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Technical Note

Sound source decomposition for two-dimensional deformable vortices

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Abstract. — The acoustic source term for a free vortical flow (Mohring, 1978) is considered for a flowfield consisting of a system of distinct vortex patches. The source description is separated into two contributions, one due to the motion of the vortex patches themselves (the "bulk" contribution) and one due the deformations of the vortex cores (the "core" contribution). Two examples using corotating vortex pairs are studied to show that when the core fluctuations are small, they may radiate an appreciable fraction of the sound, if they occur at higher frequencies than the bulk motions.

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Efforts in estimating the sound produced by an unsteady low speed flow based on the vorticity dynamics have largely focussed on flow models based on the motion of point vortices (in 2D) and vortex filaments (in 3D) (see Powell, 1964; Mohring, 1978, Kambe, 1986). While this work has shown both the validity and the utility of the vortex theory of sound generation, reliance on singular or non-deformable vortex cores limits the study of sound generation to some simple cases. Two notable exceptions are the work of Shariff et al. (1988) and Collorec et al. (1993) who explicitly considered the effect of core deformations on the radiated sound. Both of these studies have shown that the sound computed from a flow for which core deformations are present may lead to high noise levels at frequencies higher than those associated with the motion of an equivalent system of vortices with non-deforming cores. These results suggest strongly that even small vortex core deformations may contribute an appreciable percentage of the overall sound power for these flows. However, the question remains whether the higher frequency motions present in the simulations are the result of core deformations alone or whether the bulk motion of each vortex also contains high frequency components which also contribute to the high-frequency sound. In order to facilitate addressing this question, we show that it is possible to decompose the sound source expression into two contributions: one due to the motion of the centroids of distinct vortex regions and another due to deformations of these regions. In this way the contribution to the noise of each type of motion may be estimated directly from a deformable vortex flow computation instead of relying on a comparison to an equivalent nondeformable vortex flow.

Consider the two-dimensional inviscid flow composed of a collection of N distinct regions of vorticity in an otherwise still fluid of infinite extent. The coordinates x_1 and x_2 lie in the plane of motion while x_3 is normal to the flow plane, in the direction of the vorticity vector. The Mach number of the flow is very small and the length scales of the flow are much smaller than the wavelengths of the radiated sound, so that the flow may be considered as a compact acoustic source. The expression for the farfield sound radiated from this flow is given by (Mitchell et al., 1992) for an observer located at $\mathbf{x}$:

$$p(\mathbf{x}, t) = \int_{-\infty}^{\infty} \frac{\rho\Omega^3}{8c^2} \frac{x_i x_j}{|\mathbf{x}|^2} \hat{Q}_{ij}(\Omega) H_2^{(1)}\left(\frac{\Omega\,|\mathbf{x}|}{c}\right) \mathrm{e}^{-i\Omega t}\mathrm{d}\Omega \tag{1}$$

where $H_2^{(1)} = J_2 + iY_2$ is the second order Hankel function representing outgoing waves, c is the speed of sound in the ambient fluid, and $\hat{Q}_{ij}(\Omega)$ are the Fourier transforms of the second moments of the vorticity per unit length in the direction normal to the flow plane, given by:

$$Q_{ij}(t) = \sum_{k=1}^{N} \int_{\mathrm{D}^{(k)}} \left(\mathbf{x}^{(k)} \times \omega^{(k)}\hat{\mathbf{e}}_3\right)_i x_j^{(k)} \mathrm{d}A \tag{2}$$

where $D^{(k)}$ is the region containing the k^{th} vortex. (In this paper, unless otherwise noted, the superscript (k) indicates that the quantity in question is associated with the k^{th} vortex.) Here we have used the linearity of Q_{ij} in ω to write the space integral as the sum of the integrals over

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each vortex. Note that equation (1) is for the contribution of the quadrupole sound which is generally the leading-order contribution in low Mach number free vortex flows.

In order to decompose equation (1) into contributions of vortex core fluctuations and of the motion of each region as a whole, we write the coordinate $\mathbf{x}^{(k)}$ as the sum of the coordinate of the vorticity centroid of each vortex region, $\mathbf{X}^{(k)}$:

$$X_j^{(k)} = \frac{\int_{D^k} \omega^{(k)} x_i \mathrm{d}A}{\int_{D^{(k)}} \omega^{(k)} \mathrm{d}A} \tag{3}$$

and the coordinate of the point relative to the centroid, $\mathbf{r}^{(k)}$, as shown in Figure 1:

$$x_i^{(k)} = X_i^{(k)} + r_i^{(k)} \tag{4}$$

where x_i is the i^{th} component of $\mathbf{x}$. Substituting this expression into equation (2), we obtain:

$$\begin{aligned} Q_{ij} &= \sum_{k=1}^{N} \int_{D^k} \left[\left(\mathbf{X}^{(k)} + \mathbf{r}^{(k)} \right) \times \hat{\mathbf{e}}_3 \right]_j \omega^{(k)} \left[X^{(k)} + r(k) \right]_i \mathrm{d}A \\ &= \sum_{k=1}^{N} \int_{D^{(k)}} \left[\left(\mathbf{X}^{(k)} \times \hat{\mathbf{e}}_3 \right)_j X_i^{(k)} + \left(\mathbf{X}^{(k)} \times \hat{\mathbf{e}}_3 \right)_j r_i^{(k)} \right. \\ &\left. + \left(\mathbf{r}^{(k)} \times \hat{\mathbf{e}}_3 \right)_j X_i^{(k)} + \left(\mathbf{r}^{(k)} \times \hat{\mathbf{e}}_3 \right)_j r_i^{(k)} \right] \omega^{(k)} \mathrm{d}A \end{aligned} \tag{5}$$

In Cartesian tensor notation, the two middle terms are of the form:

$$\epsilon_{im3} X_j \hat{\mathbf{e}}_3 \int r_m \omega \mathrm{d}A \tag{6}$$

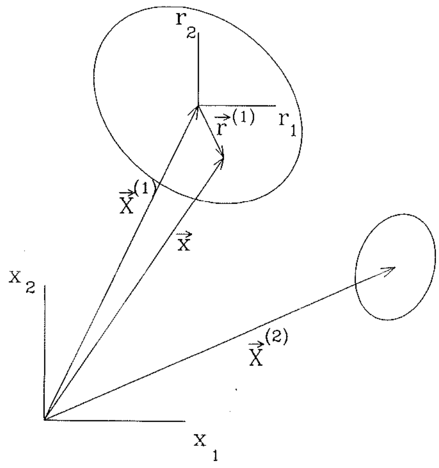


Figure 1. Coordinate decomposition used for splitting the source term. $\mathbf{x}$ is the external coordinate system, and $\mathbf{r}^{(k)}$ is the coordinate system centered on the centroid $\mathbf{X}^{(k)}$ of the k^{th} vortex.

where ϵ_{ij3} is the permutation tensor. Using equations (3) and (4), these terms can be shown to be identically zero, leaving:

$$\begin{aligned} Q_{ij} &= \sum_{k=1}^{N} \int_{D^{(k)}} \left[\left(\mathbf{X}^{(k)} \times \hat{\mathbf{e}}_3 \right)_j X_i^{(k)} \right. \\ &\left. + \left(\mathbf{r}^{(k)} \times \hat{\mathbf{e}}_3 \right)_j r_i^{(k)} \right] \omega^{(k)} \mathrm{d}A \\ &= \sum_{k=1}^{N} \left[B_{ij}^{(k)} + C_{ij}^{(k)} \right] \end{aligned} \tag{7}$$

where

$$B_{ij}^{(k)} = \left(\mathbf{X}^{(k)} \times \hat{\mathbf{e}}_3 \right)_j X_i^{(k)} \Gamma^{(k)}$$

$$C_{ij}^{(k)} = \int_{D^{(k)}} \left(\mathbf{r}^{(k)} \times \hat{\mathbf{e}}_3 \right)_j r_i^{(k)} \omega^{(k)} \mathrm{d}A$$

$\Gamma^{(k)}$ is the circulation of the k^{th} vortex region, and is defined by:

$$\Gamma^{(k)} = \int_{D^{(k)}} \omega^{(k)} \mathrm{d}A$$

$B_{ij}^{(k)}$ is the contribution from bulk motion and $C_{ij}^{(k)}$ is the contribution of the core deformations. The farfield sound is proportional to the third time derivative of these quantities. We call $\mathrm{d}^3 B_{ij}/\mathrm{d}t^3$ the "bulk noise" and $\mathrm{d}^3 C_{ij}/\mathrm{d}t^3$ the "core noise". For the case of point vortices, or when deformations of the vortex cores are neglected, $\mathrm{d}^3 C_{ij}^{(k)}/\mathrm{d}t^3$ is zero and $\mathrm{d}^3 Q_{ij}/\mathrm{d}t^3$ reduces to $\mathrm{d}^3/\mathrm{d}t_3(\sum B_{ij}^{(k)})$. Thus the expression for the radiated sound may be decomposed into a contribution due to the bulk motion of the vortices, and a contribution due to the deformations of the vortex cores.

The splitting of the source term does not imply the independence of the two types of motion. In fact the splitting is a consequence of the linearity of the sound source expression in vorticity. It is well-known that the evolution of vorticity is not a linear problem, so the total problem of solving for the sound is not linear in vorticity. To move beyond this general statement we consider two examples based on the elliptical-core vortex models of Melander et al. (1986) for two-dimensional flows (which was then extended to axisymmetric vortex rings by Shariff et al. (1988)). Although these authors consider only vortex structures with cores of uniform vorticity distribution, which is thus less general than the above results, they have nevertheless conveniently demonstrated the coupling between the motions of the vortex centroid and the core. The fact of coupling should also hold true even for vortices whose core vorticity distribution is not uniform. In the two-dimensional case considered by Melander et al. (1986), the velocity of a given vortex centroid may be expressed, to second order, as the sum of the velocity given by an equivalent system of point vortices plus a correction term expressing the effect of core distortions of the other vortices in the system on their induced velocity field. To the same order, the rate of change of the vorticity moments of each vortex are dependent on both self rotation due to distortion from a circular shape and straining due to the induced velocity field. For each vortex the core shape and orientation are both then dependent on the unsteady induced strain field. The core orientation is dependent not only on changes in core shape due to an externally induced strain, but also self-rotation due to self-strain. In these ways the core dynamics and the centroid motion are coupled.

The source decomposition allows the effect of this coupling on the radiated sound to be studied by splitting the expression for the source into the separate contributions of the vortices' bulk motions and their deformations relative to their vorticity centroids. We now use the source decomposition to briefly show the contributions of each type of motion to the source strength for two corotating elliptical vortex pair solutions of the model of Melander et al. (1986). The vortices in the pair are denoted vortex 1 and vortex 2. Each of these elliptical vortex patches is characterized by its vorticity $\omega^{(k)}$, the lengths of its major and minor axes, $a^{(k)}$ and $b^{(k)}$, respectively, and the angle ϕ of the ellipse major axis to the x-axis. From these quantities we may also define the area of each vortex ellipse $A^{(k)} = \pi a^{(k)} b^{(k)}$ and the equivalent radius $r_0 = (ab)^{1/2}$ of the vortex when it has a circular shape ($a = b$).

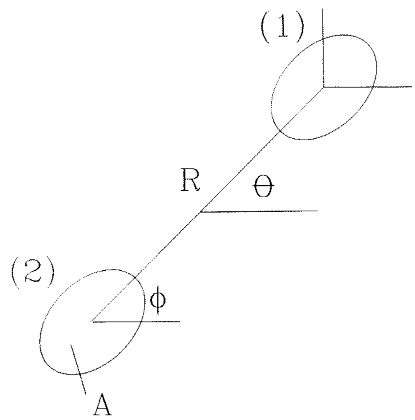


Figure 2. Symmetric corotating pair of Melander et al. (1986). $d\lambda/dt = 0$, $dR/dt = 0$, $\lambda = \lambda(\mu)$, $\mu = R(\pi/A)^{1/2}$, $A = \pi ab = \pi r_0^2$. For non-merging solution $\mu > 3.50054$.

The first example we consider is the "steady-EV state" for identical vortices, i.e. with equal vorticity and area. For this particular solution the vortex cores have a time-invariant ellipse aspect ratio $\lambda = a/b$ (major axis length/minor axis length). The ellipses rotate about the vortex centroid at the same rate as the centroids rotate about each other. Thus the bulk motion consists of circular orbits around a point halfway between the two vortices, and the core motion consists of pure rotations of the elliptical vortex cores. A schematic of this system is shown in Figure 2. The core vorticity moments are thus expressed in terms of the aspect ratio of the ellipse and the orientation angle ϕ of the major axis to the external coordinate system. The value of the aspect ratio is a function of the ratio μ, of the separation distance between the centroids, R, and the equivalent radius, r_0, of the two identical vortex cores:

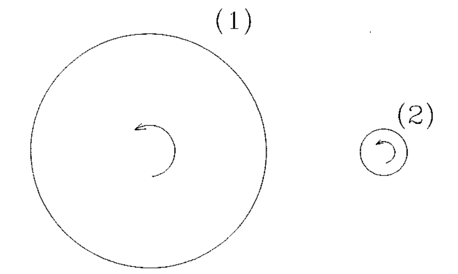
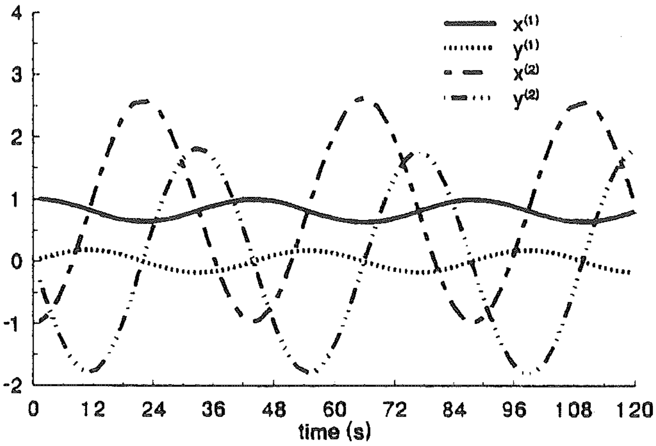


Figure 3. Initial condition for asymmetric pair (Melander et al., 1986). $A^{(1)} = \pi$, $A^{(2)} = 0.04\pi$, $\omega^{(1)} = 1.0$, $\omega^{(2)} = 2.5$. Cores initially circular and initial separation $R = 2.0$.

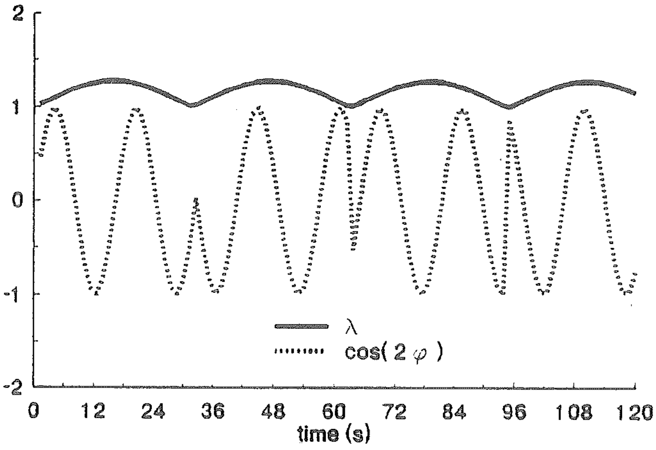
$$\mu = R\left(\frac{\pi}{A}\right)^{0.5} = \frac{R}{r_0}$$

From this solution we can obtain, for $\mu = 3.5054$, $\lambda = 1.2908$, and $d\phi/dt = 0.08309$. Knowing the motion, we can compute the vorticity moments B_{ij} and C_{ij}. For the particular value $\mu = 3.5054$, we obtain $B_1 = 3.60 \times 10^{-2}\sin(2t d\phi/dt)$ and $C_1 = 1.89 \times 10^{-4}\sin(2t d\phi/dt)$. Note that both the core and centroid motions fluctuate at the same frequency, $d\phi/dt$. It is also apparent that the amplitude of the source strength for the bulk motion is much greater than that of the core.

The second corotating vortex pair solution presented by Melander et al. (1986) that we consider is for two vortices of different size and strength. Vortex 1 has area $A^{(1)} = \pi$ and vorticity $\omega^{(1)} = 1.0$ while Vortex 2 has area $A^{(2)} = 0.04\pi$ and vorticity $\omega^{(2)} = 2.5$. The centroid separation is $R = 2.0$. The initial aspect ratio is $\lambda = 1.0$ for both vortices. A schematic of this case is shown in Figure 3. The model solution compared favorably to a more accurate contour dynamics solution for this system and initial condition (Melander et al., 1986). The motion of this system is quasiperiodic as shown in Figure 4. Figure 5a shows power spectra of the vorticity moments B and C and Figure 5b the power spectra source strengths d^3B/dt^3 and d^3C/dt^3. Notice that both the peaks of the spectra for both motions occur for the most part at the same frequencies. This matching is directly attributable to the coupling between the bulk and core motions. Where large peaks occur for the core spectrum, they are generally of higher amplitude than the corresponding peaks in the bulk motion spectrum. The exceptions to this behavior are the peaks at $f = 0.045$, 0.15, 0.225, and 0.26. Because the source strength is the third derivative of the vorticity moments, the highest frequency fluctuations of Q_{ij} are the most efficient in generating sound. In this particular case, the lowest frequency parts of the spectrum are dominated by the bulk motion contribution while the highest frequencies are dominated by the core. The three largest peaks of the core spectrum, at $f = 0.08$, 0.16, and 0.32 are larger



a)



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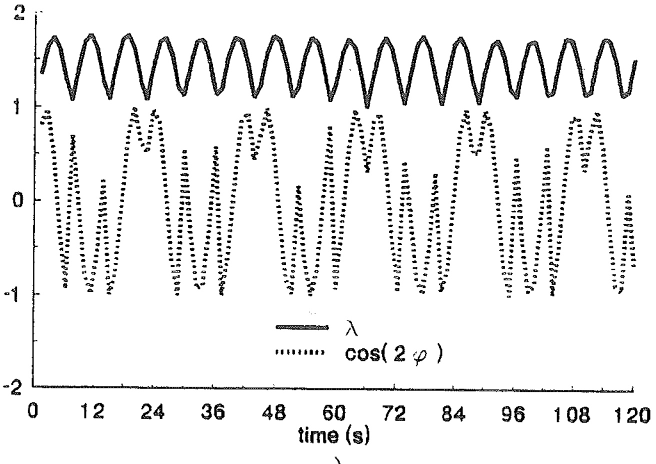
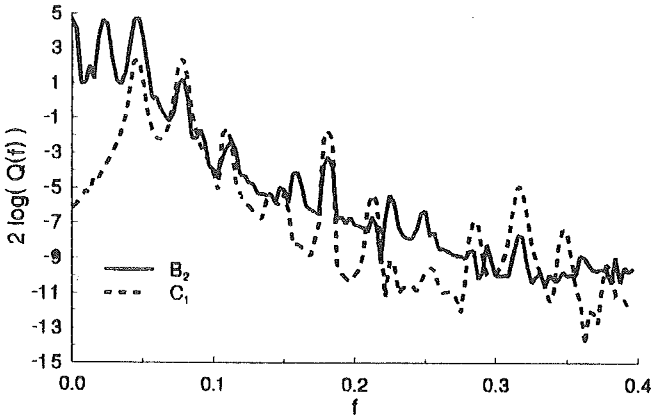


c)

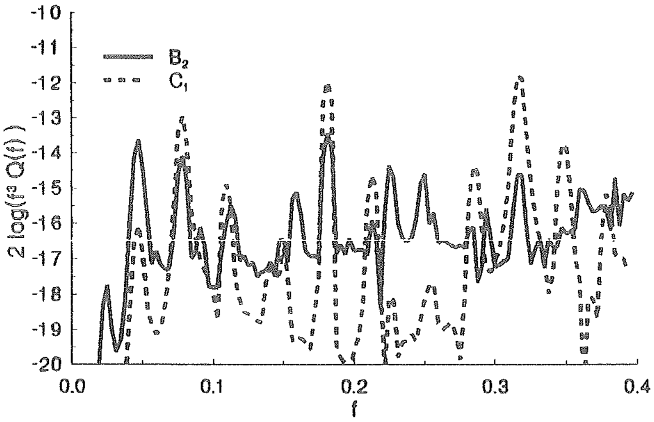
Figure 4. Motion of asymmetric vortex pair of Melander et al. (1986). (a) centroid motion. The evolution of the core aspect ratio, λ and $\cos(2\phi)$ for (b) vortex 1 and (c) vortex 2.

in magnitude than the corresponding peaks in the bulk motion spectrum.

These two relatively simple examples demonstrate the utility of the source splitting presented in this article for



a)



b)

Figure 5. Power spectra of (a) vorticity moments and (b) source strength. Resolution of these estimates is approximately 0.015 Hz.

separating the contributions of the bulk vortex motion and core deformations to the radiation of sound from vortex systems. This decomposition has been presented here for the quadrupole source term, generally considered the leading term in the noise from flow where solid surfaces are absent. The same decomposition is possible for the dipole term and higher-order terms as well. In fact, for a complete study of vortex noise, it may be necessary to consider higher-order terms when the core deformations contribute a large share of the noise at high frequencies. This work represents a first step in this direction.

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