ACUSTICA · acta acustica
Vol. 83 (1997) 455–460

Derivation of a wide-angle parabolic equation for sound waves in inhomogeneous moving media

V. E. Ostashev*
Fachbereich Physik, University Oldenburg, D-26111 Oldenburg, Germany

D. Juvé, P. Blanc-Benon
LMFA, URA CNRS 263, Ecole Centrale de Lyon, BP. 163, 69131 Ecully Cedex, France

Summary
It is shown that several problems in outdoor sound propagation cannot be studied by using a small-angle parabolic equation, and need a wide-angle parabolic equation for their solution. On the other hand, the wide-angle parabolic equations for sound propagation in the atmosphere, used previously in the literature, do not allow one to describe the effects of regular and random inhomogeneities in the density and wind velocity vector on sound propagation and scattering. The main result of the paper is the derivation of the wide-angle parabolic equation and its Padé (1,1) approximation which enable one to describe these effects. Several effects in outdoor sound propagation, which are proposed to be studied numerically on a basis of the derived wide-angle parabolic equation, are enumerated.

PACS no. 43.20.Bi, 43.20.Fn, 43.20.Fp

1. Introduction

The classical "small-angle" parabolic equation was first introduced by Leontovich and Fock in 1940s for the problem of electromagnetic wave propagation near the ground. Since that time, the equation has widely been used for investigating the propagation of electromagnetic and acoustical waves in various media. This equation employs the idea that there is only one preferred direction of wave propagation. But for many problems of atmospheric and oceanic acoustics and geoacoustics, preferred directions of wave propagation are in a cone with an angle greater than $30 - 40°$. To solve such problems, "wide-angle" parabolic equations were derived in 1970s and then used in different fields of physics.

In underwater acoustics, the wide-angle parabolic equation derived by Claerbout [1] is used for numerical computations of sound fields in the ocean with deterministic profiles of the adiabatic sound speed and density, e.g. see [2, 3, 4].

In the field of wave propagation in random media, Ostashev and Tatarskii [5] derived the equation which describes all waves multiply scattered forward. This equation is usually written as an integral equation or an integro-differential one, however it is actually analogous to the wide-angle parabolic equation. This equation has widely been used for investigating waves in random media, e.g. see [6, 7, 8], and the review article [9] recently published.

The effects of impedance ground on outdoor sound propagation is practically always significant. That dictates us a preferred direction which must be parallel to the ground. For many problems of atmospheric acoustics, source and receiver are only a few meters above the ground while the distance between them is greater than one hundred meters. For such problems, the preferred direction of wave propagation is also nearly horizontal and, hence, the small-angle parabolic equation can be applied. Nevertheless, there are also many problems where this direction significantly differs from the horizontal one so that a wide-angle parabolic equation should be used. Examples of geometry of the latter problems are shown in Figure 1. In Figure 1a, the horizontal distance between the source and receiver is comparable to their heights above the ground; in Figure 1b, the source (helicopter, airplane, etc.) is much higher above the ground than the receiver located near it; and in Figure 1c, sound waves scattered by atmospheric turbulence propagate in directions which may significantly differ from the horizontal one. Note that the latter problem of sound scattering into the refractive shadow zone attracts a great attention by scientists nowadays and still needs a theoretical explanation.

In order to handle with the problems, the geometries of which are shown in Figure 1, the wide-angle parabolic equation has been used in atmospheric acoustics in the last few years, e.g. see [10, 11, 12, 13]. In these and other papers dealing with the wide-angle parabolic equation, the approximation of the effective sound speed is used, in which the real moving atmosphere is replaced by a hypothetical motionless medium with the effective sound speed

$$c_{\text{eff}} = c + v_x \,. \tag{1}$$

Here, c is the adiabatic sound speed, v_x is the wind velocity component in the direction of the x-axis which is hereinafter assumed to be parallel to the ground and in the vertical plane containing the source and receiver. This approximation was reasonable as a first stage for studying outdoor sound propagation. But, as we shall show in section 2, strictly speaking, this approximation is not valid for the problems, the geometries of which are shown in Figure 1. Furthermore, the effects of density fluctuations on sound scattering in the atmosphere were ignored in the previous investigations.

Received 12 September 1995,
accepted 10 April 1996.

* Present address: Department of Physics, New Mexico State University, Las Cruces, NM 88003-8001, U.S.A.

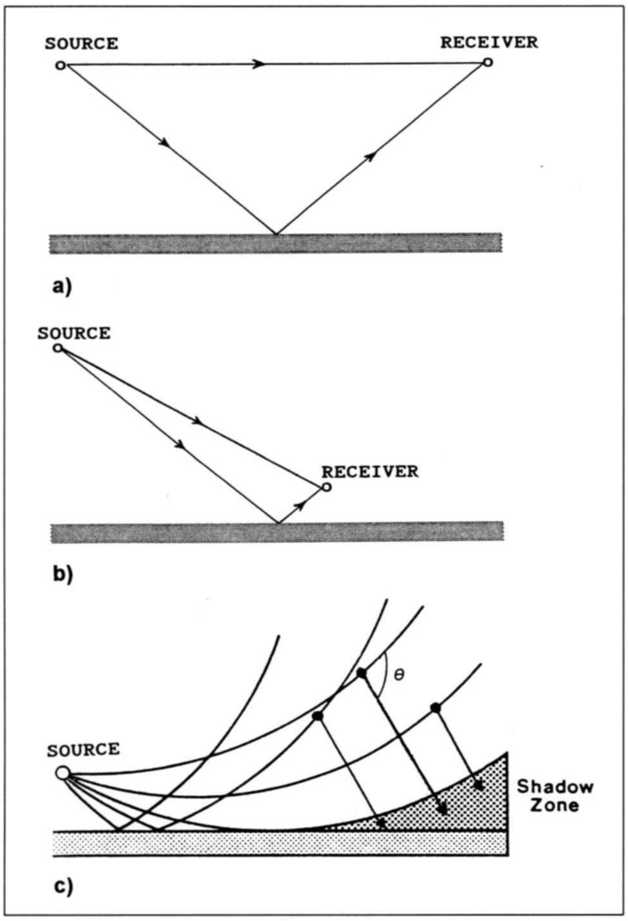

Figure 1. Geometries of the problems, the solutions of which can be obtained by using a wide-angle parabolic equation. Arrows indicate the direction of sound propagation, points indicate sound scattering by atmospheric turbulence.

The main object of the present paper is to give a derivation of a correct wide-angle parabolic equation for sound propagation in media with regular and random inhomogeneities in the adiabatic sound speed c, density ρ and medium velocity $\boldsymbol{v}$. In subsequent papers, the derived equation will be used for numerical simulation of various problems of sound propagation and scattering in the atmosphere. It should be noted here that we are mainly interested in outdoor sound propagation; however the wide-angle parabolic equation derived is valid for a moving medium with an arbitrary equation of state and, hence, can also be applied in other fields of acoustics, for instance, in underwater acoustics.

In section 2, we consider a starting equation for deriving the wide-angle parabolic equation and compare it with the starting equation used in previous studies. In section 3, we present the derivation of the wide-angle parabolic equation. The Padé (1,1) approximation of a pseudo-differential operator is used in section 4 in order to obtain a relatively simple form of the wide-angle parabolic equation. It is shown in section 5 that in limiting cases, the equation derived in the present paper coincides with the equations known in the literature. And in Conclusions we summarize the results obtained.

2. Starting equation

The sound field p propagating in an inhomogeneous moving medium obeys the following equation [14]

$$\Big[\nabla^2 + k^2(1+\epsilon) - (\nabla \ln(\rho/\rho_0)) \cdot \nabla - \frac{2i}{\omega}\frac{\partial v_i}{\partial x_j}\frac{\partial^2}{\partial x_i \partial x_j} + \frac{2ik}{c_0}\boldsymbol{v}\cdot\nabla\Big] p(\boldsymbol{R}) = 0 \,. \quad (2)$$

Here, $\boldsymbol{R} = (x,y,z) = (x_1,x_2,x_3)$ are the Cartesian coordinates with the z-axis in the vertical direction, $\nabla = (\partial/\partial x_1, \partial/\partial x_2, \partial/\partial x_3)$, $\boldsymbol{v} = (v_1,v_2,v_3)$, $k = \omega/c_0$ is the wave number, ω is the frequency, $\epsilon = c_0^2/c^2 - 1$, c_0 and ρ_0 are mean values of the sound speed and density, c, ρ and $\boldsymbol{v}$ are functions of $\boldsymbol{R}$, and repeated subscripts are summed from 1 to 3 (summation convention). Equation (2) is derived from the full set of linearized fluid-dynamic equations provided that $\nabla \cdot \boldsymbol{v} = 0$ and terms of the order of $\mu^2 = \max\left(v^2/c^2, |v\tilde{c}/c_0^2|, |v\tilde{\rho}/c_0\rho_0|\right)$ are neglected, where $\tilde{c} = c - c_0$ and $\tilde{\rho} = \rho - \rho_0$ are deviations of the sound speed and density from their mean values. Note that the equality $\nabla\cdot\boldsymbol{v} = 0$ is usually valid for turbulent media and becomes an identity for the stratified moving media where $\boldsymbol{v} = (v_x(z), v_y(z), 0)$. Equation (2) is one of the most general equations describing the propagation of sound waves in inhomogeneous moving media and, in what follows, it is used as a starting equation for deriving the wide-angle parabolic equation.

When deriving the wide-angle parabolic equation for sound propagation in the atmosphere, the following starting equation was previously used

$$\left[\nabla^2 + k^2(1+\epsilon_{\text{eff}})\right] p(\boldsymbol{R}) = 0 \,. \quad (3)$$

Here, $\epsilon_{\text{eff}} = c_0^2/c_{\text{eff}}^2 - 1$, and c_{eff} is determined by (1). (Note that in the paper [10], ϵ_{eff} also contains terms proportional to the density ρ which is assumed to be dependent only on z.) Equation (3) may be represented in the form

$$\left(\frac{\partial^2}{\partial x^2} + k^2 Q_{\text{eff}}^2\right) p(\boldsymbol{R}) = 0 \,. \quad (4)$$

Here, the pseudo-differential operator Q_{eff} is given by

$$Q_{\text{eff}} = \left(1 + \epsilon_{\text{eff}} + k^{-2}\Delta_\perp\right)^{1/2} , \quad (5)$$

where $\Delta_\perp = \partial^2/\partial y^2 + \partial^2/\partial z^2$. If ϵ_{eff} and, hence, Q_{eff} do not depend on x, equation (4) can be written as

$$\left(\frac{\partial}{\partial x} + ikQ_{\text{eff}}\right)\left(\frac{\partial}{\partial x} - ikQ_{\text{eff}}\right) p = 0 \,. \quad (6)$$

From this equation, we obtain the wide-angle parabolic equation for sound waves

$$\frac{\partial p}{\partial x} = ikQ_{\text{eff}}\, p \,. \quad (7)$$

If ϵ_{eff} depends on x, equation (4) cannot be reduced to (6). However, in this case, equation (7) is still used in the literature as a wide-angle parabolic equation.

It follows from (7) and (5) that the sound field p is affected only by the wind velocity component v_x in the horizontal direction from the source to the receiver. But it follows from geometric acoustics in a moving medium that to an accuracy of v/c, the sound field is affected by the component v_R of the medium velocity in the direction of sound propagation, see, e.g. [15]. For the problems, the geometries of which are shown in Figure 1a and 1b, v_x and v_R differ significantly so that equations (7) and (5), strictly speaking, are not applicable. On the other hand, equation (2) is applicable for solving these problems.

Furthermore, equations (7) and (5) do not allow us to study the effects of the mean crosswind v_y on sound propagation, while equation (2) allows us to do it.

Finally, in the turbulent atmosphere the sound scattering is mainly caused by fluctuations in c and v_x, entering into (5) via ϵ_{eff}, only if the scattering angle θ is small. Here, θ is the angle between a given sound wave and a wave scattered by atmospheric turbulence, see Figure 1c. But if $\theta \geq 15-20°$, the scattering of a sound wave is also significantly affected by fluctuations in density and wind velocity components v_y and v_z, which are not included in the wide-angle parabolic equation (7). On the other hand, equation (2) enables us to describe the sound scattering by turbulence for arbitrary values of θ.

To illustrate this difference in scattering of sound waves, let us compare the sound scattering cross-sections $\sigma(\theta)$ and $\sigma_{\text{eff}}(\theta)$ calculated on the basis of (2) and (3), respectively. Equation for $\sigma(\theta)$ is well-known in the literature (e.g. [14])

$$\sigma(\theta) = 2\pi k^4 \left[\frac{\beta^2(\theta)\Phi_T(q)}{4T_0^2} + \frac{\cos^2\theta\cos^2\frac{\theta}{2}\,F(q)}{c_0^2} \right].$$

Here, $\beta(\theta) = \beta_c + 2\beta_\rho \sin^2\frac{\theta}{2}$, $\beta_c = \frac{2T_0}{c_0}\frac{\partial c}{\partial T}$, $\beta_\rho = \frac{T_0}{\rho_0}\frac{\partial \rho}{\partial T}$, T_0 is the mean value of the temperature T, $\Phi_T(q)$ and $F(q)$ are the three-dimensional spectral densities of temperature and medium velocity fluctuations, where $q = 2k\sin\theta/2$. This equation for $\sigma(\theta)$ is derived from (2) assuming that fluctuations in c and ρ are caused by fluctuations in T.

On the other hand, if one starts from (3), the following equation for the sound scattering cross-section $\sigma_{\text{eff}}(\theta)$ can be derived

$$\sigma_{\text{eff}}(\theta) = 2\pi k^4 \left[\frac{\beta_c^2\Phi_T(q)}{4T_0^2} + \frac{\cos^2\frac{\theta}{2}\,F(q)}{c_0^2} \right].$$

A comparison between equations for $\sigma(\theta)$ and $\sigma_{\text{eff}}(\theta)$ reveals the difference in sound scattering cross-sections calculated on the basis of (2) and (3). For sound propagation in an atmosphere, $\beta_c = -\beta_\rho = 1$ (see [14]) and it can be shown from presented above equations for $\sigma(\theta)$ and $\sigma_{\text{eff}}(\theta)$ that

$$\sigma(\theta)/\sigma_{\text{eff}}(\theta) = \cos^2\theta\,.$$

It evidently follows from this equation that (3) and, hence (7), correctly describe the scattering of sound waves by atmospheric turbulence only if θ is small.

Thus, equation (7) cannot be considered as a wide-angle parabolic equation for sound waves propagating in media with regular and random inhomogeneities in c, ρ and $\boldsymbol{v}$. To derive such an equation we shall start from equation (2).

3. Derivation of the wide-angle parabolic equation

Equation (2) contains the term $\Omega = -(\partial v_x/\partial x)(\partial^2 p/\partial x^2)$ which should be removed when deriving the desired equation. To do that, we note that (2) may be rewritten in the form

$$\frac{\partial^2 p}{\partial x^2} = -\left(\Delta_\perp + k^2 + \eta\right)p\,, \tag{8}$$

where $\eta = O\left(v/c, |\tilde{c}/c_0|, |\tilde{\rho}/\rho_0|\right)$. Using (8), we obtain the following formula: $\Omega = (\partial v_x/\partial x)(\Delta_\perp + k^2 + \eta)p$. But the product of v_x and η is of the order of μ^2 and should be neglected in this formula in accordance with the accuracy to which (2) is derived. That yields the formula $\Omega = (\partial v_x/\partial x)(\Delta_\perp + k^2)p$. Substituting Ω into (2) yields

$$\left(\frac{\partial^2}{\partial x^2} + k^2 Q^2\right)p = 0\,. \tag{9}$$

Here, Q is the pseudo-differential operator given by

$$Q = (1+L)^{1/2}, \tag{10}$$

where

$$L = F + M\frac{\partial}{\partial x}\,, \tag{11}$$

and

$$\begin{aligned} F = {} & \epsilon + \frac{2i}{\omega}\frac{\partial v_x}{\partial x} + \left(\frac{2i}{\omega}\boldsymbol{v}_\perp - k^{-2}\nabla_\perp \ln(\rho/\rho_0)\right)\cdot\nabla_\perp \\ & + k^{-2}\left(1 + \frac{2i}{\omega}\frac{\partial v_x}{\partial x}\right)\Delta_\perp \\ & - \frac{2i}{\omega k^2}\sum_{j=2}^{3}(\nabla_\perp v_j)\cdot\nabla_\perp\frac{\partial}{\partial x_j}\,, \end{aligned}$$

$$\begin{aligned} M = {} & \frac{2i}{\omega}v_x - k^{-2}\frac{\partial \ln(\rho/\rho_0)}{\partial x} \\ & - \frac{2i}{\omega k^2}\left(\nabla_\perp v_x + \frac{\partial \boldsymbol{v}_\perp}{\partial x}\right)\cdot\nabla_\perp\,. \end{aligned} \tag{12}$$

Note that the operator L does not contain the second order derivative with respect to x, but contains the operator $M\partial/\partial x$, where M is proportional to the small parameter $\alpha = \max\left(v/c_0, |\tilde{\rho}/\rho_0|\right) \leq \eta$.

Starting from (9) and using the standard approach described in the previous section, we derive the wide-angle parabolic equation for sound waves in moving media

$$\frac{\partial p}{\partial x} = ikQp\,, \tag{13}$$

where the operator Q is given by (10)-(12). Equation (13) is one of the two main results obtained in the paper.

The explicit form of Q in (13) is given by the Taylor series of the right-hand side of equation (10)

$$Q = 1 + \frac{1}{2}L - \frac{1}{8}L^2 + \frac{1}{16}L^3 + \dots . \tag{14}$$

Since L is determined by (11), the third term on the right-hand side of equation (14) contains the operator $M^2\partial^2/\partial x^2$ which must not be present in a wide-angle parabolic equation. But in accordance with (8), $M^2\partial^2/\partial x^2$ can be expressed as $-M^2(\Delta_\perp + k^2 + \eta)$. Neglecting the term $M^2\eta$ in the latter operator, as we already did when deriving equation (9), yields the operator $-M^2(\Delta_\perp + k^2)$ which does not contain $\partial^2/\partial x^2$. The fourth term on the right-hand side of equation (14) contains $M^3\partial^3/\partial x^3$ which can be represented as $-M^3(\Delta_\perp + k^2)\partial/\partial x$ neglecting the term $M^3\eta\partial/\partial x$, etc. Thus, using (8), the operator Q can be represented in a form which does not contain the operators $\partial^2/\partial x^2, \partial^3/\partial x^3$, etc. Hence, (13) can be considered as a wide-angle parabolic equation.

4. Padé approximation

For certain cases, the pseudo-differential equation (13) is not very convenient for numerical computations of sound fields. To do effectively such computations, the Padé (1,1) approximation of the operator Q is frequently used in the literature

$$Q = (1+L)^{1/2} \cong \frac{p_1 + p_2 L}{q_1 + q_2 L} . \tag{15}$$

Here, p_1, p_2, q_1 and q_2 are numerical coefficients given by Claerbout [1]

$$p_1 = 1, \quad p_2 = 3/4, \quad q_1 = 1, \quad q_2 = 1/4 . \tag{16}$$

Note that in some papers, e.g. [16, 2], the values of p_1, p_2, q_1, q_2 are chosen in a slightly different form in order to achieve a better approximation of the operator Q. The Padé (1,1) approximation allows one to treat wave propagation in a cone with an angle up to 80° .

Substituting (15) into equation (13) yields

$$\frac{\partial p}{\partial x} = ik\frac{p_1 + p_2(F + M\partial/\partial x)}{q_1 + q_2(F + M\partial/\partial x)}p . \tag{17}$$

Multiplying both sides of (17) by the operator $q_1 + q_2(F + M\partial/\partial x)$, we obtain

$$\begin{aligned}(q_1 + q_2F - ip_2kM)\,\frac{\partial p}{\partial x} \\ = ik\left(p_1 + p_2F + \frac{iq_2M}{k}\frac{\partial^2}{\partial x^2}\right)p .\end{aligned} \tag{18}$$

We now use (8) to replace the operator $\partial^2/\partial x^2$ on the right-hand side of (18) by $-(\Delta_\perp + k^2 + \eta)$. Moreover, in the equation obtained we neglect the term proportional to $M\eta$ since such a term has already been neglected when deriving (9). Finally, representing p in equation (18) in the form

$$p(\boldsymbol{R}) = \exp(ikx)\psi(\boldsymbol{R}) , \tag{19}$$

we obtain the following equation for the complex amplitude ψ of a sound field

$$\begin{aligned}&(q_1 + q_2F - ip_2kM)\,\frac{\partial \psi}{\partial x} \\ &= ik\Big[p_1 - q_1 + (p_2 - q_2)F \\ &\qquad + i(p_2 - q_2)kM - \frac{iq_2M}{k}\Delta_\perp\Big]\psi .\end{aligned} \tag{20}$$

Substituting the operators F and M given by (12) into equation (20) yields the desired wide-angle parabolic wave equation

$$A\frac{\partial \psi}{\partial x} = ikB\psi , \tag{21}$$

where

$$\begin{aligned}A = {}& q_1 + q_2\epsilon + \frac{2iq_2}{\omega}\frac{\partial v_x}{\partial x} + \frac{ip_2}{k}\frac{\partial \ln(\rho/\rho_0)}{\partial x} + \frac{2p_2v_x}{c_0} \\ &+ k^{-1}\Big[\frac{2iq_2\boldsymbol{v}_\perp}{c_0} - \frac{2p_2}{\omega}\left(\nabla_\perp v_x + \frac{\partial \boldsymbol{v}_\perp}{\partial x}\right) \\ &\qquad - \frac{q_2}{k}\nabla_\perp \ln(\rho/\rho_0)\Big]\cdot\nabla_\perp \\ &+ q_2k^{-2}\Big[\left(1 + \frac{2i}{\omega}\frac{\partial v_x}{\partial x}\right)\Delta_\perp \\ &\qquad - \frac{2i}{\omega}\sum_{j=2}^{3}(\nabla_\perp v_j)\cdot\nabla_\perp\frac{\partial}{\partial x_j}\Big] ,\end{aligned} \tag{22}$$

and

$$\begin{aligned}B = {}& p_1 - q_1 + (p_2 - q_2)\Big[\epsilon + \frac{2i}{\omega}\frac{\partial v_x}{\partial x} \\ &\qquad - \frac{i}{k}\frac{\partial \ln(\rho/\rho_0)}{\partial x} - \frac{2v_x}{c_0}\Big] \\ &+ (p_2 - q_2)k^{-1}\Big[\frac{2i\boldsymbol{v}_\perp}{c_0} - k^{-1}\nabla_\perp \ln(\rho/\rho_0) \\ &\qquad + \frac{2}{\omega}\left(\nabla_\perp v_x + \frac{\partial \boldsymbol{v}_\perp}{\partial x}\right)\Big]\cdot\nabla_\perp \\ &+ k^{-2}\Big[(p_2 - q_2)\left(1 + \frac{2i}{\omega}\frac{\partial v_x}{\partial x}\right) \\ &\qquad + q_2\left(\frac{i}{k}\frac{\partial \ln(\rho/\rho_0)}{\partial x} + \frac{2v_x}{c_0}\right)\Big]\Delta_\perp \\ &- \frac{2i(p_2 - q_2)}{\omega k^2}\sum_{j=2}^{3}(\nabla_\perp v_j)\cdot\nabla_\perp\frac{\partial}{\partial x_j} \\ &- \frac{2q_2}{\omega k^3}\left(\nabla_\perp v_x + \frac{\partial \boldsymbol{v}_\perp}{\partial x}\right)\cdot\nabla_\perp\Delta_\perp .\end{aligned} \tag{23}$$

Equation (21) is the second main result of the paper. It allows us to describe the propagation of sound waves in a cone with an angle, at least, up to 80°. Therefore, equation (21) can be applied for solving the problems, the geometries of which are shown in Figure 1a, 1b and 1c.

Note that the derived equations (21) and (13) seem to be rather complicated. They can, however, be solved numerically by a marching technique without any principal problems because they are first order differential equations with respect to x. On the other hand, equation (2) does not contain so many terms as do (21) and (13). Nevertheless, a numerical solution of (2) is much more complicated since it is a second order differential equation with respect to x.

5. Comparison with equations known in the literature

In this section, we shall show that for certain particular cases, equation (21) coincides with equations known in the literature.

To get the small-angle parabolic equation for sound waves in moving media, we put

$$\rho = \rho_0, \quad \boldsymbol{v}_\perp = 0, \quad \partial v_x/\partial x = 0, \quad \nabla_\perp v_x = 0\,, \tag{24}$$

and

$$p_1 = 1, \qquad p_2 = 1/2, \qquad q_1 = 1, \qquad q_2 = 0\,. \tag{25}$$

In this case, equation (21) becomes

$$\left(1 + \frac{v_x}{c_0}\right)\frac{\partial\psi}{\partial x} = \frac{ik}{2}\left(\epsilon - \frac{2v_x}{c_0} + k^{-2}\Delta_\perp\right)\psi\,. \tag{26}$$

Dividing both sides of equation (26) by $1 + v_x/c_0$ and neglecting terms of the order of μ^2 yields

$$\frac{\partial\psi}{\partial x} = \frac{ik}{2}\left[\epsilon - \frac{2v_x}{c_0} + k^{-2}\Delta_\perp - \frac{v_x}{c_0}k^{-2}\Delta_\perp\right]\psi\,. \tag{27}$$

Note that the term $k^{-2}\Delta_\perp\psi \sim (kl)^{-2}\psi$, where l is the scale of inhomogeneities in the medium. Since $v/c \ll 1$, and $kl \ll 1$ in the small-angle approximation, the last term in the square brackets in (27) should be neglected in comparison with the second and third ones. This yields the equation for ψ

$$\frac{\partial\psi}{\partial x} = \frac{ik}{2}\left[\epsilon - \frac{2v_x}{c_0} + k^{-2}\Delta_\perp\right]\psi\,, \tag{28}$$

which coincides with the small-angle parabolic equation used in atmospheric and oceanic acoustics , e.g. [17].

To get the wide-angle parabolic equation used in underwater acoustics, we put

$$\boldsymbol{v} = 0, \qquad \rho = \text{const}\,. \tag{29}$$

In this case, equation (21) becomes

$$\begin{aligned}(q_1 + q_2\epsilon + q_2k^{-2}\Delta_\perp)\frac{\partial\psi}{\partial x}\\ = ik[p_1 - q_1 + (p_2 - q_2)\epsilon + (p_2 - q_2)k^{-2}\Delta_\perp]\psi\,.\end{aligned} \tag{30}$$

This equation agrees with equation (10) of the paper by Knightly et al. [2].

6. Conclusions

In the present paper, we have enumerated several problems in atmospheric acoustics, the solutions of which cannot be based on a small-angle parabolic equation but can be obtained by making use of a wide-angle parabolic equation. We have also shown that the wide-angle parabolic equation used in the literature cannot describe the effects of regular and random inhomogeneities in the wind velocity $\boldsymbol{v}$ and density ρ on sound propagation and scattering in the turbulent atmosphere.

The main results of the paper are the derivation of the wide-angle parabolic equation (13) for sound waves in moving media and its Padé (1,1) approximation (21). In limiting cases, the latter equation coincides with equations known in the literature.

In subsequent papers, we will use the derived equations (13) and (21) for a numerical simulation of sound propagation and scattering in the stratified and turbulent atmosphere. We expect to reveal and describe numerically several effects which can not obtained on the basis of a small-angle parabolic equation. Among those are the following:
the effects of density fluctuations and fluctuations in all components of the wind velocity vector on sound scattering into the refractive shadow zone;
the effects of the mean cross wind component v_y on sound propagation near the ground;
the effects of the component v_R of the wind velocity in the direction of sound propagation on the sound field in the problems the geometries of which are shown in Figure 1a and 1b.

Acknowledgement

This material is based upon work supported in part by the U.S. Army Research Office under contract number DAAH04-95-1-0593. It has been started while V.E.Ostashev was a Visiting Professor of Physics at the Ecole Centrale de Lyon with a financial support of the French Ministry of Education. The authors would like to thank Prof. G.Comte-Bellot for her interest in the work and for her constructive comments.

References

[1] J. F. Claerbout: Fundamentals of geophysical data processing. McGraw-Hill, New York, 1976, 206-207.
[2] G. H. Knightly, D. Lee, D. F. S. Mary: A higher-order parabolic wave equation. J. Acoust. Soc. Am. **82** (1987) 580–587.
[3] L. Halpern, L. N. Trefethen: Wide-angle one-way equations. J. Acoust. Soc. Amer. **84** (1988) 1397–1414.
[4] M. D. Collins, E. K. Westwood: A higher-order energy-conserving parabolic equation for range-dependent ocean depth, sound speed and density. J. Acoust. Soc. Amer. **89** (1991) 1068–1075.
[5] V. E. Ostashev, V. I. Tatarskii: A series based on multiple backscattering in problems of wave propagation in inhomogeneous media. Radiophysics and Quantum Electronics **21** (1978) 504–513.
[6] R. Große: Ausbreitung von Wellen in stochastischen Medien. Bibl. Inform. System Universität Oldenburg, Oldenburg, 1992.
[7] V. G. Gavrilenko, S. S. Petrov: Correlation characteristics of waves in lossy media with smooth fluctuations of the refractive

index with oblique incidence on boundary. Waves Random Media **2** (1992) 273–287.

[8] B. Li: Sound propagation in turbulent media. Dissertation. Oldenburg University, Germany, 1992.

[9] V. E. Ostashev, V. I. Tatarskii: Representation of the Helmholtz equation solution in the form of a series based on backscattering multiplicity. Waves Random Media **5** (1995) 125–135.

[10] K. E. Gilbert, M. J. White: Application of the parabolic equation to sound propagation in a refracting atmosphere. J. Acoust. Soc. Amer. **85** (1989) 630–637.

[11] D. Juvé, P. Blanc-Benon, P. Chevret: Numerical simulations of sound propagation through a turbulent atmosphere. Proc. 5th Internat. Symp. on Long Range Sound Propagation, England, 1992. 282–296.

[12] D. Juvé, P. Blanc-Benon, P. Chevret: Sound propagation through a turbulent atmosphere: Influence of the turbulent model. Proc. 6th Internat. Symp. on Long Range Sound Propagation, Ottawa, Canada, 1994. 270–282.

[13] R. A. Sack, M. West: A new solution of the propagation field close to a spherical acoustic source obtained with the parabolic approximation. Proc. 5th Internat. Symp. on Long Range Sound Propagation, England, 1992. 103–114.

[14] V. E. Ostashev: Sound propagation and scattering in media with random inhomogeneities of sound speed, density and medium velocity. Review article. Waves Random Media **4** (1994) 403–428.

[15] V. E. Ostashev: Geometric acoustics in a moving medium. (Review). Izv. Acad. Scienc. USSR. Atmos. Ocean. Phys. **25** (1989) 899–916.

[16] R. R. Greene: The rational approximation to the acoustic wave equation with bottom interaction. J. Acoust. Soc. Amer. **76** (1984).

[17] L. Nghiem-Phu, F. Tappert: Parabolic equation modeling of the effects of ocean currents on sound transmission and reciprocity in the time time domain. J. Acoust. Soc. Amer. **78** (1985) 642–648.