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Perturbed convective wave equation for low-to-medium Mach number subsonic flows

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ABSTRACT

A generalization of the perturbed convective wave equation is proposed. It allows extracting the acoustic field and the vortical dynamics from a compressible high-fidelity flow simulation. The wave equation computes the acoustic velocity potential, with a source term based on a source potential. This source potential describes the pressure dynamics of vortices and is identical to the incompressible pressure for incompressible-modeled flows. The application to an isothermal, subsonic two-dimensional mixing layer sound generation problem validates the wave equation. The results of the acoustic far-field pressure are comparable to the direct numerical reference solution, deviating less than 1.8 dB.

1. Introduction

Computational aeroacoustics employs two primary approaches [1]: direct sound computation, which solves the full compressible Navier–Stokes equations, and hybrid approaches (acoustic analogies), which separate flow and acoustic calculations. The main aim of the hybrid approach is to achieve computational efficiency and improve the interpretability of acoustic predictions through source analysis and energy-exchange terms. Regarding the definition of the energy transformation, Lighthill's formulation [2] was the first attempt to make the energy contribution quantifiable. A closer look at Lighthill's equation (being an exact reformulation of first principles) revealed that the formulation does not lead to a closed form and that any solution of the equation only recovers the fluctuating pressure field provided by the compressible flow field. The obtained fluctuating field only converges to the acoustic field in steady flow regions. As a consequence, the model does not provide additional insight into the acoustic field compared to the direct numerical simulation.

As first recognized by Phillips [3] and Lilley [4], the source terms responsible for mean flow-acoustics interactions should be part of the wave operator. For incompressible-modeled flows, this was achieved about two decades ago (e.g. [5]), leading to the validation of the perturbed convective wave equation model in [6]. Despite recent attempts [7,8], a more general scalar wave formulation definition in the subsonic flow regime is still missing. This article derives a scalar convective wave equation. It investigates the definition of aeroacoustic sound sources for low-to-medium Mach number subsonic flows with spatially slowly varying mean fluid density. A validation is performed by comparison to a 2D direct sound simulation of a mixing layer flow.

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2. Theory

The flow field is systematically (mathematically) decomposed by field properties that are related to acoustics and the 'pure' fluid motion. This approach circumvents that sources depend on the acoustic solution and provides a rigorous definition of acoustics inside flow regions. Firstly, the field variables $(\rho, \boldsymbol{u}, p)$ are Reynolds decomposed in a temporal mean component $\langle \star \rangle$ and a fluctuating component $\star'$.

2.1. Acoustic perturbation equations

One possibility to derive the *"perturbed convective wave equation for subsonic compressible flows"* (cPCWE) is to start from the APE-1 system [5]. The variables are defined by separating the vortical and acoustical perturbations, such that the following field variable definitions are obtained

$$p = \langle p \rangle + p' \tag{1}$$

$$\rho = \langle \rho \rangle + \rho' \tag{2}$$

$$\boldsymbol{u} = \langle \boldsymbol{u} \rangle + \boldsymbol{u}' = \langle \boldsymbol{u} \rangle + \boldsymbol{u}_{\mathrm{v}} + \boldsymbol{u}_{\mathrm{a}}, \tag{3}$$

with the pressure p, the density ρ, and the velocity $\boldsymbol{u}$. Considering a general compressible flow and the second law of thermodynamics in the form of $\frac{\mathrm{d}\rho}{\rho} = \frac{1}{\gamma}\frac{\mathrm{d}p}{p} - \frac{\mathrm{d}s}{c_{\mathrm{p}}}$, we arrive at the following acoustic perturbation equations

$$\frac{\partial p'}{\partial t} + c_0^2 \nabla \cdot \left(\langle \rho \rangle \boldsymbol{u}_{\mathrm{a}} + \langle \boldsymbol{u} \rangle \frac{p'}{c_0^2} \right) = -c_0^2 \nabla \rho \cdot \boldsymbol{u}_{\mathrm{v}} + \frac{c_0^2 \langle \rho \rangle}{c_{\mathrm{p}}} \left(\frac{\partial s'}{\partial t} + \langle \boldsymbol{u} \rangle \cdot \nabla s' \right) \tag{4}$$

$$\frac{\partial \boldsymbol{u}_{\mathrm{a}}}{\partial t} + \nabla \left(\langle \boldsymbol{u} \rangle \cdot \boldsymbol{u}_{\mathrm{a}} \right) + \langle \boldsymbol{\omega} \rangle \times \boldsymbol{u}_{\mathrm{a}} + \nabla \frac{p'}{\langle \rho \rangle} = \nabla \boldsymbol{\Phi}_{\mathrm{p}}, \tag{5}$$

with the speed of sound of an ideal gas $c_0^2 = \gamma \langle p \rangle / \langle \rho \rangle$, specific heat at constant pressure c_{p}, the vorticity $\boldsymbol{\omega}$, and the source potential $\boldsymbol{\Phi}_{\mathrm{p}}$. Furthermore, we assume a spatially slowly varying mean fluid density ($\nabla \langle \rho \rangle = \mathbf{0}$) and $c_0^2 \nabla \cdot \frac{\langle \boldsymbol{u} \rangle p'}{c_0^2} \approx \langle \boldsymbol{u} \rangle \cdot \nabla p'$. Please note that accounting for mean fluid density gradient effects is relevant for acoustic propagation in flow expansions [9].

$$\frac{\partial p'}{\partial t} + \langle \boldsymbol{u} \rangle \cdot \nabla p' + \langle \rho \rangle c_0^2 \nabla \cdot \boldsymbol{u}_{\mathrm{a}} = \frac{c_0^2 \langle \rho \rangle}{c_{\mathrm{p}}} \left(\frac{\partial s'}{\partial t} + \langle \boldsymbol{u} \rangle \cdot \nabla s' \right) \tag{6}$$

$$\frac{\partial \boldsymbol{u}_{\mathrm{a}}}{\partial t} + \nabla \left(\langle \boldsymbol{u} \rangle \cdot \boldsymbol{u}_{\mathrm{a}} \right) + \langle \boldsymbol{\omega} \rangle \times \boldsymbol{u}_{\mathrm{a}} + \nabla \frac{p'}{\langle \rho \rangle} = \nabla \boldsymbol{\Phi}_{\mathrm{p}}\,. \tag{7}$$

By definition of the acoustic perturbation equations, we neglect viscous effects and discard the vorticity mode [5]. The source potential can be computed by solving the Poisson equation (see eq. 38 and eq. 50 in [5])

$$\Delta \boldsymbol{\Phi}_{\mathrm{p}} = -\nabla \cdot \left[((\boldsymbol{u}_{\mathrm{v}} \cdot \nabla) \boldsymbol{u}_{\mathrm{v}}) + (\langle \boldsymbol{u} \rangle \cdot \nabla) \boldsymbol{u}_{\mathrm{v}} + (\boldsymbol{u}_{\mathrm{v}} \cdot \nabla) \langle \boldsymbol{u} \rangle + T' \nabla \langle s \rangle - s' \nabla \langle T \rangle \right], \tag{8}$$

with the specific entropy s and the temperature T.

2.2. Perturbed convective wave equation for low-to-medium Mach number subsonic flows

The Helmholtz decomposition of the fluctuating flow velocity [10] yields

$$\boldsymbol{u} = \langle \boldsymbol{u} \rangle + \boldsymbol{u}' = \langle \boldsymbol{u} \rangle + \boldsymbol{u}_{\mathrm{v}} + \boldsymbol{u}_{\mathrm{a}} = \langle \boldsymbol{u} \rangle + \nabla \times \boldsymbol{A} - \nabla \psi_{\mathrm{a}}. \tag{9}$$

The fluid dynamic perturbation velocity $\boldsymbol{u}_{\mathrm{v}} = \nabla \times \boldsymbol{A}$ and the acoustic perturbation velocity $\boldsymbol{u}_{\mathrm{a}} = -\nabla \psi_{\mathrm{a}}$ can be modeled by the vector potential $\boldsymbol{A}$ and the acoustic scalar velocity potential ψ_{a}, respectively. The vector potential describes the curly motion of vortices and turbulent structures, whereas the scalar potential describes the irrotational (compressible and respectively longitudinal acoustic) field.

Two options are possible to eliminate the compressible part of the flow velocity:

- *Curl extraction* is a direct way of computing $\boldsymbol{u}_{\mathrm{v}}$ out of the fluctuating velocity field by the curl–curl equation of the vector potential [10].
- *Divergence cleaning* is an indirect way to compute the vortical fluctuating velocity. The following Poisson equation $\Delta \tilde{\psi}_{\mathrm{a}} = \nabla \cdot \boldsymbol{u}'$ has to be solved, and the vortical fluctuating velocity can be obtained by $\boldsymbol{u}_{\mathrm{v}} = \boldsymbol{u}' + \nabla \tilde{\psi}_{\mathrm{a}}$ [10].

We can rewrite Eq. (7) by $\boldsymbol{u}_{\mathrm{a}} = -\nabla \psi_{\mathrm{a}}$ and with the definition of the Helmholtz decomposition of the interaction between the mean vorticity $\langle \boldsymbol{\omega} \rangle$ and the acoustic particle velocity $\boldsymbol{u}_{\mathrm{a}}$

$$-\langle \boldsymbol{\omega} \rangle \times \boldsymbol{u}_{\mathrm{a}} = \langle \boldsymbol{\omega} \rangle \times \nabla \psi_{\mathrm{a}} = \nabla q_{\omega \times \boldsymbol{u}_{\mathrm{a}}} + \nabla \times \boldsymbol{B}_{\omega \times \boldsymbol{u}_{\mathrm{a}}}\,.$$

This reformulation yields the definition of the fluctuating pressure

$$p' = \langle \rho \rangle \frac{\partial \psi_{\mathrm{a}}}{\partial t} + \langle \rho \rangle \langle \boldsymbol{u} \rangle \cdot \nabla \psi_{\mathrm{a}} + \langle \rho \rangle q_{\omega \times \boldsymbol{u}_{\mathrm{a}}} + \langle \rho \rangle \boldsymbol{\Phi}_{\mathrm{p}} = \langle \rho \rangle \frac{D \psi_{\mathrm{a}}}{Dt} + \langle \rho \rangle q_{\omega \times \boldsymbol{u}_{\mathrm{a}}} + \langle \rho \rangle \boldsymbol{\Phi}_{\mathrm{p}}\,. \tag{10}$$

From this, we can identify that the fluctuating pressure field decomposes into three parts. Firstly, the compressible effects, captured by the mean material derivative of the acoustic velocity potential, secondly, the interaction of the mean vorticity with the acoustic particle velocity, and thirdly, the vortical effects are contained within the source potential. In the incompressible limit, the source potential converges to the kinematic pressure fluctuations via the pressure Poisson equation. Concerning the reformulation, the field quantities read as

$$p = \langle p\rangle + p' = \langle p\rangle + p_\mathrm{v} + p_\mathrm{a} = \langle p\rangle + \langle\rho\rangle\Phi_\mathrm{p} + \langle\rho\rangle\frac{D\psi_\mathrm{a}}{Dt} + \langle\rho\rangle q_{\omega\times\boldsymbol{u}_\mathrm{a}} \tag{11}$$

$$\rho = \langle\rho\rangle + \rho' \tag{12}$$

$$\boldsymbol{u} = \langle\boldsymbol{u}\rangle + \boldsymbol{u}' = \langle\boldsymbol{u}\rangle + \boldsymbol{u}_\mathrm{v} + \boldsymbol{u}_\mathrm{a} = \langle\boldsymbol{u}\rangle + \nabla\times\boldsymbol{A} - \nabla\psi_\mathrm{a}. \tag{13}$$

Substituting (10) into (6) yields the *Perturbed Convective Wave Equation for low-to-medium Mach number Subsonic Flows*

$$\frac{D^2\psi_\mathrm{a}}{Dt^2} + \frac{Dq_{\omega\times\boldsymbol{u}_\mathrm{a}}}{Dt} - c_0^2\Delta\psi_\mathrm{a} = -\frac{D\Phi_\mathrm{p}}{Dt} + \frac{c_0^2}{c_\mathrm{p}}\frac{Ds'}{Dt} = S_{\mathrm{CPCWE-gen}}. \tag{14}$$

Assuming no presence of entropy and temperature fluctuations and neglecting the term $Dq_{\omega\times\boldsymbol{u}_\mathrm{a}}/Dt$, the wave equation can be represented by

$$\frac{D^2\psi_\mathrm{a}}{Dt^2} - c_0^2\Delta\psi_\mathrm{a} = -\frac{D\Phi_\mathrm{p}}{Dt} = S_{\mathrm{CPCWE}}, \tag{15}$$

where (8) simplifies to $\Delta\Phi_\mathrm{p} = -\nabla\cdot\left[((\boldsymbol{u}_\mathrm{v}\cdot\nabla)\boldsymbol{u}_\mathrm{v}) + (\langle\boldsymbol{u}\rangle\cdot\nabla)\boldsymbol{u}_\mathrm{v} + (\boldsymbol{u}_\mathrm{v}\cdot\nabla)\langle\boldsymbol{u}\rangle\right]$. This convective wave equation describes acoustic sources generated by compressible flow structures and their wave propagation through flowing media. In the incompressible and isothermal limits, S_{CPCWE} yields the known incompressible PCWE source term. In addition, instead of the original unknowns of the acoustic pressure p_a and the acoustic particle velocity $\boldsymbol{v}_\mathrm{a}$ (see [5]), one scalar unknown ψ_a must be computed. Consistent with the pressure correction equation in computational fluid dynamics, the fluctuating vortical pressure in the overall domain can be recovered by $p_\mathrm{v} = \langle\rho\rangle\Phi_\mathrm{p}$. Finally, we have derived a scalar wave equation that separates the source generation processes of compressible flows and the linear acoustic propagation.

3. Results

The validity of the cPCWE theory is examined using the DNS results of a two-dimensional isothermal mixing layer, building on prior investigations [11]. The mixing layer is centered at $y = 0$ and vortices are created at a frequency f. The vortex pairings occur periodically at intervals of $T_\mathrm{p} = 2\pi/\omega_\mathrm{p} = 2/f$, generating acoustic waves at a frequency of $f/2$. These pairings are the dominant sound sources in the flow. The inflow velocity profile $\boldsymbol{u} = (u_x, 0)^\mathrm{T}$ is defined by

$$u_x(y) = \frac{U_1 + U_2}{2} - \frac{U_2 - U_1}{2}\tanh\left(\frac{2y}{\delta_\omega}\right), \tag{16}$$

where $U_1 = 0.3c_0$ and $U_2 = 0.6c_0$ denote the velocities of the two free-streams. The vorticity thickness is given by $\delta_\omega = (U_2 - U_1)/\max(|\mathrm{d}u_x/\mathrm{d}y|)$, resulting in a Reynolds number of $Re_\omega = \delta_\omega(U_2 - U_1)/\nu = 2000$. In previous work [8,11], an aeroacoustic modeling technique (AWE-PO) was validated by this two-dimensional isothermal mixing layer configuration. The results obtained from the AWE-PO are in close agreement with the DNS results, except in the slow flow region at an angle of $\theta = 15°$, where a discrepancy is observed. This strong discrepancy led to a follow-up study verifying the algorithmic accuracy of the numerical implementations such that the algorithms are not the source of the error [8]. According to previous work [11], the results from Lighthill's equation are validated by the DNS results. From a numerical perspective, the simulations of the AWE-PO and the cPCWE are computed on a mesh twice as coarse as the one used to obtain Lighthill's theory results. The coarser mesh reduces the overall computational load (with all the extra computational steps of the cPCWE) in a comparable time range.

Fig. 1 shows snapshots of the vorticity, three aeroacoustic source terms of the acoustic equations, and the source potential of the cPCWE. In Fig. 1a, the vorticity $|\boldsymbol{\omega}|/(\Delta U/\delta_\omega)$ is depicted. Fig. 1b shows Lighthill's right-hand side (RHS) term. The RHS of AWE-PO (see Fig. 5b in [8]) is shown in Fig. 1c. Fig. 1d shows the RHS of the perturbed convective wave equation $S_{\mathrm{CPCWE}}/(\langle\rho\rangle\omega_\mathrm{p}^2\Delta U^2/c_0^2)$ and Fig. 1e the source potential $\Phi_\mathrm{p}/(\Delta U^2/\delta_\omega^2)$. The respective Figs. 1a-c are discussed in Ref. [8,11] and are presented for comparison with Figs. 1d and e. The source term of Fig. 1d is similar to that of Fig. 1c with a much more pronounced source strength. The pairing of the vortices cannot be inferred easily by Fig. 1d. However, in Fig. 1e the pairing clearly appears to occur at an x/δ_ω location close to 110, which is indicated by the two circular shapes merging to an elliptic formation with its primary axis in the y-direction.

In Fig. 2, the results of the acoustic intensity of the reference DNS [11], Lighthill's theory [11] , and the *Perturbed Convective Wave Equation for low-to-medium Mach number Subsonic Flows* are provided. The acoustic intensity $L_\mathrm{I} = 10\log I/I_0$ is used, with $I = \langle p'^2\rangle/(\rho_0 c_0)$ and $I_0 = 10^{-12}$ W/m^2.

In Fig. 2a (rapid flow region), for angles $-90° < \theta < -5°$, the intensity curve of the cPCWE agrees well with the DNS results with deviations below 1.8 dB. The deviations of the cPCWE and the DNS are comparable to those obtained by using Lighthill's theory. In the slow flow region (Fig. 2b), for angles $\theta < 50°$, the intensity curve is slightly underpredicted (about 2 dB), compared to the intensity from the DNS. This may be due to remaining effects of the near-field pressure fluctuations resolved by the DNS, as these

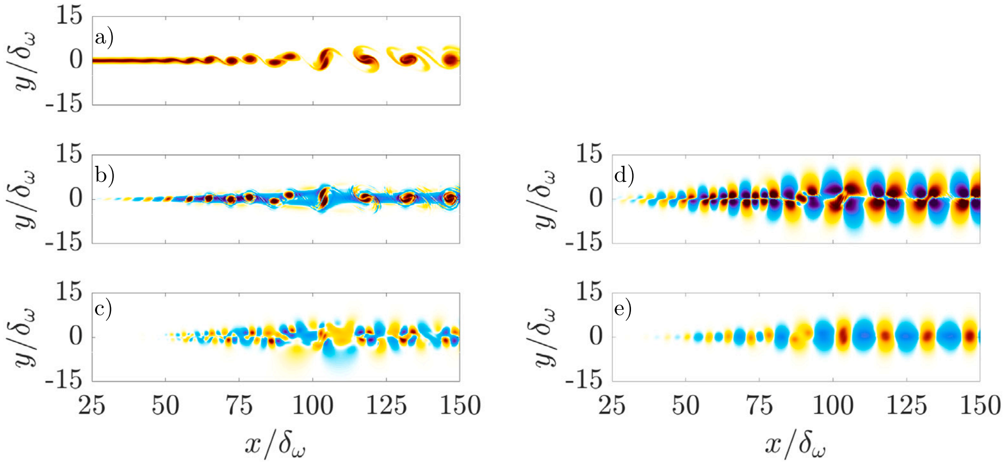

Fig. 1. Snapshots of (a) the vorticity $|\boldsymbol{\omega}|/(\Delta U/\delta_{\omega})$, (b) Lighthill's RHS term $\nabla\cdot\nabla\cdot(\rho\,\boldsymbol{u}\otimes\boldsymbol{u})/(\langle\rho\rangle\Delta U^2/\delta_{\omega}^2)$, where $\langle\rho\rangle = 1.19\,\mathrm{kg/m^3}$ is the density of the surrounding fluid and $\Delta U = U_2 - U_1$ (c) the normalized RHS of AWE-PO (see Fig. 5b in [8]) (d) the RHS of the perturbed convective wave equation $S_{\mathrm{CPCWE}}/(\langle\rho\rangle\omega_{\mathrm{p}}^2\Delta U^2)$ and (e) the source potential $\Phi_{\mathrm{p}}/\Delta U^2$. The color scales range between ± 0.2 from blue to red. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

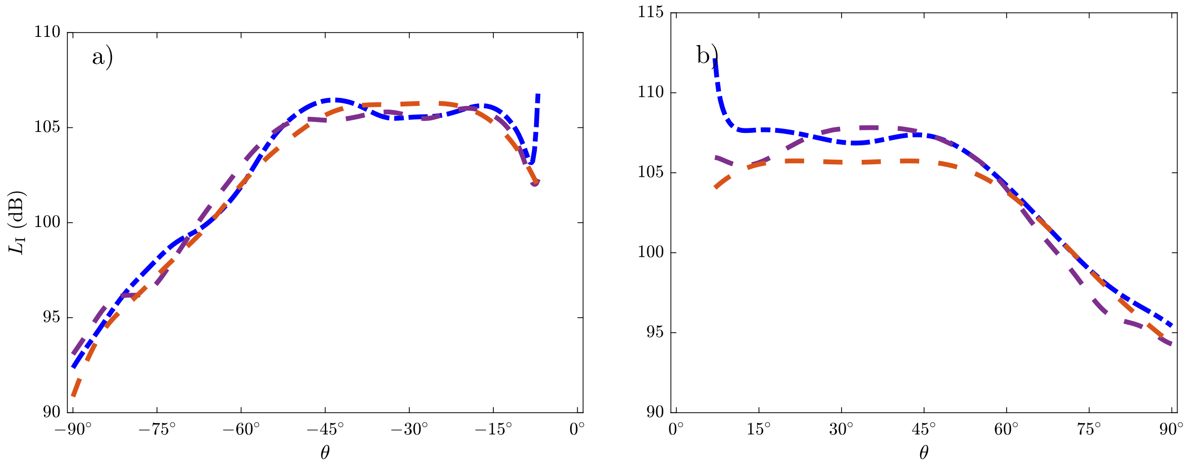

Fig. 2. Acoustic intensity L_{I} depending on the angle θ (a) in the rapid flow below the mixing layer and (b) in the slow flow region above the mixing layer. – – DNS, - - - Lighthill and - - - cPCWE.

flow fluctuations near the mixing layer are still present at the shown locations. This disparity is expected to vanish in the acoustic far field. Compared to the results obtained in [8,11], the results are a substantial improvement on the subject of aeroacoustic modeling.

Fig. 3 shows the pressure fluctuations p' obtained by the DNS and using Lighthill's wave equation, the AWE-PO [11], and the cPCWE. The fluctuating pressure results of the DNS serve as a reference in the following discussion (see Fig. 3a). The fluctuating pressure results of Lighthill's theory (see Fig. 3b) are in very good agreement with the DNS results. The fluctuating pressure results using the AWE-PO can be seen in Fig. 3c, where a substantial discrepancy of the results in the slow flow region at an angle of $\theta = 15°$ is visible. The results computed by the cPCWE (only plotted p_{a}) are in excellent agreement with the DNS results. Compared to the AWE-PO [11], there is no substantial discrepancy at an angle of $\theta = 15°$ visible.

4. Discussion

Regarding the state-of-the-art of aeroacoustic equations [5], we showed the connection of Helmholtz decomposition and the respective aeroacoustic source terms. The derived equation is consistent with previously derived equations for incompressible flows. Conceptually, it is a physical generalization of the perturbed convective wave equation to subsonic compressible flows. The wave operator only excites longitudinal wave modes (acoustic modes) and includes mean convection effects. Furthermore, the simplified wave operator (15) is self-adjoint as presented in [7].

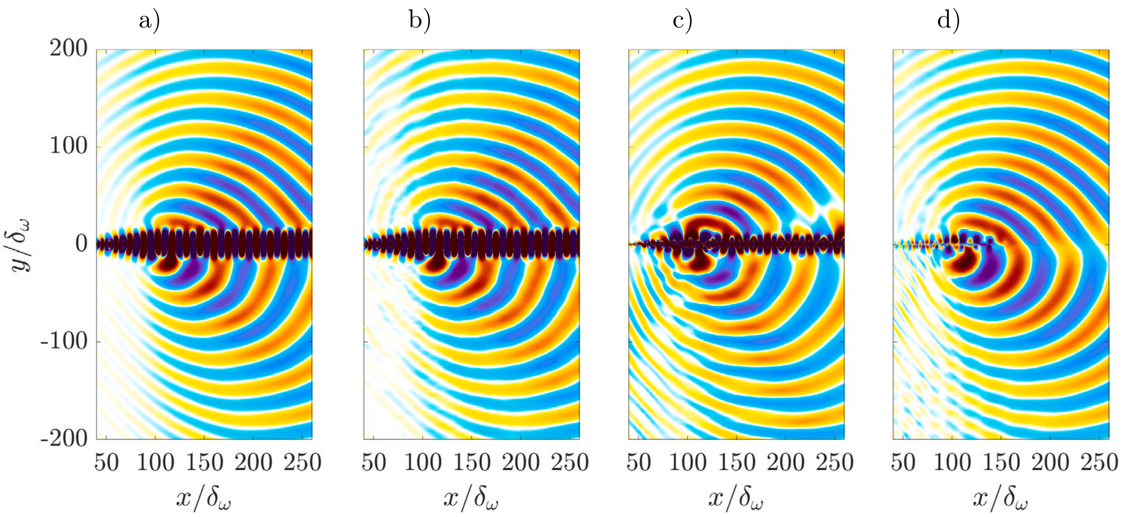

Fig. 3. Fluctuating pressure $p'/(\langle\rho\rangle c_0^2)$ from (a) DNS, (b) Lighthill's equation (c) AWE-PO [11] and (d) the cPCWE (15). The color scales range between $\pm 1.5 \cdot 10^{-4}$, from blue to red. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

In contrast to the AWE-PO [8], the source term of the cPCWE includes the mean flow convection terms of the fluctuating vortical velocity modes. Conceptually, this might be a missing part of the source term of the AWE-PO, which is responsible for the discrepancy in the slow flow region at an angle of $\theta = 15°$ (see Fig. 3c). Furthermore, the convective wave operator of the AWE-PO and the cPCWE are identical, and theoretical considerations regarding the convective wave operator apply to both.

5. Conclusions

In conclusion, the article proposes a consistent approach by using Helmholtz decomposition (both for the velocity field and the source term) and a convective wave equation to model field quantities in the context of aeroacoustics. This results in an acoustic formalism called *Perturbed Convective Wave Equation for low-to-medium Mach number Subsonic Flows*. The advantage of this approach can be summarized as follows:

- It provides equations that generalize known theories into the subsonic regime and explain the underlying mechanisms of aeroacoustic source generation in low-to-medium Mach number subsonic compressible flows, as well as their relation to the vortical field.
- In relation to the well-known Lighthill's theory, a direct relation from the source term Eq. (8) can be drawn to the structure of the source term of Lighthill's theory.
- In Eq. (14), thermal effects are included, which will be investigated in the future.
- From a numerical perspective, this equation presents a promising approach for modeling aeroacoustics. Typically, numerical schemes used to model fluid flows dissipate the acoustic pressure. Therefore, for a direct noise method, the dissipation of acoustic energy due to numerical schemes can be significant as we move away from the near field. However, the fluid dynamic field is much less sensitive to numerical diffusion and dissipation. From a numerical point of view, we can perform a compressible flow simulation, taking care to accurately model the source potential without much care about the dissipation of the acoustic wave due to the numerical scheme.

The theory is validated by comparison with the acoustics field computed by a direct numerical simulation. The results of the acoustic far-field pressure are compared, showing an accuracy of the results in the range of Lighthill's theory. The spatial resolution of the cPCWE was twice as coarse as that used for obtaining the results with the Lighthill theory. In particular, the value of this wave equation formalism is twofold:

- Using the cPCWE, the DNS simulation can be post-processed into the vortical and acoustic field quantities. The cPCWE algorithm extends the splitting of the Helmholtz decomposition for the fluid velocity to the fluid pressure.
- Using the cPCWE, the relevant source terms for acoustic wave propagation can be analyzed, and a consistent wave propagation formulation can be derived — without suffering the ambiguity of Lighthill's theory.

CRediT authorship contribution statement

S. Schoder: Writing – review & editing, Writing – original draft, Visualization, Validation, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **E. Bagheri:** Writing – review & editing, Investigation. **C. Bogey:** Writing – review & editing, Validation, Data curation. **C. Bailly:** Writing – review & editing, Methodology, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Data will be made available on request.

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