Acoustique/*Acoustics*

# Propagation of finite amplitude sound through turbulence: a geometric acoustics approach

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*Abstract* – A propagation model is reported that describes the nonlinear propagation of sound through turbulence in the geometric acoustics limits. Individual realizations of a two-dimensional Gaussian correlated turbulent field are generated. Linear geometric acoustics is used to trace the rays through each realization of the turbulent field. A nonlinear transport equation is derived that describes the propagation of the sound along the eigenrays that connect source and receiver. The equation is solved by a Pestorius type algorithm. It is shown that the equivalent nonlinear distorsion after propagation through turbulence is always less than that for the homogeneous case. The effect is more pronounced for random velocity fields than for temperature fields. Results from a numerical experiment that simulates the propagation of spark-produced plane $N$ waves through turbulence are presented. When the turbulence results are compared with the no-turbulence data, it is observed that turbulence decreases on average the peak pressure of the $N$ waves and increases the rise time. These observations confirm the results from the model experiment (Lipkens, 1993) in which spark-produced $N$ waves are used to simulate sonic boom propagation through a turbulent atmosphere.

## Effet de la turbulence sur la propagation d'une onde acoustique d'amplitude finie : simulation numérique

*Résumé* – *Dans ce travail nous présentons une simulation numérique de la propagation d'une onde en N à travers une turbulence bidimensionnelle cinématique ou thermique. Le milieu de propagation est décrit comme un ensemble de réalisations d'un champ aléatoire scalaire ou vectoriel. Pour chacune d'entre elles on calcule d'abord les rayons propres à partir des équations de l'acoustique géométrique; on résout ensuite l'équation de transport associée au moyen d'un algorithme de Pestorius afin de prendre en compte les effets non linéaires, d'absorption et de relaxation. La signature temporelle de l'onde transmise est alors obtenue en sommant les contributions de chaque rayons propres. Une analyse statistique sur une centaine de réalisations permet de caractériser le pic de surpression ainsi que les variations de temps de montée.*

*Version française abrégée* – La propagation dans l'atmosphère d'une onde acoustique d'amplitude finie et intense, produite par exemple par le bang sonore d'un avion supersonique, est très fortement perturbée par les fluctuations turbulentes du milieu. Des travaux récents (Niedzwiecki *et al.*, 1978; Leatherwood *et al.*, 1992) montrent que sont notamment affectés l'amplitude de l'onde transmise et le temps nécessaire pour que la pression acoustique passe de 10 à 90% de sa valeur maximale. Ces interactions entre une onde sonore très intense et la turbulence atmosphérique ne sont pas encore très bien comprises et elles ont souvent été omises dans les simulations numériques.

Nous présentons ici un modèle non linéaire de propagation en milieu turbulent. On suppose d'abord que la turbulence est homogène et isotrope, et qu'elle est gelée pendant le temps de traversée de l'onde acoustique. Il est alors possible de représenter le milieu turbulent comme un ensemble de réalisations d'un champ aléatoire scalaire ou vectoriel. Chaque réalisation est obtenue par sommation d'un nombre fini de modes de Fourier aléatoires de propriétés connues. Ensuite pour chacune de ces réalisations, on identifie les rayons propres joignant la source au récepteur choisi. Pour cela, en utilisant une méthode de Runge–Kutta, on intègre le système d'équations différentielles donnant l'évolution des rayons dans le cadre de l'approximation linéaire de l'acoustique géométrique. Ces deux phases du modèle ont été détaillées dans des travaux antérieurs (Blanc-Benon *et al.*, 1991; Karweit *et al.*, 1990). Dans une dernière étape on écrit une équation de transport non linéaire pour la propagation de l'énergie acoustique le

long des rayons propres calculés précédemment [équation (1)]. En effectuant un changement de variables pour l'amplitude de l'onde $K$ [équation (2)] et pour la distance de propagation $Z$ [équation (4)], l'équation (1) est mise sous la forme d'une équation de Burgers [équation (5)]. Pour chacun des rayons propres l'équation (6) est intégrée au moyen d'un algorithme (Pestorius, 1973; Pestorius et al., 1973) qui prend en compte les effets non linéaires dans le domaine temporel et les effets d'absorption et de relaxation dans le domaine fréquentiel. La signature temporelle de la pression acoustique au point d'écoute est alors obtenue en combinant tous les rayons propres allant de la source au récepteur. Cette démarche est appliquée pour chaque réalisation du champ turbulent, une analyse statistique est ensuite effectuée avec un ensemble d'une centaine de réalisations.

Les résultats présentés ici concernent un milieu turbulent bidimensionnel avec des fluctuations de température ou de vitesse dont les corrélations spatiales sont gaussiennes. Les champs turbulents sont obtenus avec 60 modes de Fourier équirépartis entre 0,1 et 10 $L$, avec une échelle de corrélation spatiale $L$ de 2,5 cm. La valeur efficace des fluctuations de température $T'$ est de 4,27 K et celle des fluctuations de vitesse dans la direction de propagation $v_1'$ est de 2,5 m/s. Ces valeurs ont été choisies de façon à avoir des fluctuations d'indice de même valeur efficace pour le cas scalaire $[\mu = -(T'/2T_0)$, où $T_0$ est la température ambiante] et pour le cas vectoriel $[\mu = -(v_1'/c_0)$, où $c_0$ est la célérité du son, Blanc-Benon et al., 1991]. Les paramètres de l'onde en $N$ utilisée pour initialiser nos calculs sont donnés sur la figure 1.

La figure 2 $a$ et $b$ fournit les évolutions moyennes respectives de la distance équivalente $Z$ et du facteur d'amplitude $K$ avec la distance normalisée $X/L$. Trois configurations ont été étudiées : sans turbulence, avec une turbulence thermique et avec une turbulence cinématique. La présence de la turbulence fait décroître la distance équivalente $Z$ ce qui traduit une atténuation des effets de distortions non linéaires. Le facteur d'amplitude $K$ augmente très rapidement avec la distance, et ceci dès la rencontre de la première caustique. Les effets sont plus marqués pour une turbulence cinématique que pour une turbulence thermique. La figure 3 illustre le calcul de la signature temporelle de l'onde pour un cas de propagation où cinq rayons propres contribuent au champ de pression acoustique (fig. 3 $b$). On notera les variations importantes du maximum de pression et du temps de montée d'un rayon à l'autre (fig. 3 $a$). Les valeurs moyennes du maximum de pression et du temps de montée sont tracées en fonction de $X/L$ sur la figure 4. Sur la figure 4 $a$ on observe une décroissance du maximum de pression avec la distance $X/L$ en présence de turbulence. Ainsi, pour $X/L = 40$ on atteint 25 % de réduction par rapport au cas sans turbulence. Il convient de noter qu'il n'existe pas de différences essentielles entre le cas vectoriel et le cas scalaire. Sur la figure 4 $b$ on note que le temps moyen de montée augmente très fortement avec la distance de propagation et que cette augmentation débute à des distances plus courtes pour la turbulence cinématique ($X/L = 8$ au lieu de $X/L = 16$). Ce résultat s'explique par le fait que le temps de montée, qui est calculé à partir de la fraction de signal comprise entre 10 et 90 % du maximum de pression est fortement influencé par le nombre de rayons propres contribuant au signal total (voir fig. 3), et par leurs différences de temps d'arrivée. L'apparition de rayons propres supplémentaires est quant à elle relative à la formation des caustiques (Karweit et al., 1990). Les travaux antérieurs (Blanc-Benon et al., 1991) ont montré que la distance moyenne de formation de la première caustique est plus courte pour une turbulence cinématique que pour une turbulence thermique.

En conclusion, la présence de la turbulence se traduit par une double modification des effets non linéaires : en moyenne le pic de surpression est réduit et le temps de montée augmente. Il convient de noter que les résultats présentés dans ce travail pour la propagation d'une onde plane dans une turbulence cinématique sont en accord avec les mesures effectuées à échelle réduite en laboratoire (Lipkens, 1993; Lipkens, 1995). L'approche numérique proposée peut

être étendue à des champs turbulents dont les caractéristiques spectrales sont plus réalistes (loi en puissance pour les spectres) avec une prise en compte d'un profil moyen de vitesse et/ou de température et d'un sol.

INTRODUCTION. – Recent investigations have shown that turbulence strongly affects the perceived loudness of sound, mainly by changing its amplitude (peak pressure) and rise time (time portion between 10 and 90% of peak pressure). The results have been of great importance for sonic booms because perceived loudness of the sonic boom when heard outdoors (Niedzwiecki *et al.*, 1978; Leatherwood *et al.*, 1992) is a large factor in determining the acceptability of supersonic flight. Missing in these investigations is the effect of turbulence on the propagation of the sonic boom through the atmosphere. The interaction of finite amplitude sound with turbulence is still not very well understood, although much research has been done in the 1960s and 70s. For a literature rewiew see Lipkens (1993, 1995). Up to now the interaction between sound and turbulence has been studied only as a linear phenomenon. It is therefore important to investigate the influence of acoustical nonlinearity on the interaction of intense sound with turbulence.

The numerical model described here consists of the following parts. First, a turbulence model is used to generate individual realizations of a homogeneous, isotropic turbulent field. Temperature and velocity random fields are generated as a superposition of a finite number of random Fourier modes (Blanc-Benon *et al.*, 1991). Second, linear geometric acoustics is used to trace rays through each realization of the turbulent field. Third, a nonlinear transport equation is derived for the propagation along the rays. The transport equation is solved by a Pestorius type algorithm. Absorption and dispersion are included in the model. The first two parts have already been used in a study of the occurrence of caustics for high-frequency acoustic waves propagating through turbulent fields (Blanc-Benon *et al.*, 1991; Karweit *et al.*, 1990), and are not treated in detail here. In this study we focus entirely on the nonlinear transport equation.

NONLINEAR TRANSPORT EQUATION. – A lossless nonlinear transport equation is derived for the propagation of the waves along the eigenrays. Linear geometric acoustics is used to trace rays through each realization of the turbulent field and find the eigenrays that connect source and receiver (Candel, 1977). The step size $ds$ in the integration of the geometric acoustics equations is a function of the maximum wave number value $K_m$ that we consider in the turbulence energy spectrum and equals $ds = 1/2\,K_m$. A Runge Kutta fourth order numerical scheme is used to solve the system of differential equations.

Several assumptions are made in the development of the nonlinear equation and are listed here: 1) parameters of inhomogeneity vary slowly on a characteristic signal duration, 2) the medium is frozen, *i.e.*, during passage of the acoustic wave the turbulent field is constant in time, 3) self-refraction is not accounted for, *i.e.*, first order terms are sufficient in the ray equations while second order terms are only important in describing the transport equation, 4) loss terms are neglected in the development of the ray path and transport equations, but absorption is added in the numerical algorithm and is dominated by thermoviscous absorption and dispersion and absorption of oxygen and nitrogen, 5) the fluid motion is isentropic, 6) only quasi-linear terms are retained in the development of the transport equation, *i.e.* a weak shock approximation.

Starting from the fundamental laws of fluid mechanics and making use of the above stated assumptions, we derive a transport equation for the propagation through turbulence.

The derivation is lengthy and not presented here, but can be found in Guiraud (1965); Hayes *et al.* (1969); Pierce (1981) or Robinson (1990). The transport equation for the acoustic pressure $p$ at point $\vec{x}$ takes the form:

$$(1) \qquad \frac{\partial}{\partial s}\left[\frac{|A|}{\rho_0\, c}\,|\vec{\nu}+\vec{M}|\,(1+\vec{M}\cdot\vec{\nu})\,p^2\right] - \frac{2\,\beta\,|A|}{\rho_0^2\, c_0^4}\,p^2\, p_{t'} = 0,$$

where $s$ is the arclength along the ray, $A$ is the ray tube area, $\beta$ is the coefficient of nonlinearity, $\vec{\nu}$ is the unit vector tangent to the ray path, $\vec{M} = \vec{v}/c$ is the Mach number vector, $\vec{v}$ is the fluid velocity vector, $c$ is speed of sound, subscripts 0 denote ambient conditions, $t'$ is the retarded time coordinate given by $t' = t - \psi(\vec{x})$, where $\psi(\vec{x})$ represents the eikonal, and $p_{t'}$ is the derivation of $p$ takes at time $t'$. This equation reduces to Blokhintsev's equation for a steady atmosphere (Hayes *et al.*, 1969). The next step is to transform the equation into the form of the homogeneous plane wave case. First, let $\Pi = K\,p$, where

$$(2) \qquad K = \sqrt{\frac{|A|\,\rho_{0s}\,c_{0s}}{|A_s|\,\rho_0\,c}\,|\vec{\nu}+\vec{M}|\,(1+\vec{M}\cdot\vec{\nu})}.$$

where $\rho_{0s}$, $A_s$, and $c_{0s}$ are respectively the density, initial ray tube area, and speed of sound at the source. Equation (1) is now written as:

$$(3) \qquad \frac{\partial\Pi}{\partial s} - \frac{\beta\,|A|\,\rho_{0s}\,c_{0s}}{|A_s|\,\rho_{0s}^2\,c_0^4\,K^3}\,\Pi\Pi_{t'} = 0.$$

Next, a transformation of the independent variable $s$, $Z(s)$, is introduced (Robinson, 1990):

$$(4) \qquad \frac{dZ}{ds} = \frac{\beta\,|A|\,\rho_{0s}\,c_{0s}}{|A_s|\,\rho_0^2\,c_0^4\,K^3}\,\frac{\rho_{0s}\,c_{0s}^3}{\beta_{0s}},$$

and the transport equation now takes the well-known form of Burgers equation (Blackstock, 1985) for lossless plane wave propagation of finite amplitude:

$$(5) \qquad \frac{\partial\Pi}{\partial Z} - \frac{\beta_{0s}}{\rho_{0s}\,c_{0s}^3}\,\Pi\Pi_{t'} = 0.$$

The distortion distance variable $Z(s)$ is given by equation (4), which can be written as

$$(6) \qquad \frac{dZ}{ds} = \sqrt{\frac{|A_0|}{|A|}\,\frac{\beta^2}{\beta_0^2}\,\frac{\rho_0\,c_0^5}{\rho\,c^5}\,(1+\vec{\nu}\cdot\vec{M})^{-3}\,|\vec{\nu}+\vec{M}|^{-3}}.$$

The distortion distance variable $Z$, or age variable, describes the equivalent plane wave distortion for a wave propagating in a random medium. Equation (6) is integrated numerically along the eigenrays and yields the equivalent distortion value $Z(s)$. The rate at which $Z$ varies with actual distance is a measure of how rapidly nonlinear distortion develops. For a plane wave in a quiet, homogeneous medium, $Z$ varies linearly with $s$. Integration of equation (6) then shows the effect of turbulence on nonlinear distortion. The Burgers equation [equation (5)] has an exact analytical solution and describes the nonlinear distortion along the ray. The overall model that we apply is the Pestorius model (Pestorius, 1973; Pestorius *et al.*, 1973). This algorithm alternately applies a nonlinear distortion step

in the time domain and an absorption step in the frequency domain and is able to handle discontinuities through the use of weak shock theory.

The computational strategy is then: 1) create realizations of the turbulent field, 2) find eigenrays that connect source and receiver, 3) apply Pestorius algorithm to solve nonlinear transport equation, 4) combine eigenrays to find the waveform at the receiver.

Fig. 1. – Input waveform for the numerical experiment.

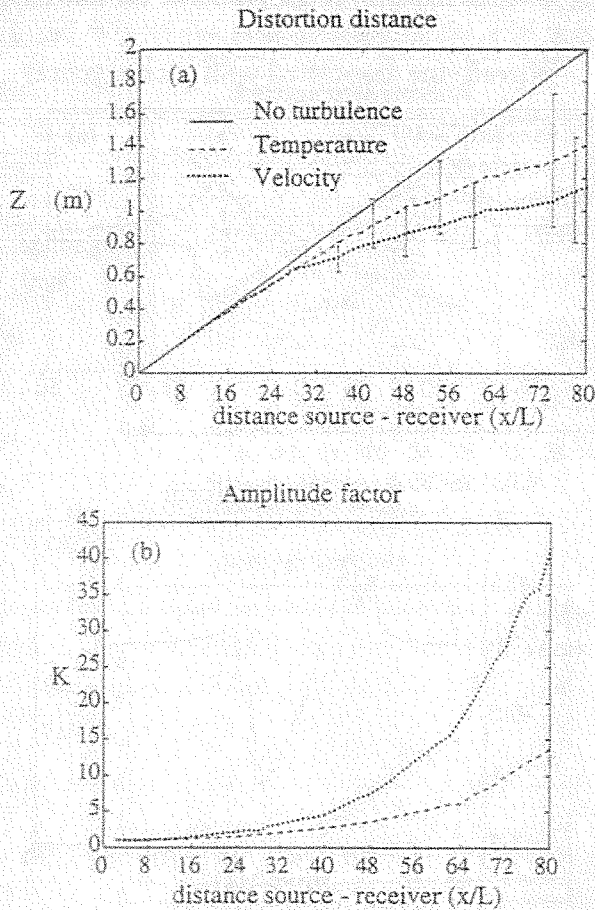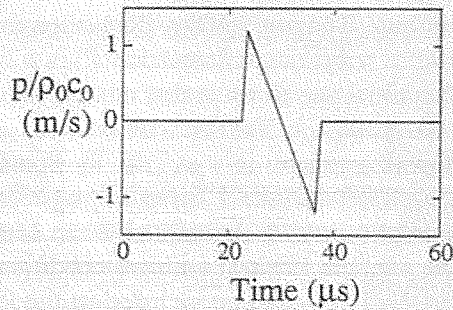*Fig. 1. – Signature temporelle de l'onde en N initialisant la simulation numérique.*

Fig. 2. – Effects of velocity and temperature fluctuations on the averaged distortion distance Z (a), and the averaged amplitude factor K (b).

*Fig. 2. – Influence des fluctuations de vitesse et de température sur les moyennes de la distance Z (a), et du coefficient d'amplitude K (b).*

RESULTS. – Results are presented for the case of 2D, temperature and velocity turbulent fields with a Gaussian longitudinal correlation function. The turbulent fields are constructed as a sum of 60 Fourier modes with wave numbers equally spaced between a lower value of $0.1 L$ and a maximum value of $10 L$, where $L$ is 2.5 cm. The refraction index of the two random fields is given by $n = 1 + \mu$, where $\mu$ is the fluctuating part of the refraction index, $\mu = -(T'/2T_0) - (v_1'/c_0)$, where $T_0$ is the ambient temperature (Blanc-Benon *et al.*, 1991). We choose the amplitude of the velocity and temperature fields so that the index is the same. The rms velocity fluctuation in the propagation direction $v_1'$ is 2.5 m/s, and hence the rms temperature fluctuation $T'$ is 4.27 K. Statistics are calculated over 100 realizations. The initial waveform is that of an $N$ wave similar to that produced by an electrical spark. Peak pressure is 500 Pa, duration is 15 $\mu$s, and rise time (time portion between 10 and 90% of peak pressure) is 1 $\mu$s (*fig.* 1). Spark-produced $N$ waves have been used successfully to simulate the sonic boom propagation through a turbulent atmosphere. For a detailed description of the design and setup of the model experiment, *see* Lipkens (1993, 1995). The medium is air at ambient conditions and classical thermoviscous absorption and $O_2$ and $N_2$ relaxation are included in the Pestorius' algorithm (Norm Ansi, 1978).

In figure 2 the values of $Z$ and $K$ averaged over 100 realizations are presented. In graph (a) the value of $Z$ is shown. Curves are presented for three cases, not turbulence,
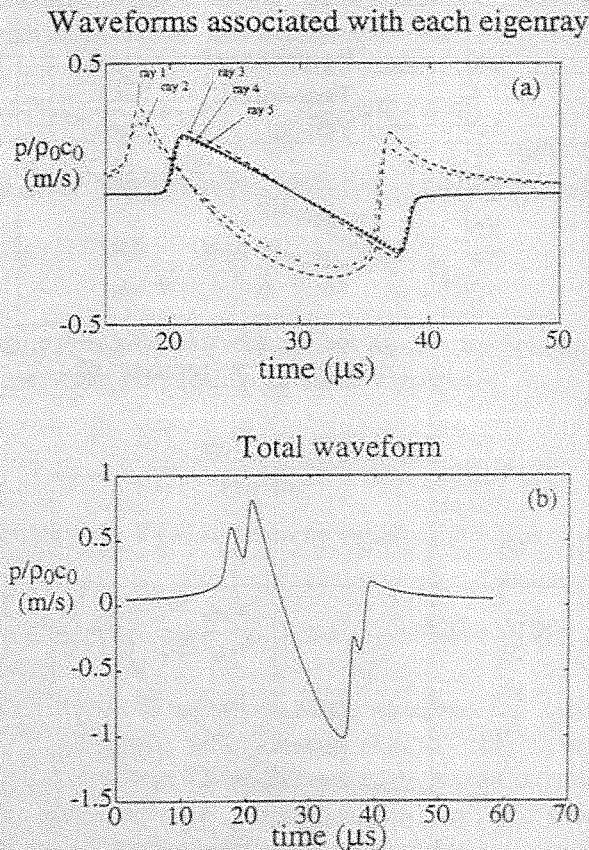


Fig. 3. – Example of waveform calculation at the receiver:
(a) individual contribution of eigenrays, (b) total resulting waveform.

*Fig. 3. – Exemple du calcul de la signature temporelle de pression acoustique :*
*(a) contribution individuelle des rayons propres, (b) signature résultante après sommation.*

temperature turbulence, and velocity turbulence. For the turbulence cases a bar mark shows the values of the average $\pm$ the standard deviation $\sigma$. It is seen that the presence of turbulence always results in a lower value for the equivalent distortion distance, *i.e.* the nonlinear distortion is weaker when turbulence is present. The effect is more pronounced when propagation distance increases, and stronger for the velocity turbulence. At a propagation distance of $x/L = 80$ the equivalent distortion distance for the velocity turbulence is slightly more than 50% that of the no turbulence case. Graph (b) presents the values of the amplitude factor $K$. The factor increases rapidly with propagation distance, and the effect is more pronounced for the velocity turbulence. Once the rays pass through a first caustic, $K$ starts to increase. However, increasing values of $K$ do not imply necessarily a decrease in peak pressure because of the increased number of eigenrays that contribute to the total signal at the receiver.

In figure 3 an example of a waveform calculation is presented. In this particular case five eigenrays were found that connect source and receiver. The waveform associated with each eigenray is shown in graph (a), while graph (b) shows the waveform of the total signal. As is seen, the waveform variation is caused mainly by the multipaths effects.

The average values of peak pressure and rise time of the total waveform calculated at different receiver distances are shown in figure 4. The rise time is calculated as the time portion between 10 and 90% of the peak pressure of the total waveform. According to
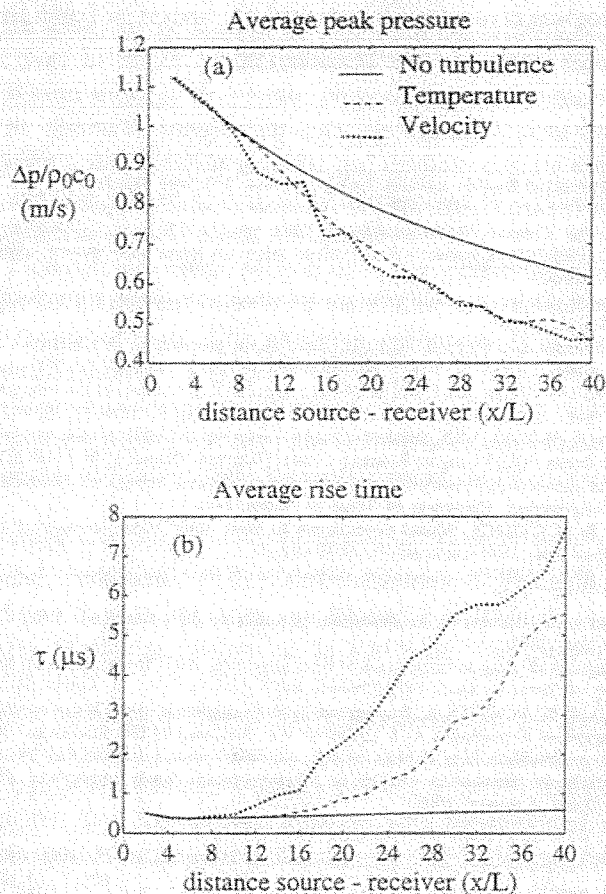


Fig. 4. – Average values of peak pressure (a) and rise time (b) of the total waveform.
*Fig. 4. – Évolution avec la distance $X/L$ de la surpression (a) et du temps de montée (b).*

this definition, we notice that the rise time is essentially determined by the differences in arrival time of the eigenrays when more than one eigenray is present. The average is taken over 100 realizations. As is observed in graph (a), peak pressure is always decreased in the presence of turbulence. There is no notable difference between the temperature and velocity turbulence. At a distance of $x/L = 40$ a decrease of about 25% is observed. This result confirms the model experiment results (Lipkens, 1993) that peak pressure on average is always decreased by turbulence. Graph (b) presents the average value of rise time. On average, rise time is always increased by turbulence. At a propagation distance of 1 m, a tenfold increase is observed. Again, this result confirms that of the model experiment. The effect is more pronounced for the velocity fields. The curves start to deviate from the no turbulence case when the rays pass through the first caustic. The shorter distance to the first caustic for the velocity fields explains the quicker departure from the no turbulence values (Blanc-Benon et al., 1991).

CONCLUSION. – In conclusion, the effect of turbulence on the waveform distortion, rise time, and peak pressure is calculated. It is shown that the nonlinear distortion in the presence of turbulence is weaker than without it. On average, peak pressure decreases in the presence of turbulence, and rise time increases. The results confirm the observations of the model experiment. Future work includes the construction of turbulent fields with a more realistic spectrum, e.g., a power spectrum, and the inclusion of boundaries.

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