Formation of Columnar Structure in Rotating Turbulence

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What is responsible for the formation?

Taylor-Proudman theorem

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{u} = -\frac{1}{\rho}\boldsymbol{\nabla} p - 2\boldsymbol{\Omega} \times \boldsymbol{u} + \nu \triangle \boldsymbol{u}$$

$$\nabla \times \Big\Downarrow$$

$$2\Omega \frac{\partial}{\partial z}\mathbf{u} = \frac{\partial}{\partial t}(\nabla \times \mathbf{u}) + \nabla \times (\mathbf{u} \cdot \nabla \mathbf{u}) - \nu \nabla^2 (\nabla \times \mathbf{u})$$

In the limit of large Ω → $\frac{\partial}{\partial z}\mathbf{u} = 0$

The steady flow is two-dimensional, i.e.
no variation in the direction parallel to the rotation axis (// z-zxis)

Two Key Elements of the Dynamics

Nonlinear dynamics due to the convection of fluid

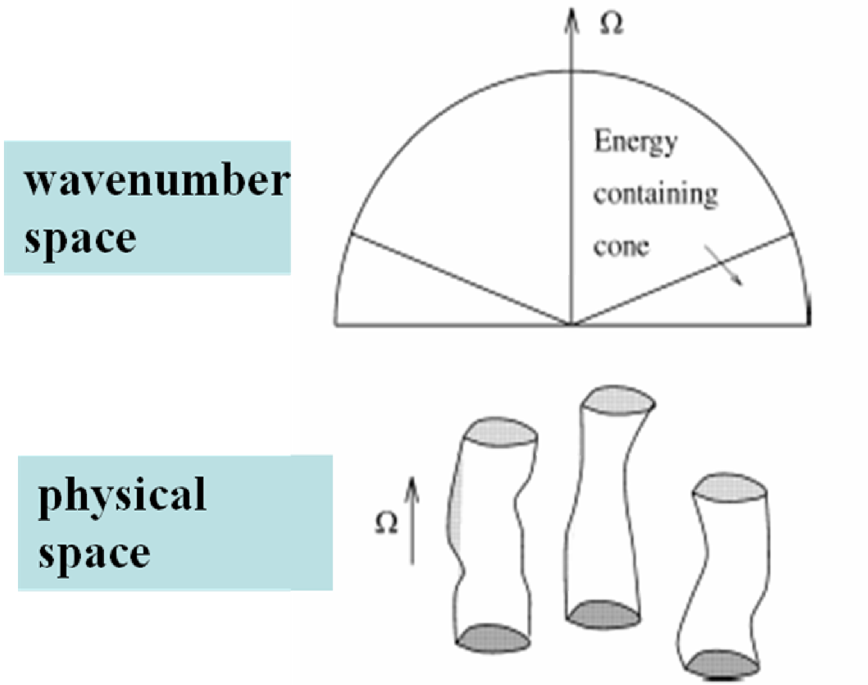
… **selective energy transfer/decay in spectral space**

Linear dynamics due to the Coriolis force

… **anisotropic linear effects due to inertial waves**

Nonlinear (advection and Coriolis terms)

The nonlinear energy transfer under the existence of the Coriolis force gives rise the concentration of energy close to the plane k_3=0

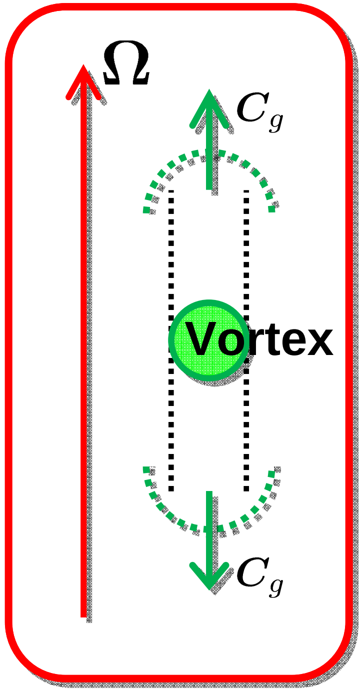


Claude Cambon *Eur. J. Mech. B - Fluids 20 (2001) 489–510*

- **Cambon and Jacquin (***J. Fluid Mech***., 202,** *295-317***)**
- **Waleffe**（1993 *Phys. Fluids A* **5**）
- **Cambon, Mansour and Godeferd**（1997 *J.Fluid Mech***. 337,** 303-332）
- **Bellet, Godefeld, Scott and Cambon**（2006 *J.Fluid Mech***. 562,** 83-121）

etc…

Linear Davidson et al: inhomogeneous turbulence $Ro \approx 1$



isolated compact eddy

The axial components of its linear impulse and angular momentum disperse only along the rotation axis.

C_g **Group velocity of inertial wave**

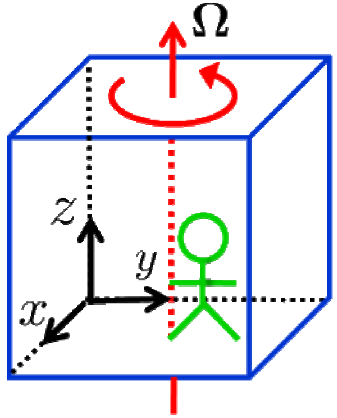
- **Davidson, Staplehurst and Dalziel** (2006 *J.Fluid Mech*. **557,** 135-144)
  (inhomogeneous rotating turbulence)
- **Staplehurst, Davidson and Dalziel** (*2008 J. Fluid Mech.* **598**, *81-105*)
  (homogenous rotating turbulence)

Objective

The role of Nonlinear dynamics & Linear dynamics=Coriolis force

- **governing equations**

$$\begin{cases} \dfrac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{u} = -\dfrac{1}{\rho}\boldsymbol{\nabla} p - 2\boldsymbol{\Omega} \times \boldsymbol{u} + \nu \triangle \boldsymbol{u} \\ \boldsymbol{\nabla} \cdot \boldsymbol{u} = 0 \end{cases}$$



Nonlinear **Linear/Coriolis**

Coordinate system subjected to constant rotation

Ω // z =x_3 axis

CASE 1

“Formation of Columnar Structure”

“Elongation” along …

Quantification ? -- Length Scale

The answer depends on the definition of the length scale

Case 1

Dynamics = Linear & Inviscid
at t=0, a periodic array of compact eddies

- **governing equations**

Linear/Coriolis

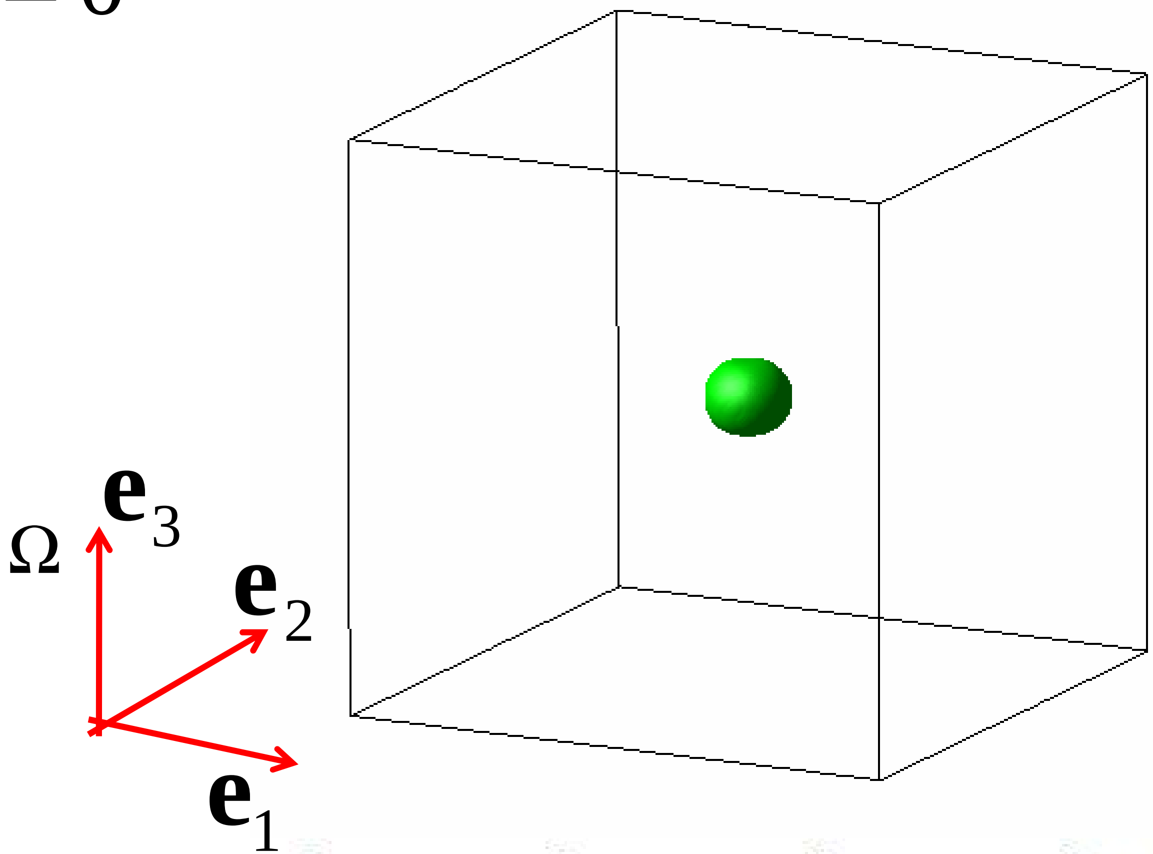
$$
\begin{cases}
\dfrac{\partial \boldsymbol{u}}{\partial t} + \cancel{(\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u}} = -\dfrac{1}{\rho}\boldsymbol{\nabla} p - 2\boldsymbol{\Omega}\times\boldsymbol{u} + \cancel{\nu\triangle\boldsymbol{u}} \\[2ex]
\boldsymbol{\nabla}\cdot\boldsymbol{u} = 0
\end{cases}
$$

$$
\frac{\partial \boldsymbol{u}}{\partial t} + 2\boldsymbol{\Omega}\times\boldsymbol{u} = -\frac{1}{\rho}\nabla p.
$$

Initially Compact Gaussian-like eddy in a periodic box

- **Iso-surface of modulus of vorticity**

$$\Omega t = 0$$



$$|\omega| = m + 3\sigma$$

$$(u_1, u_2, u_3) = \Lambda \exp\left[-\{(x-\pi)^2 + (y-\pi)^2 + (z-\pi)^2\}/\delta^2\right](-y+\pi, x-\pi, 0),$$

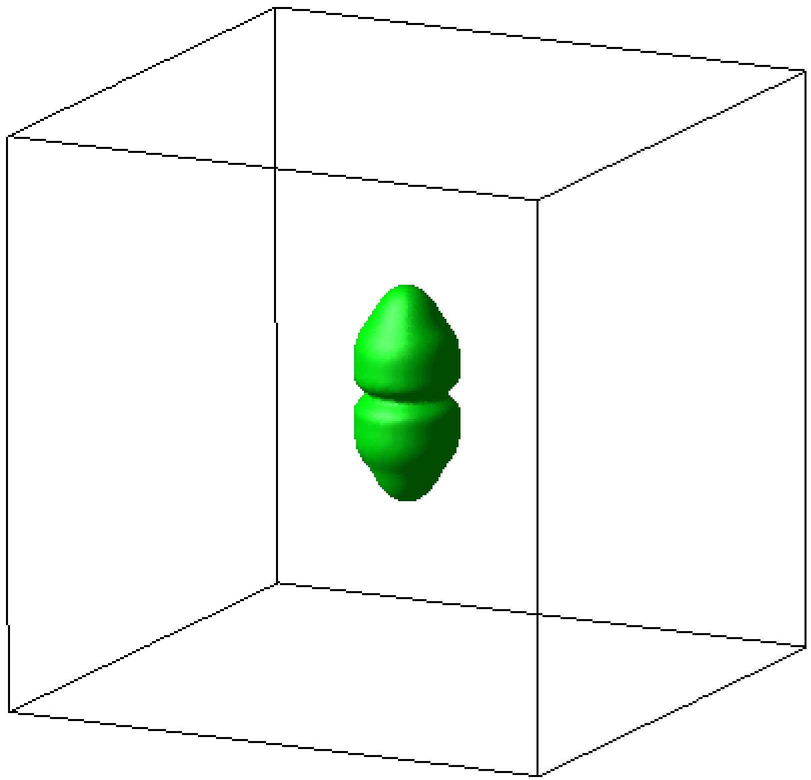
$\Lambda = 5 \qquad \delta = 0.2$

cf. Davidson, Staplehurst and Dalziel（2006 J.Fluid Mech.）

Linear — Gaussian-like eddy in a periodic box

- **Iso-surface of modulus of vorticity**

$$\Omega t = 3$$

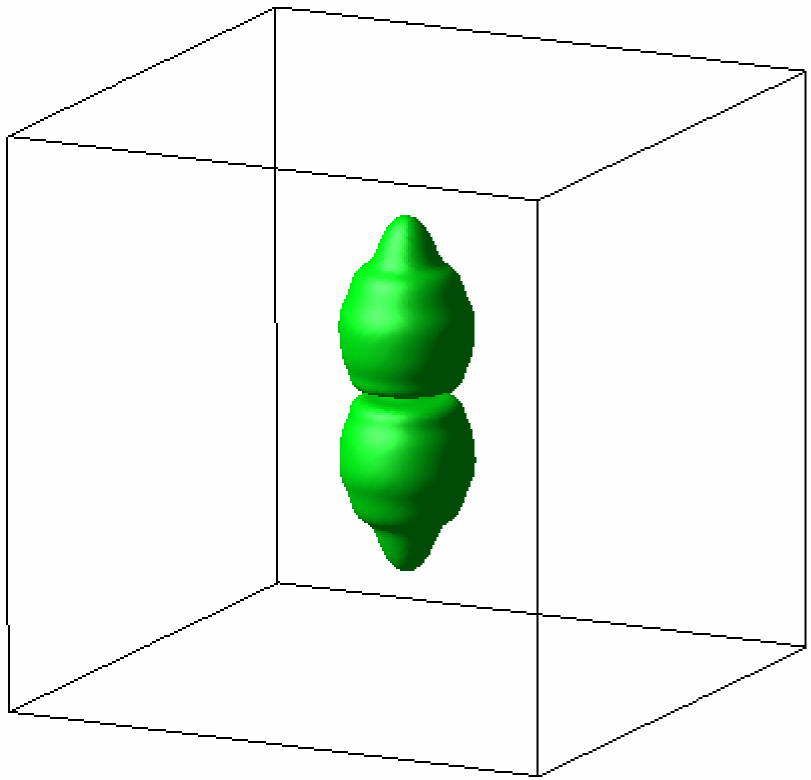


$$|\omega| = m + 3\sigma$$

Linear — Gaussian-like eddy in a periodic box

- **Iso-surface of modulus of vorticity**

$$\Omega t = 6$$



$$|\omega| = m + 3\sigma$$

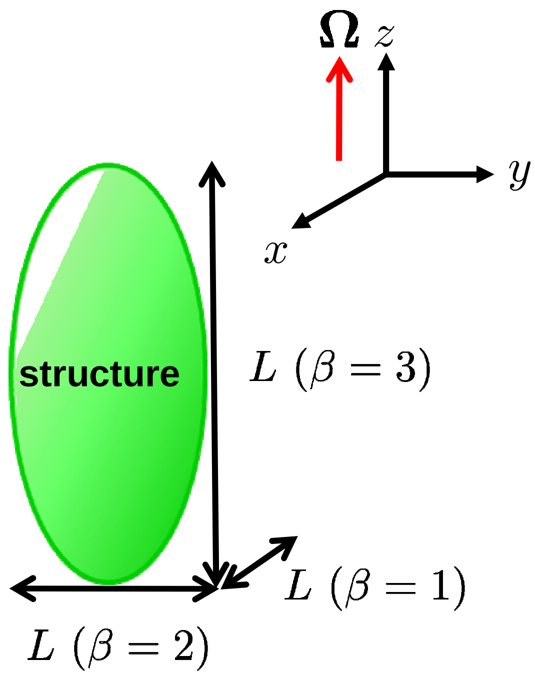
Quantification of the elongation:

Integral length scales in the β -th direction

$$L^{\beta}(\zeta) = \frac{\int_0^{\pi} \langle \bar{\zeta}(\boldsymbol{x},t)\bar{\zeta}(\boldsymbol{x}+r\boldsymbol{e}_{\beta},t)\rangle dr}{\langle \bar{\zeta}(\boldsymbol{x},t)^2\rangle}$$

$$\bar{\zeta} = \zeta(\boldsymbol{x},t) - \underbrace{\langle \zeta(\boldsymbol{x},t)\rangle}_{\text{spatial average}}$$

$$\zeta = \omega_i, u_i$$

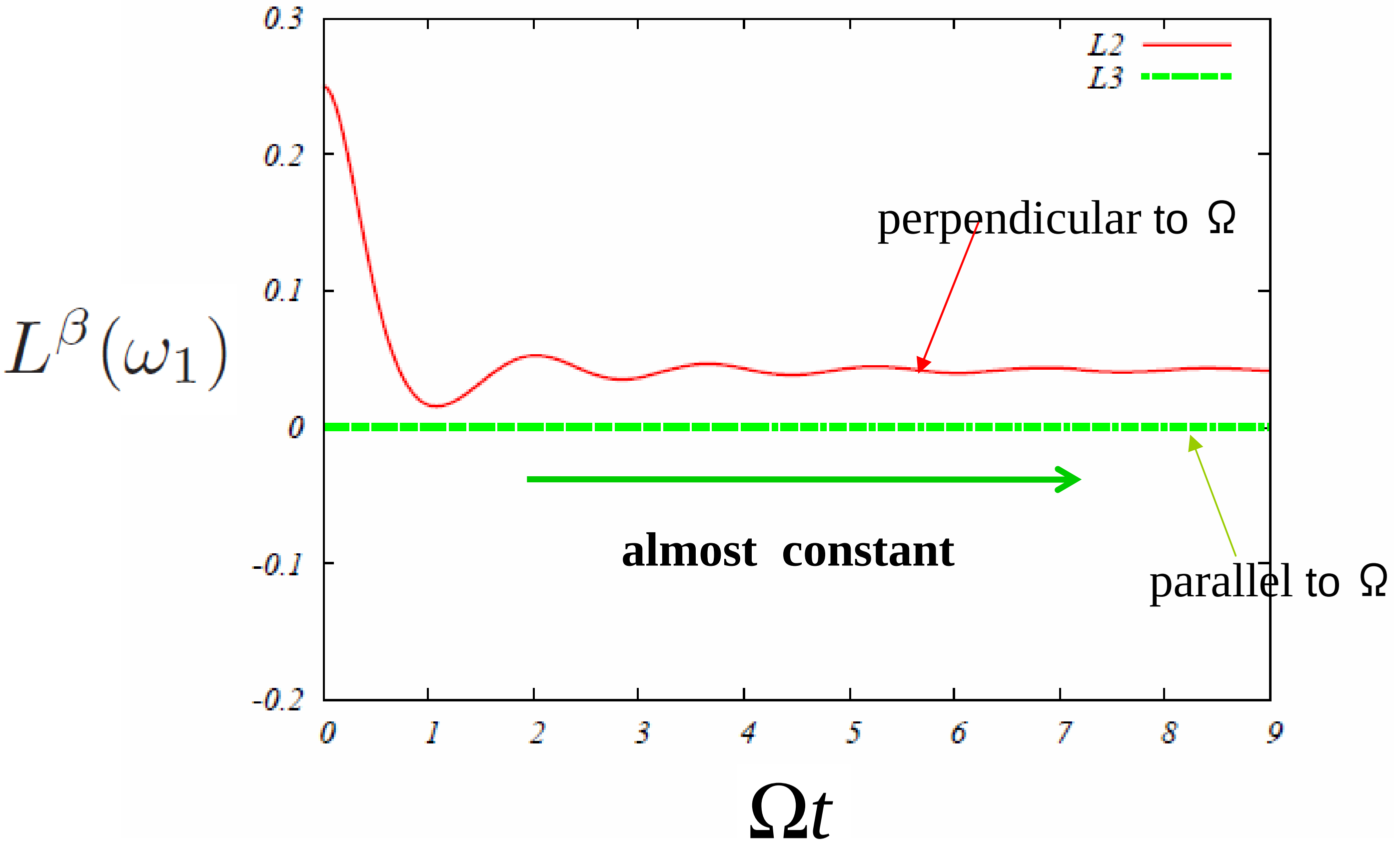


second-order moment

one-time two-point correlation

(Cambon and Jacquin (1989), Bartello, Metais and Lesieur(1994))

Linear: integral scale of ω_1 (vorticity component perpendicular to Ω)

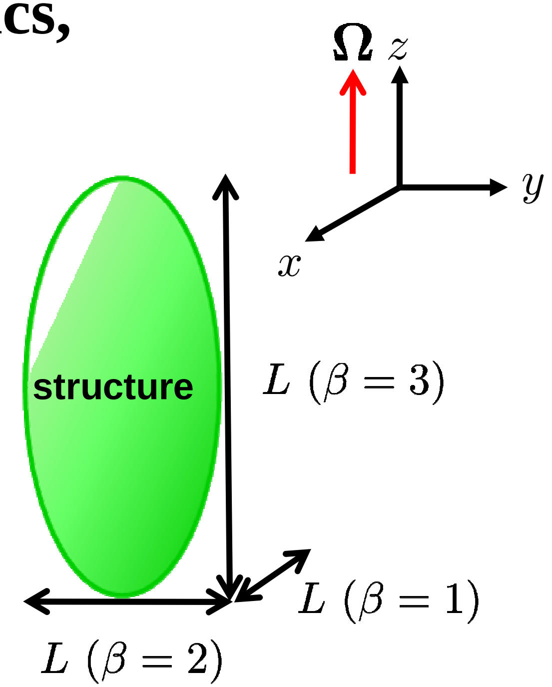


The integral scale in the axial direction does not grow
in contrast to the intuitive impression obtained from the visualization.

Integral length scales in the β direction

$$L^{\beta}(\zeta) = \frac{\int_0^{\pi} \langle \bar{\zeta}(\boldsymbol{x},t)\bar{\zeta}(\boldsymbol{x}+r\boldsymbol{e}_{\beta},t)\rangle dr}{\langle \bar{\zeta}(\boldsymbol{x},t)^2 \rangle}$$

dose not grow in time within the linear dynamics,
as far as ζ is linear in the velocity or vorticity



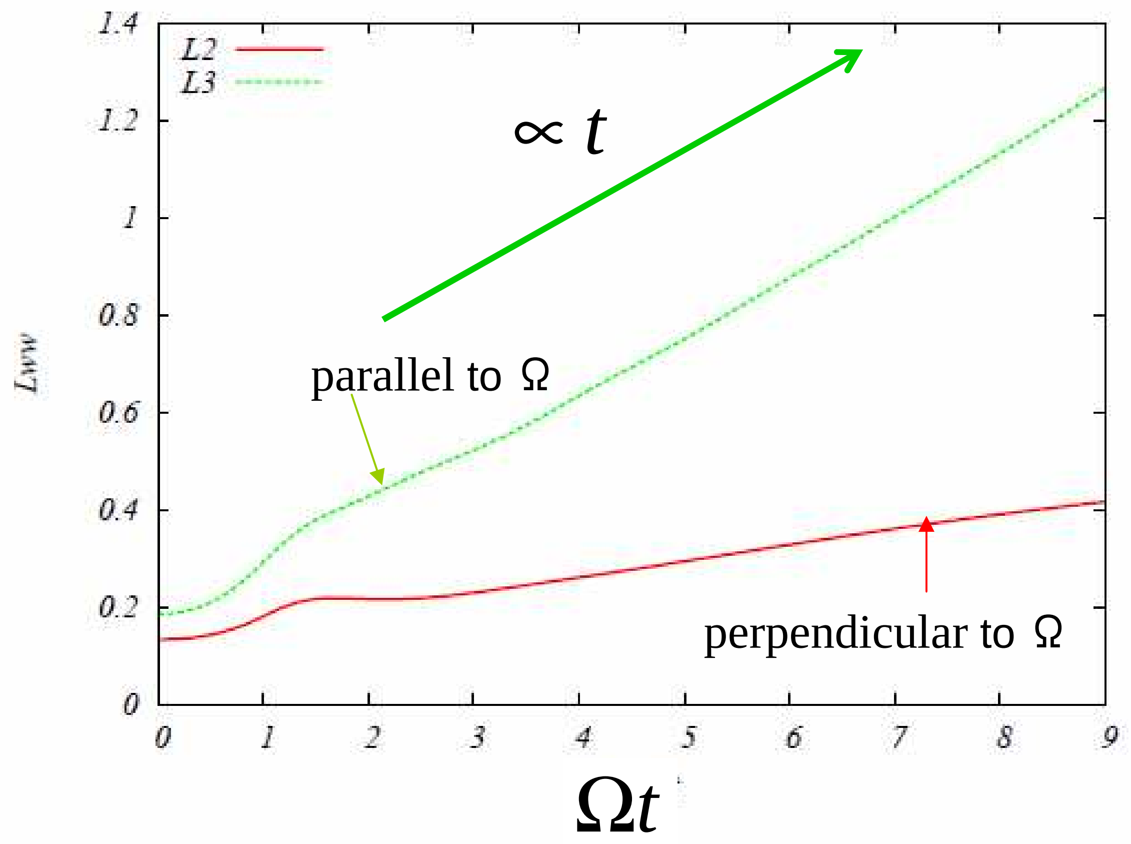
$$\zeta = \omega_1,\ \omega^2$$

$L^{\beta}(\omega_1)$ **: second-order moment**

$L^{\beta}(\omega^2)$ **: fourth-order moment**

Linear integral scale of ω^2

$L^{\beta}(\omega^2)$



- **Two conclusions for the same phenomenon:**

  1) There is no elongation in the z-direction

  2) There is elongation in the z-direction

- **The difference is due to the difference in the definitions of the length scale:** $L^{\beta}(\omega_1)$ $L^{\beta}(\omega^2)$

→ **the importance of**

1) the specification of the length scale

2) the proper choice of the definition of the length scale for quantitative discussion of the elongation.

CASE 2

Freely decaying homogeneous rotating turbulence

Dynamics: Navier Stokes Dynamics with Coriolis force

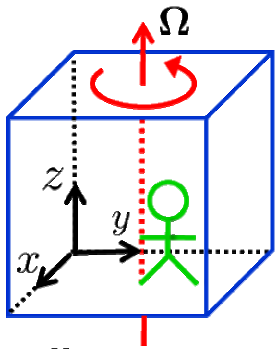
at t=0: Homogeneous developed turbulence,

Case 2: Homogeneous rotating turbulence

- **governing equations**

$$\begin{cases} \dfrac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u} = -\dfrac{1}{\rho}\boldsymbol{\nabla} p - 2\boldsymbol{\Omega}\times\boldsymbol{u} + \nu\triangle\boldsymbol{u} \\ \boldsymbol{\nabla}\cdot\boldsymbol{u} = 0 \end{cases}$$

- boundary condition : periodic box $\boldsymbol{2\pi \times 2\pi \times 2\pi}$
- number of grid points : 256*256*256
- alias-free Fourier spectral method
- removal of alias error :
  phase shift method + spherical cut
  $k < \sqrt{2}N/3, \quad N = 256$
- time marching : 4th order Runge-Kutta method



Coordinate system subjected to constant rotation

Run Conditions:
DNS of freely-decaying homogeneous rotating turbulence

	Ω	Initial field	$R_L(t=0)$	$R_\lambda(t=0)$	$Ro^\omega(t=0)$	$R_O^L(t=0)$
Run2-1	2.0	Ini1	93.2	48.0	4.48	0.24
Run5-1	5.0	Ini1	93.2	48.0	1.79	9.65×10^{-2}
Run10-1	10.0	Ini1	93.2	48.0	0.90	4.82×10^{-2}
Run5-2	5.0	Ini2	56.6	33.5	1.92	0.148

TABLE 1. The constant angular velocity Ω and initial Reynolds and Rossby numbers of freely-decaying homogeneous rotating turbulence.

$Ro^\omega = \omega'/(2\Omega)$ and $Ro^L = \langle\epsilon\rangle/(2\Omega E)$

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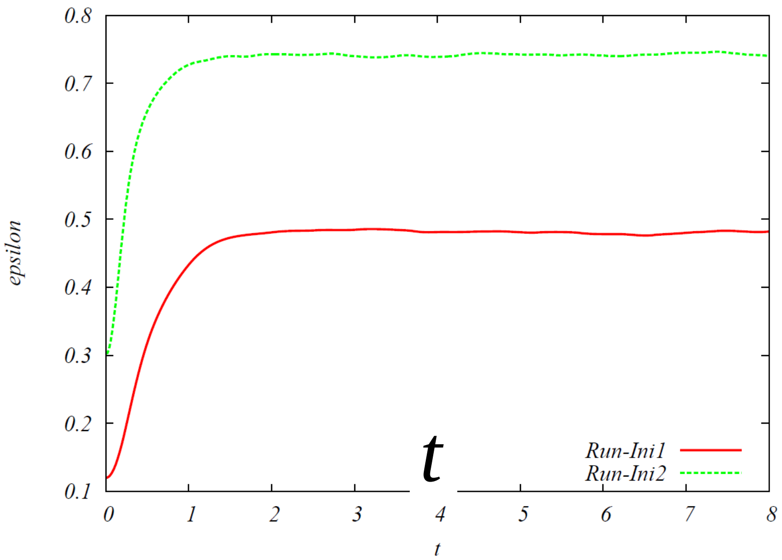
$$Ro^\omega = \omega'/(2\Omega) \text{ and } Ro^L = \langle\epsilon\rangle/(2\Omega E)$$

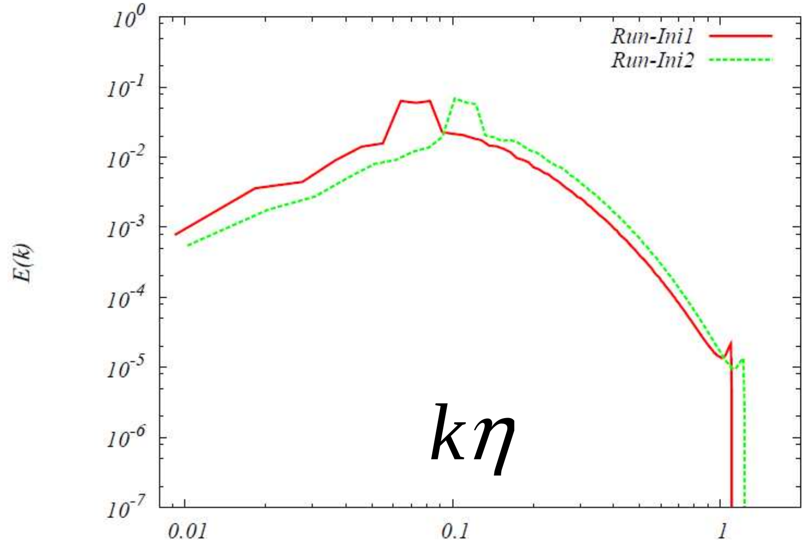
Initial filed:
3D homogeneous isotropic turbulence (forced)

in the preliminary runs to generate the initial fields

$< \varepsilon >$



$E(k)$



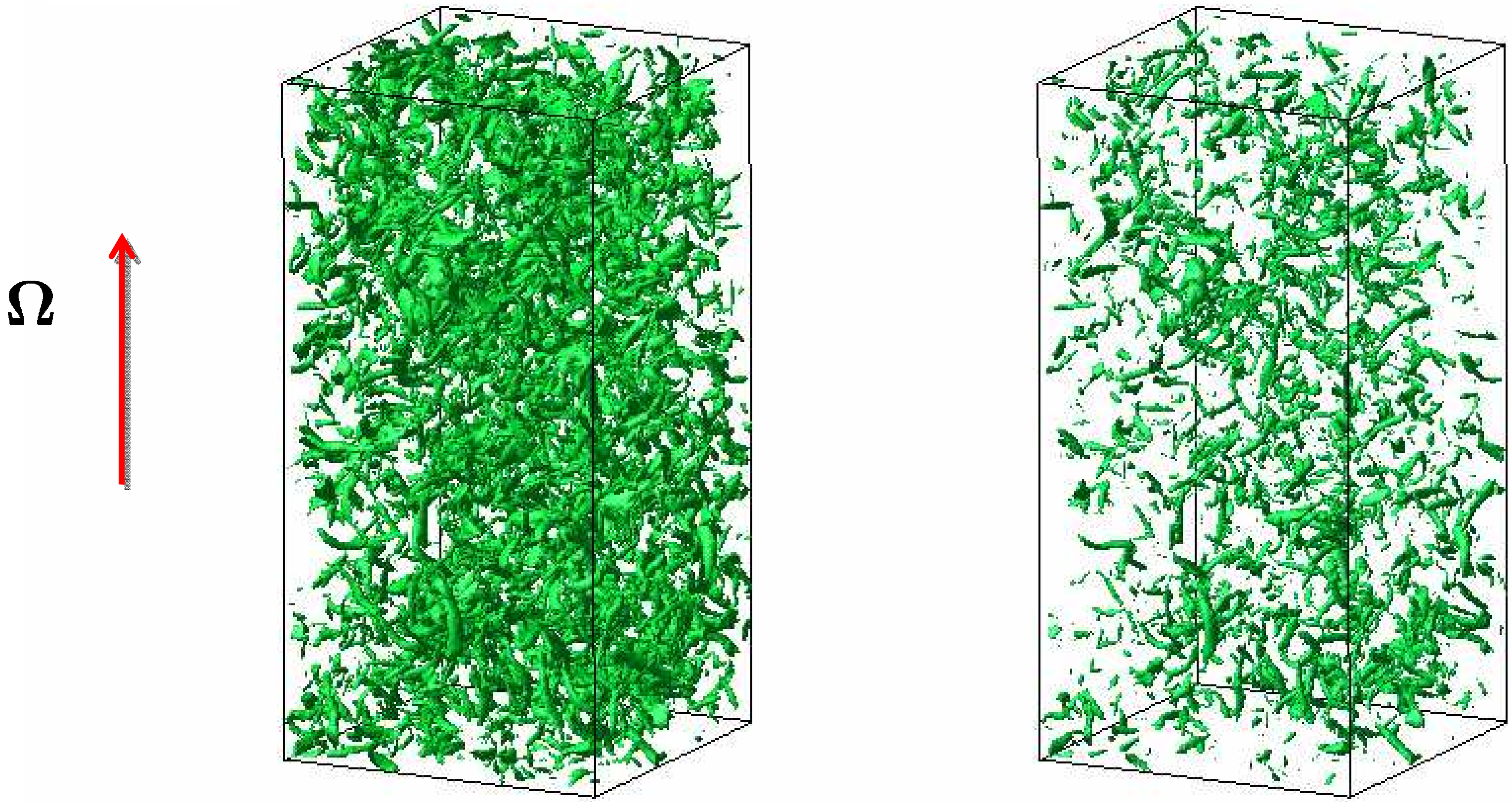
	$10^4\nu$	$f_{\min}$	$f_{\min}$	k_p	R_λ	R_L	$\langle\epsilon\rangle$	$10^3\eta$	L
Ini1	15.0	6.5	9.5	8.0	48.0	93.2	0.482	9.15	0.242
Ini2	20.0	9.5	12.5	11.0	33.5	56.6	0.740	10.2	0.196

TABLE 3. DNS parameters and characteristics of homogeneous isotopic turbulence at $t = 8$.

Nonlinear - rotating turbulence (128*128*256 domain)

- **Iso-surfaces of modulus of vorticity**

Ini1



$|\omega| = m + 2\sigma$ $|\omega| = m + 3\sigma$

Isotropic turbulence

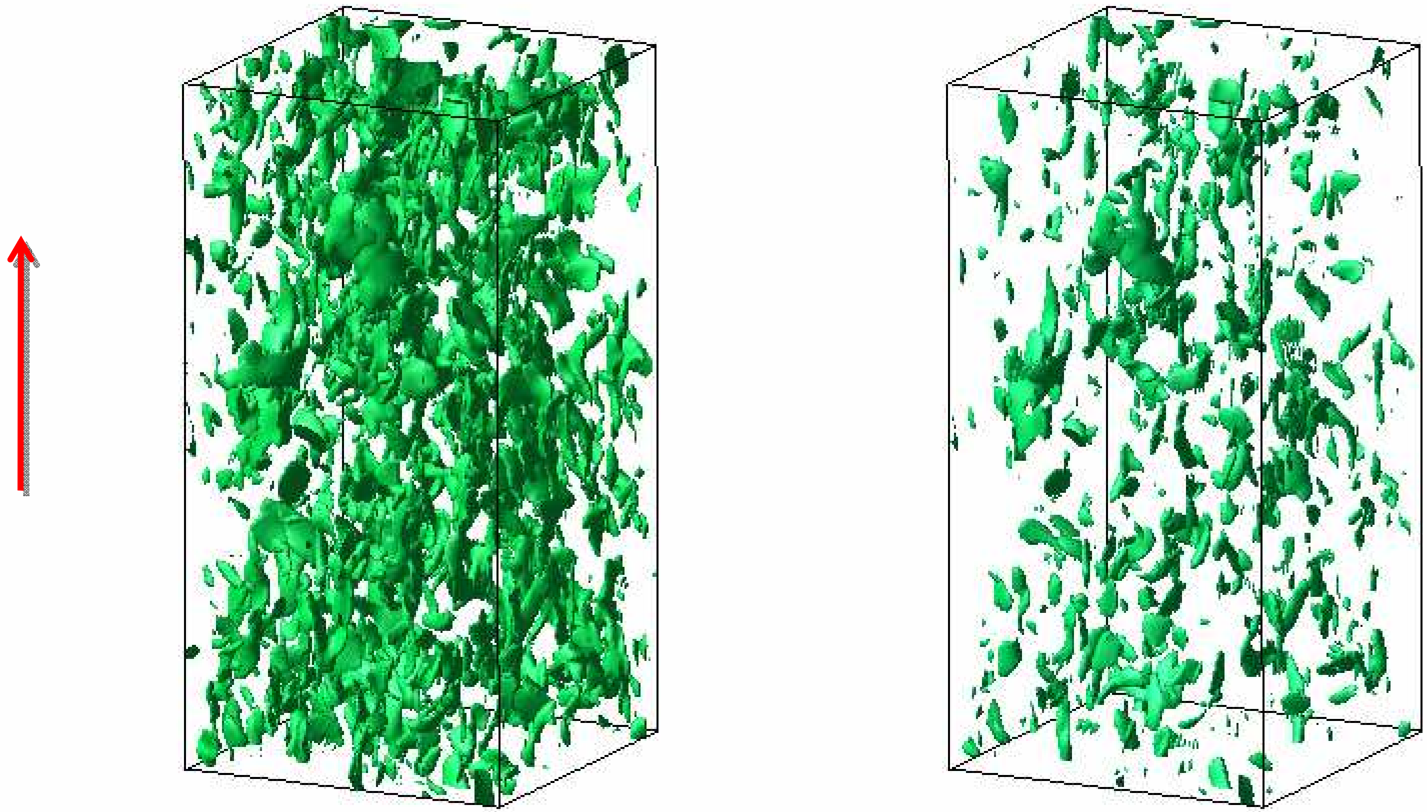
Nonlinear - rotating turbulence (128*128*256 domain)

- **Iso-surfaces of modulus of vorticity**

$\Omega t = 10$

$\Omega = 5, \mathrm{Ini1}$

Ω



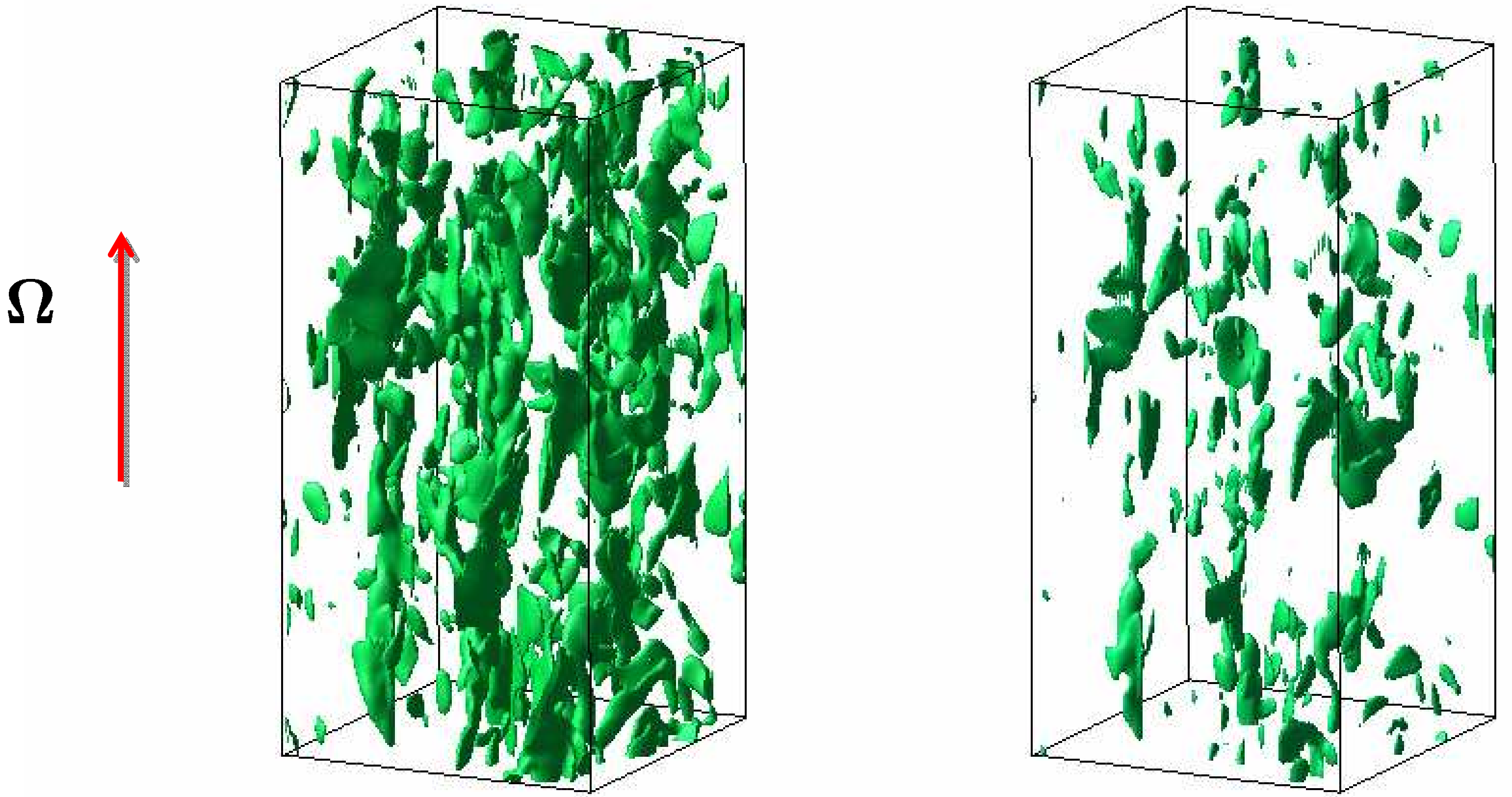
$|\omega| = m + 2\sigma$ $\qquad$ $|\omega| = m + 3\sigma$

Vortices becomes elongated along the rotational axis.

Nonlinear - rotating turbulence (128*128*256 domain)

- **Iso-surfaces of modulus of vorticity**

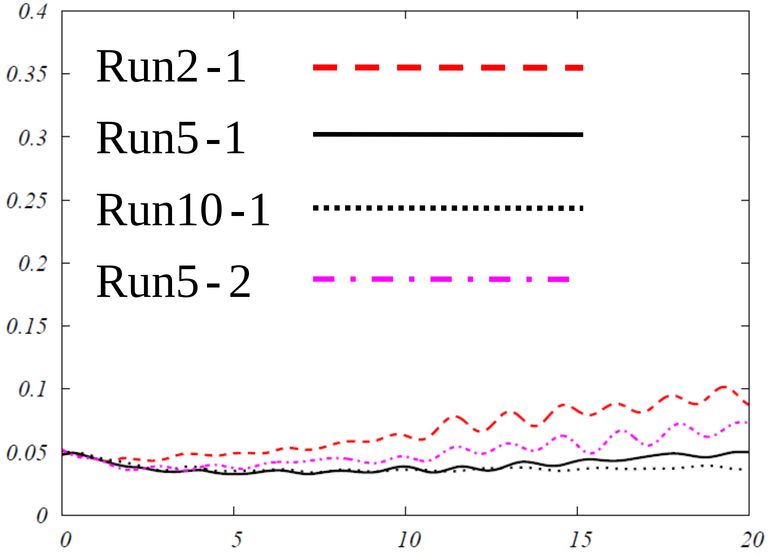
$$\Omega t = 20$$



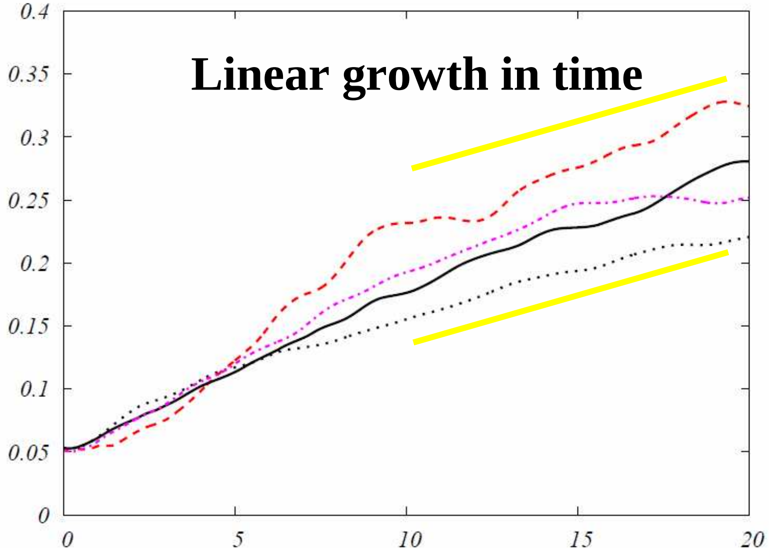
$$|\omega| = m + 2\sigma \qquad |\omega| = m + 3\sigma$$

integral scale of ω_1 (vorticity component perpendicular to **Ω**)

$L^2(\omega_1)$ perpendicular **to** Ω



$L^3(\omega_1)$ parallel to Ω



Ωt **Selective growth of** $L^3(\omega_1)$ Ωt

The growth of $L^3(\omega_1)$ **in the axial direction is much faster than** $L^2(\omega_1)$

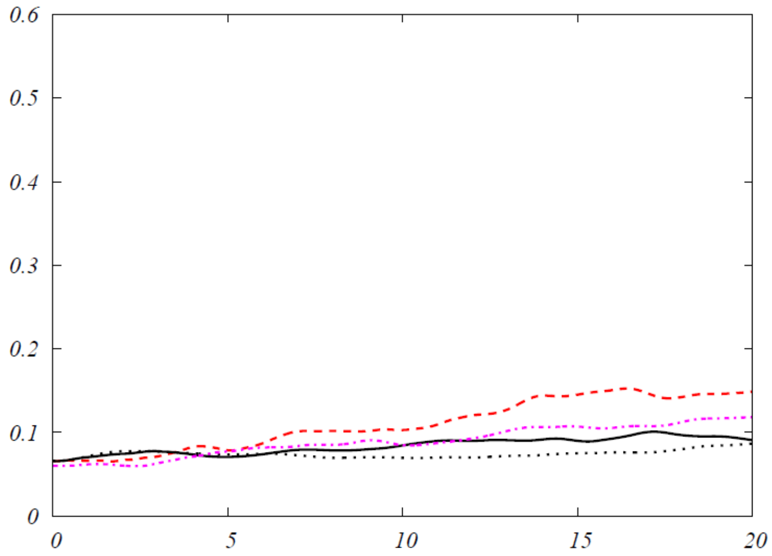
→ selective growth

The growth rates averaged over an appropriate time interval are similar, and linear in t.

This is in agreement with the collapse of corresponding correlation function reported by Staplehurst et al. (*2008 J. Fluid Mech.* **598**, *81-105*)**.**

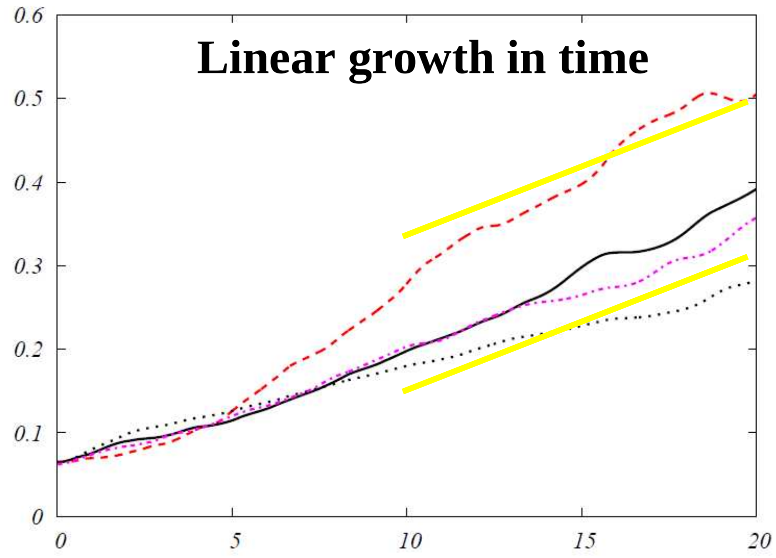
integral scale of ω^2

$L^2(\omega^2)$ perpendicular to Ω



Ωt

$L^3(\omega^2)$ parallel to Ω



Ωt

Selective growth of $L^3(\omega^2)$

Run2-1

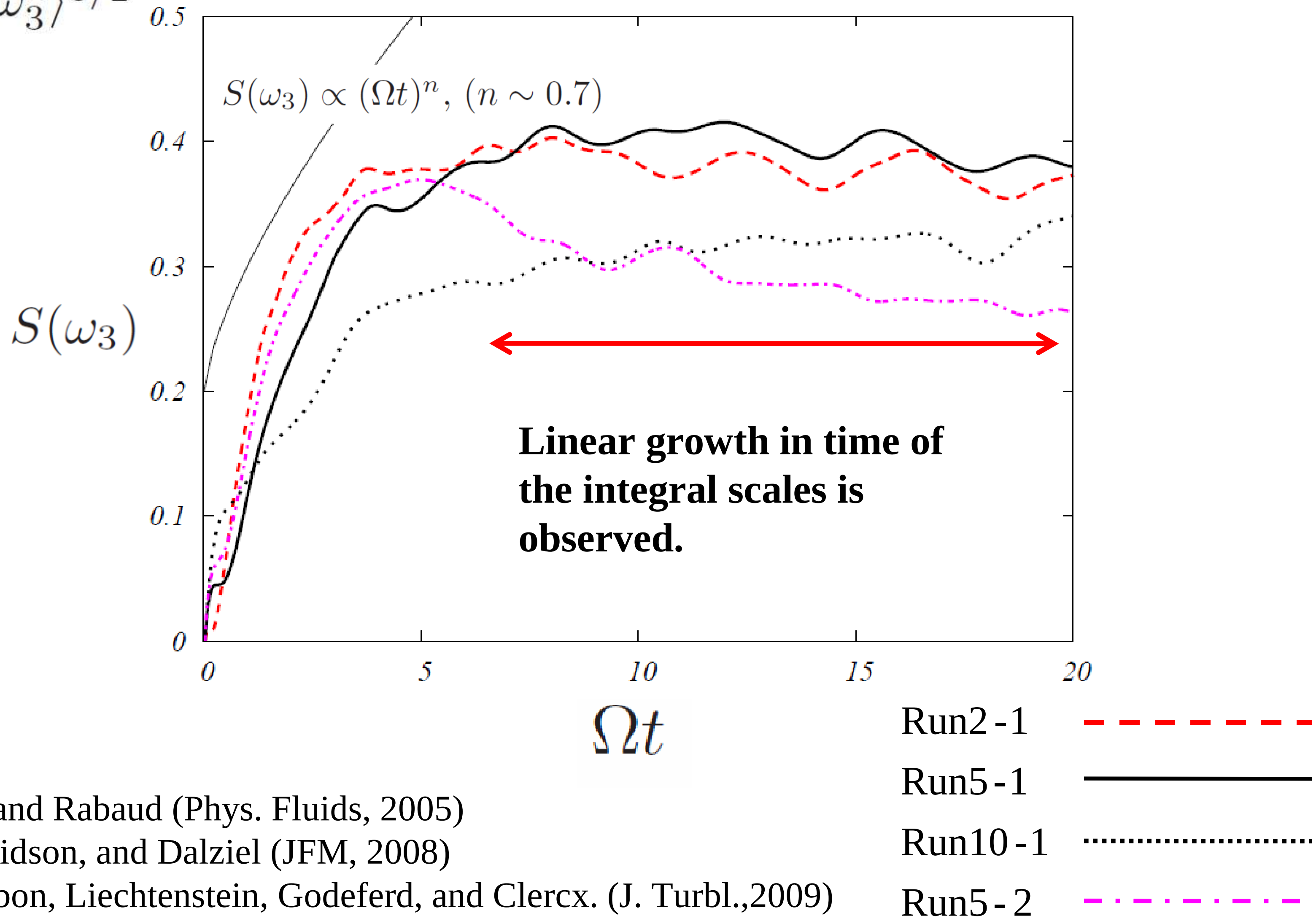
Run5-1

Run10-1

Run5-2

Asymmetry between cyclonic and anti-cyclonic vortices

$$S(\omega_3) = \frac{\langle \omega_3^3 \rangle}{\langle \omega_3^2 \rangle^{3/2}}$$



Morize, Moisy, and Rabaud (Phys. Fluids, 2005)
Staplehurst, Davidson, and Dalziel (JFM, 2008)
Bokhoven, Cambon, Liechtenstein, Godeferd, and Clercx. (J. Turbl.,2009)

Computational Surgery/Experiment

(Thought Experiment)

Remove a certain effect/term

→ some idea on the effect/term

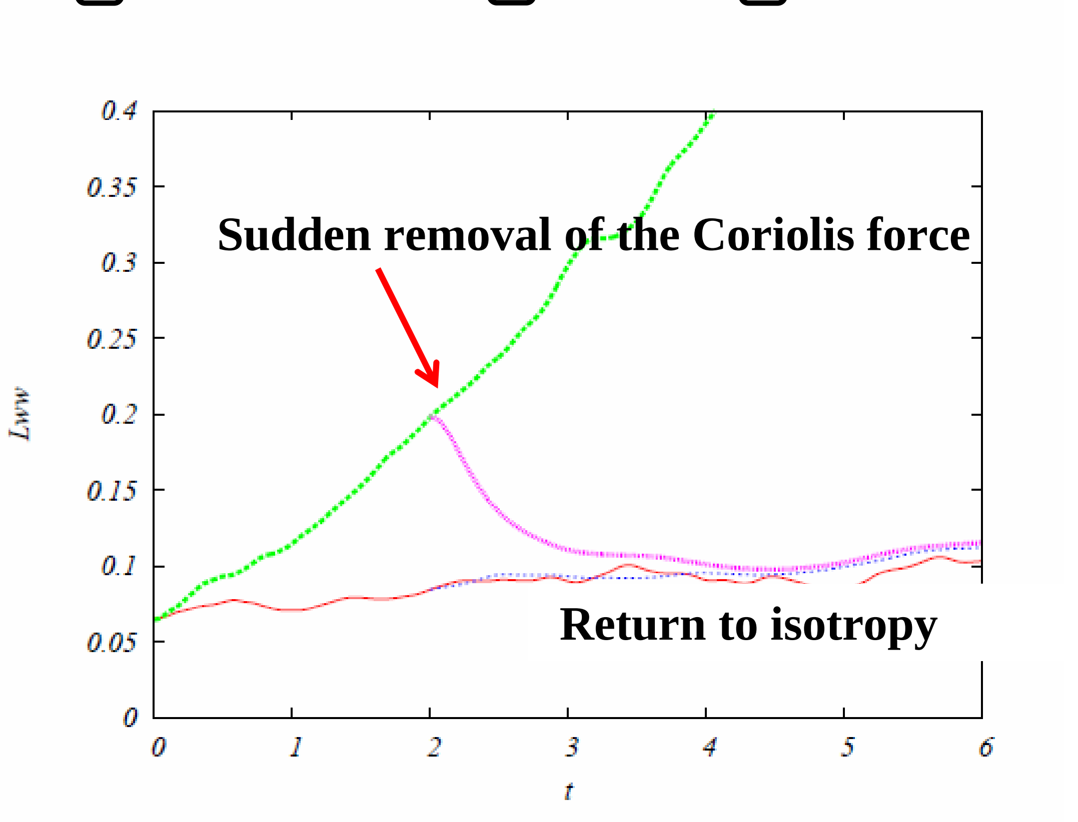
Computational Surgery 1

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u} = -\boldsymbol{\nabla} p - \cancel{2\boldsymbol{\Omega}} \times \boldsymbol{u} + \nu \triangle \boldsymbol{u}$$

$\boxed{\boldsymbol{N}}$ $\boxed{\boldsymbol{\Omega}}$ $\boxed{\boldsymbol{\nu}}$

$\Omega = 5$, Ini1

$L^{\beta}(\omega^2)$



Computational Surgery 2

$$\begin{cases} \dfrac{\partial \boldsymbol{u}}{\partial t} + \cancel{(\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{u}} = -\dfrac{1}{\rho}\boldsymbol{\nabla} p - 2\boldsymbol{\Omega} \times \boldsymbol{u} + \cancel{\nu \triangle \boldsymbol{u}} \\ \boldsymbol{\nabla} \cdot \boldsymbol{u} = 0 \end{cases}$$

	Initial field	R^ω
A	Run5-1 at $\Omega t_n = 5$	0.975
B	Run5-1 at $\Omega t_n = 10$	0.581
C	Run5-1 at $\Omega t_n = 15$	0.407

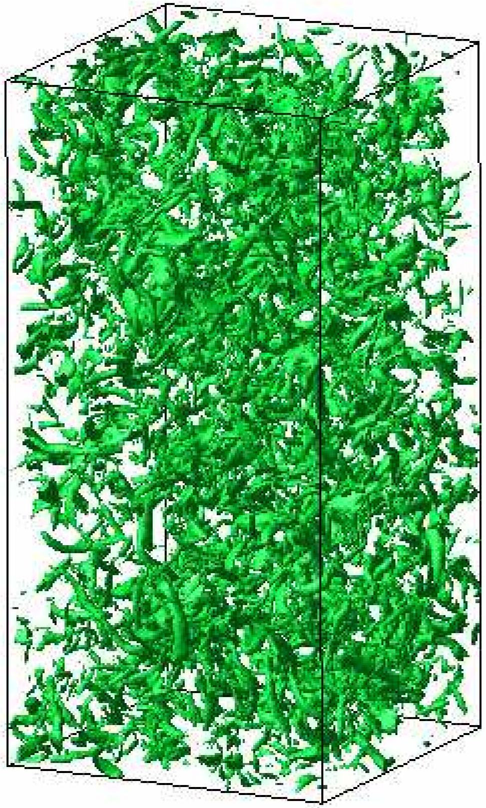
Nonlinear - rotating turbulence (128*128*256 domain)

- **Iso-surfaces of modulus of vorticity**

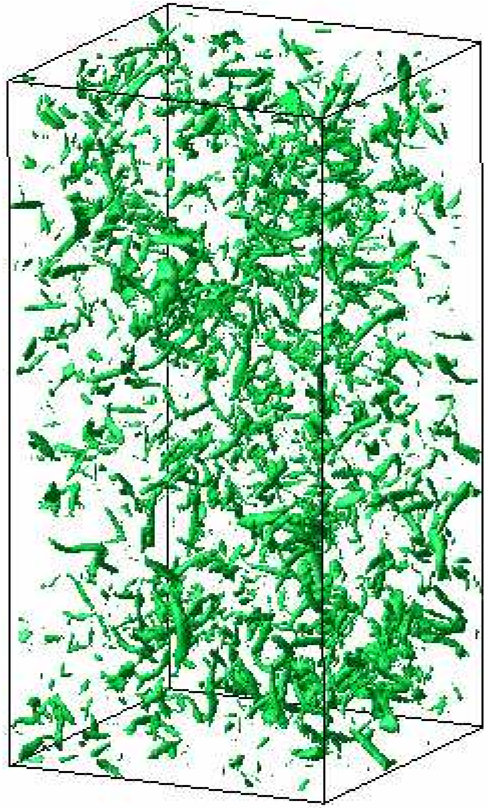
$$\Omega = 5, \mathrm{Ini1}$$

$$\Omega t = 0$$

Ω



$$|\omega| = m + 2\sigma$$



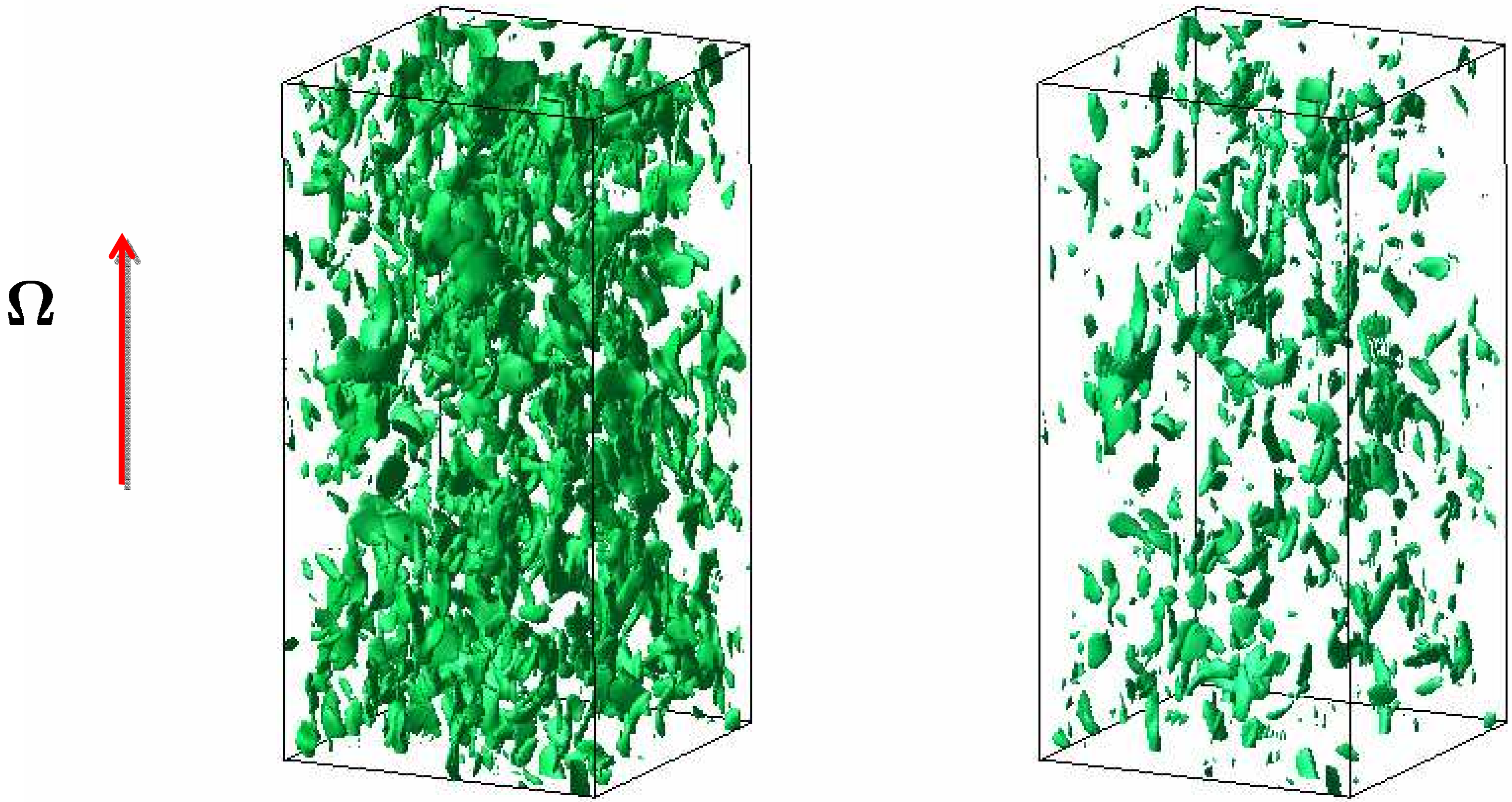
$$|\omega| = m + 3\sigma$$

Isotropic turbulence

Nonlinear - rotating turbulence (128*128*256 domain)

- **Iso-surfaces of modulus of vorticity**

$$\Omega t = 10$$

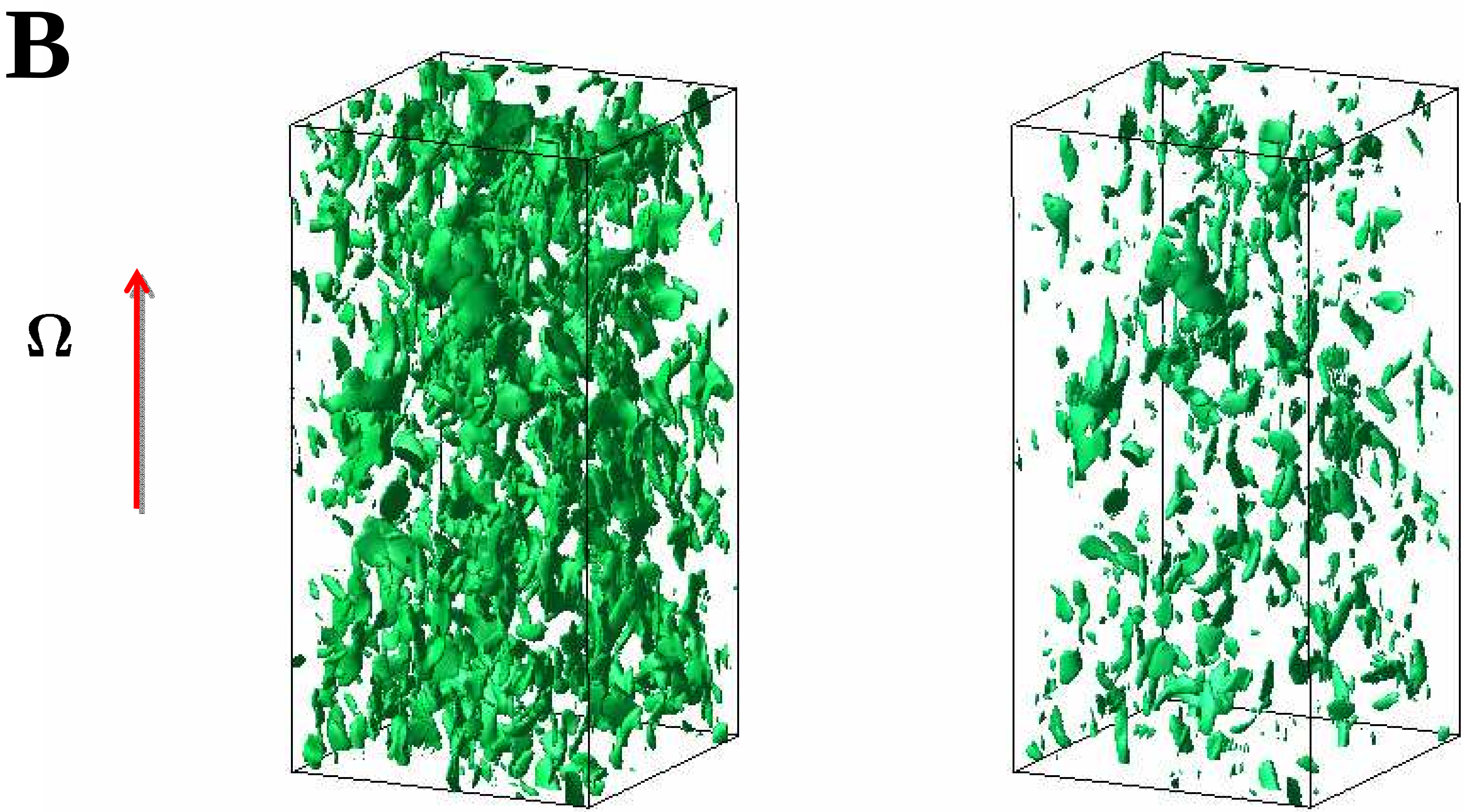


$|\omega| = m + 2\sigma$ $\qquad$ $|\omega| = m + 3\sigma$

Vortices becomes elongated with the rotational axis.

Linear - rotating turbulence (128*128*256 domain)

- **Iso-surfaces of modulus of vorticity** $\Omega = 5$, Ini1 at $\Omega t = 10$,

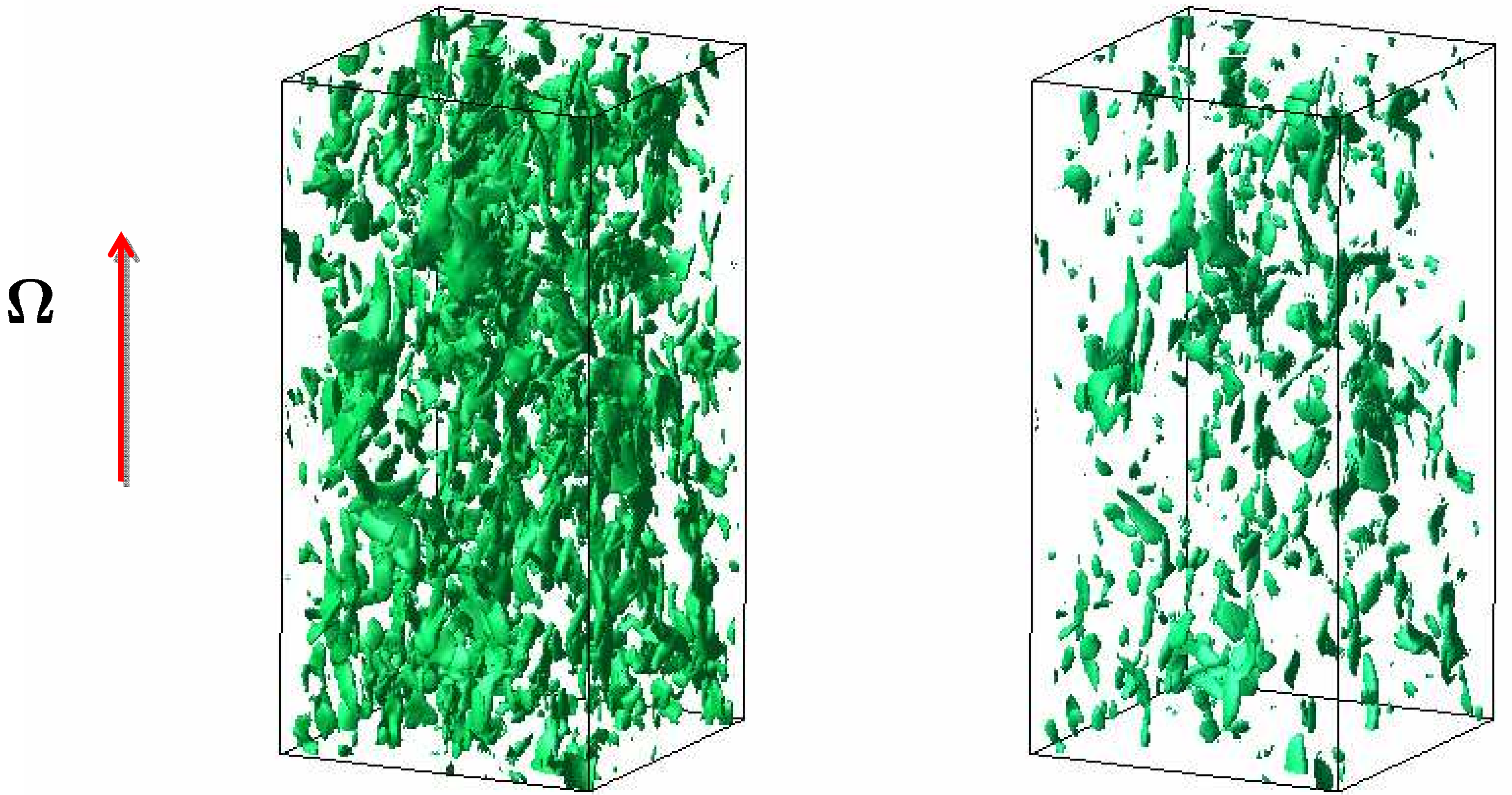


$|\omega| = m + 2\sigma$ $|\omega| = m + 3\sigma$

Linear - rotating turbulence (128*128*256 domain)

- **Iso-surfaces of modulus of vorticity**

$\Omega t = 11$

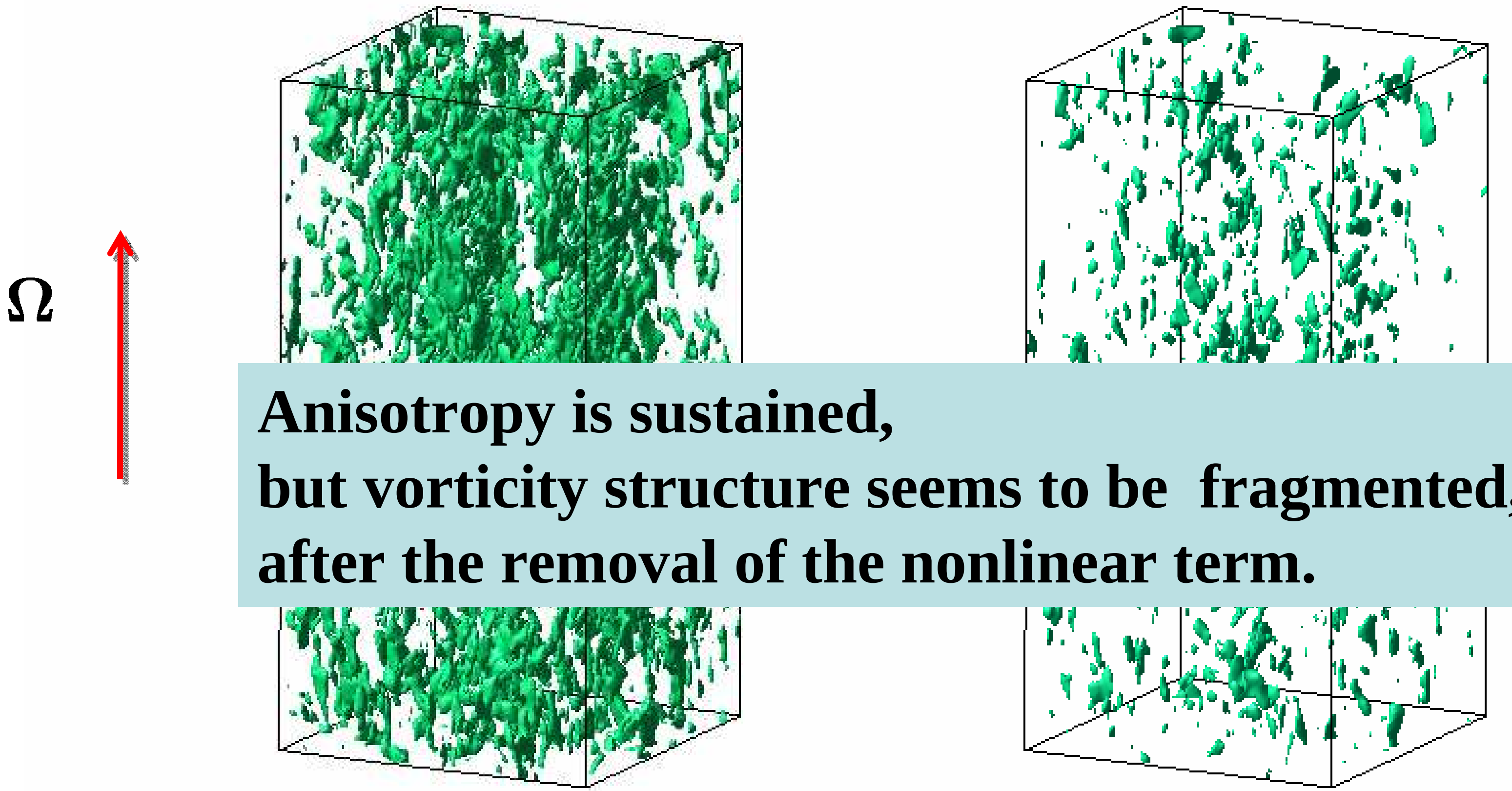


$|\omega| = m + 2\sigma$ $|\omega| = m + 3\sigma$

Linear - rotating turbulence (128*128*256 domain)

- **Iso-surfaces of modulus of vorticity**

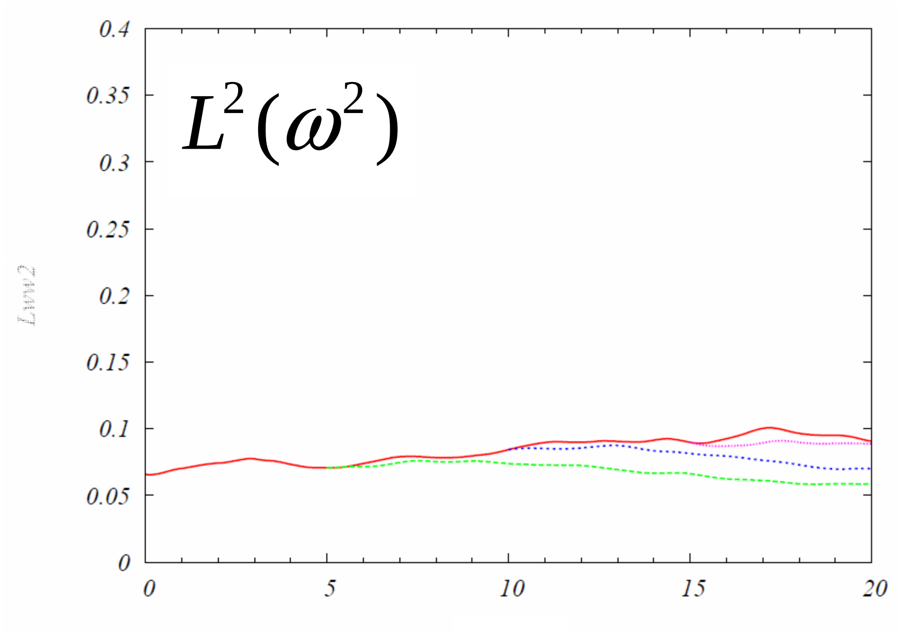
$\Omega t = 20$



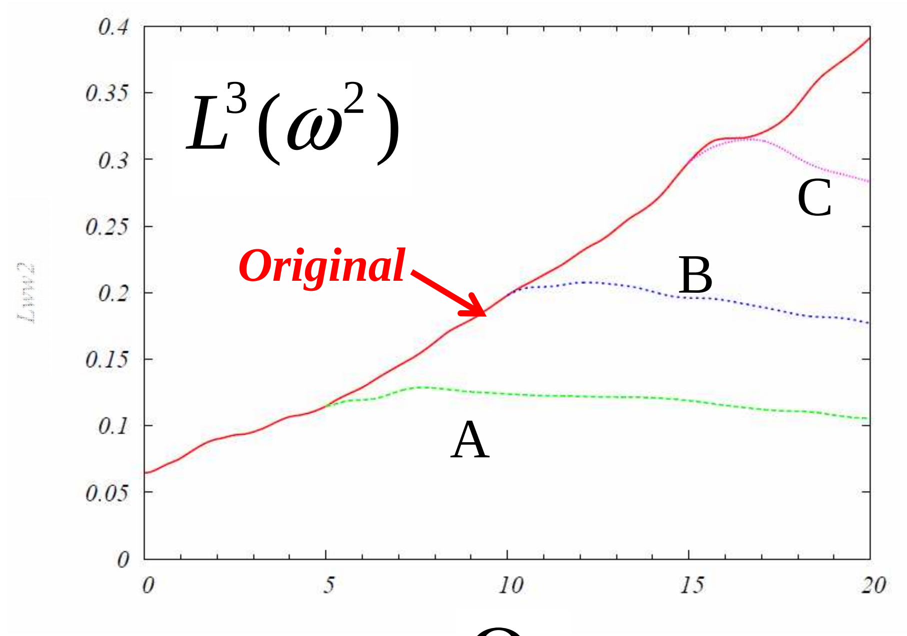
Anisotropy is sustained,
but vorticity structure seems to be fragmented,
after the removal of the nonlinear term.

$|\omega| = m + 2\sigma$ $\qquad$ $|\omega| = m + 3\sigma$

Linear integral scale of ω^2



Ωt



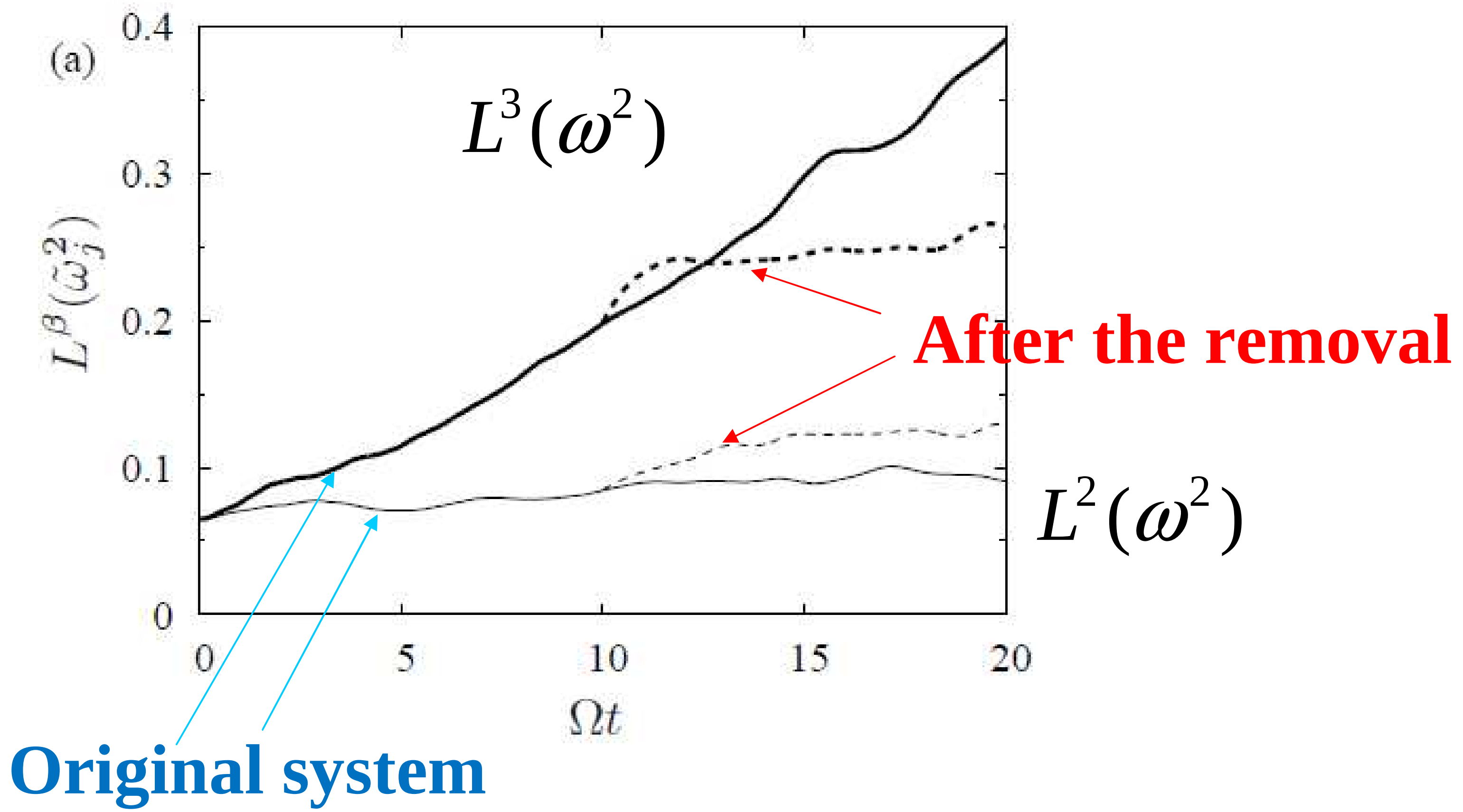
Ωt

the Coriolis force, if it is without the nonlinear convection effect, cannot sustain the increase of the length scale
i.e., the formation of the columnar structure

Linear but with the viscous term

$$\frac{\partial \boldsymbol{u}}{\partial t} + \cancel{(\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u}} = -\frac{1}{\rho}\boldsymbol{\nabla} p - 2\boldsymbol{\Omega}\times\boldsymbol{u} + \nu\triangle\boldsymbol{u}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{u} = 0$$



Conclusions

We considered 2 cases:

Case 1: Linear dynamics,

initially an array of compact eddies

Case 2: Nonlinear Dynamics,

Homogeneous, freely decaying homogeneous turbulence

Case 1

$L^3(\omega_1)$ **doesn't grow** in contrast to the impression by the visualization,

$L^3(\omega^2)$ **does grow**.

→ the answer depend on the definition, or on what we are talking.

we need aware of the difference depending on **the definition**

of the length scale for quantitative discussion of the elongation.

Case 2

the Coriolis force, if it is without the nonlinear convection effect,

cannot sustain the increase of the length scale

i.e., the formation of the columnar structure