Nonlinear propagation of spark-generated *N*-waves in air: Modeling and measurements using acoustical and optical methods

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(Received 10 March 2010; revised 29 September 2010; accepted 29 September 2010)

The propagation of nonlinear spherically diverging *N*-waves in homogeneous air is studied experimentally and theoretically. A spark source is used to generate high amplitude (1.4 kPa) short duration (40 μ s) *N*-waves; acoustic measurements are performed using microphones (3 mm diameter, 150 kHz bandwidth). Numerical modeling with the generalized Burgers equation is used to reveal the relative effects of acoustic nonlinearity, thermoviscous absorption, and oxygen and nitrogen relaxation on the wave propagation. The results of modeling are in a good agreement with the measurements in respect to the wave amplitude and duration. However, the measured rise time of the front shock is ten times longer than the calculated one, which is attributed to the limited bandwidth of the microphone. To better resolve the shock thickness, a focused shadowgraphy technique is used. The recorded optical shadowgrams are compared with shadow patterns predicted by geometrical optics and scalar diffraction model of light propagation. It is shown that the geometrical optics approximation results in overestimation of the shock rise time, while the diffraction model allows to correctly resolve the shock width. A combination of microphone measurements and focused optical shadowgraphy is therefore a reliable way of studying evolution of spark-generated shock waves in air. (© 2010 Acoustical Society of America, [DOI: 10.1121/1.3505106]

PACS number(s): 43.25.Cb, 43.25.Zx, 43.28.Mw, 43.28.Js [ROC]

Pages: 3321–3333

I. INTRODUCTION

Pressure *N*-waves with durations of the order of tens of microseconds can be generated in air by electric sparks or focused laser beams.^{1,2} Close to the source, the amplitude of the initial spark-generated *N*-wave can be quite high (>1000 Pa), and nonlinear propagation effects are of importance not only in the close vicinity of the spark (where they are the reason for *N*-wave formation) but also at some distance from the origin.³ Such experimental conditions that provide a nonlinearly propagating *N*-wave have been used in downscaled laboratory experiments on sonic boom propagation in order to study the distortion of *N*-waves by atmosphere inhomogeneities.^{4–12} Short duration *N*-waves are also used in laboratory experiments to simulate problems of architectural acoustics,¹³ street acoustics,¹⁴ and outdoor sound propagation,¹⁵ where wide bandwidth and high frequencies of pressure waves are desirable.

The validity of experimental data analysis and the validity of comparison between experimental and theoretical data strongly depend on the accurate knowledge of the initial *N*-wave parameters such as peak pressure, duration, and shock rise time. It is thus of the utmost importance to accurately measure and model the propagation of *N*-waves in homogeneous air before investigating more complex configurations. However, it has been shown that it is technically difficult to resolve the fine structure of the shocks that have rise times less than 1 μ s, mainly because of the limited frequency bandwidth of commercially available microphones.¹⁶ This complicates experimental validation of the theoretical predictions of shock fine structure and high frequency spectrum.

Propagation of spark-generated *N*-waves in homogeneous atmosphere has been studied experimentally by several

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authors, particularly by Wright and co-workers.^{1,17,18} Recently, the relative roles of nonlinear effects, thermoviscous absorption, and molecular (oxygen and nitrogen) relaxation in N-waveform distortion were studied both experimentally and theoretically.^{3,19} A numerical model based on the generalized Burgers equation was developed, and the ability of the model to predict peak pressures and duration of the wave was established. However, strong discrepancies between modeling and measurement were observed in estimation of the front shock rise time which was attributed to the limited bandwidth of the microphone. Better resolution was achieved using handmade microphones, but overestimated values of the shock rise time were still recorded.^{1,6,17,18} More accurate experimental techniques are therefore necessary to measure frequencies above 150 kHz and to resolve fine shock structure. This problem is of particular importance for sonic boom propagation studies because shock rise time is one of the important parameters responsible for the boom annoyance.²⁰

An alternative approach to measure pressure shocks in a laboratory environment is to use optical methods instead of acoustic microphones. Measurement of shock thickness using optical methods has been widely treated in literature.^{21–27} However, although weak shocks have been addressed,²¹ most of the effort was generally focused on measuring strong shocks created by supersonic flows; the thickness of strong shocks was usually estimated using indirect methods based on the shock speed measurements.^{26–28} It was also shown that a laser interferometer can be used to measure spark-generated pressure waves, but no quantitative analysis has yet been done.²⁹

The goal of this paper is to demonstrate that the combination of optical and acoustical methods allows complete characterization of short duration and high amplitude N-waves, including rise time measurements. An N-wave is recorded by a microphone along the propagation path from a spark source in air. The fine structure of the shock is also deduced from the shadowgraphy images with a resolution that cannot be obtained via acoustic measurements.^{21,22} The distributions of light intensity in the measured images are compared to the results of modeling of light propagation through inhomogeneities of refraction index caused by the presence of the shock. To reveal whether diffraction of light has to be considered in simulations, two models are built. The first model is based on the geometrical optics approximation,³⁰ while the second one is based on scalar diffraction theory.³¹ Experimental data for N-wave parameters are compared with the results of acoustic modeling, where spherical spreading, thermoviscous dissipation, oxygen and nitrogen relaxation, and nonlinear propagation are included.³

II. EXPERIMENTAL ARRANGEMENT FOR ACOUSTICAL AND OPTICAL MEASUREMENTS

The experimental setup designed for acoustical and optical measurements of spark-generated *N*-waves in homogeneous air is presented in Fig. 1. High amplitude pressure pulses are produced by a 15 kV spark source (1) with 15 mm gap between tungsten electrodes. The repetition rate is of the order of 1 Hz. Spherically diverging pulses are measured along the *x* coordinate with a broadband microphone [Brüel & Kjær



FIG. 1. (Color online) Experimental setup: 1-spark source, 2-microphone in a baffle, 3-nanolight flash-lamp, 4-focusing lens, 5-camera, 6-lens.

(B&K) (Nærum, Denmark), 3 mm (1/8 in.) diameter, type 4138] coupled with an adapted preamplifier (B&K 2670) and amplifier (B&K Nexus amplifier with extended bandwidth -3 dB at 200 kHz). The microphone is mounted into a baffle (2) in order to avoid diffraction effects on its edge. The recorded signal is digitized (12 bit, 5 MHz) using a data acquisition card (National Instruments PCI 6610, Austin, TX). The sensitivity of the microphone is defined using the calibration method presented in Refs. 3, 11, and 19 and is equal to 1.46 mV/Pa at low frequencies. The measurements are performed at increasing source-microphone distances from 15.8 to 105 cm. Optical equipment includes a flash-lamp (3) (Nanolite KL-L model, 3.5 kV tension), light filter, lens (4) (4 cm diameter, 16 cm focal length), a digital charge-coupled device (CCD) camera (5) (Dantec Dynamics, FlowSense 2M, Skovlunde, Denmark) with acquisition system, Nikon lens (6) with 60 mm focal length, and calibration grid. Optical equipment is mounted on a rail and aligned coaxially. The flash-lamp generates short duration (20 ns) light flashes that allow to have a good resolution of the front shock shadow. According to theoretical estimations, the minimum rise time expected in experiments is as small as 0.1 μ s, that is five times longer than the flash duration. Note that longer light flash duration would result in long signal integration times, so that the shock image would be blurred. The focusing lens is used to collimate the flash light in order to have a parallel light beam. The dimension of the CCD camera is 1600 pixels along the horizontal coordinate and 1186 pixels along the vertical coordinate. The lens is used to focus the camera at a given observation plane perpendicular to the optical axis. Before each optical measurement, the CCD camera and lens are calibrated by capturing an image of a calibration grid placed at the chosen observation plane. Light source and camera are triggered by the acoustic source.

Choosing t = 0 as the time when a spark is generated, photographs are taken at the instant t when the front shock position is tangent to the optical axis. At this time, a local coordinate system shown in Fig. 1 is defined in the following way: y is the coordinate along the optical axis, which is the light propagation direction from the optical source to the camera; x is the axis in the acoustic source-microphone direction, perpendicular to the optical axis; and z is the axis perpendicular to the xy plane. The origin O thus corresponds to the point where the light beam grazes the front shock of the *N*-wave. The acoustic source coordinates are $(x_S, y_S = 0, z_S = 0)$, with $x_S < 0$. The coordinate *r* corresponds to the radial distance from the acoustic source, it is related to the local cartesian coordinates by $r = \sqrt{(x - x_S)^2 + y^2 + z^2}$. The wavefront radius at the observation time *t* is named *R*; with this definition $R = |x_S|$. Optical measurements of the shock front are performed for different distances from 15.8 to 65.9 cm between the spark source and the optical axis.

III. THEORETICAL BACKGROUND: PROPAGATION OF A SPHERICAL *N*-WAVE IN AIR

In this section, the propagation of a high amplitude pulse is studied theoretically, simulating the experimental conditions of spark-generated N-waves in air. The propagation model is first reviewed,³ then the effects of thermoviscous absorption, nonlinear propagation, and relaxation on distortion of the N-wave parameters are discussed.

A. Sound propagation model

Consider a point source that generates high amplitude spherically divergent pulsed waves. To model the propagation of the pulse in the relaxing homogeneous atmosphere a generalized Burgers equation^{3,32} is used:

$$\frac{\partial p}{\partial r} + \frac{p}{r} = \frac{\beta}{\rho_0 c_0^3} p \frac{\partial p}{\partial \tau} + \frac{b}{2\rho_0 c_0^3} \frac{\partial^2 p}{\partial \tau^2} + \sum_{\nu=1}^{\nu=2} d_\nu \frac{\partial}{\partial \tau} \int_{-\infty}^{\tau} \exp\left(-\frac{\tau - \tau'}{\tau_\nu}\right) \frac{\partial p}{\partial \tau'} d\tau'.$$
(1)

Here p is the acoustic pressure, r is the radial propagation coordinate with the origin at the source, $\tau = t - (r - r_0)/c_0$ is the retarded time, c_0 is the ambient sound speed at low frequencies, r_0 is the reference distance to set boundary conditions, ρ_0 is the density, β is the coefficient of nonlinearity, and b is the coefficient of viscosity in the air. Each relaxation process v is characterized by two parameters: Relaxation time τ_{v} and coefficient $d_v = (c_{\infty}^v - c_0)/c_0^2 = c_v/c_0^2$, where c_{∞}^v is the so-called frozen sound speed of an acoustic signal propagation through the medium with relaxation time much longer than the effective duration of the signal $T_s \ll \tau_{\nu}$. The second term on the left-hand side of Eq. (1) accounts for the spherical divergence of the wavefront, while the right-hand side accounts for nonlinear effects (first term), dissipation due to thermoviscous absorption (second term), and relaxation processes associated with the excitation of oscillatory energy levels of oxygen O_2 and nitrogen N_2 in the air (last term).^{32–34} Equation (1) is valid if $\lambda/r \ll 1$, where λ is the wavelength associated with the length of the N-wave. This condition is satisfied in the current laboratory-scaled experiments ($\lambda \sim 15 \text{ mm}, r > 150 \text{ mm}$).

To study sound propagation theoretically, Eq. (1) is solved numerically in finite differences using a previously developed algorithm.³ The initial waveform is set at the distance r_0 from the source in the form of an ideal *N*-wave with thin shocks that are smoothed according to the quasi-stationary solution of the classic Burgers equation. The initial rise time of the shock is thus defined by the balance of the nonlinear and dissipation effects, which better approximates the experimental conditions. Equation (1) is solved using a method of fractional steps with an operator splitting procedure.^{3,35} At each step in the propagation distance the calculations of the nonlinear term are performed in the time domain using the central flux-conservative Godunov-type algorithm³⁶; thermoviscous absorption and relaxation terms are calculated in the frequency domain using the exact solution for each spectral component; spherical divergence of the wave is taken into account by introducing specific dimensionless variables in Eq. (1). Both oxygen and nitrogen relaxation effects are included in further simulations presented in the paper.

B. Effect of nonlinearity, thermoviscous absorption, and relaxation on *N*-wave propagation

In order to evaluate the relative roles of acoustic nonlinearity, thermoviscous absorption, and relaxation on the distortion of the spherically diverging N-wave in air, numerical simulations based on Eq. (1) are performed with alternative inclusion of each term, responsible for the corresponding effect. The parameters used to model the propagation medium correspond to the experimental conditions: $\beta = 1.2$, $b = 4.86 \times 10^{-5}$ Pa s, $\rho_0 = 1.19$ kg/m³, and $c_0 = 343.77$ m/s. For a relative humidity of 34%, a temperature of 293 K, and an ambient pressure level of 1 atm, the parameters c_v and τ_v of oxygen and nitrogen relaxation processes are calculated using empirical expressions: $c_1 = 0.11$ m/s, $\tau_1 = 6.0 \ \mu s$ (O₂), $c_2 = 0.023$ m/s, and $\tau_2 = 531 \ \mu s \ (N_2).^{34}$ The amplitude of the initial N-wave is $p_0 = 1000$ Pa and the half duration is T_0 = 15 μ s, which corresponds to the typical values measured at $r_0 = 15$ cm from the acoustic source. The shock rise time of the initial N-wave, defined here as the time during which the acoustic pressure increases from 0.1 to 0.9 level of the peak positive pressure,⁶ is chosen according to the quasi-stationary solution of the Burgers equation as $\tau_{sh0} = 0.18 \ \mu s$.

A summary of the results of modeling N-wave propagation is presented in Fig. 2. While propagating in air, the shape of the initial N-wave is distorted by the combined effects of spherical divergence, acoustic nonlinearity, absorption, and relaxation. Shown in Fig. 2(a) are the waveforms obtained at the distance r = 6 m from the source. This distance is chosen because it is sufficiently long to distinguish clearly the changes in the waveform due to all the effects discussed above. The waveforms are multiplied by the ratio r/r_0 in order to compensate for spherical divergence of the wave that dominates over other physical effects. The solid curve with bold circles is obtained if the right-hand side of Eq. (1) is set to zero, i.e., if only spherical divergence is considered. In this case, the waveform remains identical to the initial waveform at r_0 (no distortion). Other physical effects are further accounted separately: Nonlinear effects (dashed curve), thermoviscous absorption (dashed-dotted curve), and oxygen and nitrogen relaxation (dotted curve). Finally, all the mentioned effects are accounted together (black solid curve).

In accordance to the weak shock theory, the presence of nonlinear effects only (dashed curve) results in increased duration and decreased peak pressure level of the wave.^{32,34} Linear thermoviscous absorption (dashed-dotted curve) has



FIG. 2. (Color online) Relative effects of nonlinearity, thermoviscous absorption, and molecular (O₂ and N₂) relaxation on *N*-wave propagation. (a) Simulated waveforms at the distance r = 6 m, (b) peak positive pressure, (c) duration, and (d) rise time along the propagation path.

no effect on the wave duration measured between half peak pressure levels but increases the rise time and attenuates the peak pressures. Relaxation phenomena (dotted curve) have negligible effects on the rise time of the shocks and pulse duration, but they attenuate the peak pressures and distort the waveform that becomes asymmetric; the front and rear shock fronts are shifted equally in time because of faster propagation of high frequencies in a relaxing medium. All effects except molecular relaxation keep the wave symmetric. When all effects are included together in the propagation model (black solid curve), the duration and the rise time increase while the peak pressure decreases and the waveform becomes asymmetric. Note, however, that for propagation distances shorter than 1 m the waveform retains its symmetric shape almost unchanged (not shown here) in contrast to the waveform calculated at 6 m from the source.

Propagation curves for the peak positive pressure p_{max} and half duration T of the wave are presented in Figs. 2(b) and 2(c). The peak positive pressure depends on both nonlinear effects, relaxation, and thermoviscous absorption [Fig. 2(b)]. No pulse lengthening is observed when only thermoviscous absorption or relaxation are present (dotted and dashed-dotted curves coincide with the solid curve with bold circles); it is mainly determined by the nonlinear effects [Fig. 2(c)]. The difference between the black solid curve (all effects) and the dashed curve (nonlinear effects only) at propagation distances longer than 1 m comes from the fact that molecular relaxation and thermoviscous absorption attenuate the pressure level, which in turn weakens the nonlinear effects. For distances up to 1 m, pulse lengthening can be estimated by accounting only for nonlinear propagation effects, as relaxation and thermoviscous absorption have little influence. This justifies the method proposed by Wright¹ to estimate the sensitivity of microphones from the pulse lengthening. However, for propagation distances longer than 1 m the effect of molecular relaxation $(O_2 \text{ and } N_2)$ cannot be neglected.

The dependence of the front shock rise time τ_{sh} (from 0.1 to 0.9 of the peak pressure) on the propagation distance is presented in Fig. 2(d). If only spherical divergence is considered, there is no change in the rise time. If the relaxation effects are taken into account, the rise time is calculated by considering the front shock amplitude [from zero pressure up to the cross marker in Fig. 2(a)] rather than using the maximum pressure of the wave (circle marker). The rise time coincides with the rise time of the initial wave showing that relaxation effects have no influence on smoothing the shock. On the contrary, the thermoviscous absorption significantly increases the rise time. If nonlinear effects only are considered (dashed curve), the rise time tends to decrease to zero, as described by the weak shock theory.³² However, the minimum rise time of 0.05 μ s is achieved in numerical simulations due to internal viscosity of the numerical algorithm. This internal viscosity is much weaker than the real air viscosity and hence does not introduce significant error to the results of simulations. Finally, if all the effects are taken into account (black solid curve), the shock front rise time is governed by the competition between the nonlinear and absorption effects. In this competition, the role of nonlinear effects diminishes with the propagation distance because the pressure level decreases.

In the next section, theoretical predictions obtained from the model that include all previously mentioned effects are compared to experimental data measured using a 3 mm condenser microphone.

Acoustically measured pressure waveforms are analyzed first to define the parameters of the *N*-wave in the recorded signals and to set an accurate boundary condition to the model. Simulations and acoustic measurements of the peak pressure, *N*-wave duration, and shock rise time are then performed along the propagation path, and the accuracy of the rise time measurements are improved using optical methods.

IV. PRESSURE MEASUREMENT USING ACOUSTIC MICROPHONES

The goal of this section is to compare, for increasing propagation distances, theoretical predictions to pressure measurements using a microphone. Accurate characterization of the *N*-wave which is used as an input to the model is a critical point. First we detail how the duration and the peak pressure of the measured wave can be determined despite the microphone-induced distortion of the waveform. Simulations and acoustic measurements of the peak pressure, *N*-wave duration, and shock rise time are then compared along the propagation path, and the accuracy of the rise time measurements is improved using optical methods.

A. Characterization of the *N*-wave parameters in experiment

1. Estimation of the N-wave duration from measurement

The straightforward definition of the *N*-wave duration in the modeling is taken as the time between the points of the half peak positive and negative pressure values. However, in the measurements, the waveform is distorted due to the limited bandwidth of the microphone; therefore, the definition is not so obvious. This section details how the duration of the *N*-wave can be accurately estimated from the spectrum of the measured waveform even if it is filtered by the microphone. Accurate estimation of the *N*-wave duration is also critical since this parameter is used to determine the sensitivity of the microphone by analyzing the pulse lengthening due to nonlinear propagation effects.¹

The pressure waveforms obtained from the microphone output are typically different from an *N*-wave: They have a much more complex and asymmetric shape with oscillations.^{2,3} This distortion is due to the limited frequency response of the microphone and amplifier and also due to the diffraction effects on the edge of the residual small gap between the microphone and the baffle.¹ Even if an ideal pressure *N*-wave is incident on the membrane of the microphone, the output voltage from the amplifier v(t) is not a symmetric *N*-wave. Consequently, the estimation of the pulse duration is significantly influenced by the frequency response of the measuring system. In contrast to other papers,^{1,6} the duration of the pulse is estimated here from the spectrum of the wave rather than from the waveform itself.

The definition of pulse duration in the frequency domain is based on the analysis of the frequencies where the amplitude of the spectrum is minimum (close to zero).³ If the microphone response and diffraction effects can be represented in the frequency domain as a smooth transfer function, the frequencies of the spectrum minima (which are zeros for an ideal *N*-wave) should be the same for the *N*-wave spectrum and for the corresponding output voltage from the microphone. If $\hat{p}(f)$ is the spectrum of the pressure *N*-wave incident on the microphone (in Pa), and $\hat{H}_M(f)$ is the measurement chain frequency response (in V/Pa), then the spectrum of the output voltage from the amplifier is $\hat{v} = \hat{p}\hat{H}_M$. If the high frequency cutoff of the response \hat{H}_M is above the frequency of the second minimum in the spectrum \hat{p} , the frequencies of the first two minima of the N-wave spectrum \hat{p} and of the measured spectrum \hat{v} will be the same and well detectable. If the incident pressure wave is close to an ideal N-wave, then its duration can be deduced from the frequencies f_1 and f_2 of the first two minima in the spectrum \hat{v} , even if the voltage signal v(t) is not an N-wave. For an ideal N-wave, its half duration T is related to f_1 and f_2 by $T = 0.718/f_1$ and $T = 1.226/f_2$. In this paper, it is assumed that pressure waveforms are close to ideal N-waves. The duration of the measured wave v(t) is therefore considered to be equal to that of the ideal N-wave whose frequencies of the first two minima in the amplitude spectrum \hat{p} match the frequencies of the first two minima in the spectrum \hat{v} of the measured signal.

2. Setting initial peak pressure and duration of the N-wave in the modeling

To proceed with the numerical modeling, the initial peak pressure and duration of the N-wave should be set at $r_0 = 15.8$ cm from the source to use as a boundary condition to the model. The half duration of the pulse is deduced from the spectrum as explained in the previous section. The peak pressure at r_0 is determined according to the following method proposed in the earlier papers.^{1,3,6} As it was shown in Fig. 2 the effects of nonlinear propagation are dominant close to the source (r < 1 m); thus, it is assumed that the lengthening of the *N*-wave with the propagation distance *r* is linked to the initial peak pressure according to the weak shock theory.³⁴ The mean half duration of the experimental wave is estimated at different distances from the source and the linear dependence of T^2 on $\ln(r/r_0)$ is then fitted to find the initial peak pressure of the wave. The following values are obtained: Peak pressure $p_{\text{max}} = 1400 \pm 80$ Pa and half duration $T_0 = 19.0 \pm 0.1 \ \mu s$.

B. Results

In this section the results of the microphone measurement of the pressure waveform, peak pressure, half duration, and rise time of the front shock at different distances are compared to the theoretical data obtained with medium and initial wave parameters identical to that in the experiment [temperature = 291 K, relative humidity = 34%, $r_0 = 15.8$ cm, $p_{max}(r_0) = 1400$ Pa, $T_0(r_0) = 19.0 \ \mu$ s].

1. Waveform and spectrum

The pressure waveform obtained from the microphone output voltage at 48.8 cm from the source and the corresponding amplitude spectrum (dashed curves) are compared in Fig. 3 to the results of numerical simulations (solid curves). The experimental and the calculated waveforms are quite different. The shape of the calculated waveform is almost symmetric which confirms that oxygen and nitrogen relaxation has little effect at short propagation distances

J. Acoust. Soc. Am., Vol. 128, No. 6, December 2010



FIG. 3. (Color online) Measured (dashed) and calculated (solid) waveforms at the distance r = 48.8 cm from the source (a); corresponding normalized amplitude spectra (maximum at 0 dB) (b).

(Fig. 2). Contrary to that, the measured waveform is much less symmetric, has oscillations on the back slope, and the rise time of both shocks is much longer. Concerning the amplitude spectrum [Fig. 3(b)], at frequencies <80 kHz the measured spectrum (dashed curve) is very close to the calculated one (solid curve) both in its shape and amplitude. At frequencies higher than 80 kHz the amplitude of the measured spectrum decreases with much higher rate than the calculated one. These differences, that are due to the limited frequency response of the measurement system, explain the fact that the rise time of the measured waveform is much longer than that of the calculated one. A good agreement between the positions of spectrum minima is achieved at all ranges of frequencies, that validates the method of estimating pulse duration from the spectrum minima. Note, that the values of half duration T calculated using only the first f_1 or only the second f_2 minima agree within an interval of 3%.

2. Peak pressure, duration, and rise time

Measured and modeled propagation curves of (a) the peak pressure, (b) the duration, and (c) the rise time are compared in Figs. 4(a)–4(c). Similar to the results shown in Fig. 2, the peak pressure in Fig. 4(a) is multiplied by the ratio r/r_0 in order to compensate for the decrease of the pressure amplitude due to the spherical divergence of the wavefront.



FIG. 4. (Color online) Theoretical (solid line) and experimental (markers with error bars) data for peak positive pressure (a), half duration (b), and rise time (c) of the wave.

The error bars in Fig. 4 are deduced from statistical processing of 100 measured waveforms. The experimental and theoretical decreases of the peak pressure with the propagation distance are in accordance within an interval of 4%. Concerning the *N*-wave duration, experimental and theoretical data are also in good agreement, within an interval of 2% [Fig. 4(b)]. On the contrary, it is clear from Fig. 4(c) that there is no agreement between the rise time of the shock front estimated from the output voltage of the microphone and the rise time deduced from the model. The huge overestimation of the rise time in the experimental signal is due to the limited frequency response of the measurement chain (microphone and amplifier) at high frequencies.¹⁹ Note that some authors used handmade microphones with higher frequency cutoff,^{1,6} however they were still limited in frequency to accurately measure the shock front.

Very good agreement between the experimental and theoretical data for the peak pressure and duration tends to show that the model is very accurate, but the ability of the model to predict shock structure has to be confirmed. Since microphones are not able to measure correctly high frequencies, optical measurements based on shadowgraphy have been performed to measure the fine structure of the front shock with higher accuracy.

V. MEASUREMENT OF THE FRONT SHOCK RISE TIME USING SHADOWGRAPHY

Using optical methods it is possible to reconstruct the spatial variation of the pressure wave with a good resolution.²¹ The rise time of the shock front can then be calculated from the pressure rise in space (shock width). Among numerous visualization methods for compressible flows (schlieren, interferometry, etc.), the shadowgraphy technique^{21,22} is chosen here because it is relatively simple in design but still sufficiently sensitive to obtain images of the acoustic shock wavefront. According to this method the distribution of light intensity in space is photographed and then analyzed. The pattern of the light intensity is formed due to the light refraction on inhomogeneities of the refraction index caused by variations are produced by the acoustic wave.

The shadowgraphy technique has been used in literature to capture hydrodynamic shocks in supersonic flows (Mach number > 1.12), but it was reported to be unsuitable for the direct measurement of shock thickness.²⁶ The difficulty is related to strong diffraction effects (shock thickness δx ~ 0.5 μ m was close to light wavelength),²⁷ which dominate light focusing and complicate interpretation of light intensity patterns.²² However, it was detected that the width of the central dark stripe in the shadow depends on a jump in refraction index across the shock, but this information was not correlated with the shock thickness. To assess the shock amplitude and shock structure of such strong shocks, other methods based on measurements of light reflection coefficient have been proposed.^{24,25}

The shocks investigated in the present work are very weak in comparison with those studied in Refs. 26 and 27: Typically, a measured acoustic wave has a peak pressure $p_{\text{max}} < 1500$ Pa, i.e., a Mach number M < 1.0045, and the theoretically estimated shock thickness is greater than 30 μ m. In this case a "focused" shadowgraphy technique can be used. Shadow images called *shadowgrams* are captured by a camera at some distance from the shock wave by changing the position of the objective focal plane. Shadowgrams are interpreted by comparison with simulation of light propagation through the inhomogeneity of the refraction index induced by the front shock of the *N*-wave. In this way, the front shock thickness and its rise time can be obtained. Modeling of light propagation and the analysis of the shadowgrams are detailed in the next sections.



FIG. 5. (Color online) Positions of the camera observation plane along the y axis that correspond to the shadowgrams given in Fig. 6: 32 mm (a), 12 mm (b), 2 mm (c), -4 mm (d), -13 mm (e), and -24 mm (f). Shock wavefront is plotted schematically as a solid curve.

A. Measurement and processing of shadowgrams

First, six photographs of the shock are recorded for the acoustic source-optical axis distance $|x_S| = 15.8$ cm (Fig. 1) and for six different positions of the focal plane of the camera on the y-axis (Fig. 5): y = 32 (a), 12 (b), 2 (c), -4 (d), -13 (e), and -24 mm (f). The position of the focal plane was changed by moving the camera along the light propagation direction y while keeping the focal distance of the camera fixed. The shadowgrams are shown in Figs. 6(a)-6(f); the dimensions of each image are 1 mm (97 pixels) in x direction and 12.4 mm (1186 pixels) in z direction; and the image scale is 0.0104 mm per pixel. The shadows induced by the front shock have a spherical curvature with a radius of 15.2 \pm 1.3 cm, which is in good agreement with the chosen acoustic source—optical axis distance $|x_S| = 15.8$ cm. Distribution of the light intensity in the focal plane of the camera is recorded in the shadowgram image. Each image contains one dark and one bright stripe that represents a shadow of the shock front. The bright stripe is formed by converging and the dark stripe by diverging light due to bending of the light while passing through the shock front. It can be seen in Fig. 6 that



FIG. 6. Shadowgrams captured at different positions of observation plane along the light propagation path y = 32 mm (a), 12 mm (b), 2 mm (c), -4 mm (d), -13 mm (e), and -24 mm (f).



FIG. 7. Normalized intensity distributions $(I - I_0)/I_0$ along the x-axis obtained from shadowgrams given in Fig. 6. I_0 is the background intensity. In the subplot (c) a zoom is plotted.

the bright and dark stripes are inverted if the focal plane of the camera is set upstream [y < 0 for Figs. 6(d)-6(f)] or downstream [y > 0 for Figs. 6(a) and 6(b)] the grazing point O. The images recorded with the camera focused downstream the grazing point (y > 0) are *real* or *normal* shadowgrams formed at the observation plane, while images recorded with the focus upstream the grazing point (y < 0) are so-called imaginary or virtual shadowgrams, which are inverted as if one looks across a diverging lens.^{21,22} The inversion of the position of the dark and bright stripes in real and virtual shadowgrams will be illustrated in more detail later in the paper. In shadowgraphy measurements here the inversion effect is used to determine the position of the y = 0 mm plane. The error in the localization of the origin (y = 0) is about ± 1 mm. The shock shadows in the images that are taken close to the position y = 0 [Fig. 6(c)] are thinner and the contrast between the dark and bright stripes is lower; the shadows become wider when the focal plane is moved away from the origin. To quantify the width of the shadow of the front shock, the following image processing is applied: (1) Spherical curvature of the shock shadow is compensated in order to obtain straight vertical stripes, (2) two dimensional (2D) images with vertical stripes are averaged along the z direction to obtain the variation of light intensity along the x-axis, and (3) background noise intensity is estimated and subtracted from the signal.

The resulting one dimensional intensity distributions calculated from the 2D images are presented in Figs. 7(a)–7(f). To increase the signal/noise ratio, 15 shadowgrams are recorded at each y position, and the average intensity is computed after compensation of small random shifts in the shock arrival time due to small variations of spark position. The width of the front shock shadow is further referred as the *shadow width* and is defined as the distance Δx between maximum and minimum values of the light intensity distribution [Fig. 7(a)].

B. Calculation of the rise time from shadowgrams

The shadow width Δx [Fig. 7(a)] is related to the front shock thickness, which is equal to $c_0 \tau_{sh}$, and thus is also related to the rise time τ_{sh} . In order to establish the relation between the shadow width Δx and the shock thickness $c_0 \tau_{sh}$, propagation of light through the inhomogeneity of refraction index induced by the pressure shock is simulated. Two models that govern light propagation through the shock are built and compared. The first model is based on the geometrical optics approximation.³⁰ This approximation is very often used to explain image properties in optical methods.²⁸ However, diffraction effects were reported to have strong influence on shadowgraphy images.²⁶ In order to reveal the effect of light diffraction on image formation in our experiments, a second model based on the parabolic diffraction equation is built³¹ and the results obtained using these two models are compared.

1. Inhomogeneity of refraction index induced by the front shock

Acoustic pressure *p* can be related to the perturbations of the refraction index *n* of the light. The relationship between the refraction index and air density $\rho_0 + \rho$ is linear: $n = 1 + k(\rho_0 + \rho)$, where $k = 0.00023 \text{ m}^3/\text{kg}$ is a Gladstone– Dale constant,²² $\rho_0 = 1.19 \text{ kg/m}^3$ is the ambient density, and ρ is the density perturbation caused by the acoustic wave. Under our experimental condition, the density perturbation can be regarded as a linear function of acoustic pressure $p: \rho = p/c_0^2$; higher order terms can be neglected as the acoustic pressure is small compared to the ambient atmospheric pressure: $p_{\text{max}}/p_{\text{atm}} \sim 0.01$. Hence, the refraction index can be expressed as

$$n = 1 + k \left(\rho_0 + \frac{p}{c_0^2} \right).$$
 (2)

In the experiments, shadowgrams are recorded at the time *t* when the front shock grazes the optical axes (Fig. 1). At this time, the radius of curvature of the spherically diverging front shock is $R = |x_S|$, where x_S is the acoustic source position on the *x*-axis. Since the radius of front shock curvature *R* is much greater than both shock thickness $c_0 \tau_{sh}$ and the shadow width Δx , no significant gradient in distribution of the refraction index is expected along the *z*-axis which is perpendicular to the *y* coordinate of light path and tangent to the shock front. The deflection of light therefore will mainly occur in the *x*-direction and simplified 2D geometry can be used in simulating light propagation in the z = 0 plane.

As mentioned in Sec. III A, the front shock of the acoustic wave in homogeneous air is well described by the quasistationary solution of the classical Burgers equation. The shock has the form of hyperbolic tangent and the rise time is determined by the shock amplitude and thermoviscous absorption in the medium.³² The refraction index at the time when the shadowgram is recorded thus can be written as

$$n = n_0 + \frac{\Delta n_{\rm sh}(r - R + d)}{2d} \times \left[\tanh\left(2.2\frac{r - R}{c_0\tau_{\rm sh}}\right) - \tanh\left(2.2\frac{r - R + 2d}{c_0\tau_{\rm sh}}\right) \right].$$
(3)

Here $r = r(x, y) = \sqrt{(x - x_S)^2 + y^2}$, $n_0 = 1 + k\rho_0$ is the refraction index of ambient air, $\Delta n_{\rm sh} = kp_{\rm max}/c_0^2$ is the magnitude of the refraction index variations, and $d = c_0 T$ is the half length of the acoustic pulse.



FIG. 8. Formation of shadows. Spatial variations of refraction index caused by the acoustic shock front in the *xOy* plane. Contour dashed-dotted lines show refraction index levels corresponding to values 0.1 $\Delta n_{\rm sh}$, 0.5 $\Delta n_{\rm sh}$, and 0.9 $\Delta n_{\rm sh}$. Refraction of optic rays is schematically shown by arrows (solid lines), back propagation of rays is shown by dashed lines.

Spatial variations of the refraction index $n - n_0$ around the front shock of the acoustic wave, defined by Eq. (3), are plotted in Fig. 8. This picture is calculated using the parameters of the shock pulse modeled at R = 15.8 cm from the spark (Fig. 4): Peak pressure $p_{\text{max}} = 1400$ Pa, half duration $T = 19 \ \mu\text{s}$, and rise time $\tau_{\text{sh}} = 0.15 \ \mu\text{s}$. Dark-gray color $(n - n_0 = 0)$ corresponds to the ambient refraction index; light deflection on the refraction index inhomogeneity is schematically shown with white arrows. Black dashed contours correspond to the levels of 0.9, 0.5, and 0.1 (from left to right) of the jump in the refraction index at the shock front.

2. Light refraction model based on geometrical optics

Light refraction on the shock front (refraction index inhomogeneity shown in Fig. 8) is first calculated based on the geometrical optics approximation using a Hamilton– Jacobi type system of equations,³⁰

$$\frac{d\mathbf{s}}{d\theta} = \mathbf{k} ; \qquad \frac{d\mathbf{k}}{d\theta} = \frac{1}{2}\nabla n^2.$$
(4)

Here vector **k** is the wave vector of the light, $\mathbf{s}(x, y)$ is the ray trajectory, θ is the propagation path along the ray, and n = n(x, y) is the refraction index. Initial conditions correspond to the parallel beam incident from the negative direction of the *y* axis [Fig. 8(a)]. The system of Eq. (4) is solved numerically using a fourth order accurate Runge–Kutta algorithm. In this method no diffraction effects are included, and light intensity in the observation plane is inversely proportional to the cross-sectional area of the light ray tubes.

3. Light diffraction model based on the parabolic approximation

A second light propagation model is built to account for diffraction effects. This model is based on the parabolic



FIG. 9. (Color online) Intensity distributions calculated along the *x*-axis for three observation positions y = [0, 4, 8] mm using (a) geometrical optics model and (b) parabolic equation model.

approximation of scalar diffraction theory.³¹ For the harmonic wave propagation the equation has the form

$$\frac{\partial E}{\partial y} = \frac{i}{2k_0} \frac{\partial^2 E}{\partial x^2} + \frac{i\Delta n(x,y)}{n_0} k_0 E.$$
(5)

Here $\Delta n(x, y) = n(x, y) - n_0$ is the variation of the refraction index, *E* is the electric component of the light, $k_0 = 2\pi/\lambda$ is the wave number, λ is the light wavelength, and *i* is the imaginary unit. Equation (5) is solved in finite differences using an operator splitting procedure.³⁵ At each grid step in the propagation direction *y* the diffraction operator [first term of the right-hand side in Eq. (5)] is calculated using the Crank–Nicholson algorithm. Variations of the refraction index [second term of the right-hand side in Eq. (5)] are taken into account using the exact solution.

To model the white light flash used in the experiment, a set of wavelengths in the range from 370 to 780 nm is superposed. Light propagation of 15 wavelengths uniformly distributed in the given range is calculated based on Eq. (5). The initial condition remains the same for each light wavelength and is given by $E(x, y = y_0) = 1$. It is verified that a greater number of wavelengths does not significantly affect the modeling results. The light intensities are calculated for each wavelength as proportional to $|E|^2$ and then superposed to obtain the total intensity distribution.

4. Shadow width predicted with two light propagation models

To determine if light diffraction is important in formation of shock shadows, light propagation through the refraction index distribution (Fig. 8) is simulated using the geometrical and the parabolic models. Intensity distributions are then calculated along the horizontal dashed lines in Fig. 8(a) (y = 0, 4, and 8 mm) and presented in Fig. 9.

The results of both models show good qualitative agreement with the intensity distributions obtained from the shadowgrams (Fig. 7). At the y = 0 position, the contrast is very low, the bright component of the shadow is not well pronounced, and the shadow width Δx cannot be measured. With increasing distance of the observation plane (y = 4 and 8 mm) from the origin y = 0 the bright component in the



FIG. 10. (Color online) Shadow width modeled for images taken at different positions *y* of the camera focus. Dashed curves are obtained using the geometrical optics model and solid curves—using the parabolic equation. The change of the contrast in both sets of curves from dark to light corresponds to increasing shock rise time in the modeling. Points with vertical bars are shadow widths and corresponding standard deviations measured from experimental shadowgrams.

image and the contrast increase. The quantitative agreement between the two models is however not so good in both maximum intensity level and in the shadow width. In particular, significant broadening of the shadow for increasing distances *y* is observed only if diffraction is accounted for. This disagreement is more pronounced at longer distances *y*.

The summary of the modeling results for the shadow width Δx versus the position of the observation plane y is shown in Fig. 10. Simulations are performed for different rise times τ_{sh} of the shock front, i.e., for different variations of refraction index obtained using Eq. (3). Dashed curves are obtained with the geometrical optics model [Eq. (4)], and solid curves—with the diffraction model [Eq. (5)]. Pairs of curves (dashed and solid) correspond to the values of the shock rise time τ_{sh} varying over the range from 0.1 to 0.3 μ s. The contrast of both set of curves varies from dark to light when the rise time increases.

If the observation plane is too close to the position y = 0where the light just grazes the front shock, the light intensity variation is too small to estimate the shadow width and both models give erroneous estimations. Away from the position y = 0, the comparison of dashed and solid curves in Fig. 10 shows that the results obtained using geometrical optics or the parabolic approximation differ strongly. According to geometrical optics, the shadow width Δx does not increase when the observation plane is moved away from y = 0, while the parabolic approximation model predicts an increase of the shadow width by a factor of roughly 3. This difference outlines that diffraction effects cannot be neglected in the analysis of shadowgrams.

In order to discuss the ability of the two models to predict experimental data, shadow widths obtained from shadowgrams measured at different observation plane positions y are plotted in Fig. 10 with dots. Error bars indicate standard deviations on the estimation of Δx . Sources of error are mainly finite pixel size and small variations of the shadow width from one shot to another. The comparison of the mod-



FIG. 11. Light intensity distribution on (a) real shadows and (b) real and imaginary shadows.

eling and experimental data confirms that light diffraction cannot be neglected: At observation positions y > 2 mm, the shadow width obtained in the parabolic approximation modeling grows in a similar way as in experiment, while the geometrical optics results strongly differ from the experimental data. The only region where geometrical optics could be used is the region within 2 mm near the y = 0 position where diffraction effects are not important, but even in this region geometrical optics overestimates Δx . At longer distances y a geometrical optics propagation model is not appropriate to predict the shadow width Δx . Consequently, only results of the parabolic approximation modeling will be further used to deduce shock rise time from shadowgraphy images.

However, note that even if diffraction is accounted for (solid curves), the difference between the shadow widths calculated for different shock rise times depends on where the observation plane is positioned. If the coordinate of the observation plane is beyond y = 10 mm, the difference between shadow widths obtained for different rise times $\tau_{\rm sh}$ decreases significantly and becomes almost negligible beyond the distance y = 20 mm. Shadowgrams taken at these positions therefore would lead to strong uncertainty in the estimation of the rise time $\tau_{\rm sh}$.

A diffraction propagation model is also used to explain inverted patterns of *real* and *virtual* shadows. First, consider a light intensity field of parallel beams passed through the shock, Fig. 11(a). The shock configuration is the same as in Fig. 8. The intensity of *real* shadows is uniform upstream the grazing point. Nevertheless, the optical system focused upstream this point perceives the image formed by the light propagated backward through *homogeneous* air. The intensity field of backward propagated light is shown in Fig. 11(b). It indicates inversion of the shadow's pattern between upstream and downstream areas that is in accord with experimental results (Fig. 6).

5. Estimation of the shock rise time from shadowgrams

Shock rise time is deduced from the measured shadowgrams based on the comparison between shadow widths Δx in the recorded image and in the results of the light diffraction modeling. As the shadow width can be modeled for a given front shock rise time τ_{sh} and observation plane position y (Fig. 10), it is possible to find which rise time of the shock in simulations corresponds to a measured shadow width.

An interval of observation plane positions y for results obtained using the parabolic approximation (Fig. 10) is chosen where it is possible to estimate the rise time by comparing predicted and measured shadow widths. At observation planes too close to y = 0, the contrast of theoretical shadowgrams is too low. At observation planes positioned beyond y = 10 mm the difference between the theoretical Δx obtained for different rise times τ_{sh} is smaller than the experimental error bars. A wide range of rise times could thus be associated to a small range of measured shadow width, which would lead to a huge uncertainty in rise time estimation. The shadow width is most sensitive to τ_{sh} at positions 2 mm < y < 5 mm. For example, among shadowgrams given in Fig. 6, only shadowgrams (c) and (d) can be used. The front shock rise time thus is obtained by fitting the rise time in the model to produce the same shadow width Δx as in the corresponding measured shadowgram, with the condition that the observation plane position y is in the range [2, 5] mm. Note that the virtual shadowgrams with focus at positions $y \in [-5, -2]$ can be also used. Note also that the shadow width theoretical prediction is not sensitive to fluctuations in peak pressure: Possible error in the shock width estimation due to peak pressure uncertainty is less than 1% (only shadow contrast is affected, lower p_{max} produces lower contrast). An interval of confidence of τ_{sh} can be obtained by determining the rise times that correspond to the higher and lower values of the Δx error bars.

Using this method with the spark source positioned at 15.8 cm from the optical axis, the analysis of shadowgrams measured with the camera focused at the y = 2 mm [Fig. 6(c)] gives an estimation of the rise time $\tau_{\rm sh} = 0.15 \pm 0.03 \ \mu s$. This value agrees very well with the result predicted by the acoustic propagation model and is about 17 times smaller than deduced from the microphone output voltage.

C. Shock rise time in acoustic modeling and optical measurements

In order to test the ability of the sound propagation model to predict the increase of the front shock rise time τ_{sh}



FIG. 12. (Color online) Comparison between the optically measured rise time (circles) and the rise time predicted by using the acoustic wave propagation (crosses) at different distances from the spark source.

with the sound propagation distance r [Fig. 2(d)], shadowgrams are recorded for increasing spark source-optical axis distances $|x_S|$ over the interval [15.8, 69.5] cm, that is for increasing wavefront radius R. The rise time of the front shock is deduced from the images by fitting the shadow width in the image and in the results of the diffraction model. Note, that at distances longer than 65.9 cm the shadowgrams contrast was not sufficient to allow accurate analysis. A first reason is the spherical divergence of the wave that induces amplitude decreases, that itself leads to the lowering of image contrast. A second reason is the front shock rise time increase with the distance, which also decreases the contrast of the image.

In Fig. 12, the front shock rise time deduced from the optical method and the prediction using the wave propagation model [Eq. (1)] are compared. There is a very good agreement between the results obtained in acoustic modeling and measured optically. The rise times deduced from the experiments are slightly higher than those predicted by the Burgers equation, but the relative error does not exceed 10%. This result confirms that the rise time of the front shock is much smaller than it could be deduced on the basis of the microphone output voltage.

Note that, successfully applied to measure the front shock, shadowgraphy appeared to be not sufficiently sensitive to detect the rear shock of the acoustic pulse. One explanation could be that the rear shock is smoother than the front shock due to relaxation effects and diffraction on the spark source electrodes. To measure the rear shock with precision using optical methods, a more sensitive experimental setup should be developed; laser interferometry could be a possible technique.²⁹

VI. SUMMARY AND CONCLUSION

Propagation of nonlinear *N*-waves in homogenous relaxing air is studied theoretically and experimentally. Numerical simulations are performed based on a Burgers-type equation generalized to describe spherically divergent nonlinear sound waves in a medium with thermoviscous absorption and oxygen and nitrogen relaxation. The relative influence of all mentioned phenomena on *N*-wave distortion is studied numerically. Simulations show that the decrease of the peak pressure of the *N*-wave with propagation distance is affected by both nonlinear dissipation and linear absorption as well as the relaxation phenomena. The pulse lengthening is mainly determined by nonlinear effects. The rise time lengthening is mainly determined by the thermoviscous absorption.

Numerical results are compared to results from two experiments. The initial wave parameters for simulations are deduced by fitting the dependence of pulse duration on the propagation distance in modeling and acoustic measurements. As proposed in Ref. 3, the duration of the *N*-wave is deduced from the wave spectrum rather than from the time domain waveform.

Comparison of numerical simulation results with microphone pressure measurements shows quite good agreement for peak positive pressure values and duration of the pulse. However, the rise time of the front shock deduced from the microphone output is largely overestimated because of the limitations in the frequency response of the measuring system.

In order to estimate rise time of the front shock more accurately, optical measurements based on a focused shadowgraphy technique are performed. Shock front shadowgrams are captured and analyzed. To interpret shadowgrams, the refraction index inhomogeneity produced by the shock is modeled and two light propagation models are tested: The first is based on the geometrical optics approach, and the second is based on the scalar diffraction theory and the parabolic approximation. It is shown that geometrical optics results in overestimation of the acoustic shock rise time. The rise times deduced from shadowgrams using the parabolic model are in a very good agreement with the acoustic modeling, thus validating the acoustic wave propagation model [Eq. (1)] and the accuracy of the optical measurements. However, only shadow images captured close to the point where the light grazes the front shock are useful for the rise time estimation. The results from measurements of short duration N-waves by microphones should be analyzed with care because the response of the measurement system has a great influence on shock rise time estimation.

The combination of modeling, acoustical, and optical measurements provides accurate characterization of high amplitude shock pulses in laboratory experiments in air. Now after the validity of the prediction of *N*-wave propagation in homogeneous atmosphere has been established, comparison between theoretical and experimental data can be done for more complex atmospheric conditions, including the presence of turbulence in air and the boundaries.¹²

ACKNOWLEDGMENTS

This work has been partially supported by Russian Foundation for Basic Research (RFBR) and Projet International de Coopération Scientifique (PICS) grants. Computer simulations were performed using "Chebyshev" SKIF–Moscow State University (MSU) computer complex. The authors are grateful to Jean-Michel Perrin for his help with the fabrication of the experimental setup and to Olivier Marsden for the helpful comments on the manuscript. ¹W. M. Wright, "Propagation in air of *N* waves produced by sparks," J. Acoust. Soc. Am. **73**, 1948–1955 (1983).

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3332 J. Acoust. Soc. Am., Vol. 128, No. 6, December 2010

Yuldashev et al.: Nonlinear propagation of N-waves in air

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