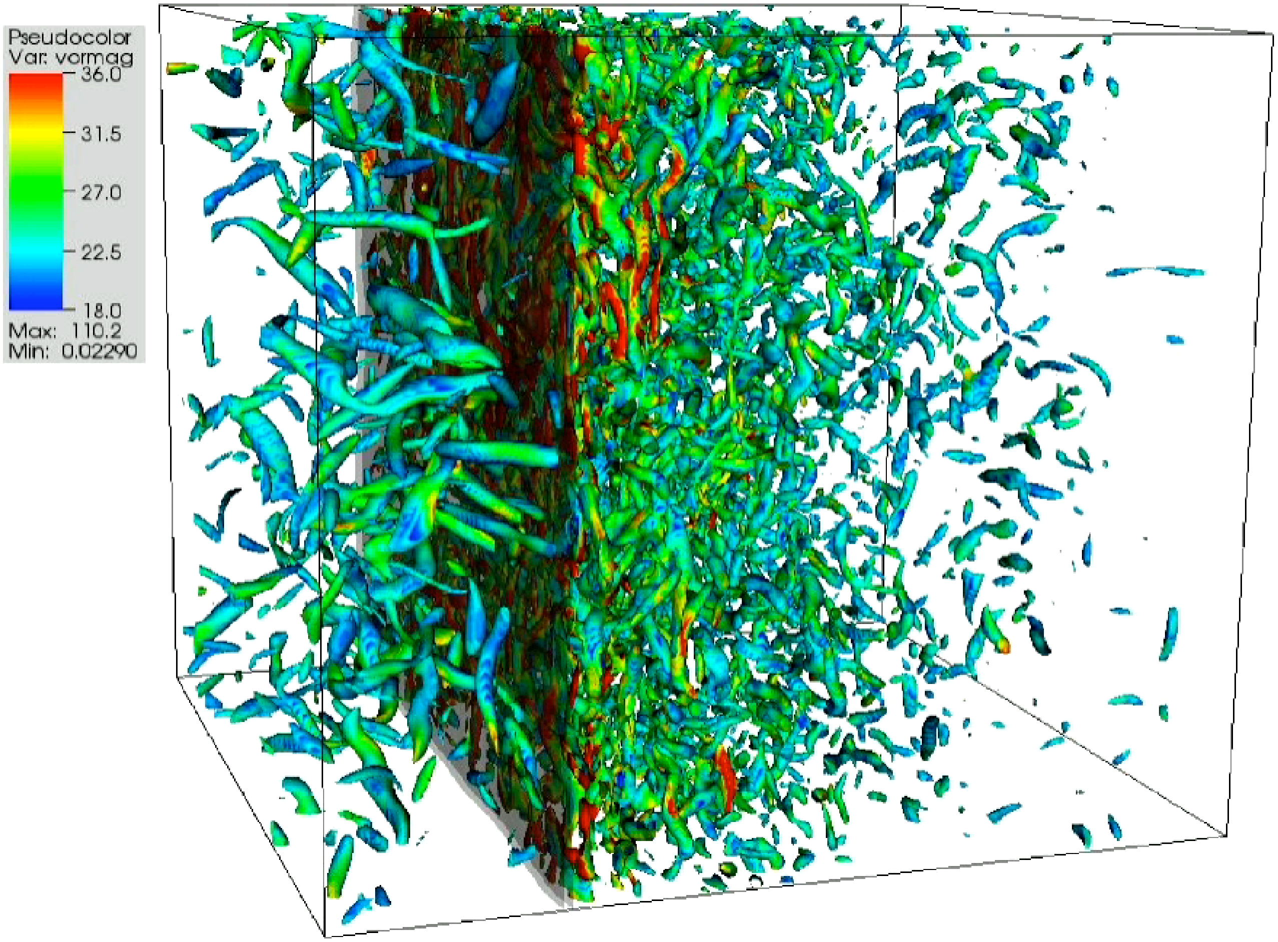
(a) Taylor-Green vortex at $t = 5$.

(b) Isotropic turbulence at $t/\tau = 4$.

Figure 15: Compensated spectra on a 64^3 grid. As a reminder, *Stan* is dashed red, *Stan-I* is dashed magenta (thin), *Hybrid* is solid blue, *WENO* is solid cyan (thin), and *ADPDIS3D* is dash-dotted green.

Canonical shock/turbulence interaction

- Isotropic turbulence passing through a normal shock in a perfect gas
- Isolates the core interaction between turbulence and shock



Eddies visualized by Q-criterion, colored by vorticity. Shock visualized by dilatation contour.

Some past work on canonical shock/turbulence interaction

- Theoretical
    - Ribner (NACA 1953, NACA 1954, AIAA J 1987), Moore (NACA 1954): linear interaction analysis including Rankine-Hugoniot relations
    - Lele (PhysFl 1992): turbulent shock relations using RDT, predicted modified mean shock jumps
- Experimental: wind tunnels
    - Barre et al (AIAA J 1996): grid turbulence passing through a Mach 3 shock, hot-wire and LDV measurements
- Experimental: shock tubes
    - Hesselink & Sturtevant (JFM 1988): found “peaked” and “rounded” instantaneous pressure profiles in random medium
- Computational (DNS):
    - Lee et al (JFM 1993): resolved viscous shock structure, $M = 1.2$, $Re_\lambda = 20$, $M_t = 0.1$, found modified instantaneous profiles of dilatation
    - Lee et al (JFM 1997): captured shock up to $M = 3$, found good agreement with Ribner’s linear theory
    - Mahesh et al (JFM 1997): influence of entropy fluctuations
    - Ducros et al (2000): LES; Jamme et al (FTC 2002)

Post-shock Kolmogorov length scale: Scaling Estimate

- From the (incompressible) definition of the Kolmogorov length scale

$$\eta = \left(\frac{\nu^3}{\varepsilon/\rho}\right)^{1/4} \sim \left(\frac{\mu^2}{\rho^2\omega^2}\right)^{1/4} \sim \frac{T^{3/8}}{\rho^{1/2}\omega^{1/2}}$$

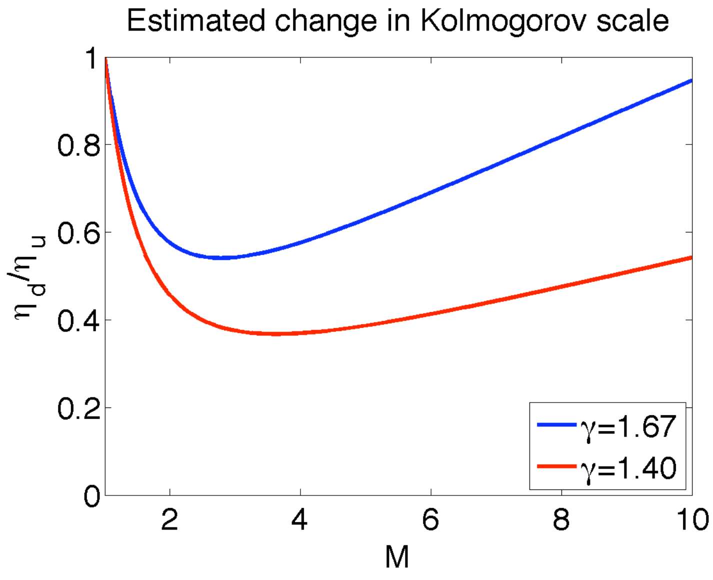
where we used

$$\varepsilon \approx \mu\omega^2 \ , \quad \nu = \mu/\rho \ , \quad \mu \sim T^{3/4}$$

- Across the shock, assume that the vorticity changes as the density ratio; gives

$$\frac{\eta_\mathrm{d}}{\eta_\mathrm{u}} \approx \left(\frac{T_\mathrm{d}}{T_\mathrm{u}}\right)^{3/8} \left(\frac{\rho_\mathrm{d}}{\rho_\mathrm{u}}\right)^{-1}$$

in terms of Rankine-Hugoniot relations



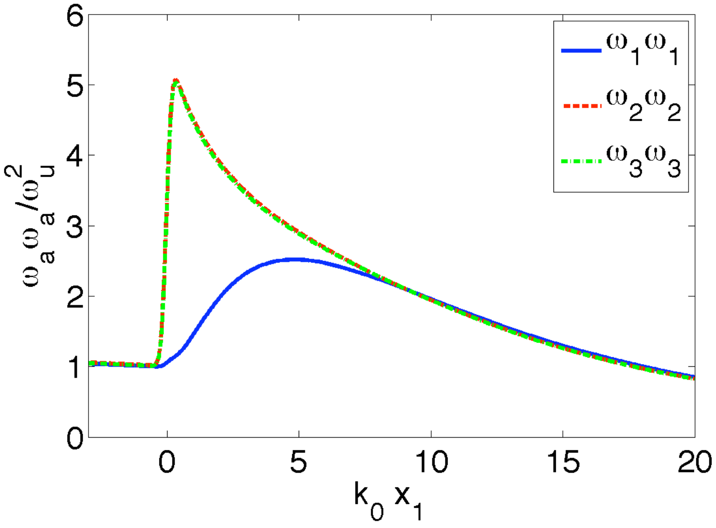
Post-shock Kolmogorov Scale determines the resolution requirement for turbulence

Difference compared to previous studies

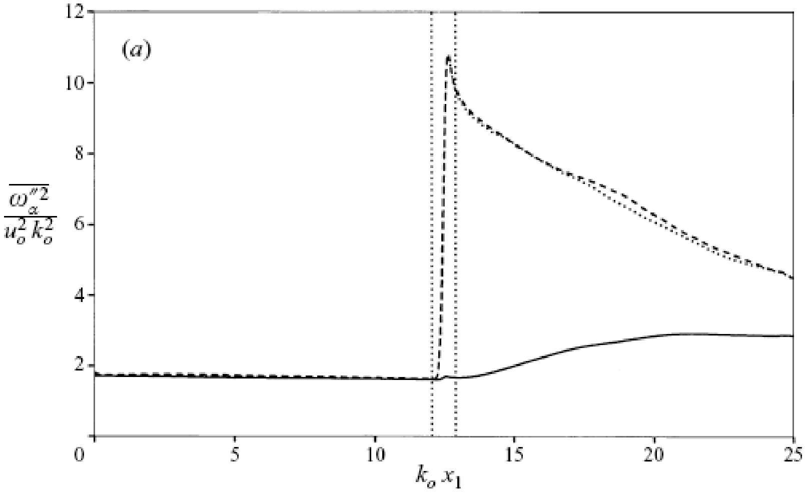
- Past studies: $Re_\lambda = 20$ on $129*64^2$ grids (Lee et al, 1993, 1997)
- Present: $Re_\lambda = 40$ on $1040*384^2$ grids; $Re_\lambda = 60$ on $1675*512^2$ grids

- Return to local (small-scale) isotropy in present DNS, but not in past work
- Vorticity redistribution is a nonlinear process
    - Past work did not fully capture this due to lack of grid resolution
    - Has led to persistent misunderstanding of vorticity evolution at shock

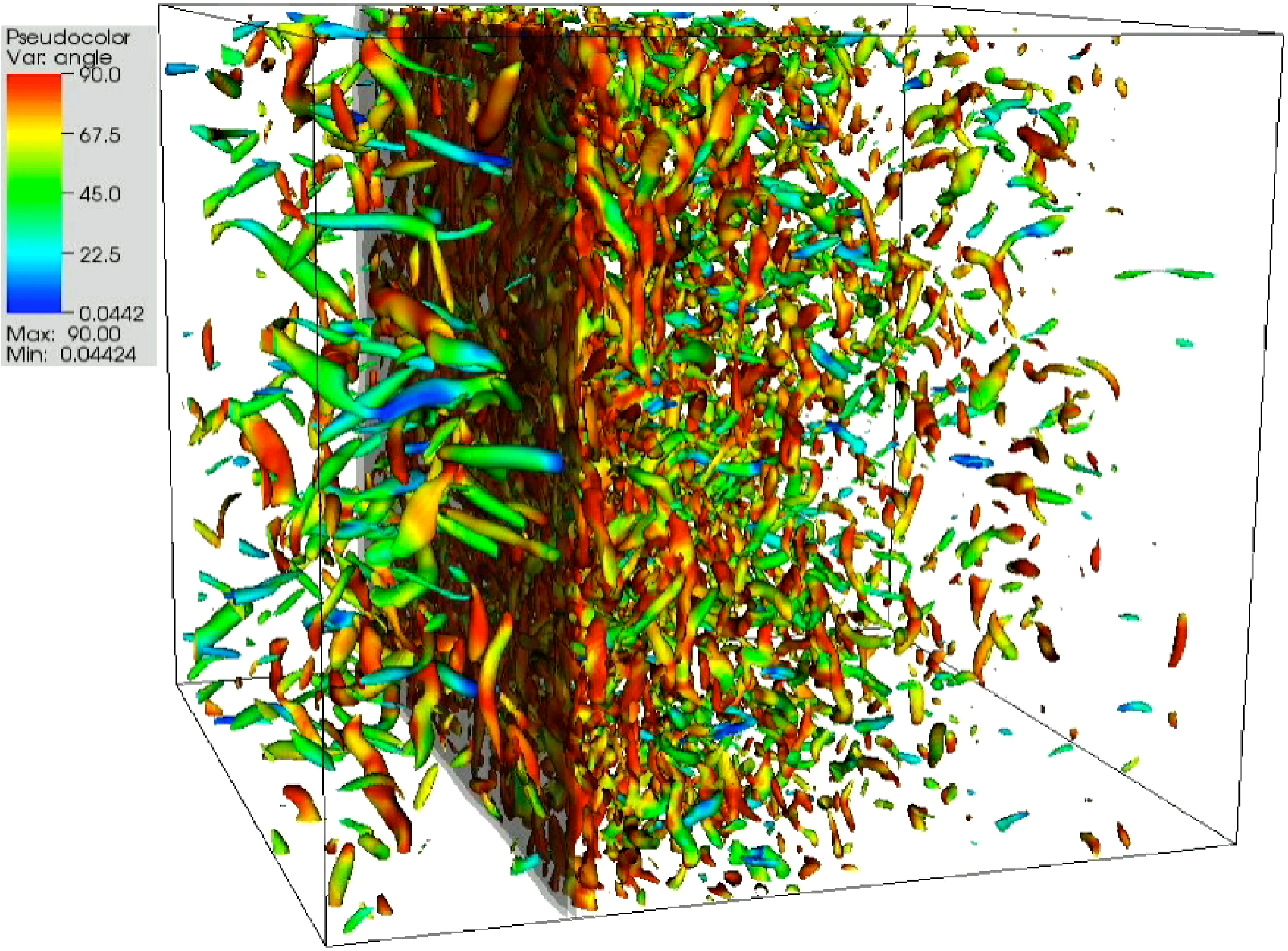
Vorticity variances at $(M,M_t)=(1.87,0.22)$



Lee et al (1997)

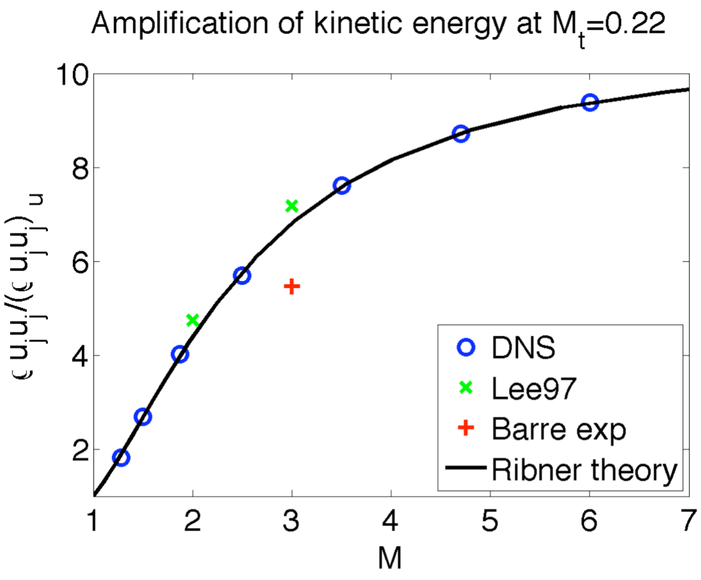


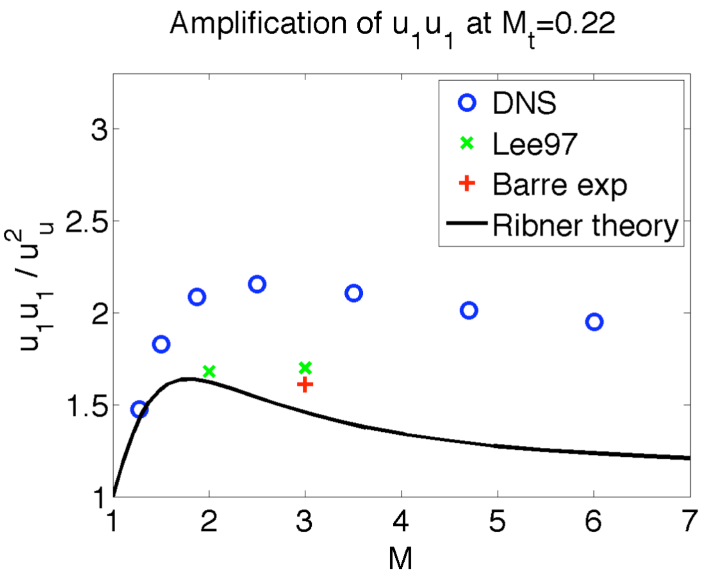
Anisotropic post-shock turbulence



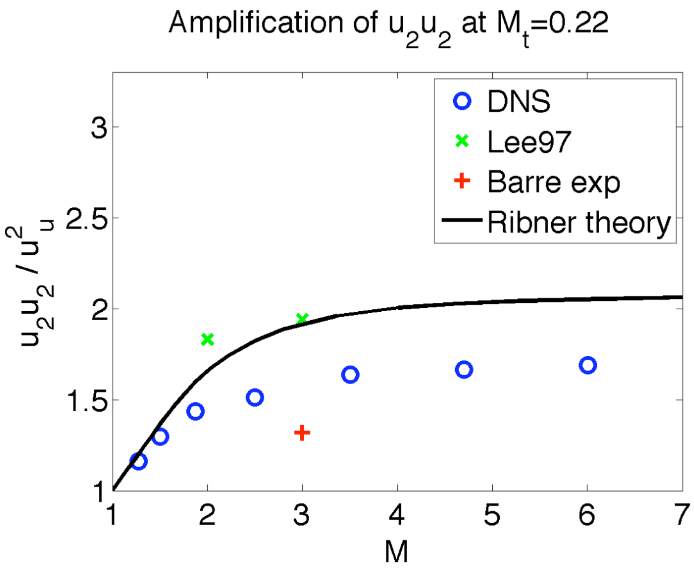
Eddies visualized by Q-criterion, colored by angle between vorticity vector and x-axis.
Shock visualized by dilatation contour.

Reynolds stresses: amplification across the shock vs Mach number

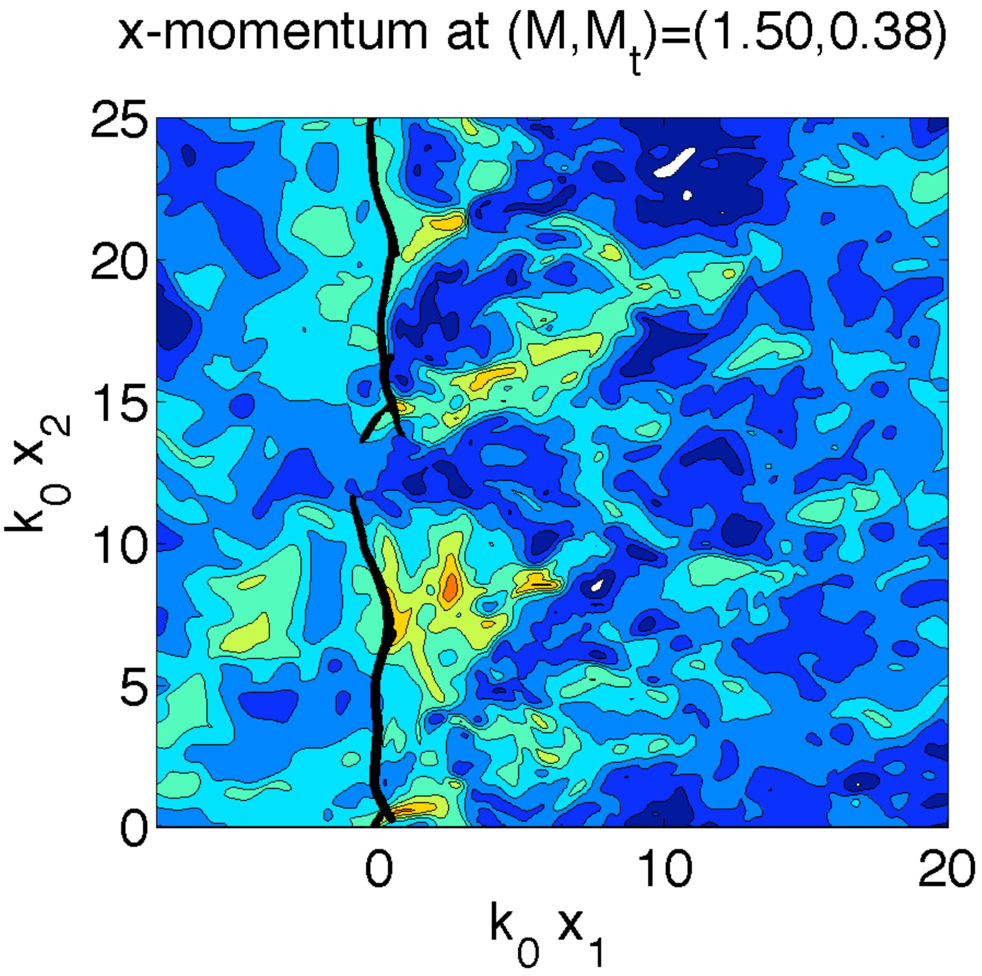

Amplification of kinetic energy at M_t=0.22


Amplification of u_1u_1 at M_t=0.22

- Amplification of kinetic energy agrees very well with Ribner's linear theory (and Lee97)
  - Kinetic energy must be governed largely by linear processes
- DNS (and experiment) yields much larger anisotropy
  - Nonlinear redistribution processes must be important


Amplification of u_2u_2 at M_t=0.22

Complex shock structure in nonlinear regime

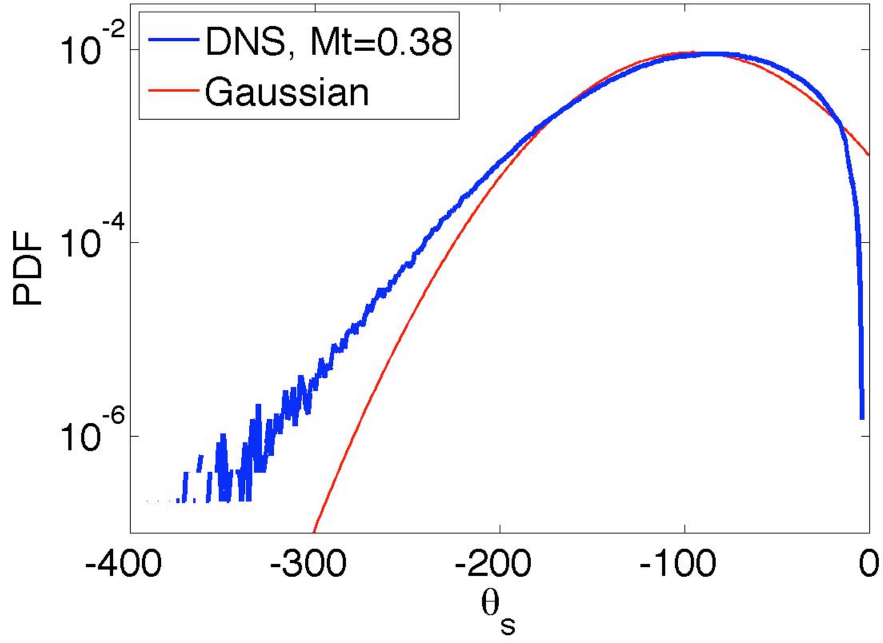


- Shock no longer a simple object
    - Qualitative agreement with Zank et al (PhysFl 2002)

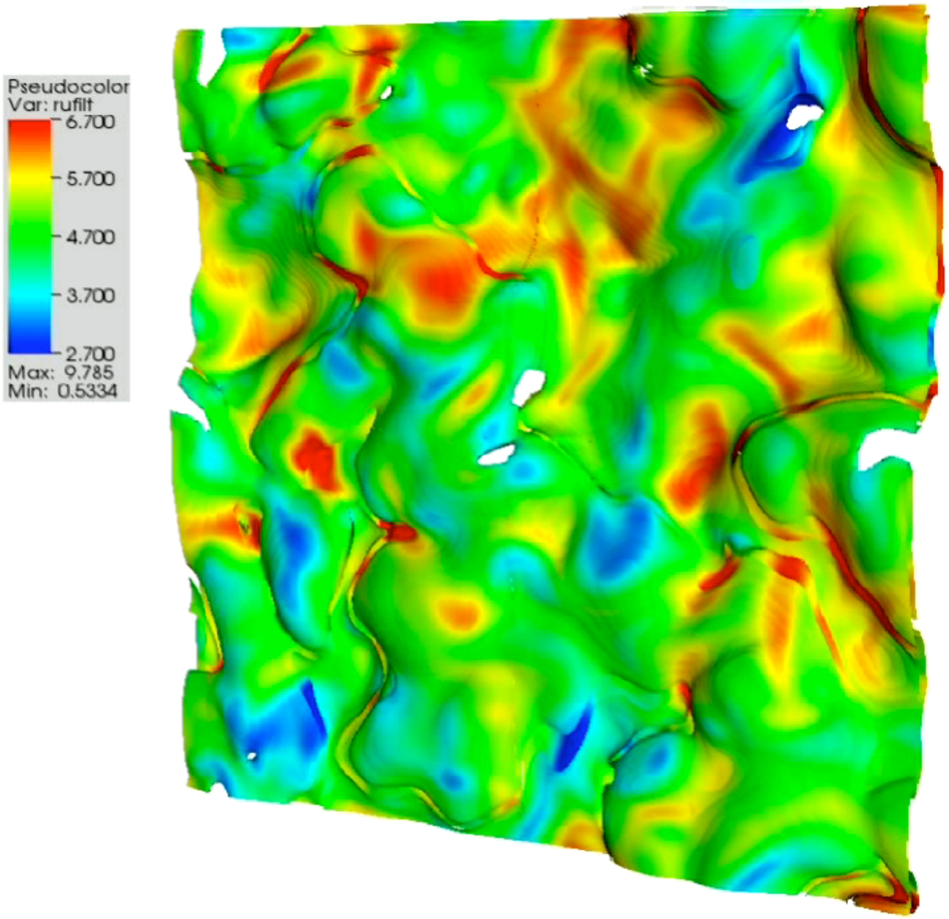
Extracting "extreme" interaction events

- Define shock-dilatation as $\theta_{\text{shock}}(y,z) = \min_{x} \theta(x,y,z)$
- Study instantaneous profiles through the shock at extreme values of shock-dilatation:
  - "Strong" interactions: most compressive events
  - "Weak" interactions: least compressive events

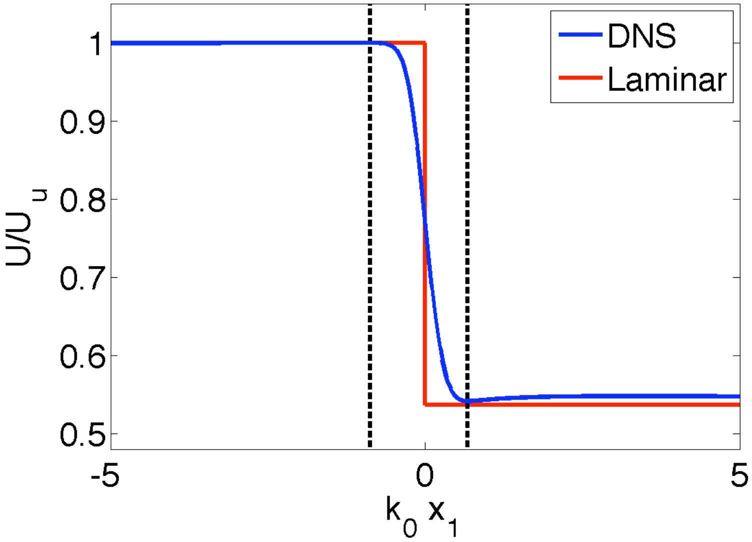
PDF of shock dilatation



Unsteady shock motion



Mean velocity at $(M,M_t)=(1.50,0.22)$



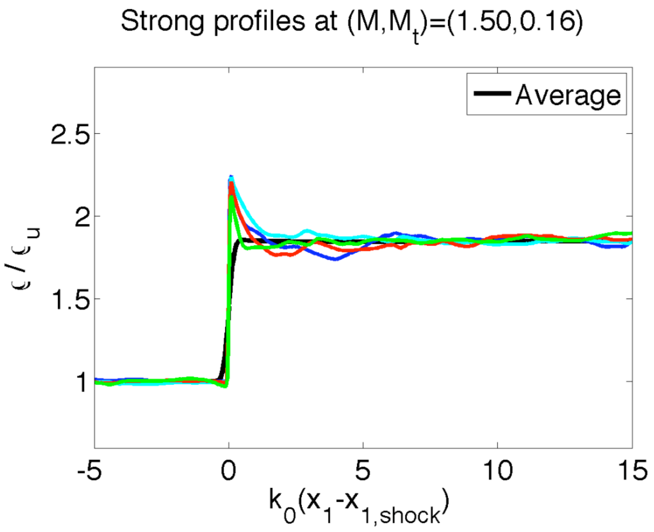
Shock visualized by dilatation contour, colored by streamwise momentum.

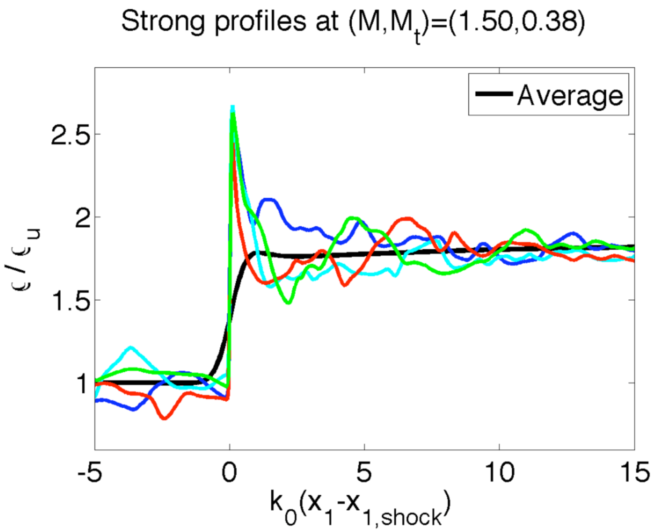
“Strong” events: over-compression and post-shock expansion

Low M_t

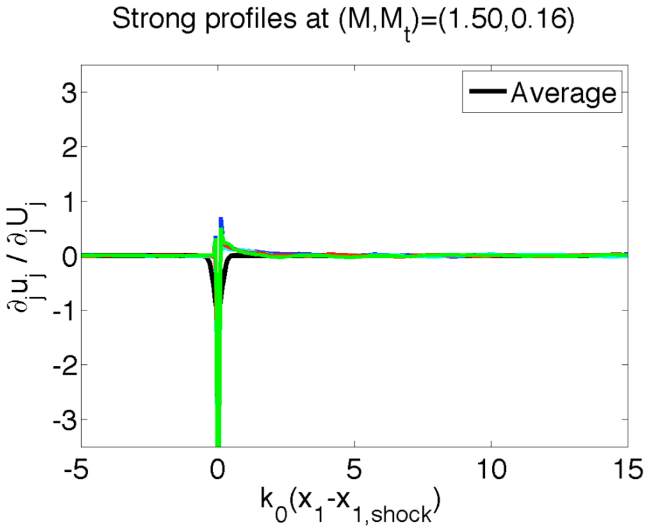
High M_t

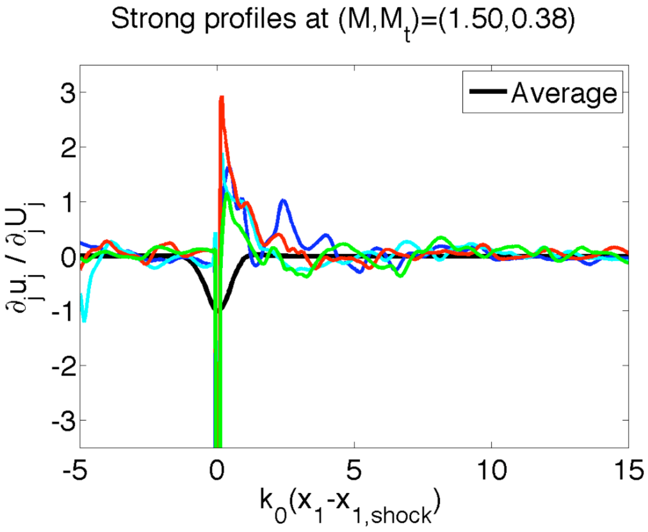
Density


Strong profiles at $(M,M_t)=(1.50,0.16)$


Strong profiles at $(M,M_t)=(1.50,0.38)$

Dilatation


Strong profiles at $(M,M_t)=(1.50,0.16)$

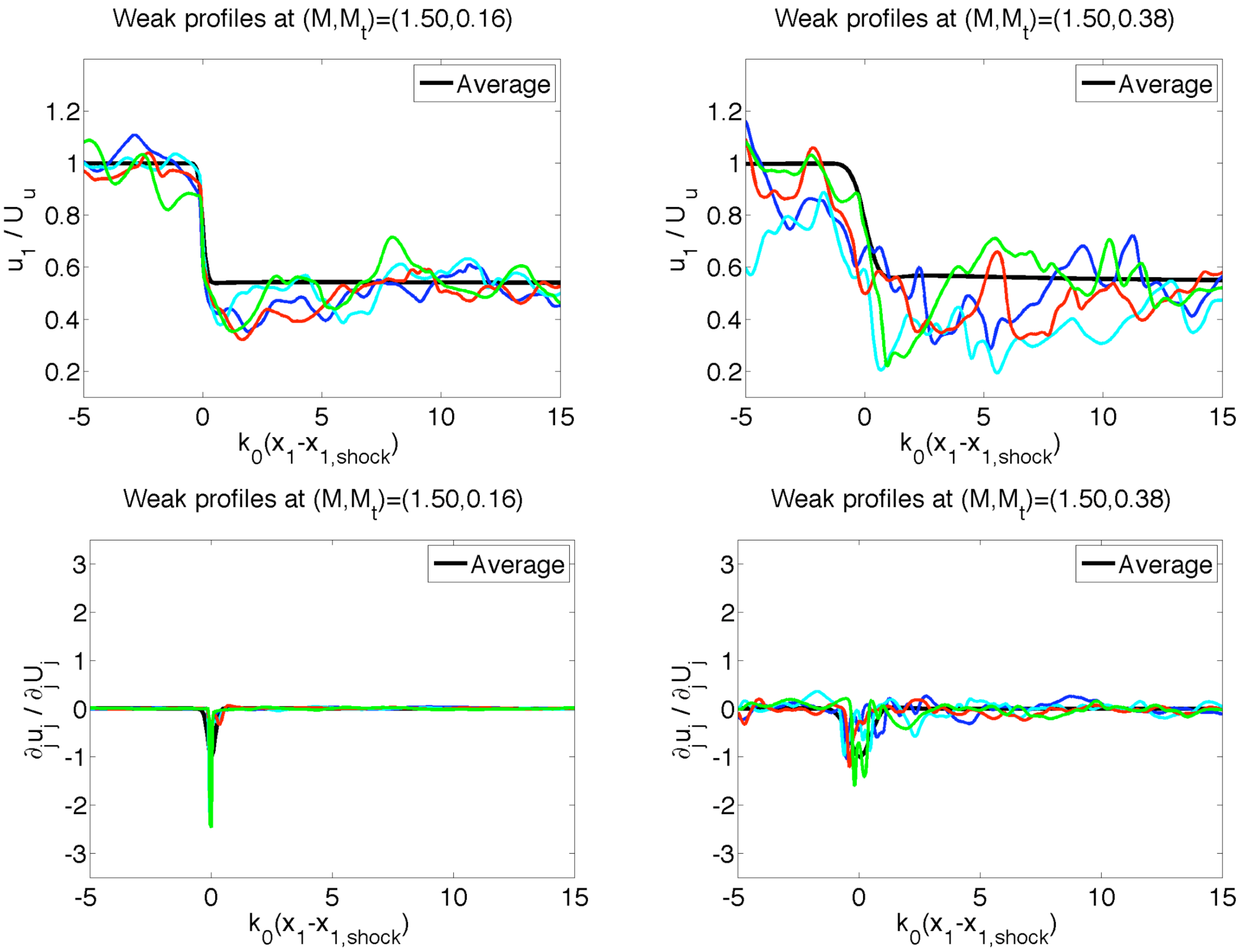

Strong profiles at $(M,M_t)=(1.50,0.38)$

“Weak” events: smooth profiles in nonlinear (high M_t) regime

Low M_t High M_t

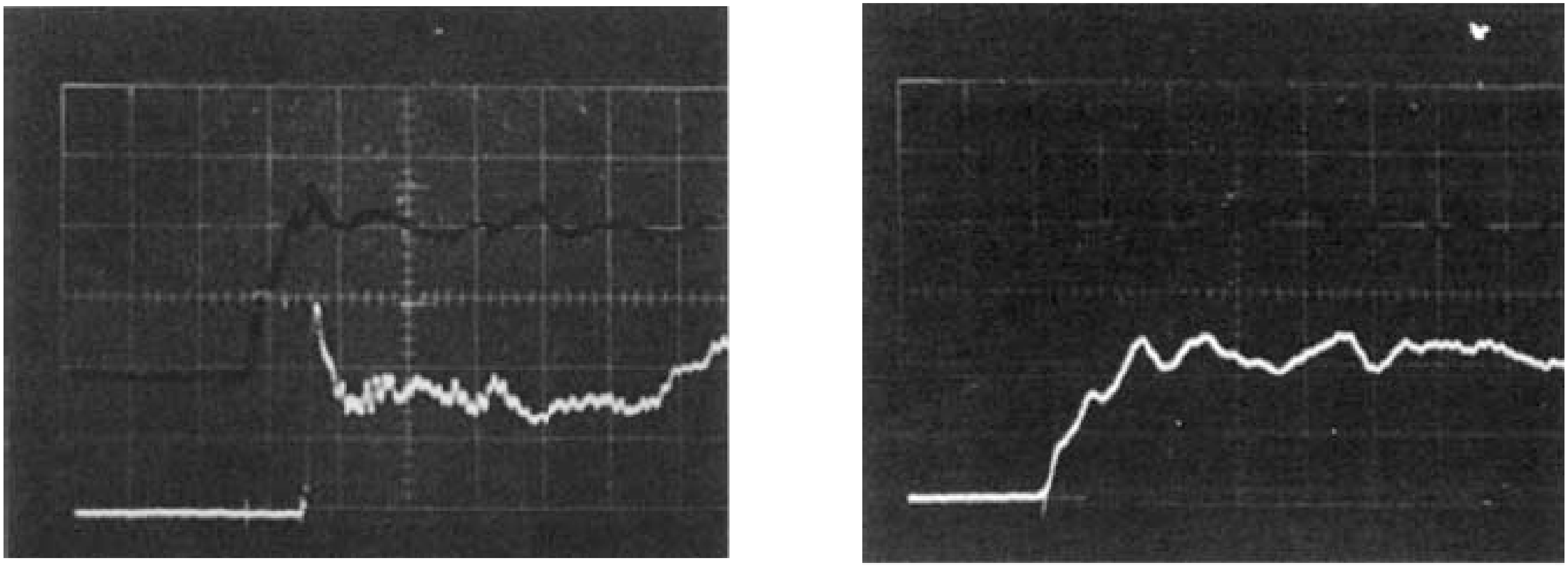
Velocity

Dilatation



Hesselink & Sturtevant (JFM 1988)

- “Peaked” and “rounded” pressure traces for shock propagating through a random mixture of He and R12 at M = 1.007 - 1.1



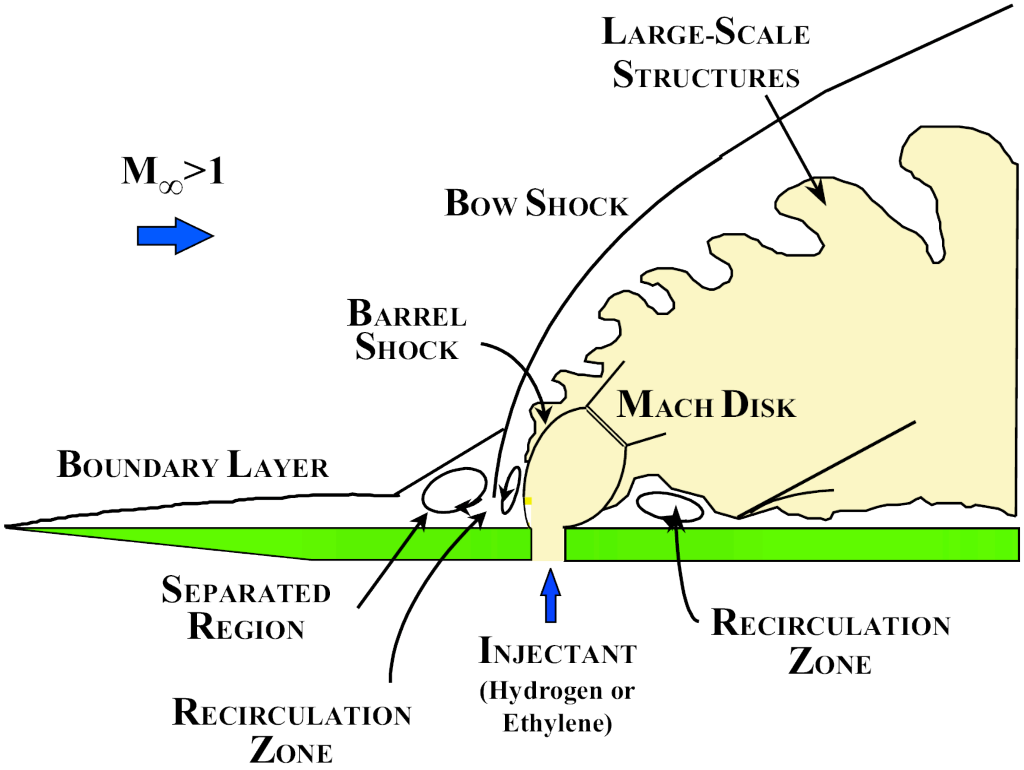
Summary -- what we've learned so far

- Direct numerical simulations of canonical shock/turbulence interaction that truly resolves the post-shock turbulence

- Disagreements with linear theory and past computations:
  - Vorticity components return to isotropy
  - Qualitatively different, and much larger, stress anisotropy
- Agreement with linear theory and past computations:
  - Kinetic energy amplification -- governed by linear processes

- Instantaneous shock/turbulence interaction dynamics:
  - Shock weakens (to the point of disappearing!) locally -- smooth profiles through shock in nonlinear regime
  - "Strong" shock events with up to twice the average compression

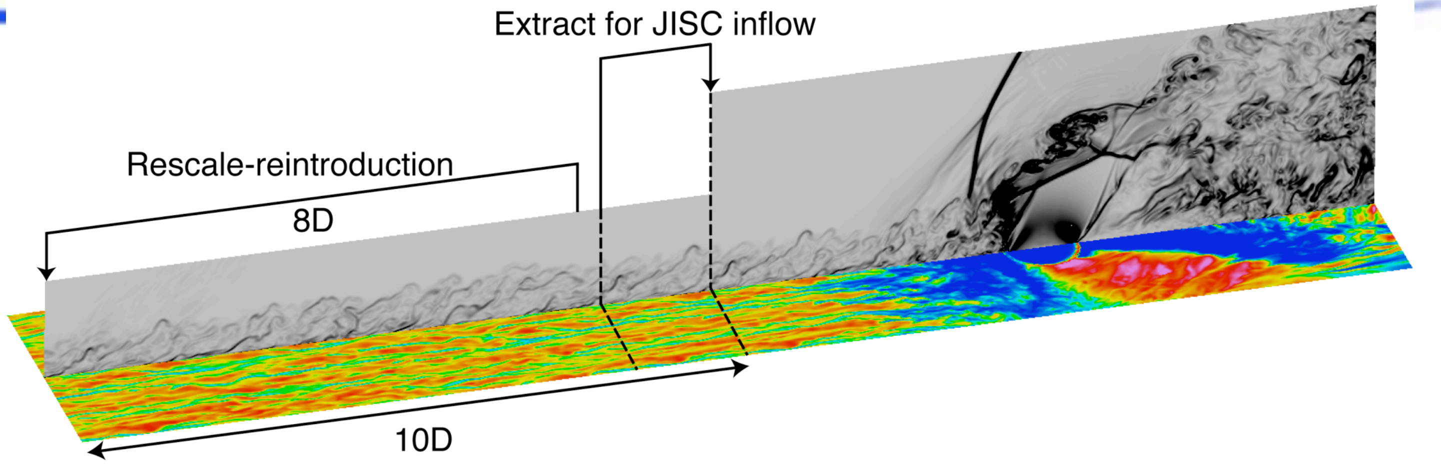
A more complex flow with shock-turbulence interaction

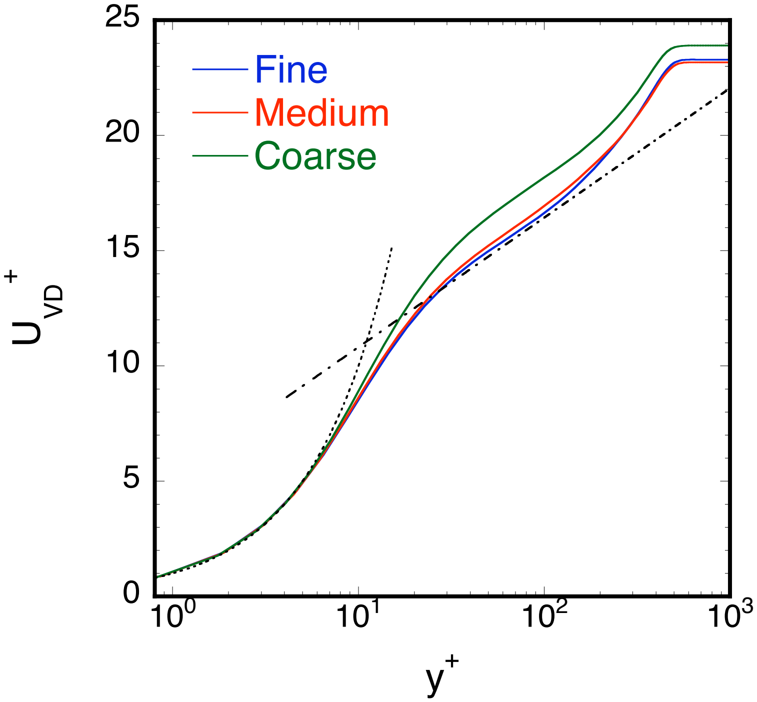


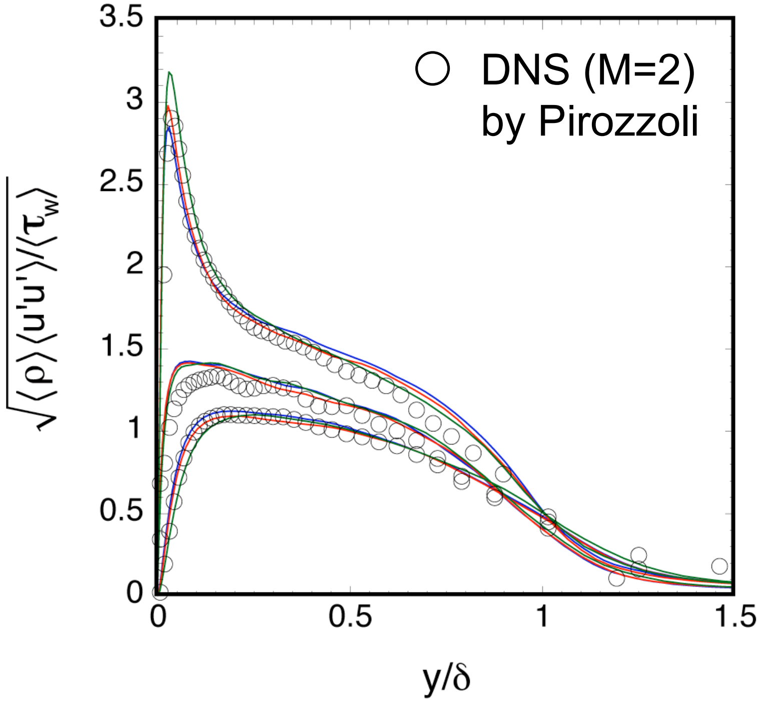
Expt. By Santiago & Dutton, 1997, JPP; Ben Yakar et al. 2006, PoF

LES by Kawai & Lele, AIAA 2009-3995

JISC with Turbulent Crossflow

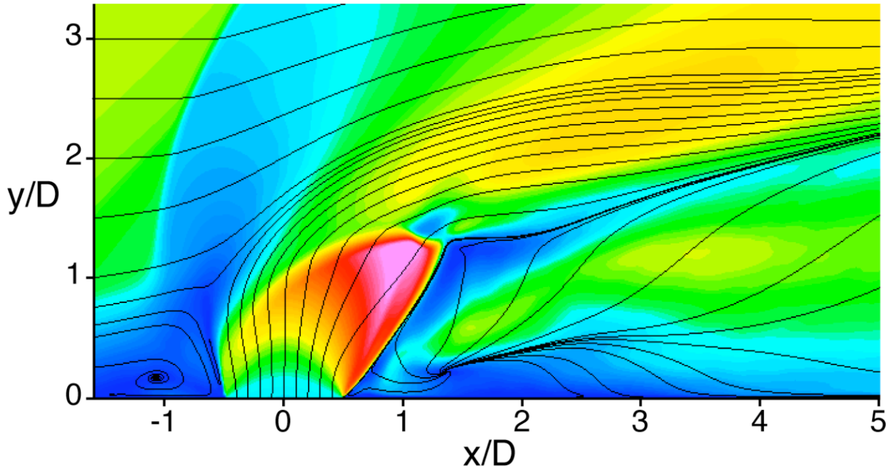


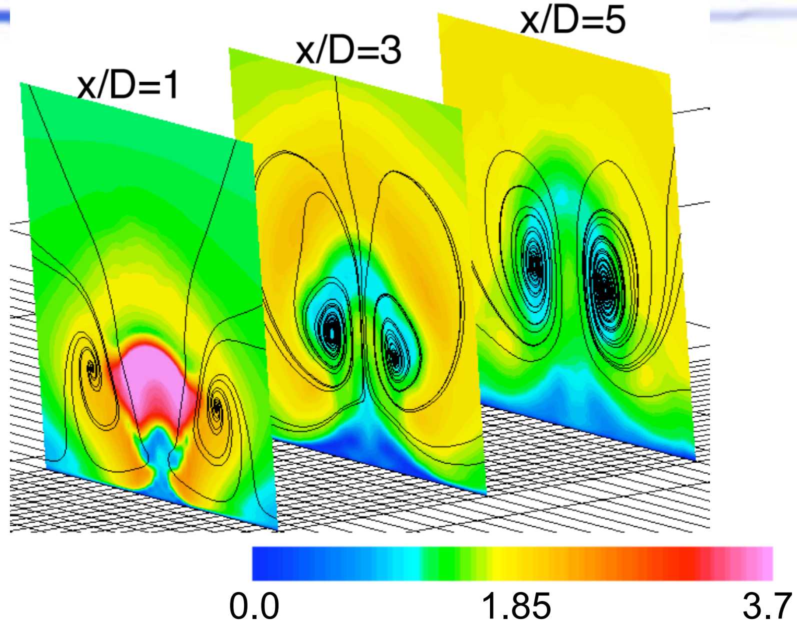




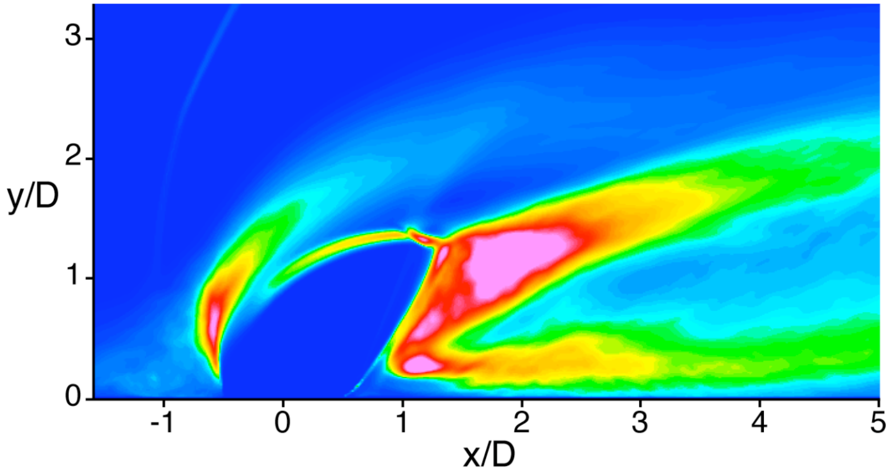
Time-Averaged Flowfields

Mach number

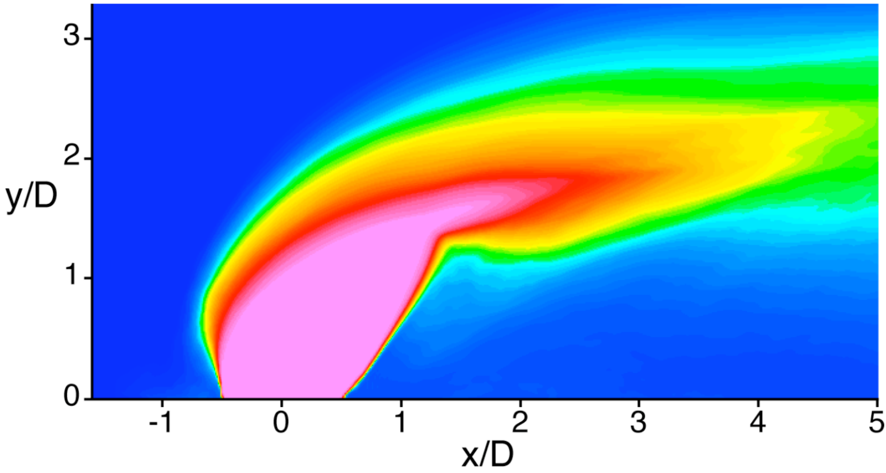




Turbulent kinetic energy



Jet fluid (passive scalar)

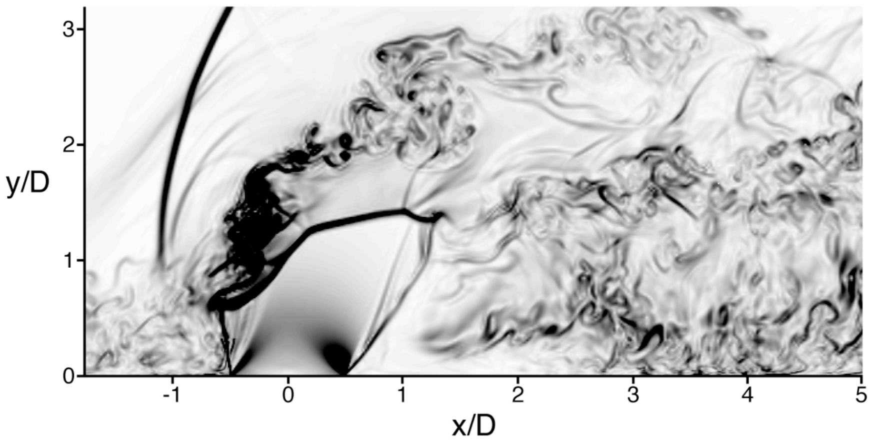


Animation

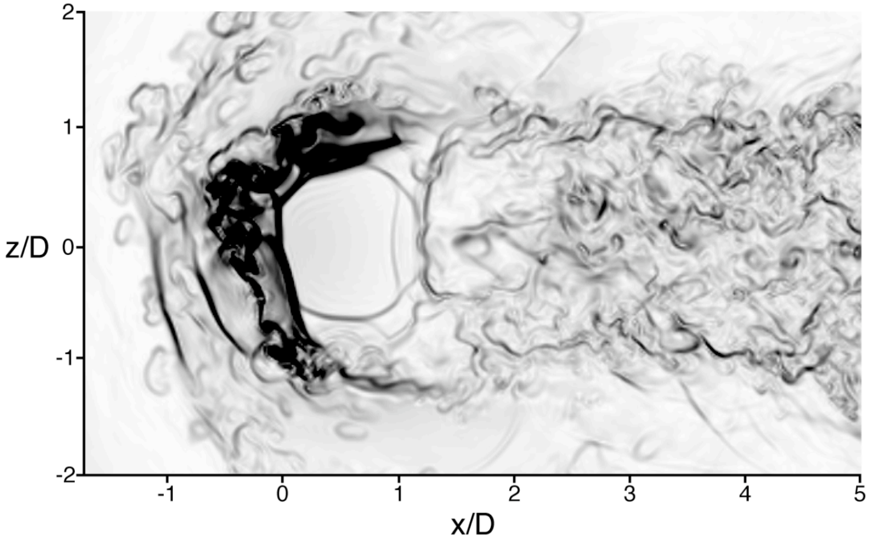
Norm of density gradient

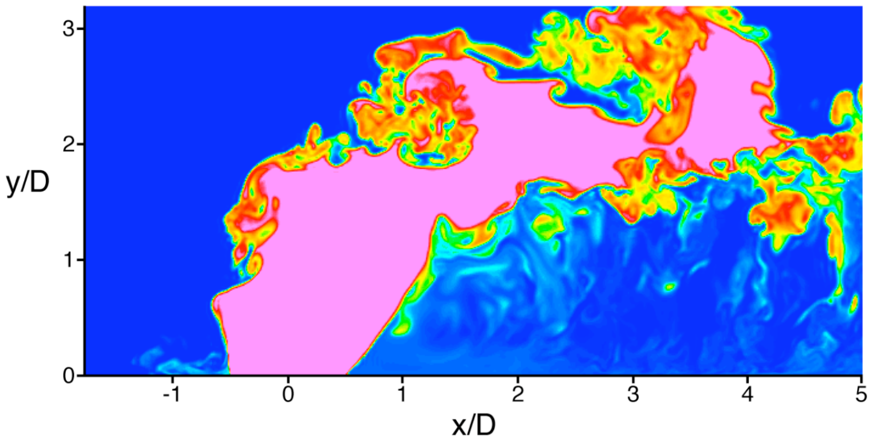
Jet fluid (scalar)

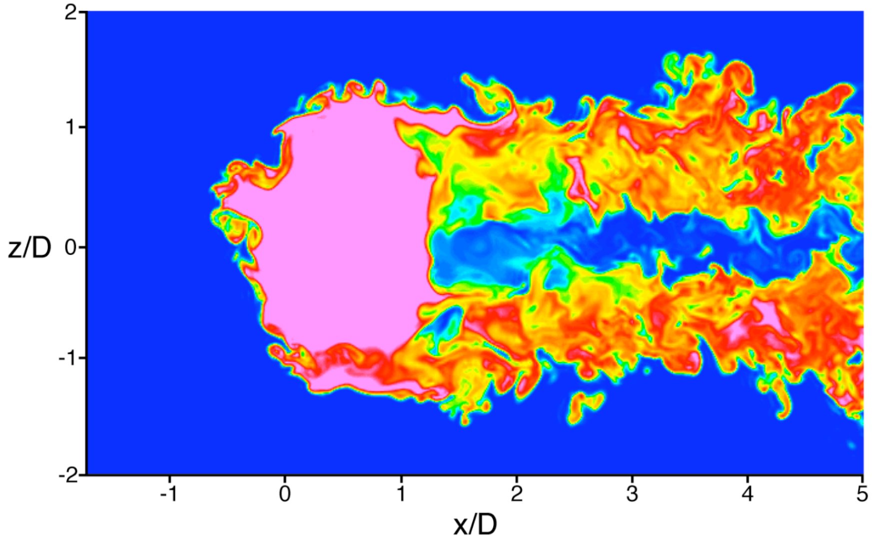
Sideview at z/D=0



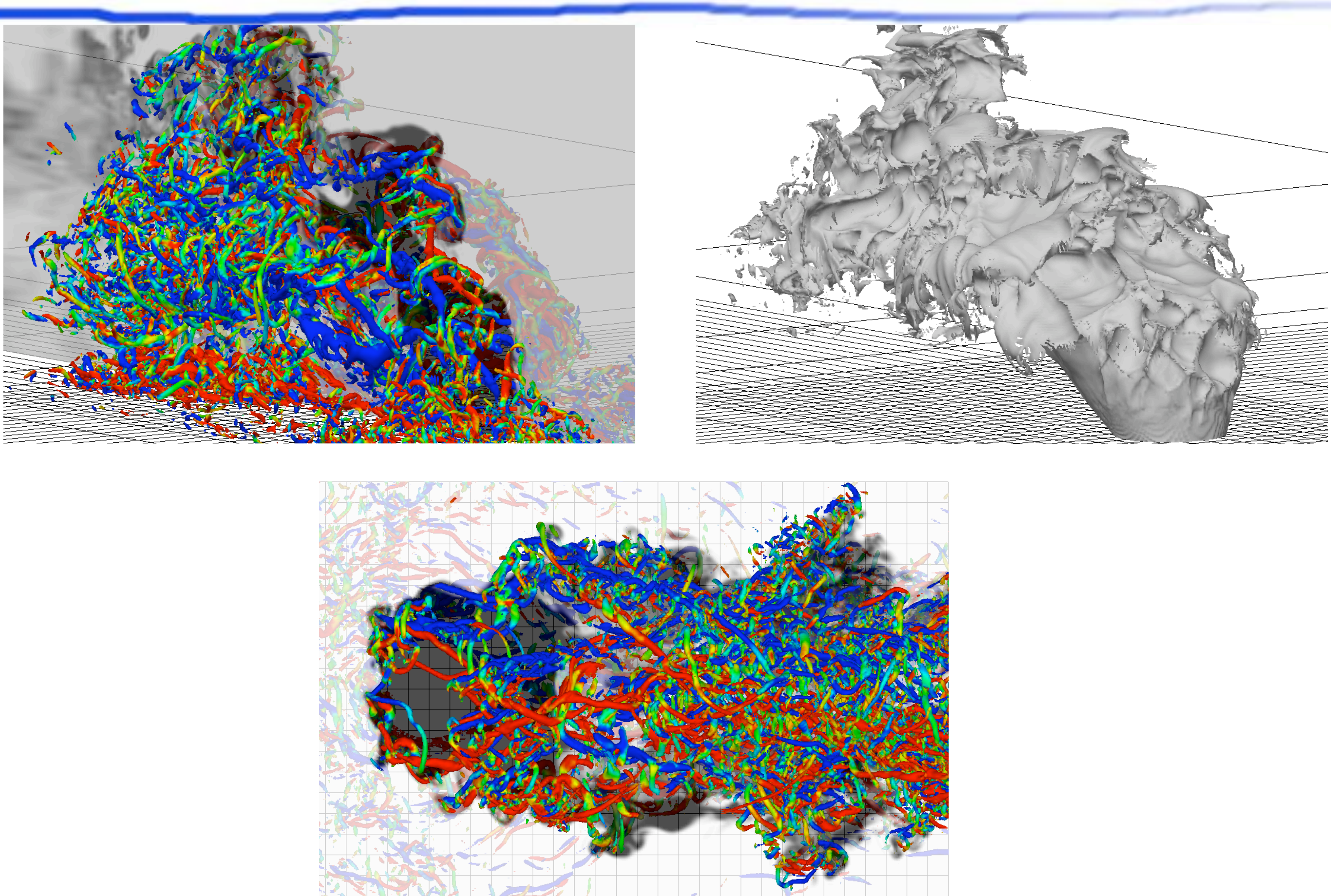
Topview at y/D=1







3D Flow Structures



Closure …

- Revisit to canonical shock-turbulence interaction (DNS)
- Other studies (DNS/LES) of shock-turbulent BL interaction, compression ramp..
- More complex engineering flows involving injection, mixing and combustion in high-speed flows

These are good laboratories for improving our understanding of turbulence behavior, its engineering modeling (RANS & LES), and for studies of aeroacoustics and flow control.

What will the next 50 years bring ?

Exciting science and collaborative research for sure. Happy 50 ! And Thanks.

Additional slides

Summary -- new and unanticipated questions

- Effect of turbulent Mach number on amplification of turbulence -- viscous effects make conclusions difficult
    - New Re_λ = 60 runs should clarify this

- Why is peak shock-motion at larger length scales than peak turbulence energy?
    - Theory suggests shock-motion "enslaved" to incoming turbulence
    - Need runs with higher k_0 to allow for deeper analysis

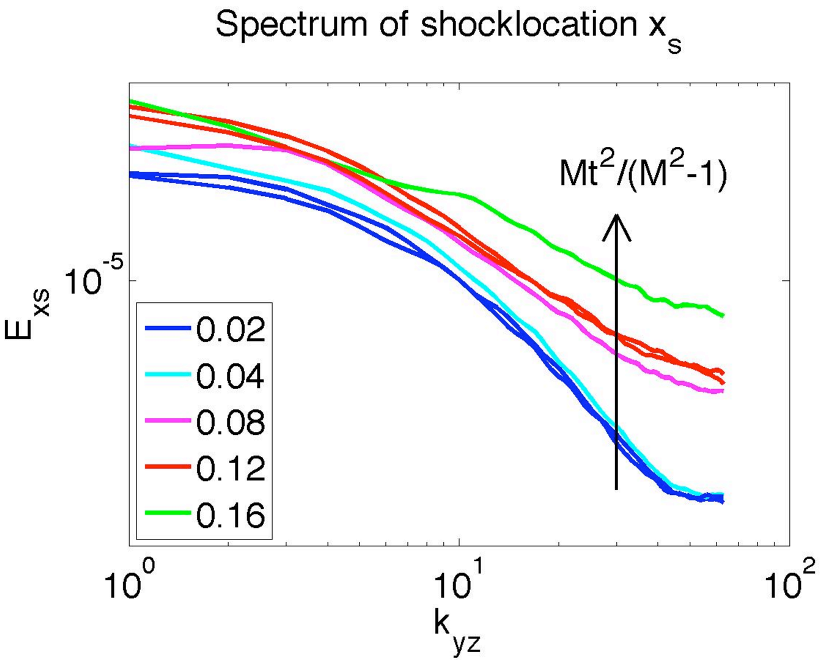
- What is the relation between shock-motion and Reynolds number? Or is the shock-motion dependent on some unknown parameter that was not matched?

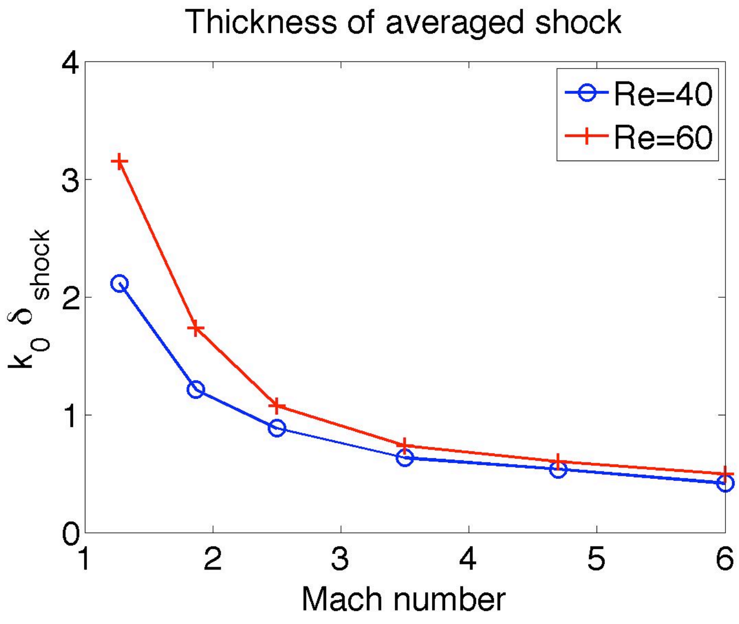
- What is the turbulence structure behind the shock -- statistics show elongation in streamwise direction, but what does it look like?
    - More, and more intelligent, visualization needed

Unsteady shock motion

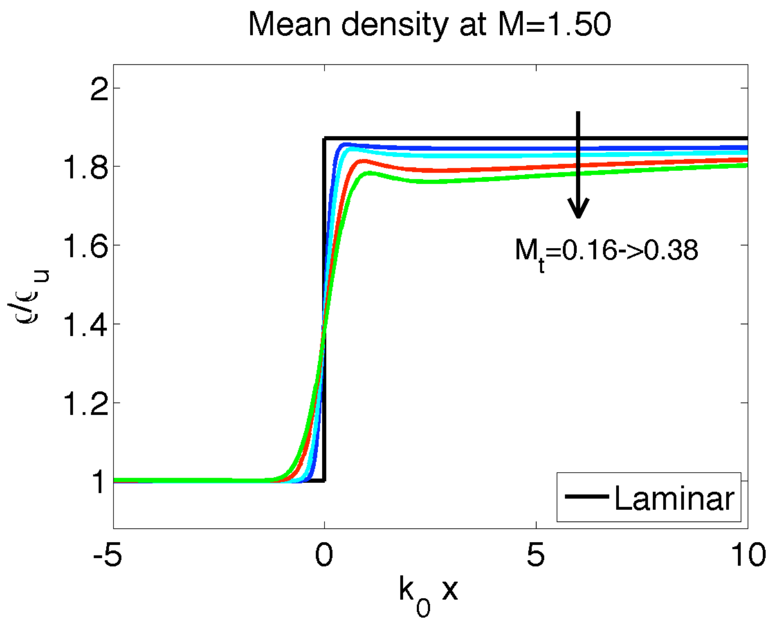
- Interesting finding: spectrum of shock motion seems to scale with ratio of turbulent pressure fluctuation to shock-induced pressure jump: M_t^2 / (M^2-1)

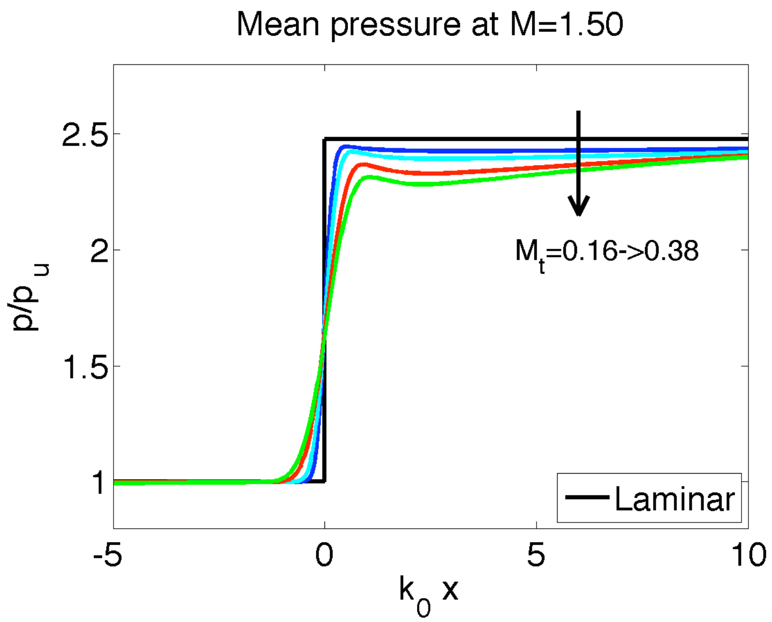
- Puzzle #1: turbulence has peak energy at k=4, but shock motion has peak at k=1-2
- Puzzle #2: thickness of averaged shock increases with Re





Mean profiles


Mean density at M=1.50


Mean pressure at M=1.50

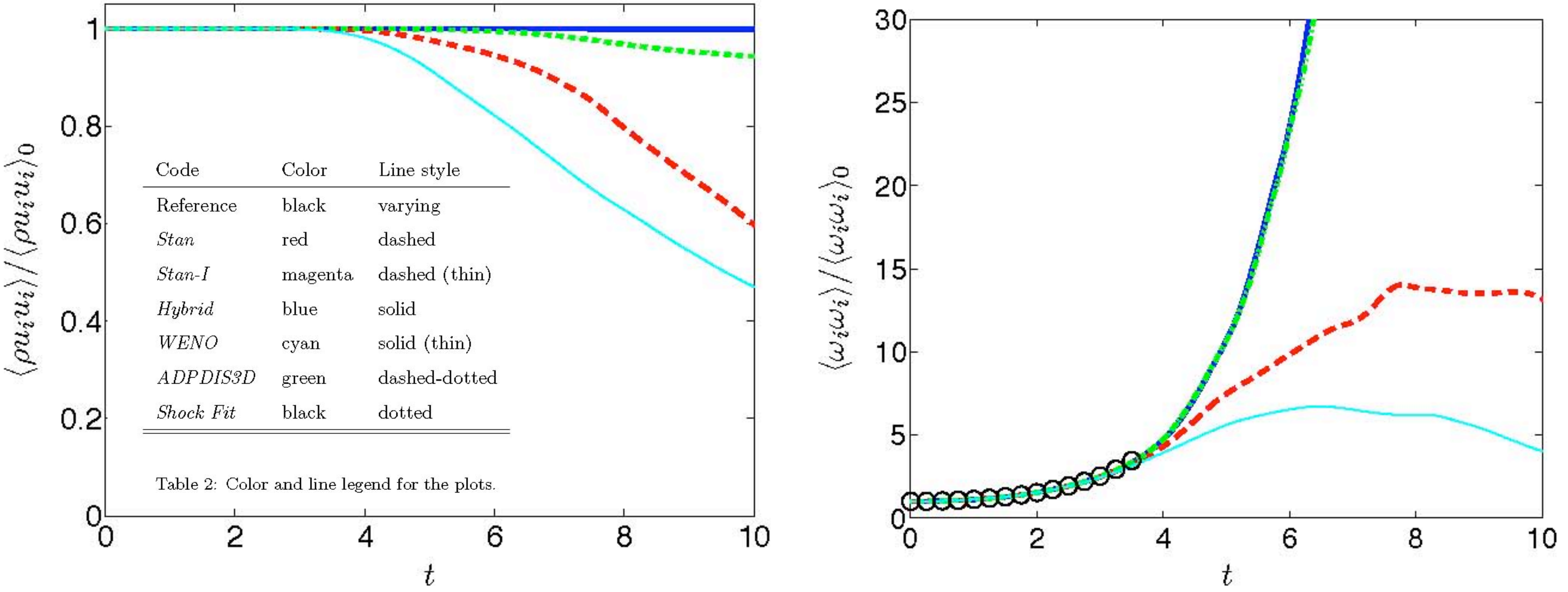
- Turbulence modifies the Rankine-Hugoniot relations (Lele, PhysFl 1992):

$$\begin{aligned}
\overline{\rho}\widetilde{u} &= \mathrm{const} \\
\overline{\rho}\widetilde{u}^2 + \overline{p} + \overline{\rho}\widetilde{u''u''} &= \mathrm{const} \\
\overline{\rho}\widetilde{h_0}\widetilde{u} + \overline{\rho}\widetilde{h_0''u''} &= \mathrm{const}
\end{aligned}$$

Numerical Algorithm Assessment: Taylor-Green Problem

- Inviscid problem with strong vortex-stretching

  Kinetic energy conservation



Code	Color	Line style
Reference	black	varying
Stan	red	dashed
Stan-I	magenta	dashed (thin)
Hybrid	blue	solid
WENO	cyan	solid (thin)
ADPDIS3D	green	dashed-dotted
Shock Fit	black	dotted

Table 2: Color and line legend for the plots.

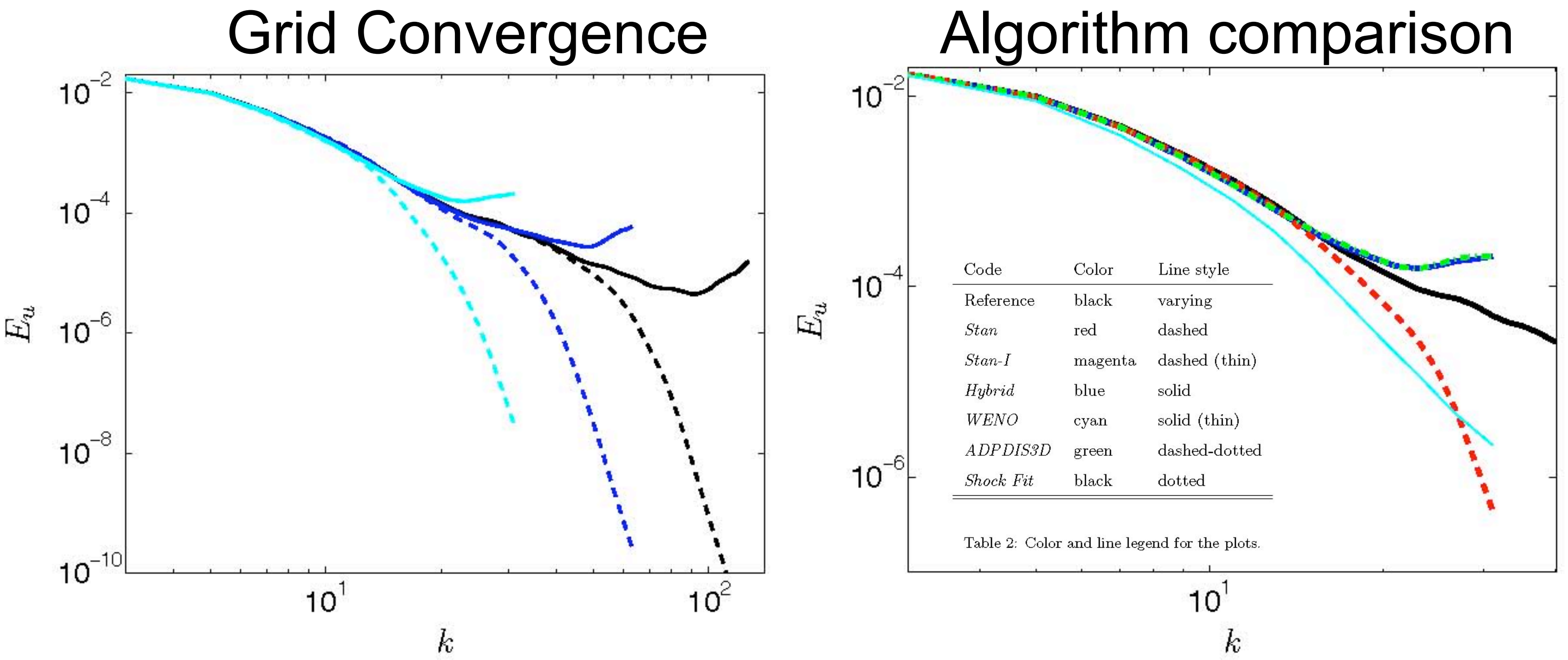
(a) Kinetic energy.

(b) Enstrophy. The semi-analytical result of Brachet *et al.* (1983) are the black symbols.

Figure 1: Mean quantities for the Taylor-Green vortex on a 64^3 grid. The zero subscript denotes the initial value.

Numerical Algorithm Assessment: Taylor-Green Problem

- Kinetic Energy Spectrum


Grid Convergence

Algorithm comparison

Code	Color	Line style
Reference	black	varying
Stan	red	dashed
Stan-I	magenta	dashed (thin)
Hybrid	blue	solid
WENO	cyan	solid (thin)
ADPDIS3D	green	dashed-dotted
Shock Fit	black	dotted

Table 2: Color and line legend for the plots.

(a) Convergence of the reference solution using the *Hybrid* code in standard mode (solid) and with eight-order accurate dissipation (dashed), on 256^3 (black), 128^3 (blue), and 64^3 (cyan).

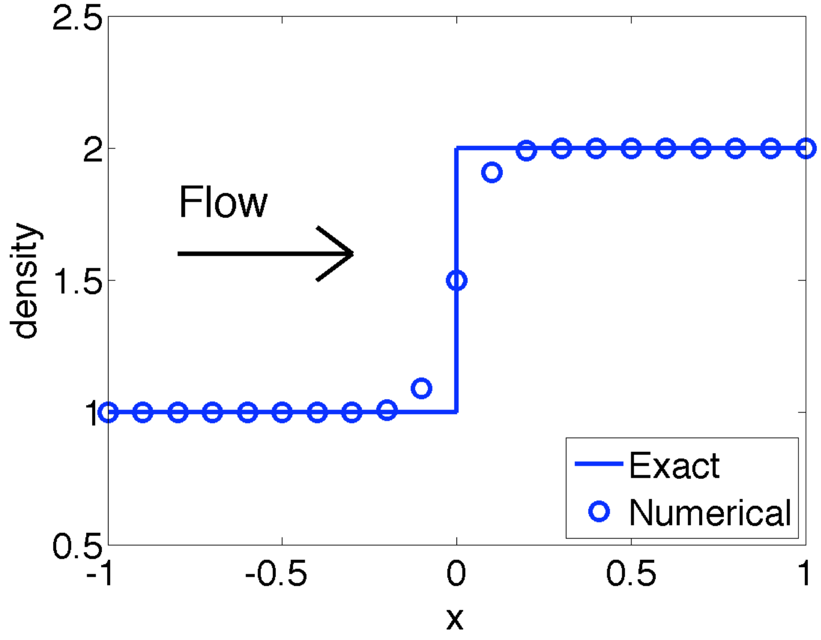
(b) Comparison between the different schemes. The reference solution is that obtained on the 256^3 grid using the *Hybrid* code.

Figure 2: Velocity spectra for the Taylor-Green vortex on 64^3 grid at $t = 5$.

Direct numerical simulation (DNS)

- Solve the Navier-Stokes equations without modeling
- Numerical challenges:
  - Shock-capturing requires numerical dissipation to smear shock
  - Numerical dissipation kills turbulence

Numerically captured shock



Temperature variance