Oscillation Modes in Screeching Jets

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I. Introduction

Nonideally expanded supersonic jets generate three basic noise components, namely, the turbulent mixing noise, the broadband shock-associated noise, and the screech noise [1]. The mixing noise, obtained for both subsonic and supersonic jets, is most intense in the downstream direction, and it occurs at Strouhal numbers of around 0.15. The broadband shock-associated noise is radiated mainly in the radial direction, and it has a central frequency varying with the emission angle. The screech noise consists of tones measured in the upstream direction. These tones are due to an aeroacoustic feedback mechanism establishing between turbulent structures propagating downstream and acoustic waves propagating upstream, which was described by Powell [2] and then by Raman [3], among others. According to these authors, the structures developing in the jet shear layers interact with the quasi-periodic shock-cell structure of the jet, which creates upstream-propagating acoustic waves. The sound waves are reflected back at the nozzle lips, which excites the shear layers and closes the feedback loop.

At the screech tone frequencies, the jets undergo strong oscillations. For round jets, Powell [2] identified four oscillation modes (A, B, C, and D) on the basis of the variations of the screech frequency with the jet ideally expanded Mach number $M_j$. Each mode was observed over a specific range of Mach number, and frequency jumps were noted between the modes. Later, Merle [4] showed that mode A could be divided into modes A1 and A2. Davies and Oldfield [5] studied the oscillation modes of the jets associated with the screech modes. They found that modes A1 and A2 corresponded to axisymmetric oscillation modes, mode C corresponded to helical modes, and modes B and D corresponded to flapping and sinusoidal modes. Powell et al. [6] and Tam et al. [7] suggested that the mode switching phenomenon was due to the change in the growth rate of the shear-layer instability waves as $M_j$ varied. Using numerical simulations, Shen and Tam [8] also provided evidence that two screech tones could coexist. As an explanation, they proposed that there were two ways by which the screech feedback loop was accomplished. In the first one, the feedback acoustic waves were generated by the interactions between the jet instability waves and the shock-cell structures, as was done for the broadband shock-associated noise. Using the formula [9] giving the frequency of this noise in the upstream direction, good agreement was thus noticed for the frequencies of screech modes A1 and B. In the second way to complete the screech loop, the feedback link was provided by upstream-propagating acoustic wave modes of the jets. By comparing the screech tone wavelengths predicted when the frequency of these modes was calculated using a vortex sheet model of the ideally expanded equivalent jet with experimental data, Shen and Tam [8] showed that this was most probably the case for screech modes A2 and C. However, the acoustic modes of the ideally expanded equivalent were not directly observed in the jet.

In this Note, the origin of the oscillation modes of screeching round jets is investigated by assuming that the feedback part of the aeroacoustic loop responsible for screech noise can be modeled by considering the neutral acoustic wave modes of the equivalent ideally expanded jets, as in the work of Shen and Tam [8]. Such modes are characterized in Sec. II by a simple wave analysis using a vortex sheet model. The validity of the present assumption is assessed from reference experimental data in Sec. III, as well as from simulation results in Sec. IV. In the latter section, an attempt is made to detect upstream-propagating waves in a screeching jet, as was done recently in ideally expanded impinging jets [10]. In particular, based on the frequency-wave-number spectra of density fluctuations, high-amplitude patterns corresponding to neutral acoustic wave modes of the ideally expanded equivalent jet are observed inside the jet, for the first time (to the best of our knowledge), in nonideally expanded jets. Concluding remarks are given in Sec. V.

II. Modeling of the Feedback Waves in Screeching Jets

In the present Note, as in the work by Shen and Tam [8], it is assumed that the feedback loop causing screech noise in nonideally expanded jets is closed by acoustic waves belonging to the family of the upstream-propagating acoustic wave modes of the equivalent ideally expanded jet. This hypothesis seems natural, given that, experimentally [2,4,5,11,12], the screech frequencies and the corresponding oscillation modes of the jets depend primarily on the ideally expanded Mach number of the jets.

The dispersion relations of these instability waves are approximated by using a vortex sheet model of the jets as proposed by Lessen et al. [13]. By starting from the linearized governing equations for a compressible inviscid fluid, and noting that the waves are neutral for a vortex sheet model (i.e., that they have both a real wave number $k$ and angular frequency $\omega$), these authors obtained the following relations for an ideally expanded round jet of diameter $D$, exit velocity $u_e$, and Mach number $M_j$:

$$
\frac{\xi_+}{\xi_-} \left| J_n(\xi_+,\alpha) \right| K_{n-1}(\xi_+,\alpha) + K_{n+1}(\xi_+,\alpha) \frac{1}{\alpha_0 C/\alpha_j - M_j^2} \left( J_{n-1}(\xi_+,\alpha) - J_{n+1}(\xi_+,\alpha) \right) = 0
$$

where $\alpha_0$ and $\alpha_j$ are the sound speeds in the ambient medium and in the jet, $C = \omega/\left(\alpha_0 \alpha_j\right)$, $J_n$ is the nth order Bessel function of the first kind,
III. Assessment of the Modeling Using Experimental Data

The allowable frequency ranges obtained for the axisymmetric and the helical upstream-propagating neutral wave modes are represented in Fig. 3 as a function of the Mach number $M_j$. They are determined from the maximum and minimum Strouhal numbers of each mode, which are taken from the dispersion curves in the first case, and are calculated in the second case as in the work of Tam and Ahuja [15]. To assess the validity of the modeling proposed by Shen and Tam [8] and examined in the present Note, the tone frequencies acquired in the reference experiments of Powell et al. [6] and Ponton and Seiner [16] for different screech modes are also depicted. For Ponton and Seiner [16], the results represented were those for jets with a nozzle thickness equal to 0.625D, where $D$ was the exit diameter. The semiempirical relation developed by Tam et al. [9] was also plotted in the figure. The relations are written as

$$St = \frac{0.62}{(M_j^2 - 1)^{1/2}} \left[ 1 + 0.65M_j \left( 1 + \frac{\gamma - 1}{2}M_j^2 \right)^{-1/2} \left( \frac{T_0}{T_r} \right)^{-1/2} \right]^{-1}$$

where $T_0$ and $T_r$ are the ambient temperature and the total temperature of the jet, respectively; $\gamma = 1.4$ is the ratio of specific heats; and a mean convection velocity equal to 65% of the ideally expanded jet velocity is considered.

Overall, as observed by numerous researchers [11,12,17], the screech frequency decreases with the ideally expanded Mach number $M_j$ [9], following the semiempirical relation developed by Tam et al. [9]. Moreover, the variations of the screech frequencies with $M_j$ closely follow those of the allowable frequency ranges. This is the case for the dominant screech mode $C$ in both experiments and for the secondary screech mode $c$ in the experiment of Powell et al. [6], for which the frequency strikingly fall on the line corresponding to the lower limit of the band for the helical mode $H1$. This is also true for dominant screech modes $A1$ and $A2$ and secondary screech mode $u$, for which the frequencies are located near the lower limit of the band for axisymmetric mode $S2$. These results suggest that the former screech modes are helical and that the latter are axisymmetric, which is in agreement with experimental observations for screech modes $A1$, $A2$, $C$, and $c$, as well as with expectations [6] for screech mode $u$. It is interesting to note that the screech tones are located mainly near the lower limits of the allowable frequency ranges. At these positions, the neutral upstream-propagating acoustic waves have group and phase velocities both close to the ambient speed of sound. In the present
model, the waves closing the feedback loop therefore appear to propagate at the same speed as the acoustic waves considered to complete the loop outside the jet flow in the classical screech mechanism model [2, 3].

Similar observations have been made by Tam and Norum [18], Gojon et al. [19], and Bogey and Gojon [10] for impingement tones in supersonic ideally expanded jets. It thus appears that the present modeling of the feedback waves allows us to predict the variations of the frequencies of these screech modes with \( M_j \), as well as their axisymmetric or helical natures. Note that the frequencies of screech modes \( B \) and \( D \), as well as of modes \( b \) and \( d \), lie outside the allowable frequency ranges for the axisymmetric and helical upstream-propagating neutral acoustic wave modes. This can be explained by the fact that these modes are associated with more complex flapping/sinusoidal jet oscillation modes.

IV. Further Assessment of the Modeling Using Numerical Data

The results obtained for a round screeching jet in a recent study [20] using large-eddy simulation (LES) are now examined.

A. Parameters and Methods

At the exit of a straight pipe nozzle with a diameter of \( D = 2r_0 = 2 \text{ mm} \) and a thickness of 0.1\( r_0 \), the jet is characterized by a nozzle pressure ratio of \( p_j/p_0 = 4.03 \) and a temperature ratio of \( T_j/T_0 = 1 \), where suffixes \( r \) and \( 0 \) denote, respectively, stagnation and ambient values. The jet is underexpanded; has a fully expanded Mach number of \( M_j = u_j/a_j = 1.56 \); an exit Mach number of \( M_0 = 1 \) (i.e., it simulates a jet exiting from a convergent nozzle); and a Reynolds number of \( Re_j = u_j D / \nu_j = 6 \times 10^4 \), where \( u_j \) and \( a_j \) are the velocity and the speed of sound in the equivalent ideally expanded jet with a diameter of \( D_j = 2.2 \text{ mm} \); and \( \nu_j \) is the kinematic molecular viscosity.

The LES is performed by solving the unsteady compressible Navier-Stokes equations on a cylindrical mesh \((r, \theta, z)\). The time integration and spatial derivation are performed using an explicit six-stage Runge-Kutta algorithm and low-dispersion and low-dissipation explicit finite differences [21,22], respectively. At the end of each time step, a high-order filtering is applied to the flow variables in order to remove grid-to-grid oscillations and to dissipate subgrid-scale turbulent energy [23–25]. The boundaries are treated with the radiation conditions of Tam and Dong [26], and a sponge zone is employed to damp the turbulent fluctuations. No-slip adiabatic conditions are imposed at the nozzle walls. Finally, a shock-capturing filtering is applied in order to avoid Gibbs oscillations near shocks [27]. The simulation has been carried out using an OpenMP-based in-house code solving the unsteady compressible Navier-Stokes equations in cylindrical coordinates \((r, \theta, z)\) using low-dispersion and low-dissipation finite difference schemes [21,27–29]. The mesh contains 400 million points, and it exhibits a minimal spacing of 0.0075\( r_0 \) in the jet shear layers and at the nozzle exit, as well as a maximum spacing of 0.06\( r_0 \), allowing properly propagated acoustic waves with Strouhal numbers up to \( St = f D_j / u_j = 5.3 \). The simulation time, after the transient regime, is equal to 500\( D_j / u_j \).

B. Flow Snapshot and Near-Nozzle Pressure Spectrum

Three isosurfaces of density in the jet flow and the pressure fluctuations in the planes \( \theta = 0 \) and \( \theta = \pi \) are represented in Fig. 4. In the jet, a shock-cell structure and the jet mixing layers are well visible. Outside, strong acoustic waves clearly appear to propagate in the upstream direction. More information about the hydrodynamic and acoustic fields of the jet can be found in a dedicated paper [20].

The pressure spectrum obtained near the nozzle exit at \( z = 0 \) and \( r = 2r_0 \) is displayed in Fig. 5 as a function of the Strouhal number. Two tones emerge 15 dB above the broadband noise at Strouhal numbers of \( St_1 = 0.28 \) and \( St_2 = 0.305 \), and a secondary tone is found at \( St_3 = 0.38 \). Such a result is typical of a screeching jet; see, for instance, in the work of Westley and Woolley [11], Panda [12], and André [17]. Moreover, Eq. (2) gives \( St = 0.285 \) for the simulated jet, which is in good agreement with the frequency of the two dominant tones. The present jet thus generates screech tones. According to the classification of the screech modes used in Fig. 3, the two dominant tones belong to mode \( C \), and the secondary tone belongs to mode \( u \). An in-depth analysis of those tones was given in a previous paper [20].

C. Near-Pressure Field

A Fourier transform has been applied to the near-pressure field recorded in the \((z, r)\) plane in order to determine the amplitude and

![Fig. 4 Density isosurfaces for 0.8 and 2.5 kg · m⁻³ (purple and red) and 1.25 kg · m⁻³ colored by the Mach number; pressure fluctuations at \( \theta = 0 \) and \( \pi \) using a color scale from –2000 to 2000 Pa from blue to red; nozzle in gray.](attachment:image.png)
phase fields associated with the different screech tones. These fields are represented in Fig. 6.

At the three tone frequencies, the amplitude fields in Figs. 6a–6c exhibit cell structures. Such structures have been observed, for instance, experimentally by Panda et al. [30] in screeching jets and numerically by Gojon et al. [19] in ideally expanded planar impinging jets. They are due to the formation of hydrodynamic–acoustic standing waves. The levels on the jet axis are negligible at the two first frequencies but strong at the third one. The phase fields of Figs. 6d–6f, as well as the results obtained using Fourier decomposition in the azimuthal direction at $z = 0$ and $r = 2r_0$ (not shown here for brevity), provide additional information on the spatial organization of the pressure field. They indicate that the jet undergoes helical oscillations at $St_1 = 0.28$ and $St_2 = 0.305$ but an axisymmetric oscillation at $St_3 = 0.38$. These findings are consistent with the screech modes associated with the different tones, namely, mode $C$ for the two first tones and mode $U$ for the third tone. In particular, the axisymmetric nature of the mode $U$ suggested by the model in Sec. II clearly appears.

D. Wave-Number-Frequency-Density Spectra in the Jet

To detect upstream-propagating waves in the present jet, as was recently done by Towne et al. [31] in a subsonic free jet and by Bogey and Gojon [10] in ideally expanded impinging jets, a space–time Fourier transform has been performed between $r = 0$ and $r = r_0$ along a line extending from the axial position of $z = 0$ to $z = 10r_0$. The frequency-wave-number spectra obtained for azimuthal modes of $n = 0$ and $n = 1$ at $r = 0.1r_0, r = 0.5r_0$, and $r = r_0$ are shown in Fig. 7 as functions of the Strouhal number of $St = fD_j/u_j$ and of the normalized axial wave number $kD_j$. Only the negative wave number part is shown. The dispersion relations of the neutral acoustic wave modes of the equivalent ideally expanded jet are also displayed. For the azimuthal mode of $n = 0$, significant levels are found in Figs. 7a, 7c, and 7e that are slightly above the theoretical curves associated with modes $S1$ and $S2$. The discrepancies with respect to the model may be due to the presence of shock motions in the simulated jets, as well as to the use of an infinitely thin shear layer in the vortex sheet model [15, 10]. Moreover, the amplitude of those bands decreases with the radial position at which the frequency-wave-number spectrum is represented, following the eigenfunction distributions in Fig. 2a. In Figs. 7b, 7d, and 7f for the azimuthal mode of $n = 1$, strong components appear along mode $H2$ at $r = 0.1r_0$ in Fig. 7b and along modes $H1$ and $H2$ at $r = 0.5r_0$ in Fig. 7d. Those results are again in agreement with the eigenfunction distributions of the two first helical neutral acoustic wave modes in Fig. 2b.

The eigenfunction profiles presented in Fig. 2 are now compared with the dimensional amplitude profiles extracted from the frequency-wave-number spectra of density fluctuations at 11 radial positions from $r = 0$ to $r = r_0$. They are estimated at the tone Strouhal numbers of $St = 0.38$ and $St = 0.28$ for $S2$ and $H1$, respectively, and at the maximum Strouhal number of the mode for $S1$ and $H2$. In the first case, the amplitude seems to be well converged because of the high-energy content at the tone Strouhal numbers. In the latter case, the maximum Strouhal number corresponds to the position where the eigenfunction profiles have been calculated using the vortex sheet model. The comparisons are given in Fig. 8. A good agreement is obtained for all the modes. The same observation has been made for round, ideally expanded, impinging jets [10] where the amplitude fields obtained for the pressure fluctuations at the tone frequencies bear similarities with the eigenfunction distributions of the neutral acoustic wave modes of the jet. These results provide strong evidence, for the first time (to the best of the authors’ knowledge), that a vortex sheet model of the corresponding ideally expanded jet is capable of predicting the wave modes of a nonideally expanded supersonic jet. However, among those waves, the ones with a positive group velocity that have been observed in Fig. 7 cannot be involved in the proposed feedback loop model.

Peak levels are reached in the spectra on the line $k = ωa_0/a_0$ at the Strouhal number of $St_3 = 0.38$ of the axisymmetric screech tone for the azimuthal mode of $n = 0$ in Figs. 7a, 7c, and 7e as well as at the Strouhal numbers of $St_1 = 0.28$ and $St_2 = 0.305$ of the helical screech tones for the mode of $n = 1$ in Figs. 7a, 7c, and 7e. They are located very near the lower limits of the dispersion relations of the neutral acoustic wave modes of the equivalent ideally expanded jet. Therefore, the associated waves propagate upstream inside the jet with group and phase velocities both close to the ambient speed of sound.

In the spectra of Fig. 7, energy is distributed along a wide range of spatial wave numbers for each tonal frequency, which seems in contradiction with the model proposed by Shen and Tam [8] and in the present Note, in which the waves closing the feedback loop exist only at a single wave number for each frequency. This is most likely due to the interaction of the upstream-propagating acoustic waves with the turbulence in the jet shear layer and the shock cells, which leads to a redistribution of the energy to the close wave numbers. To quantify the energy redistribution in the wave number space, the spectra of the density fluctuations are plotted in Fig. 9 as a function of
Fig. 7 Frequency-wave-number spectra of density fluctuations at a,b) \( r = 0.1r_0 \), c,d) \( r = 0.5r_0 \), and e,f) \( r = r_0 \) for modes a,c,e) \( n = 0 \) and b,d,f) \( n = 1 \); (solid lines) dispersion relations of the axisymmetric (left) and helical (right) neutral acoustic modes; (×) mode lower limits; (dashed) \( k = -\omega/a_0 \); screech tone frequencies for axisymmetric (diamonds) and helical (squares) modes; color-scale levels over 60 dB from blue to red.

Fig. 8 Eigenfunction distributions of the neutral acoustic modes: a) axisymmetric modes \( S_1 \) (black line) and \( S_2 \) (red line), and b) helical modes \( H_1 \) (black line) and \( H_2 \) (red line); (solid dotted line) results from the LES.
the wave number for the main axisymmetric and helical screech mode frequencies for the radial positions of $r = 0.1r_0$, $r = 0.5r_0$, and $r = r_0$. In all cases, the energy peak is found very near the wave number of corresponding upstream-propagating neutral acoustic wave modes $S2$ and $H1$. Finally, at the peak wave numbers, energy is strongest at $r = 0.1r_0$ for the axisymmetric mode in Fig. 9a and at $r = 0.5r_0$ for the helical mode in Fig. 9b. This is consistent with the eigenfunction distributions of modes $S2$ and $H1$ in Fig. 2.

The frequency-wave-number spectra obtained at $r = r_0$ are replotted in Fig. 10 by showing both the negative and the positive wave number parts of the spectra. In this way, both the downstream- and the upstream-traveling waves can be seen. The former waves consist of aerodynamic broadband disturbances convected in the flow direction in the shear layers at an average velocity of $\bar{u}_c \approx 0.75\bar{u}_u$. The latter are acoustic waves propagating at the ambient speed of sound at the screech tone frequencies. They are very likely to close the feedback loop in the jet.

V. Conclusions

In this Note, the origin and the properties of the oscillation modes in screeching, nonideally expanded, round jets are investigated on the basis of the hypothesis of Shen and Tam [8] that the acoustic waves completing the feedback loop in these jets are linked to the upstream-propagating acoustic wave modes of the equivalent ideally expanded jets. Using a jet vortex sheet model to describe the dispersion relations of these modes, it is found that this hypothesis allows us to explain the axisymmetric or helical jet oscillations and the variations with the Mach number of the tone frequencies for screech modes $A1$, $A2$, and $u$ and modes $C$ and $c$ observed in the experiments. For an underexpanded round jet at $M_j = 1.56$ generating screech tones of modes $C$ and $u$, which was considered in a previous study [20] using large-eddy simulation, it is also shown that at the screech frequencies, there exist acoustic waves in the jet flow that are propagating in the upstream direction at the ambient speed of sound, which belongs to the neutral acoustic wave modes of the equivalent ideally expanded jet. These results provide strong evidence that a vortex sheet model of the corresponding ideally expanded jet is capable of predicting the wave modes of a nonideally expanded supersonic jet. They also suggest that the feedback path of the mechanism causing screech noise in nonideally expanded jets is achieved, at least for the screech modes mentioned previously, by these waves.

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