Frequency Averaged Injected Power under Boundary Layer Excitation: An Experimental Validation

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Summary

On one hand, energy methods usually deal with frequency averaged quantities. Statistical Energy Analysis, for example, uses energy per subsystems and power injected averaged over frequency bands. On the other hand, when a structure is excited by a turbulent boundary layer, its response is calculated using a model of boundary pressure cross spectrum (Corcos, Efimtsov...). Such calculations take into account boundary conditions and exact geometry of the excited structure which is often non necessary for energy methods. A model of Frequency Averaged Injected Power under boundary layer excitation (FAIP model [1]) has been previously proposed as a tool for vibroacoustic pre-design process. The present paper deals with an experimental validation of the FAIP model. To validate this approach, a characterization of the turbulent flow (wall pressure spectrum, velocity profiles, correlation lengths, Corcos' coefficients) have been carried out in the wind tunnel of the Ecole Central of Lyon. In a second step, the power injected into a plate placed onto a dedicated setup has been evaluated measuring the velocity field on the plate with a laser vibrometer. Thus, experimental power injected into the plate has been compared with the one predicted by FAIP model. FAIP model are in good agreement with Experiments showing that FAIP model is able to give accurate estimation of power injected into a plate excited by a turbulent boundary layer (TBL) on the whole frequency range (below, above and at the aerodynamic coincidence frequency). Four plates made in different materials (steel, copper, PVC) and of different geometries have been tested for three free stream velocities (20, 35 and 50 m/s).

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1. Introduction

Energy methods like Statistical Energy Analysis (SEA) [2, 3, 4] are useful to predict energy flows between subsystems. For those methods, excitations acting on subsystems are taken into account by their injected power averaged on frequency bands. Thus, injected powers estimation is critical in SEA and error on injected powers leads to a proportionally wrong description of energy shared by the subsystems. Consequently, even at a design stage, it is relevant to establish SEA models with realistic excitations.

A model of Frequency Averaged Injected Power (FAIP) into a rectangular plate excited by a turbulent boundary layer was previously established [1]. It is based on a model of boundary pressure cross spectrum expressed in a geometrical space (Corcos [5] or Efimstov [6]). Three similar expressions of the FAIP model were derived depending on the model of boundary pressure cross spectrum. The

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simplest one is based on Corcos' model and Davies' approximation [7], it depends only on plate characteristics (Young's Modulus, thickness, density, surface) and on the convection velocity of the flow, but not on the shape of the plate or on its boundary conditions. This model can thus be extended to non-rectangular plate with any boundary conditions.

The FAIP model was compared to the one derived by Blake [8] and simplified by Lyon and Dejong [2], demonstrating that Blake's formulas overestimate power injected into the plate for frequency below the aerodynamic coincidence frequency ω_c . At ω_c , Blake's model presents a discontinuity up to 10 dB. Above ω_c , both models are equivalent.

The present paper deals with an experimental validation of FAIP model's predictions. First, expressions of FAIP model are remembered and briefly explained. Then, the turbulent flow produced by the wind tunnel is characterized in terms of boundary layer thicknesses, mean velocity profiles and wall pressure spectral density. Then a Corcoslike model is derived from measurements and compared to the ones classically found in the literature [9, 10, 8]. The next section presents velocity vibratory fields of each of the four plates under study, measured with a laser vibrometer, each plate being successively excited by a turbulent boundary layer produced at three different free stream velocities ($U_{\infty} = 20$, 35 and 50 ms⁻¹). Then, the power injected into the plate is deduced and averaged on frequency bands. Finally, in the last section, measured and predicted frequency averaged injected power are compared.

2. The model of Frequency Averaged Injected Power (FAIP model)

The FAIP model established in [1] is given by equation 1. It was derived from a rectangular simply supported plate of length a, width b, mass per unit area M, bending stiffness D and surface A excited by a turbulent boundary layer flowing along the longitudinal direction as presented in Figure 1.

2.1. Using Corcos' model and Davies' approximation: FAIP_{CD}

Using Corcos' model and Davies' approximation, the model of Frequency Averaged Injected Power FAIP_{CD} is given by

$$FAIP_{CD}(\omega) = \frac{\langle P_{inj}(\omega) \rangle}{\langle S_{pp}(\omega) \rangle}$$
(1)
$$= \frac{U_c^2}{\alpha_x \alpha_z \pi \sqrt{MD} \omega^2} \frac{A}{2} \Psi_{CD} \left(\frac{\omega}{\omega_c}\right),$$

where $S_{pp}(\omega)$ is the wall pressure spectral density at the angular frequency ω , α_x and α_z are the coefficients of Corcos' model [5], U_c is the convection velocity. The convection velocity is often considered proportional to the free stream velocity of the fluid ($U_c = KU_{\infty}$).

 Ψ_{CD} , given by equation (2), is a characteristic function which depends on the model used to represent the wall pressure cross spectrum. Using Corcos' model and Davies' approximation, the characteristic function Ψ_{CD} is only dependent on Corcos' coefficients α_x and α_z .

$$\Psi_{CD}\left(\frac{\omega}{\omega_{c}}\right) = \int_{0}^{\sqrt{\frac{\omega_{c}}{\omega}}} \frac{1}{\sqrt{\frac{\omega_{c}}{\omega} - X^{2}}} \frac{1}{1 + (X/\alpha_{z})^{2}}$$
$$\cdot \left[\frac{1}{1 + (1/\alpha_{x}^{2})\left(1 - \sqrt{\frac{\omega_{c}}{\omega} - X^{2}}\right)^{2}} + \frac{1}{1 + (1/\alpha_{x}^{2})\left(1 + \sqrt{\frac{\omega_{c}}{\omega} - X^{2}}\right)^{2}}\right] dX,$$

where ω_c is the aerodynamic coincidence angular frequency (equation 3). At this angular frequency, the convection velocity U_c is equal to the bending wave velocity of the plate [11].

$$\omega_c = U_c^2 \sqrt{\frac{M}{D}}.$$
(3)



Figure 1. Sketch of the system under study.

Equation 2 is valid for angular frequency verifying

$$\omega \gg \frac{U_c \pi}{\min(a, b)} = \omega_{lim}.$$
 (4)

This function has to be calculated only once whatever the plate or the free stream velocity. Moreover, as this function is slowly variable with frequency, it can be interpolated using a limited number of frequency points. The calculation of frequency averaged injected power on a wide frequency band is then almost instantaneous. To evaluate this integral, Gauss-Legendre or adaptive Simpson quadrature can be used, both lead to very close results.

Frequency averaged injected power is proportional to the surface of the plate and does not depend on its exact geometry. In addition, $\langle P_{inj} \rangle_{\text{FAIP}_{CD}}$, because of frequency averaging, is independent of boundary conditions and damping loss factor of the plate.

2.2. Using Corcos' model: FAIP_C

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Using Corcos' model without Davies' approximation, the FAIP model becomes a little more complicated and depends on the length and width of the plate. However, as it was demonstrated in [1], for $\omega \gg \omega_{lim}$ differences between FAIP_C model and FAIP_{CD} model are often not significant.

Even if the characteristic function Ψ_C has to be calculated for each plate, it needs only a few calculations points to describe it in a wide frequency band thanks to its smoothness.

$$\mathbf{P}_{C}\left(\frac{\omega}{\omega_{c}}, \frac{\omega a}{U_{c}}, \frac{\omega b}{U_{c}}\right) = \frac{\omega a}{U_{c}} \frac{\omega b}{U_{c}} \alpha_{x} \alpha_{z} \\
\cdot \int_{\frac{U_{c}\pi}{\omega b}}^{\sqrt{\frac{\omega_{c}}{\omega} - \left(\frac{U_{c}\pi}{\omega a}\right)^{2}}} \frac{1}{\sqrt{\frac{\omega_{c}}{\omega} - X^{2}}} F_{1}\left(\frac{\omega b}{U_{c}}(-\alpha_{z} + iX)\right) \\
\cdot \left[F_{2}\left(\frac{\omega a}{U_{c}}(-\alpha_{x} + i(\sqrt{\frac{\omega_{c}}{\omega} - X^{2}} - 1))\right) + F_{2}\left(\frac{\omega a}{U_{c}}(-\alpha_{x} + i(\sqrt{\frac{\omega_{c}}{\omega} - X^{2}} + 1))\right)\right] dX,$$
(5)



Figure 2. Wind tunnel in the anechoic chamber of the Acoustic Center of Ecole Centrale de Lyon.

where $F_1(z)$ and $F_2(z)$ are two functions defined by

$$F_{1}(z) = -\frac{\Re(z)}{|z|^{2}} + \frac{\Re(z^{*2}(e^{z} - 1))}{|z|^{4}} + \frac{\Im(z^{*}(e^{z} - 1))}{\frac{\omega b}{U_{c}}X|z|^{2}}$$
(6)

$$F_{2}(z) = -\frac{\Re(z)}{|z|^{2}} + \frac{\Re(z^{*2}(e^{z} - 1))}{|z|^{4}} + \frac{\Im(z^{*}(e^{z} - 1))}{\frac{\omega a}{U_{c}}\sqrt{\frac{\omega_{c}}{\omega} - X^{2}}|z|^{2}}.$$
(7)

3. Characterization of the flow

To verify that the turbulent flow in the wind tunnel was fully developed, the mean velocity profiles, the boundary layer thicknesses and the mean wall pressure spectral densities were analyzed.

3.1. The wind tunnel

Measurements were made in the wind tunnel of the Acoustic Center of Ecole Centrale de Lyon, it is located in a large anechoic chamber, is 6 meters long and has a square section $(50cm \times 50cm)$. The air is firstly propelled at low speed by a centrifugal blower located in a room disconnected from the anechoic chamber to ensure vibration insulation. In order to minimize acoustic contamination coming from machinery, acoustic mufflers are located upstream and downstream from the ventilator. The air is then accelerated into a convergent equipped with one honeycomb section and two sections of grids. Finally, the air passes trough the wind tunnel in the anechoic chamber. The chamber is open toward outside to evacuate the flow. As it can be seen in Figure 2, one side of the tunnel is made of plexiglass and others are made of wood panels.

3.2. Mean flow profiles

Measurements of the mean flow profiles have been done with a DANTEC 55P11 hot-wire probe and a TSI-IFA 100



Figure 3. Picture of the hot-wire probe, the pitot tube and the Preston tube.



Figure 4. Mean velocity profiles for each free stream velocity. +: $U_{\infty} = 20 \text{ m/s}; \circ: U_{\infty} = 35 \text{ m/s}; \triangle: U_{\infty} = 50 \text{ m/s}; \longrightarrow: \text{power law}$ proposed by Blake [8].

anemometer at 2.5m downstream from the beginning of the wind tunnel. The hot-wire probe has been calibrated with a Pitot tube as shown in Figure 3. Figure 4 presents the mean velocity profiles for each free stream velocity in a dimensionless form. The three mean flow profiles follow a power law form given by equation 8 with n = 7 as proposed in [8].

$$U/U_{\infty} = (y/\delta)^{1/n},\tag{8}$$

where y is the distance from the wall and δ is the boundary layer thickness.

Mean velocity profiles presented in Figure 4 follow classical profiles for turbulent boundary layer. Far form the wall, the mean flow velocity tends to the free stream velocity and strongly decreases to zero near the wall.

3.2.1. Boundary layer thicknesses

The mean velocity profiles allow to evaluate the boundary layer thicknesses δ but it is not precise. It is much more

Table I. Boundary layer thickness δ , displacement thickness δ^* (mm), momentum thickness θ (mm), friction velocity u_{τ} (m/s) and shear stress at the wall τ_w (Pa) for each free stream velocity U_{∞} (m/s).

U_{∞}	δ	δ^{*}	θ	$H=\delta^*/\theta$	u_{τ}	$ au_w$
50	85	8.8	6.7	1.31	1.96	4.58
35	55	6.4	4.9	1.3	1.4	2.34
20	50	7.7	5.8	1.33	0.87	0.87

preferable to evaluate the displacement thickness δ^* and the momentum thickness θ . These thicknesses are given by equations (9) and (10).

$$\delta^* = \int_0^\infty \left(1 - \frac{U}{U_\infty} \right) \,\mathrm{d}z \tag{9}$$

$$\theta = \int_0^\infty \frac{U}{U_\infty} \left(1 - \frac{U}{U_\infty} \right) \, \mathrm{d}z. \tag{10}$$

The calculated values are listed in table I. The ratio $H = \delta^*/\theta$ is approximately equal to 1.3 which is close to the value given by Schlichting *et al.* [12] for a fully developed turbulent boundary layer without pressure gradient. For values close to 2.6 the flow is laminar, for values close to 1.4 it is turbulent. Between these values there is a transient flow. The friction velocity u_{τ} and the shear stress at the wall τ_w were also measured using a Preston's tube.

3.3. Wall pressure spectral density

The turbulent boundary layer was fully developed for each free stream velocity ($U_{\infty} = 20$, 35 and 50 m/s). The wall pressure spectral density was measured with a fixed microphone (1/8 inch, protection grid removed, see Figure 5). This setup is usually used to measure wall pressure cross spectrum for different longitudinal and transversal separation between two microphones (the fixed and the moving one in Figure 5). It is here supposed that the pressure field is homogeneous on the whole plate and can be represented by the pressure on this small scanned surface. To evaluate transversal and longitudinal dependency of wall pressure cross spectrum, measurements points have been located on two transversal and one longitudinal lines as presented in Figure 6.

The acoustic component has been canceled from the wall pressure spectral densities using the signal processing method described by Durant [13]. This is based on the principle that the acoustic component is more coherent than the turbulence in the spanwise direction. Figure 7 presents the decontaminated wall pressure spectral density $\bar{S}_{pp}(f)$ in a dimensionless form for each free stream velocity.

In addition, small microphone (1/8 inch) have been chosen to minimize the spatial filtering effect due size of the measuring surface [5].

As it can be seen in Figure 7(a), the acoustic component of the wall pressure spectral density is dominating below



Figure 5. Picture of panel with two moving circular parts and three flush-mounted microphones: a fixed microphone gives the reference pressure; a moving microphone for correlation measurements and a microphone for acoustic decontamination.



Figure 6. Measuring points for the wall pressure cross spectrum. +: position of the fixed microphone, o: 65 positions of the moving microphone.

100 Hz and is negligible above 100 Hz. As the acoustic decontamination can't be done easily on plate velocity measurements, the comparison model/experiments presented in the following has been done on the frequency band 100-2000 Hz, where the acoustic component is negligible.

3.4. A Corcos-like model

The Corcos' model of wall pressure cross spectrum can be expressed as equation 11.

$$S_{pp}(r_x, r_z, \omega) = S_{pp}(\omega)C(r_x, r_z, \omega)e^{i\theta(r_x, \omega)}, \quad (11)$$

where $C(r_x, r_z, \omega)$ and $\theta(r_x, \omega)$ are respectively a coherence and a phase function and where r_x (resp. r_z) is the longitudinal (resp. transversal) separation. Corcos' model is based on the hypothesis of space variable separation which implies that correlation function $C(r_x, r_z, \omega)$ can be split into two independent functions $A(r_x, \omega)$ and $B(r_z, \omega)$. Classically, functions $A(r_x, \omega)$ and $B(r_z, \omega)$ are expressed as in equations 12 and 13 as a function of longitudinal and transversal correlation lengths $L_x(\omega)$ and



Figure 7. (a): influence of the acoustic component on the wall pressure spectral density. Black: contaminated wall pressure; gray: decontaminated wall pressure. (b): Decontaminated wall pressure spectral density $\bar{S}_{pp}(f)$ for each free stream velocity. —: $U_{\infty} = 20 \text{ m/s}; -: U_{\infty} = 35 \text{ m/s}; ...: U_{\infty} = 50 \text{ m/s}$

 $L_z(\omega).$

$$A(r_x,\omega) = e^{-\frac{r_x}{L_x(\omega)}},$$
(12)

$$B(r_{z},\omega) = e^{-\frac{1}{L_{z}(\omega)}}.$$
 (13)

The phase function $\theta(r_x, \omega)$ depends on the longitudinal separation, frequency and convection velocity U_c (equation 14).

$$\theta(r_x,\omega) = -\frac{\omega r_x}{U_c}.$$
(14)

Finally, Corcos' coefficients α_x and α_z are related to correlation lengths $L_x(\omega)$ and $L_z(\omega)$ and convection velocity U_c by equations 15 and 16.

$$L_x(\omega) = \frac{U_c}{\alpha_x \omega},\tag{15}$$

$$L_z(\omega) = \frac{U_c}{\alpha_z \omega}.$$
 (16)

In the following, the convection velocity (using equation 14), the correlation lengths (using equations 12 and 13) and finally the Corcos' coefficients (using equations 15 and 16) will be evaluated and compared, when it is possible, to well established results [8, 5, 12, 11].



Figure 8. Phase function as a function of frequency for different longitudinal separations r_x . —: $r_x = 7.5$ mm; - - :: $r_x = 20$ mm;: $r_x = 40$ mm;: $r_x = 70$ mm.



Figure 9. Convection velocity U_c in a dimensionless form as a function of frequency for each free stream velocity. —: $U_{\infty} = 20 \text{ m/s}$; ---: $U_{\infty} = 35 \text{ m/s}$;: $U_{\infty} = 50 \text{ m/s}$; -+-: Chen's law (equation 17).

3.4.1. Convection velocity

Measuring phase function with the experimental setup described in 3.3, it is possible to evaluate convection velocity U_c . Figure 8 presents the measured phase function versus frequency for different longitudinal separations r_x .

The mean convection velocities obtained using equation 14 are presented in Figure 9 in a dimensionless form. As it can be seen, in the 0 - 2000 Hz frequency range, the convection velocity is of the expected form KU_{∞} (with K = 0.62) when $\omega \delta^* \ge 1.5U_{\infty}$, but deviates from this value when $\omega \delta^* \le 1.5U_{\infty}$. In order to fit with experimental data, the simple law $U_c = KU_{\infty}$ can be replaced by equation 17 proposed by Chen *et al.* [14] as mentioned in [11], that is realistic in the whole frequency range.

$$\frac{U_c}{U_{\infty}} = 0.6 + 0.4 \times \exp\left(-2.2 \times \frac{\omega \delta^*}{U_{\infty}}\right).$$
(17)

3.4.2. Correlation lengths

Thanks to the grid used for pressure measurements (longitudinal and transversal lines, see Figure 6), it is possible to



Figure 10. Longitudinal (a) and transversal (b) exponential dependency of wall pressure cross spectrum. •: 304 Hz; +: 1104 Hz. Solid and dotted: fitted exponential functions.

evaluate longitudinal $(A(r_x, \omega))$ and transversal $(B(r_z, \omega))$ dependencies of the wall pressure cross spectrum.

As presented in Figure 10, both longitudinal and transversal dependencies can be fit with exponential functions for each angular frequency ω as suggested by Corcos' model (equations 12 and 13).

These exponential functions $A(r_x, \omega)$ and $B(r_z, \omega)$ depends only on longitudinal and transversal separations r_x and r_z and on longitudinal and transversal correlation lengths L_x and L_z . Estimating coefficients of exponential functions for each angular frequency allows to evaluate L_x and L_z .

Figures 11(a) and 11(b) present the deduced correlation lengths compared to those obtained with classical expressions of L_x and L_z for Corcos' model (equations 12 and 13 with $\alpha_x = 0.116$ and $\alpha_z = 0.7$). As it can be seen in Figures 11, L_x measured with the dedicated setup (Figure 5) agrees well with Corcos' model. A diminution of coherence lengths in low frequency, not predicted by Corcos' theory, can be observed whatever the free stream velocity. In that frequency band (0-250 Hz), Corcos' coefficients can't be considered as constant.

3.4.3. Corcos' coefficients α_x and α_z

As convection velocity U_c and correlation lengths L_x and L_z have been previously estimated, it is possible, using



Figure 11. Longitudinal and transversal correlation lengths L_x (a) and L_z (b) as a function of frequency. $\circ: U_{\infty} = 20 \text{ m/s}$; +: $U_{\infty} = 35 \text{ m/s}; \square: U_{\infty} = 20 \text{ m/s}.$ Solid, dotted and dash-dotted lines: Corcos' correlation lengths (equations 15 and 16).



Figure 12. Coefficients α_x and α_z as a function of frequency in third octave bands. $\circ: U_{\infty} = 20 \text{ m/s}$, $+: U_{\infty} = 35 \text{ m/s}$, $\Box U_{\infty} = 50 \text{ m/s}$; Gray lines: Efimtsov's empirical expressions [6, 9]. Arrows indicate increased flow speed.

equations 15 and 16, to evaluate coefficients α_x and α_z as a function of frequency as presented in Figure 12 in third octave bands.

As already observed by Efimtsov [6] and more recently by Finnveden *et al.* [15], coefficients α_x and α_z are not constant in the whole frequency range.

At low frequency, coefficients α_x and α_z can take higher values than those proposed by Blake [8] ($\alpha_x = 0.116$ and $\alpha_z = 0.7$). In that frequency range (below 500 Hz), the measured α_x and α_z agree well with the model proposed by Efimtsov [6, 9].

Finnveden *et al.* underline that α_x and α_z also depend on flow velocity (as indicated by the arrows in Figure 12). The same observation can be made here.

4. Measurements of power injected into a plate

4.1. Methodology

The power injected $\langle P_{inj} \rangle$ into a plate is equal to the sum of the power dissipated $\langle P_{diss} \rangle$ and the power radiated $\langle P_{rad} \rangle$ by the plate as expressed in equation 18.

$$\langle P_{inj} \rangle = \langle P_{diss} \rangle + \langle P_{rad} \rangle, \qquad (18)$$

where frequency average is defined integration over the frequency band 2Δ centered on angular frequency Ω as $\langle P \rangle = \frac{1}{2\Delta} \int_{\Omega-\Delta}^{\Omega+\Delta} P(\omega) d\omega$. As classically done, the radiated and dissipated power are assumed to be proportional to the frequency averaged energy $\langle E \rangle$ of the plate (equations 19 and 20) through internal loss factor η and radiated loss factor η_{rad} (details of radiation loss factor are given in [16]).

$$\langle P_{diss} \rangle = \Omega \eta \langle E \rangle \,, \tag{19}$$

$$\langle P_{rad} \rangle = \Omega \eta_{rad} \langle E \rangle \,. \tag{20}$$

Frequency averaged energy of the plate can be expressed as a function of mean velocity spectral density $\bar{S}_{\nu\nu}(\omega)$ defined in equation 21.

$$\bar{S}_{\nu\nu}(\omega) = \int_{S} S_{\nu\nu}(x, y, \omega) \,\mathrm{d}S. \tag{21}$$

Frequency averaged energy of the plate is then given by equation (22).

$$\langle E \rangle = MA \left\langle \bar{S}_{\nu\nu}(\omega) \right\rangle. \tag{22}$$

M is the mass per unit of area and A is the surface of the plate. Finally, the frequency averaged injected power is given by

$$\left\langle P_{inj} \right\rangle = \eta_{glob} \Omega M A \left\langle \bar{S}_{\nu\nu}(\omega) \right\rangle,$$
 (23)

where $\eta_{glob} = \eta + \eta_{rad}$. To obtain the experimental estimation of frequency averaged injected power $\langle P_{inj} \rangle_{EXP}$, it is only necessary to measure mean velocity spectral density of the plate $\bar{S}_{vv}(\omega)$ and global damping loss factor η_{glob} . Usually, for light fluids, modal radiation loss factor is lower than structural loss factor (ten times lower in example presented in [16]) so that $\eta_{glob} \approx \eta$.

To compare $\langle P_{inj} \rangle_{EXP}$ to the model $\langle P_{inj} \rangle_{FAIP}$, it is also necessary to measure the wall pressure spectral density $S_{pp}(\omega)$ as one can see in equation (1).

Table II. Physical properties of the four plates. *l*: Length (m), *w*: Width (m), S: Shape, *A*: Surface (m²), *d*: Thickness (mm), *E*: Young's Modulus (Pa), ρ : Mass per unit area (kg/m³), *v*: Poisson's coefficient

	Plate (A)	Plate (B)	Plate (C)	Plate (D)
l	0.6	/	0.6	0.6
w	0.3	/	0.3	0.3
S				
Α	0.18	0.18	0.18	0.18
d	0.5	0.5	0.5	1
Ε	2.1e11	2.1e11	1.25e11	4.5e9
ρ	7800	7800	7550	1400
ν	0.3	0.3	0.3	≈ 0.3

Table III. Theoretical values of aerodynamic coincidence frequency for each plate and each free stream velocity with U_c approximated by equation (17).

	20 m/s	35 m/s	50 m/s
Plate (A)	63.6 Hz	181 Hz	313.8 Hz
Plate (B)	63.6 Hz	181 Hz	313.8 Hz
Plate (C)	77.4 Hz	218.3 Hz	374.2 Hz
Plate (D)	85.5 Hz	240.4 Hz	409.9 Hz

4.1.1. Experimental setup

Four different plates have been tested:

- plate (A): rectangular plate made of steel
- plate (B): non rectangular plate made of steel
- plate (C): rectangular plate made of copper
- plate (D): rectangular plate made of PVC.

Material properties and shapes of plates are detailed in Table II.

Each plate was excited by a turbulent boundary layer at three different free stream velocities: 20, 35 and 50 m/s. The aerodynamic coincidence phenomenon occurs when the plate bending wave speed is equal to the convection velocity U_c [11]. This phenomenon produces maximum excitation of the plate and appears at one particular frequency ω_c called aerodynamic coincidence frequency given by equation 24.

$$\omega_c = U_c^2 \sqrt{\frac{M}{D}}.$$
(24)

Table III presents the theoretical value of ω_c for each plate and each free stream velocity. The convection velocity is approximed by equation (17).

A particular setup, presented in Figure 13, has been designed to uncouple the tested plate from the vibration of the wind tunnel. The plates were glued on a heavy frame made of steel (Figure 13-(B)) and put on four flexible mountings (Figure 13-(A)) in order to have a very low resonance frequency (manufacturer's characteristic: 7 Hz). These flexible mountings are fixed to the frame made of wood and a screw (Figure 13-(D)) is used to adjust level of the plate so that the edge of the plate be flushed with the wind tunnel floor (Figure 13-(C)). Obviously, because of



Figure 13. Pictures of the setup used to uncouple the tested plate from the vibrations of the wind tunnel. A: frame in wood clamped on the wind tunnel and flexible mountings: sight of top, without the plate. B: setup of uncoupling, sight of bottom; C: plate flushed with the wind tunnel floor. D: flexible mounting with an adjusting screw.



Figure 14. Coherence between acceleration on plate (A) and acceleration on the wind tunnel for each mean flow velocity, $U_{\infty} = 50$ m/s.

the rigid body modes induced by flexible mountings and of large static pressure difference between the inside and the outside of the wind tunnel under operation, the level of the plate is adjusted for each free stream velocities (using screws).

The coherence between acceleration on plate (measured with a POLYTEC vibrometer) and acceleration on wind tunnel (measured with a Brüel & Kjær accelerometer) has been evaluated when the plate was excited by the turbulent boundary layer. Low coherence observed in Figure 14 indicates that only turbulent flow excited the plate and no Strouhal phenomenon appeared.

To evaluate the efficiency of the uncoupling setup (Figure 13) between the plate and the wind tunnel, a deterministic excitation was used (an electrodynamic shaker). As presented in Figure 15, transfer functions between acceleration of the plate and excitation on the wind tunnel (FRF p_1/wt) and between acceleration of plate and excitation on plate (FRF p_1/p_2) have been compared.

As it can be seen in Figure 16, FRF p_1/p_2 is much higher than FRF p_1/wt (at least 20 dB higher except below 25 Hz). This indicates that vibratory transmission from the



Figure 15. sketch of measurements for evaluation of efficiency of the uncoupling setup. The plate (point p_2) and the wind tunnel (point *wt*) are successively excited, each time transfer function is measured at point p_1 . Distances between p_2 and p_1 and between *wt* and p_1 are equal.



Figure 16. (a) transfer functions between deterministic excitation (shaker) and acceleration on plate. —: transfer function p_1/p_2 (excitation on plate); - - -: transfer function p_1/wt (excitation on wind tunnel). (b) difference between FRF p_1/p_2 and FRF p_1/wt .

wind tunnel to the plate is negligible and demonstrates the efficiency of the uncoupling setup.

4.2. Influence of pre-stress on vibration of plates

Static pressure induced by the pressure difference between the two sides of the plates leads to modification of the natural frequencies of plates. Indeed, as it can be seen in Figure 17, natural frequencies of plates increase with free stream velocity U_{∞} . Example of plate (B) (Figure 17) demonstrates that influence of pre-stress is sig-



Figure 17. Influence of pre-stress on natural frequencies of plate (B) up to 200 Hz. Black: frequency shifts between $U_{\infty} = 20$ m/s and $U_{\infty} = 35$ m/s; Gray: frequency shifts between $U_{\infty} = 20$ m/s and $U_{\infty} = 50$ m/s

nificant in low frequency (the first six natural frequencies, below 120 Hz) where natural frequencies increases (up to 21%). However, in higher frequency, pre-stress only slightly modify natural frequencies (3 to 4%). These conclusions agree with those of Finnveden *et al.* [15]. In the following, pre-stress will be considered as negligible above 100 Hz.

4.3. Mean velocity spectral density of plates

To obtain frequency averaged injected power $\langle P_{inj} \rangle_{EXP}$, it is necessary to measure the space mean velocity spectral density and the damping loss factor of plates. The velocity spectral density is measured using a vibrometer POLYTEC (OFV-056) with a scanning head at 72 different points distributed on the whole surface of the plate. These velocity spectral densities are then averaged for each plate and each free stream velocity. As proposed by Finnveden *et al.*[15], a nondimensional metric *R* for the response of the plate is defined as in equation (25).

$$R = \frac{\omega^2 \rho^2 h^2 \bar{S}_{\nu\nu}(\omega)}{S_{\rho\rho}(\omega)}$$
(25)

The classical response of a plate excited by a turbulent boundary layer can be observed in Figure 18: for frequency below the aerodynamic coincidence phenomenon, the mean velocity fluctuates around a constant value, then for higher frequency it decreases.

R depends also on damping loss factors of plates. Indeed, plate (D) has a metric *R* lower (-6 dB) than other metallic plates because of its relatively high damping (2% compared to 0.5%).

Finally, it can be noticed (especially in Figure 18(c)) that plate (A) and plate (B) have the same aerodynamic coincidence frequency. Plates (C) and (D) have a higher aerodynamic coincidence frequency as expected (see table III).



Figure 18. Nondimensional vibration response in 1/3-octave bands. (a): $U_{\infty} = 20 \text{ m/s}$; (b): $U_{\infty} = 35 \text{ m/s}$; (c): $U_{\infty} = 50 \text{ m/s}$. —: plate (A); - - : plate (B); ...: plate (C); -.-: plate (D).

Compared to experimental measurements done by Finnveden *et al.*[15], Figure 18 shows relatively high values for metric R particularly above aerodynamic coincidence frequency. This is probably due to boundary conditions and thickness of plates. Indeed, in [15], metric R is given for a clamped thick plate (thickness: 1.6 mm) and high free stream velocities whereas in present measurements non clamped thin plates (see table II) and low free stream velocities are used. In addition, Finnveden presents an averaged response at five positions of the plate whereas in present measurements 72 measuring points has been used to evaluate mean velocity spectral density of plates.



Figure 19. Measured damping loss factor for each plate for 62.5 Hz, 125 Hz, 250 Hz, 500 Hz and 1000 Hz octave bands. •: plate (A) made in steel; *: plate (B) made in steel; *: plate (C) made in copper; +: plate (D) made in PVC.

4.4. Damping loss factor of plates

To identify the injected power into the plate from dissipated power it is necessary to evaluate damping loss factor of each plate. Finnveden *et al.* [15] have established that the damping loss factor of a plate excited by a turbulent boundary layer slightly depends on the free stream velocity and frequency. At low frequency and high free stream velocity ($U_{\infty} = 120 \text{ m/s}$), the influence of the steady flow on the damping loss factor can be important (1% at $U_{\infty} = 80 \text{ m/s}$ and 6.5% at $U_{\infty} = 120 \text{ m/s}$) for the 125 Hz octave band). At medium and high frequency, the influence of the steady flow is negligible.

Thus, in the present measurements, the influence of turbulent flow on damping loss factors is considered to be negligible for $U_{\infty} = 20$, 35 and 50 m/s and damping loss factor is measured without flow. The damping loss factor is deduced from equation 26 measuring the applied force $F(\omega)$ by a shaker and the velocity at the excited point $V(\omega)$ in order to evaluate the injected power $\prod_{inj}^{shaker}(\omega)$ and the mean velocity spectral density $\bar{S}_{vv}(\omega)$ of the plate to evaluate plate energy $\bar{E}(\omega)$.

$$\eta = \frac{\Pi_{inj}^{shaker}(\omega)}{\bar{E}(\omega)} = \frac{\frac{1}{2}\Re(F(\omega)V^*(\omega))}{\omega\rho Ah\bar{S}_{vv}(\omega)}.$$
 (26)

Figure 19 presents measured damping loss factor for each plate in different octave bands.

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As it can be seen, metallic plates (plates (A), (B) and (C)) have approximately the same low damping loss factor (between 0.3 and 0.5%). On the other hand, plate (D) made of PVC has a much higher damping loss factor (close to 1.8%) which will allow to verify if frequency averaged power injected by a TBL in a plate is independent of damping as predicted by the FAIP model.

In addition, damping loss factors agree well with those estimated with the -3 dB bandwith method on the first modes (plate (A): 0.86%; plate (B): 0.72%; plate (C): 0.58%; plate (D): 2.16% for modes around 120 Hz).

5. Model/Experiments comparison

5.1. FAIP model and frequency-depedent laws for U_c , α_x and α_z

In the following, the measured injected power per unit of wall pressure spectral density $\langle P_{inj} \rangle_{\text{FAIP}} / \langle \bar{S}_{pp}(\omega) \rangle$ is compared to

- the FAIP_C^{exp} function (equation 28) using frequencydependent law 17 and Efimtsov' model, for U_c , α_x and α_z respectively.
- the FAIP^{exp} function (equation 27) using frequencydependent laws 17 and Efimtsov' model.
- the FAIP_C^{lit} function (equation 28) using classical values of Corcos' coefficients found in the litterature ($\alpha_x = 0.116, \alpha_z = 0.7$ [8]),

where the FAIP_{CD} and FAIP_C functions are given by equations (27) and (28).

$$\text{FAIP}_{CD} = \frac{\alpha_x \alpha_z U_c^2}{\pi \sqrt{MD} \Omega^2} \frac{S}{2} \Psi_{CD} \left(\frac{\Omega}{\omega_c}\right)$$
(27)

$$\text{FAIP}_{C} = \frac{\alpha_{x} \alpha_{z} U_{c}^{2}}{\pi \sqrt{MD} \Omega^{2}} \frac{S}{2} \Psi_{C} \left(\frac{\Omega}{\omega_{c}}, \frac{\Omega a}{U_{c}}, \frac{\Omega b}{U_{c}} \right). (28)$$

The characteristic functions Ψ_{CD} and Ψ_{C} are given by equations 2 and 5. The following results are presented in logarithmic frequency bands from 100 Hz to 2000 Hz.

Figures 20, 21, 22 and 23 present respectively the comparison between measured injected power and FAIP_{C}^{exp} model for plates (A), (B), (C) and (D). Predictions of the model are in good agreement with measurements and demonstrate that the FAIP models can be used as an input for energy methods.

As demonstrated in [1], the frequency averaged injected power is theoretically independent of damping and, in a first approximation, of shape and boundary conditions of the plate. The plates under study and the setup have been chosen to verify these remarks.

The experimental setup presented in Figure 13 does not reproduce the boundary conditions of a simply supported plate. However, the FAIP model derived from a simply supported rectangular plate correctly predict the power injected into a plate with arbitrary boundary conditions (as one can see in Figure 20 for example). The independence of the frequency averaged injected power compared to the boundary conditions is verified.

Plate (B) is made in steel as plate (A). They have the same surface but not the same shape. The model of Frequency Averaged Injected Power has been derived using a rectangular plate but as demonstrated by equations 1 and 2, $\langle P_{inj} \rangle_{\text{FAIP}_{CD}}$ does not depend on the exact geometry on the plate but only on its surface. The plate (B) was used to verify this property. Figure 21 shows that FAIP model estimates the power injected into plate (B) with the same precision than for plate (A). The frequency of the aerodynamic coincidence phenomenon is well estimated and the model has the same tendencies as measurements.

The plate (D), made of PVC, is highly damped (\approx 1.8%). The second property of Frequency Averaged Injected Power is that it does not depend on damping of the



Figure 20. Plate (A): Comparison between measured injected power $\langle P_{inj} \rangle_{EXP} / \langle \bar{S}_{pp}(\omega) \rangle$ and function FAIP^{*exp*}_{*C*}. (a): $U_{\infty} = 20 \text{ m/s}$; (b): $U_{\infty} = 35 \text{ m/s}$; (c): $U_{\infty} = 50 \text{ m/s}$. $-\circ-: \langle P_{inj} \rangle_{exp} / \langle \bar{S}_{pp}(\omega) \rangle$; —: FAIP^{*exp*}_{*C*}.

plate. Plate (D) was studied to verify this property. Figure 23 presents the comparison between models and experiments when plate (D) is excited by a turbulent boundary layer. Figure 23 shows a good agreement between models and experiments. Even if the plate is highly damped, the model correctly predict tendencies of injected power below or above aerodynamic coincidence frequency.

The results of measurements presented in Figures 20, 21 and 22 and 23 demonstrate that the FAIP model is able to estimate the power injected into a rectangular or a non rectangular plate whatever the boundary conditions and the damping, at least in a first approximation.

5.2. FAIP model and classical values for U_c , α_x and α_z

In section 3.4.3, an experimental Corcos' model has been derived. Corcos' coefficients α_x and α_z have been obtained measuring phase function and correlation lengths



Figure 21. Plate (B): Comparison between measured injected power $\langle P_{inj} \rangle_{EXP} / \langle \bar{S}_{pp}(\omega) \rangle$ and function FAIP^{exp}_C. (a): $U_{\infty} = 20 \text{ m/s}$; (b): $U_{\infty} = 35 \text{ m/s}$; (c): $U_{\infty} = 50 \text{ m/s}$. $-\circ-: \langle P_{inj} \rangle_{exp} / \langle \bar{S}_{pp}(\omega) \rangle$; —: FAIP^{exp}_C.

with a special setup. This setup presented in Figure 5 allows to measure pressure cross spectrum between two microphones for which longitudinal and transversal separations are known. These measures are complex and time-consuming. Thus, it should be interesting to know if it is sufficient to use classical values of Corcos' coefficients ($\alpha_x = 0.116$ and $\alpha_z = 0.7$ [8]) and convection velocity ($U_c = 0.7 \times U_{\infty}$) to correctly estimate frequency averaged injected power. As it can be seen in Figure 24, curves of FAIP^{exp}_C and FAIP^{lit}_C are slightly differents. However, using classical values for α_x , α_z and U_c does not lead to significant differences between both curves and such an estimation can be sufficient in a design stage.

However, for a more detailed analysis, Corcos' model may be not sufficient and frequency dependent laws for α_x , *alpha_z* and U_c can advantageously be introduced.

Indeed, it is classically assumed that the convection velocity is proportional to the free stream velocity ($U_c = KU_{\infty}$). Figure 9 demonstrates that this assumption is



Figure 22. Plate (C): Comparison between measured injected power $\langle P_{inj} \rangle_{EXP} / \langle \bar{S}_{pp}(\omega) \rangle$ and function FAIP^{exp}_C. (a): $U_{\infty} = 20 \text{ m/s}$; (b): $U_{\infty} = 35 \text{ m/s}$; (c): $U_{\infty} = 50 \text{ m/s}$. $-\circ-: \langle P_{inj} \rangle_{exp} / \langle \bar{S}_{pp}(\omega) \rangle$; —: FAIP^{exp}_C.

not verified in the frequency band under study especially when $\omega \delta^*/U_c < 1.5$ (that is to say below 620 Hz for $U_{\infty} = 20$ m/s, below 1305 Hz for $U_{\infty} = 35$ m/s and below 1356 Hz for $U_{\infty} = 50$ m/s).

Figure 24 compares FAIP_C^{exp} and FAIP_C^{lit} models to verify if the assumption $U_c = KU_{\infty}$ leads to important errors on estimation of injected power. As it can be seen in Figures 24, the aerodynamic coincidence phenomenon occurs at higher frequency with FAIP_C^{exp} model than with FAIP_C^{lit} model. Indeed, in the case of plate (A) and $U_{\infty} =$ 50 m/s, the aerodynamic coincidence frequency occurs at 248 Hz for FAIP_C^{lit} model (with K = 0.7) and at 313.8 Hz with FAIP_C^{exp} model (using law 17). However, the FAIP_C^{exp} model seems to better fit the experimental results especially close to aerodynamic coincidence phenomenon. Interest of using law 17 for the convection velocity appears in Figures 24(b) and 24(c).

Corcos' model is believed to overestimate low-wavenumber frequency domain. As it can be seen in Figure 24,



Figure 23. Plate (D): Comparison between measured injected power $\langle P_{inj} \rangle_{EXP} / \langle \bar{S}_{pp}(\omega) \rangle$ and function FAIP^{exp}_C. (a): $U_{\infty} = 20 \text{ m/s}$; (b): $U_{\infty} = 35 \text{ m/s}$; (c): $U_{\infty} = 50 \text{ m/s}$. $-\circ-: \langle P_{inj} \rangle_{exp} / \langle \bar{S}_{pp}(\omega) \rangle$; —: FAIP^{exp}_C.

Corcos' model with classical values for α_x and α_z overestimates power injected into the plate below aerodynamic coincidence frequency (up to 3 dB, Figure 24(c)) and above aerodynamic Coincidence frequency (up to 2 dB, Figure 24(c)). In addition, aerodynamic coincidence frequency is not well estimated using constant values for U_c .

Finally, modified Corcos' model (α_x , α_z and U_c depending on frequency and boundary layer thickness) gives better estimation for power injected into a plate excited by a turbulent boundary layer. Classical Corcos' model seems to suffer from its constraining assumptions.

More sophisticated model for boundary pressure cross spectrum like Chase's model was not investigated here. FAIP model have been only derived from Corcos' assumptions to have a quick and reliable estimation of power injected into a plate excited by a TBL. However, it is possible that, in some circumstances (high free stream velocities, thick plate, etc...), FAIP model based on Corcos' model leads to overestimation of power injected. Further devel-



Figure 24. Validity of the classical values for K, α_x and α_z . (a): $Plate(D), U_{\infty} = 20 \text{ m/s}$; (b): Plate (A), $U_{\infty} = 35 \text{ m/s}$; (c): Plate (B) $U_{\infty} = 50 \text{ m/s}$. $-\circ-: \langle P_{inj} \rangle_{EXP} / \langle \bar{S}_{pp}(\omega) \rangle$; $-:: \text{FAIP}_{C}^{exp}$; - $-: \text{FAIP}_{C}^{lit}$.

opments of FAIP model to use Chase's model rather than Corcos' model will be investigated.

5.3. Influence of the Davies' approximation

As already demonstrated in [1], FAIP^{*exp*}_{*CD*} model is very close to FAIP^{*exp*}_{*C*} one provided that the condition in equation 4 is satisfied which often occurs from frequency below ω_c . For the three free stream velocities used here $(U_{\infty} = 20, 35 \text{ and } 50 \text{ m/s})$, the condition is satisfied respectively above 23.3 Hz, 40.8 Hz and 58.3 Hz that is to say well below the lower bound of the frequency band under study 100-2000 Hz. The only discrepancy appears near the aerodynamic coincidence phenomenon where Davies' approximation leads to slightly underestimate the injected power. That can be seen in Figures 25.

5.4. Comparison with the Blake's model

The model of power injected into a plate excited by a turbulent boundary layer proposed by Blake [8] and simplified by Lyon and Dejong [2] has been compared to



Figure 25. Plate (B): Validity of the Davies' approximation. (a): $U_{\infty} = 20 \text{ m/s}$; (b): $U_{\infty} = 35 \text{ m/s}$; (c): $U_{\infty} = 50 \text{ m/s}$. -o-: $\langle P_{inj} \rangle_{EXP} / \langle \bar{S}_{pp}(\omega) \rangle$; --: FAIP_C^{exp}; - -: FAIP_{CD}^{exp}.

FAIP_C model and measurements. As presented in Figure 26, Blake's model clearly overestimates power injected up to 6 dB and introduces a discontinuity at $\omega = \omega_c$. FAIP_C model gives better estimation of power injected into the plate on the whole frequency range (below or above aerodynamic coincidence angular frequency ω_c) either in amplitude or in tendencies.

Figure 26 demonstrates that $FAIP_C$ model improves estimation of power injected into a plate excited by a turbulent boundary layer compared to existing models.

6. Conclusion

The present paper deals with an experimental validation of the Frequency Averaged Injected Power models established in [1]. These models are derived from vibratory response field of a simply supported rectangular plate excited by a turbulent boundary layer represented by a Corcos' model. Several remarks had been done analyzing equations of the FAIP model. First of all, the frequency



Figure 26. Comparison between FAIP_{C}^{exp} and Blake models for frequency averaged injected power, Plate (B).

(a): $U_{\infty} = 20 \text{ m/s}$; (b): $U_{\infty} = 35 \text{ m/s}$; (c): $U_{\infty} = 50 \text{ m/s}$. -o-: $\langle P_{inj} \rangle_{EXP} / \langle \bar{S}_{pp}(\omega) \rangle$; -:: FAIP_C^{exp}; - - -: Blake's model.

averaged injected power is independent of damping of the plate. Then, it can be allowed that the frequency averaged injected power is, in a first approximation, independent of the shape and the boundary conditions of the plate.

If the FAIP is realistic, it can be an useful tool to quickly estimate power injected into a subsystem of an energy method (like Statistical Energy Analysis). As energy methods, the FAIP model does not need the exact description of the modal behavior of the plate and gives an averaged value of the injected power on a frequency band. Thus, experiments have been made to compare the predictive FAIP model to the real measurement of power injected into a plate excited by a TBL. The wind tunnel of the Acoustic Center of Ecole Centrale of Lyon has been used to produce a turbulent boundary layer.

The turbulent flow has been characterized measuring the mean flow profiles and the boundary layer thicknesses. Then, a Corcos-like model of wall pressure cross spectral density has been derived from measurements of wall pressure spectral density, correlation lengths and phase function using a dedicated setup. Canceling the acoustic component of the wall pressure spectral density, it has been verified that it is negligible above 100 Hz.

The power injected into the plate is obtained measuring mean velocity response and damping of the plate. The comparison between models and experiments shows a good agreement especially when frequency-dependent laws are used for Corcos' coefficients α_x , α_z and convection velocity U_c .

The FAIP model gives accurate enough results to be an useful tool to predict the power injected into a structure excited by a TBL even if this structure is not rectangular and is not simply supported.

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