Fully Coupled Finite-Element Modeling of Active Sandwich Panels With Poroelastic Core

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Active sandwich panels are an example of smart noise attenuators and a realization of hybrid active-passive approach for the problem of broadband noise reduction. The panels are composed of thin elastic faceplates linked by the core of a lightweight absorbent material of high porosity. Moreover, they are active, so piezoelectric actuators in the form of thin patches are fixed to their faceplates. Therefore, the passive absorbent properties of porous core, effective at high and medium frequencies, can be combined with the active vibroacoustic reduction necessary in a low frequency range. Important convergence issues for fully coupled finite-element modeling of such panels are investigated on a model of a disk-shaped panel under a uniform acoustic load by plane harmonic waves, with respect to the important parameter of the total reduction of acoustic transmission. Various physical phenomena are considered, namely, the wave propagation in a porous medium, the vibrations of elastic plate and the piezoelectric behavior of actuators, the acoustics-structure interaction and the wave propagation in a fluid. The modeling of porous core requires the usage of the advanced biphasic model of poroelasticity, because the vibrations of the skeleton of porous core cannot be neglected; they are in fact induced by the vibrations of the faceplates. Finally, optimal voltage amplitudes for the electric signals used in active reduction, with respect to the relative size of the piezoelectric actuator, are computed in some lower-to-medium frequency range. [DOI: 10.1115/1.4005026]

Keywords: active sandwich panels, multiphysics, vibroacoustics, poroelasticity, piezoelectricity

1 Introduction

A sandwich panel is a special class of composite materials fabricated by attaching two thin but stiff faceplates to a lightweight but thick core. Commonly used core materials are open and closed cell foams (like polyvinylchloride, polyurethane, polyethylene or polystyrene foams), syntactic foams, and honeycombs. In case of an *active* panel some piezoelectric elements are usually fixed to one or both faceplates. In general, sandwich panels, apart from being lightweight and flexible, can be very effective in reducing vibrations, and in some instances they may even sustain high velocity impacts. An increasingly important feature of such panels is their ability to be also good sound insulators. Active sandwich panels with a core made up of acoustically insulating material are a very good example of the so-called *hybrid active-passive approach* for noise reduction, which combines passive techniques efficient in attenuation of high- and medium-frequency contributions of noise and vibration, with active techniques of the active structural acoustic control (ASAC) [1], which appear to be the only way to reduce the low frequency components.

Research on hybrid sandwich panels for acoustic insulation is obviously based on a considerable body of literature concerning — separately — passive acoustic panels and the ASAC approach [1]. However, most of the work concerning the classic research on active approach was limited to weakly damped metallic structures and it is obvious that composite structures are substantially different than their metallic counterparts. Thus, first works on active sandwich panels are mainly focused on the feasibility and performance of some dedicated algorithms for active structural acoustic control of such panels. For example, Petitjean et al. [2] conducted experiments on a square panel with a honeycomb core, to compare and evaluate various active control strategies for acoustic radiation reduction. They observed that the high level of structural damping exhibited by typical composite structures, including sandwich honeycomb panels, adds to the difficulty of dynamic analysis. Experimental tests were also performed by Lee et al. [3] to check feasibility of the concept of active-passive hybrid panels made up of a layer of porous noise-absorbing material with an aluminum faceplate, or two faceplates and an air gap in case of the so-called double panel. A piezoelectric patch was fixed to the aluminum faceplate and served as a sensor and actuator for the active approach adopted for the low frequency range, for which — instead of using a complicated controller — a simple negative feedback was used (in the passive case the circuit was simply shunt).

Accurate modeling and optimization of sandwich panels is quite complicated. Often, some substantial simplifications are practiced, especially, when modeling the material of the core. In sandwich panels for active-passive damping of vibrations the core can be often simply considered as viscoelastic (e.g., Ref. [4]); this is; however, insufficient in the case of panels for acoustic insulation. Thus, new theoretical models are still being developed, in particular, for sandwich panels for passive acoustic insulation. For example, Xin et al. [5] have recently proposed a model for the radiation of sound from an infinite orthogonally rib-stiffened sandwich structure filled with fibrous sound absorptive material in the partitioned cavity. The propagation of sound in the fibrous material is modeled by adopting an equivalent fluid model by Allard and Champoux [6,7], which assumes that the skeleton of porous medium is rigid. A more advanced approach to describe porous material, taking into consideration the elastic vibrations of skeleton, was used by Dauchez et al. [8] in their numerical investigation of the problem of a single plate with a bonded porous layer. Thus, the theory of poroelasticity [9] was applied for the porus medium, yet the problem was still purely passive. A similar example of a porous coated plate excited by a point force was also

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presented in the work by Rigobert et al. [10] on multilayered structures involving poroelastic, acoustic, and elastic materials.

An advanced fully coupled approach to model a problem of noise transmission through hybrid acoustic panels has been applied by Galland and her collaborators [11–16]. They have started to study active-passive sandwich panels with a core of poroelastic material and piezoelectric patches glued to elastic faceplates. The piezoelectric elements as well as the poroelastic core are modeled accurately, using fully coupled theories. However, the results presented in [11,12] are purely numerical and rather preliminary: there is no discussion concerning convergence nor any parametric survey. The works by Batifol et al. [13–15] bring a comparison of some selected results obtained using finite-element modeling with analytical developments, showing also some findings of a preliminary experimental test.

Similar fully coupled modeling involving poroelastic, piezoelectric, acoustic, and elastic materials will be used in the present work for the problem of active-passive reduction of acoustic transmission by a disk-shaped sandwich panel. The *h*- and *p*-convergence tests will be performed in order to check necessary mesh densities and shape functions for approximations required by subdomains of various media. This will be done: first, for the passive case, when the piezoelectric actuator is inactive and some numerical results may be compared with the analytical solutions, and next, for the active approach, where it will be shown that the requirements for the mesh density and the order of approximation are much more demanding. In that way, 2D finite-element meshes suitable for axially symmetric problems will be investigated together with a 3D finite-element mesh which may be extended for nonsymmetrical configurations. Finally, some frequency and parametric studies will be carried out using a numerically economical finite-element model chosen thanks to the convergence analyses, and some optimal voltage amplitudes for the active reduction signals will be found in a wide range of frequencies and with respect to the size of piezoelectric actuator. Although the investigated disk-shaped model is directly related to a typical experimental setup, the procedure discussed in the paper, as well as the indications concerning modeling obtained from the convergence tests, are valid for a variety of other configurations. Moreover, even the results of parametric survey are in some way relevant — though only to some degree — for a more practical setup of large sandwich panels, because the assumed boundary conditions are appropriate if one considers a typical (i.e., far from the edges) *small* repeatable fragment of a vast, uniform panel with a regular distribution of piezoelectric actuators and a macrohomogeneous porous core.

2 Fully Coupled Modeling

The following relevant theoretical models will be used for fully coupled modeling of a segment of an active sandwich panel: (a) the Biot's theory of poroelasticity – to model the vibroacoustic transmission and dissipation of acoustic waves in a porous core, (b) the linear acoustics – to model the propagation of acoustic waves in the surrounding air or in air-waveguides, (c) the linear elasticity — to model the vibrations of elastic faceplates, and (d) the theory of piezoelectricity —to model the piezoelectric actuators relevant for the feasibility and performance of the active vibroacoustic reduction. Frequency analyses of the panel will be carried out; therefore, all the involved problems are time-harmonic. Moreover, they are coupled since the consideration of such mutual interaction of various media is essential for the problem. The piezoelectric actuator will be modeled using fully coupled electromechanical variational form which can be regarded as the sum of the conventional principle of virtual mechanical displacements and the principle of virtual electric potential [17–19]. Since one of the aims of this work is to provide accurate estimations of necessary voltage amplitudes for active signals, the electro-mechanical coupling must be complete, taking into consideration the so-called induced potential. It should be also emphasized that to model a porous core the advanced model of poroelasticity will be used, which allows to consider the elastic vibrations of the solid frame (skeleton). Since this model is not so well-known as the other involved theories, some basic facts about poroelasticity will be briefly discussed, and the necessity for its use in the considered problem will be justified below.

A very important component of the considered panel's assembly is a core of porous material; the porous core should be a good passive acoustic insulator in a wide range of medium and high frequencies. Although, for many porous materials the frame can be almost motionless for large spectrum ranges — thus allowing the use of models worked out for rigid-frame materials — this is not generally true for the entire range of acoustical frequencies. Moreover, for a porous material set between two elastic plates — as in a sandwich panel — and for many other similar situations, frame vibrations are induced by the vibrations of the plates [7]. The transmission of sound through such a sandwich can be predicted only in the context of a model where the air and the skeleton move simultaneously. Such a model is provided by the Biot's theory of poroelasticity [6,7,9]. This is a rather complex model, where an advanced, *bi-phasic* approach is applied, namely, two homogeneous continua are considered in the very same place of a porous medium: the so-called *solid phase*, used to describe the behavior of elastic skeleton (frame), and the *fluid phase*, which pertains to the fluid in the pores (a pore-fluid, typically the air). Obviously, these two phases are coupled since the important interaction between the pore-fluid and the elastic frame must be taken into account. The original formulation of this theory proposed by Biot in 1956 [9] assumes that a porous medium is macroscopically isotropic which means that both pases are isotropic. In this classic formulation the solid-phase displacements, $\boldsymbol{u}^s$, and the fluid-phase displacements, $\mathbf{u}^f$, are used as primary dependent variables, which in general, gives 6 degrees of freedom in every point of poroelastic medium. The *total* displacement vector is introduced as the sum of porosity-dependent contributions of the displacements of both phases, that is

$$\mathbf{u}^t = (1-\phi)\mathbf{u}^s + \phi\mathbf{u}^f$$

where ϕ is the porosity of poroelastic medium. In practice, the porosity of many acoustically-insulating materials (PU foams, etc.) can be very high (typically, more than 90%, and often reaching 98–99%), and so the solid-phase displacements may seem *not* to contribute much to the total displacement field. This would be, however, a very unjustified statement, because of some important coupling terms present in the Biot's equations of poroelasticity: in fact, the fluid-phase displacements can be strongly influenced by the solid-phase vibrations at some lower frequencies (of several hundred hertz and less), and thus the elastic behavior of the solid-phase substantially affect —at least indirectly — the total vibrations of a poroelastic medium.

Since a time-harmonic analysis of active sandwich panels will be considered in this work the so-called *displacement-pressure* ($\mathbf{u}^s-p$) formulation of Biot's poroelasticity can be used. This mixed formulation was developed by Atalla et al. [20,21] basing on the realization that in the time-harmonic case (the complex-amplitude vector of) the fluid-phase displacements $\mathbf{u}^f$ can be expressed as a combination of the solid-phase displacements $\mathbf{u}^s$ and the gradient of pressure ∇p in the fluid in the pores. In that way, three fluid-phase displacements are eliminated from the set of primary dependent variables, and replaced with only one parameter of pore-fluid pressure p. Thus, the dependent variables are now three solid-phase displacements and the pore-fluid pressure, so the mixed model have only 4 degrees of freedom in a node – instead of 6 degrees of freedom required by the original purely displacement ($\mathbf{u}^s-\mathbf{u}^f$) formulation. In their paper form 1998 Atalla et al. [20] presented also a weak integral formulation for the mixed version of the Biot's poroelasticity equations; the general boundary conditions were discussed in [21]. In 2001, an enhanced version for this weak integral form of the mixed displacement-pressure formulation was derived [22]. In this version the coupling integrals on the interface between two poroelastic media and also between poroelastic and elastic media are zeroed, so that relevant interface-coupling conditions as well as some boundary can be easily handled. As a matter of fact, this weak form was used to carry out all the finite-element analyses given in the present work. An alternative displacement representation of Biot's theory has been recently proposed by Dazel et al. [23,24] by choosing the generalized coordinates in order to simplify the expression of the strain energy, which is expressed as the sum of two decoupled terms. In a very practical case when the material of solid skeleton is incompressible this becomes simply the solid-phase displacement–total displacement ($\mathbf{u}^s$–$\mathbf{u}^t$) representation.

A complete theoretical basis for finite element modeling of a harmonically driven fully coupled system composed of poroelastic, acoustic, elastic, and piezoelectric domains is discussed in [25,26]. The theoretical discussion provides weak, variational formulations of the Biot's poroelasticity, linear elasticity, piezoelectricity, and acoustics. Furthermore, the interface-coupling conditions between all these media are extensively discussed and the corresponding coupling integrals are derived. Finally, a discrete model is obtained using the Galerkin finite element approximation: the coefficient matrices and components of right-hand-side vector of the system of algebraic equations are given together with some comments on their physical interpretations and properties (like frequency-dependence). Some experimental validation of this system — at least partial, since no porous media were involved — was presented in Ref. [27] for a problem of active reduction of structure-borne noise for a thin aluminum plate, with actuators in the form of piezoelectric patches. This discrete, fully coupled model will be used here for the problem of vibroacoustic transmission through a disk-shaped segment of an active sandwich panel with poroelastic core, coupled to a fragment of air-waveguide. The finite-element analyses will involve some convergence tests, as well as some feasibility study and a parametric survey concerning the piezoelectric actuator. Some guidelines concerning the design of a square piezoceramic patch actuator for the implementation of a stable proportional velocity feedback control loop on a thin plate can be found in Ref. [28].

3 Finite-Element Model of a Disk-Shaped Segment of an Active Sandwich Panel

Numerical analysis will be carried out for a disk of sandwich panel with poroelastic core and a thin PZT-ceramic patch fixed to one of its faceplates. In general, such a disk of panel can be considered as a fragment of a larger regular structure of active sandwich panel; however, its disk shape was chosen so it could be fitted into a circular impedance or transmission-loss tube in order to test the acoustical efficiency of such panel. Figure 1 shows a lateral view of such configuration, that is, the panel fitted into a rigid-walled air-waveguide (like in a tube). The total thickness of panel (without PZT-patch) is 20 mm and its diameter is 100 mm. Such diameter is suitable to examine (experimentally) acoustic impedance, sound absorption, and transmission loss in a range which includes also low and very low frequencies, that is; for example, from 50 Hz up to 1.6 kHz. It is obvious that the opportunity to test the acoustical performance at low frequencies is crucial for the active panel. The PZT-patch is 0.3 mm thick. It is also circular in shape and it is fixed in the center of the circular faceplate of panel. Therefore, the whole configuration of the panel in air-waveguide is perfectly symmetric relative to its axis. This is extremely important since it permits to apply two-dimensional axially symmetric approach to modeling. Nevertheless, three-dimensional modeling will also be applied (see Fig. 2). As a matter of fact, one of the purposes of this paper is to carry out a numerical study of the problem described above to determine proper polynomial orders for approximation functions in various domains of the problem, as well as necessary mesh densities, in order to ensure convergence of finite element solutions. To this end, two-dimensional axially symmetric finite element models will be used, and eventually, the three-dimensional model shown in Fig. 2 will

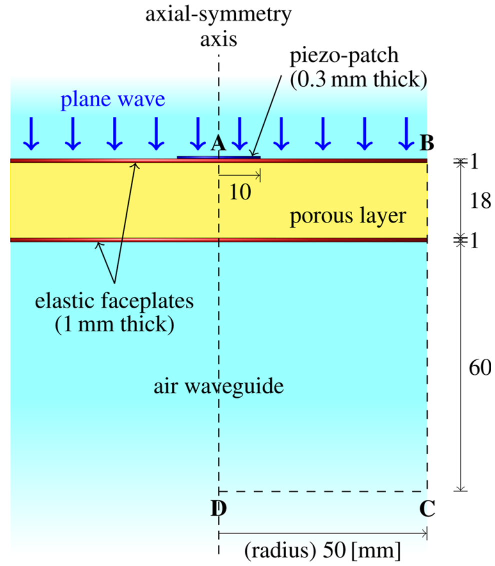

Fig. 1 A lateral view of an axially symmetric disk-shaped sandwich panel with a poroelastic core and a PZT-patch fixed to one of its faceplates, coupled to a fragment of air waveguide (the region ABCD represents a modeled domain)

be validated. Moreover, whenever it is possible, analytical solutions will also be exploited to validate finite element modeling.

It is assumed that the panel is fitted into the circular-tube waveguide in the way often required by the procedures for acoustic impedance and transmission loss tube testing; thus, it can smoothly slide along the tube, but the radial displacements on lateral faces of the ring-shaped sample of panel are blocked. A plane, time-harmonic, acoustic wave propagates in the air waveguide onto the panel's surface (its upper surface, accordingly with its orientation as presented in Figs. 1 and 2). In general, this normally incident wave is partially reflected, partially absorbed, and partially transmitted through the panel. The acoustic transmission through the panel will be analyzed for a wide frequency range of such harmonic excitation. In case of some lower frequency range the feasibility of active approach will be studied as well, which means that, apart from the acoustic excitation, an appropriate time-harmonic voltage signal is simultaneously applied to the electrodes of the piezoelectric patch fixed to the upper faceplate of panel in order to improve the total reduction of acoustic transmission which combines the effects of reflection and the actual transmission loss (TL) of the panel [29]. In that way, the passive as well as the active behavior of panel can be analyzed. In both cases, the

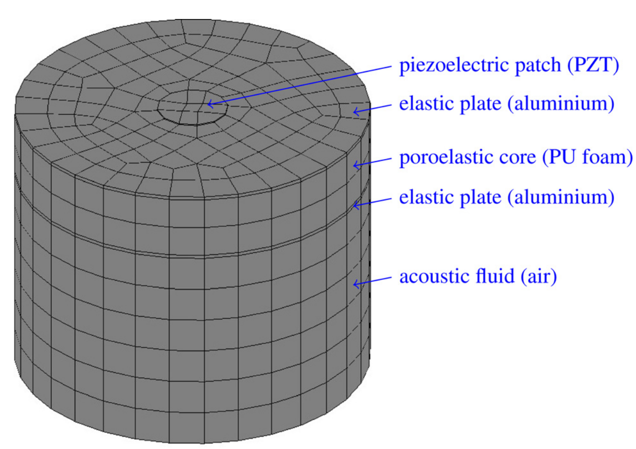

Fig. 2 3D finite-element mesh for a disk of sandwich panel with PZT-patch, coupled to a fragment of air waveguide

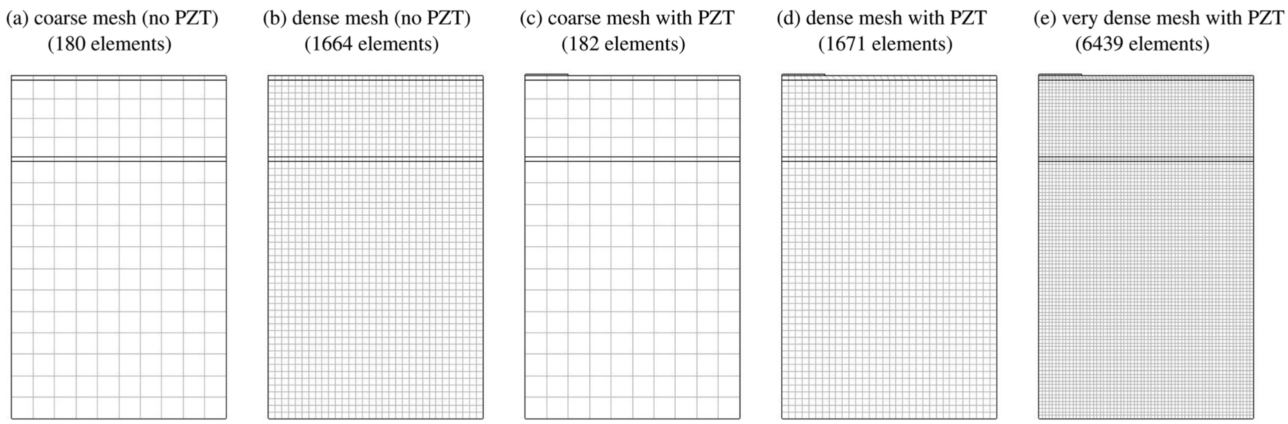

Fig. 3 Finite-element meshes for the two-dimensional axially symmetric domain of a disk of sandwich panel, with or without PZT-patch, coupled to an air waveguide

acoustic pressure, the sound pressure level (SPL), and the total reduction of acoustic transmission will be computed at point D situated on the symmetry axis, 60 mm from the lower surface of panel (see Fig. 1). Therefore, a fragment of air waveguide coupled directly to the panel is also modeled, with appropriate boundary conditions, namely: the rigid wall condition set on the boundary BC (or on the lateral surface of air domain in case of three-dimensional modeling) which means that the normal acceleration of air particles are there set to zero, and the nonreflecting condition applied to the boundary CD (or on the circular boundary surface of air domain in case of three-dimensional modeling) which for the time-harmonic analysis can be easily realized by applying the impedance boundary condition with the characteristic impedance of air.

The *h*-convergence of finite element solutions for the considered problem will be discussed below. To this end, several mesh cases of various density were used. For example, Fig. 3(*a,b*) presents two mesh cases, a coarse one and a dense one, for a purely passive sandwich panel coupled to a fragment of air waveguide, as presented in Fig. 1 as the region ABCD but with no piezoelectric element. They are two-dimensional meshes for axially symmetric panel; however, it should be noticed that the problem can be considered as one-dimensional propagation of plane waves. This is because, in that case, there is no piezoelectric patch fixed to the panel's faceplate, so that the panel is simply formed of parallel layers of aluminum faceplates and poroelastic core, and moreover, as explained above, the lateral boundary conditions block normal displacements allowing at the same time for lateral slip alongside the edges (as a matter of fact, these are the conditions usually required for samples tested in impedance tubes). Thus, no bending is involved and the panel's vibrations are homogeneous in plane. That allows for one-dimensional modeling for which analytical solutions exist. The analytical solutions will be used to validate the finite-element modeling of passive panel. On the other hand, the two meshes from Figs. 3(*a*) and 3(*b*) are compatible with coarse and dense meshes for active sandwich panel shown in Figs. 3(*c*) and 3(*d*), respectively. It will be shown that in some frequency range the presence of small piezoelectric patch, although it adds locally some stiffness and mass, has very small influence or no effect at all. Therefore, under some circumstances, the analytical solutions may also be used to validate the two- and three-dimensional finite-element meshes with respect to modeling of the *passive behavior* of active panel, that is, when the electrodes of piezoelectric patch are simply shunted, and the patch itself is not too large. Finally, the very dense mesh for active sandwich panel presented in Fig. 3(*e*) will be used, together with the mentioned coarse and dense mesh cases, to perform the *h*-convergence tests for solutions of the active behavior of panel, and to validate the *p*-convergence of active solutions obtained with the three-dimensional mesh presented in Fig. 2. Such numerical study should allow to assess necessary mesh sizes and approximation orders for the problem of active sandwich panel, where different materials and media are supposed to interact. It is obvious that different finite element discretizations and approximations are required for different subdomains, and moreover, the discretization must depend on frequency, since the wavelengths are different. Thus, the study is aimed to provide convincing, convergent solutions for two- and three-dimensional finite-element modeling of active sandwich panels, where sufficiently dense meshes combined with proper shape functions should render optimal finite element models, namely, the models with a very moderate number of degrees of freedom, yet at the same suitable to achieve accurate solutions in the desired frequency range. This is not by any means a trivial task since the wave propagation may change drastically between various media, and moreover, in the case of poroelastic medium there are in fact three waves that propagate with different speeds: a fast compressional wave and a shear wave, both originating mainly from the elastic solid of skeleton, and a slow compressional fluid-borne wave. Moreover, the poroelastic medium is dispersive, so that the velocities of waves may depend on frequency. This can be observed in Fig. 4, which shows the wavelengths and wave speeds for the open-cell PU foam used for the core. The material properties for the Biot-Johnson-Allard model [7] of this foam are as follows: the porosity $\phi = 0.99$, the tortuosity 1.98, the permeability $2.81 \times 10^{-10}\ \mathrm{m}^2$, the characteristic dimension of pores for viscous forces 37×10^{-6} m, and for thermal forces 121×10^{-6} m, the solid-phase mass density $16\ \mathrm{kg/m^3}$, the shear modulus $18(1 + 0.1i)$ kPa, the bulk Poisson ratio 0.3.

The actuator patch is made up of a transversally isotropic piezoelectric ceramic PZT, polarized along the x_3-axis, with the following material constants: the mass density $7500\ \mathrm{kg/m^3}$; the elastic constants $C_{1111} = C_{2222} = 127$ GPa, $C_{3333} = 117$ GPa, $C_{1122} = C_{2211} = 80.2$ GPa, $C_{1133} = C_{3311} = C_{2233} = C_{3322} = 84.7$ GPa, $C_{2323} = C_{3131} = 23.0$ GPa, $C_{1212} = 23.5$ GPa; the piezoelectric constants $d_{311} = d_{322} = -274 \times 10^{-12}\ \mathrm{m/V}$, $d_{333} = 593 \times 10^{-12}\ \mathrm{m/V}$, $d_{223} = d_{131} = 741 \times 10^{-12}\ \mathrm{m/V}$; the relative dielectric constants in the absence of stress $\varepsilon_{11}^{\mathrm{rel}} = \varepsilon_{22}^{\mathrm{rel}} = 3130$, $\varepsilon_{33}^{\mathrm{rel}} = 3400$.

4 Results of the Passive Behavior

A series of finite element computations was conducted for the described above problem of the active-passive sandwich panel subjected to harmonic excitations. The main objective was to analyze how acoustic waves of different frequencies are transmitted through the panel or how they are (at least partially) absorbed by it. As a matter of fact, these various results of acoustic transmission were also utilized to validate the convergence of finite element solutions. Both, passive and active cases were considered.

First, several finite element analyses were carried out to test passive behavior of sandwich panel. For example, Fig. 5 presents

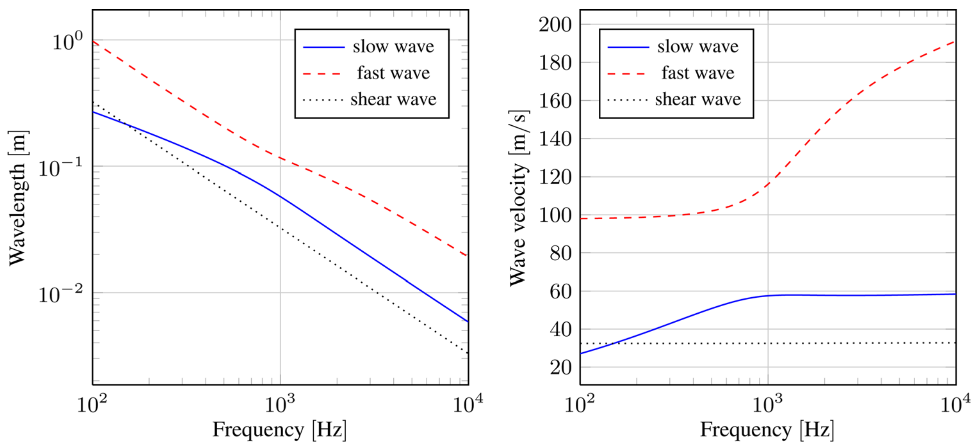

Fig. 4 Frequency-dependent wavelengths and wave velocities in the PU foam used for the core of panel

the sound pressure level calculated for purely passive panel at point D (60 mm from the lower surface of panel, see Fig. 1) for harmonic acoustic excitation with the amplitude of 1 Pa as described in the previous Section. Four results are plotted in the form of SPL curves. Three of them, namely, (a), (b), and (c), were obtained for two-dimensional, axially-symmetric, finite-element models using coarse or dense meshes shown in Fig. 3(*a*,*b*), and linear or quadratic approximations in poroelastic domain. The elastic faceplates were always approximated using the second-order shape functions. The fourth curve (*d*) is a result of one-dimensional analytical solution. Since no piezoelectric patch was present in the finite element models the one-dimensional modeling is fully appropriate under the circumstances described above (that is, the applied boundary conditions and uniformly distributed acoustic pressure excitation), and thus, the analytical solution is exact for such configuration of purely passive panel. From Fig. 5 it can be seen that the results for the proposed two-dimensional finite-element models are very accurate even for the coarsest of meshes with linear approximation in poroelastic domain (notice that some discrepancies visible for this model in the range above 3 kHz are for SPL below 10 dB).

Figure 6 compares the transmission reduction — which is a total reduction combining the effects of wave reflection and transmission loss (TL) — and the amplitude of acoustic pressure at point D computed using the three-dimensional finite-element model (see Fig. 2) with the results calculated analytically and

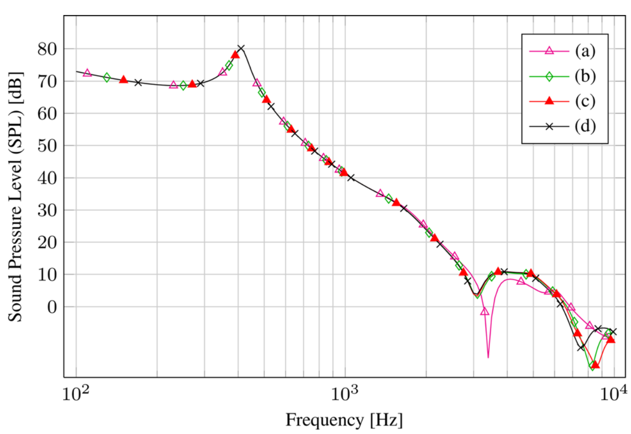

Fig. 5 Transmission results for the passive sandwich panel — the sound pressure level (SPL) at point D: (*a*) 2D coarse mesh with linear approximation in the poroelastic domain, (*b*) 2D dense mesh with linear approximation, (*c*) 2D coarse mesh with quadratic approximation, (*d*) analytical solution

using the two-dimensional axial models. In the considered frequency range below 3 kHz, all these results are nearly identical although the passive piezo-patch is present in the three-dimensional model and in one of the two-dimensional axial models, whereas it is absent in the case of the other two-dimensional model and the one-dimensional analytical analysis. Thus, the general conclusion is that all the considered models are in fact valid for passive analyses, so that the very economical coarse-mesh model with linear approximation in poroelastic domain may be used. It will be demonstrated; however, that it is not true in the case of the active approach.

5 Feasibility and Convergence Tests of the Active Approach

The results of passive analysis presented in Figs. 5 and 6 show that the panel exhibit a resonance behavior at the frequency slightly above 400 Hz. Around this frequency the acoustic insulation is very poor, and in general, it is not very effective at lower frequencies. To alleviate this problem the active approach should be used.

The passive and active behavior of panel at 400 Hz is compared in Fig. 7. In the passive case, the excitation is by a plane harmonic wave with the pressure amplitude of 1 Pa impinging the upper faceplate of panel. In the active case, apart from the acoustic excitation, a harmonic voltage signal of the same frequency of 400 Hz is applied to the electrodes of the piezo-patch actuator. The actuator is in that way harmonically expanded and contracted, which induces some bending deformation of the upper faceplate, affecting the vibrations of the panel. It is possible to choose the amplitude (and phase) of the signal necessary to better attenuate the acoustic wave transmitted through the panel. To this end, the acoustic pressure below the panel should be observed. It should be noticed that in the passive case the transmitted wave is plane, yet in the case of the active approach it may not be plane in the vicinity of the lower faceplate because of the additional, nonuniform, electric excitation induced on the upper faceplate. Thus, in the active case, the vibrations of the lower faceplate may not be planar, depending on the frequency of excitations, the thickness and material properties of poroelastic core, etc. Nevertheless, at some distance from the lower faceplate the transmitted acoustic wave is plane. This is certainly on the line CD situated across the waveguide, 60 mm from the lower faceplate. Therefore, the acoustic pressure of the transmitted wave will be observed at point D — which should be in practice equal to the acoustic pressure at any other point on the line CD — or, alternatively, a measure of the acoustic pressure defined as an integral of the pressure along the line CD may be used. As a matter of fact, one should notice that this approach complies actually with the experimental practice: during the measurements the microphones are

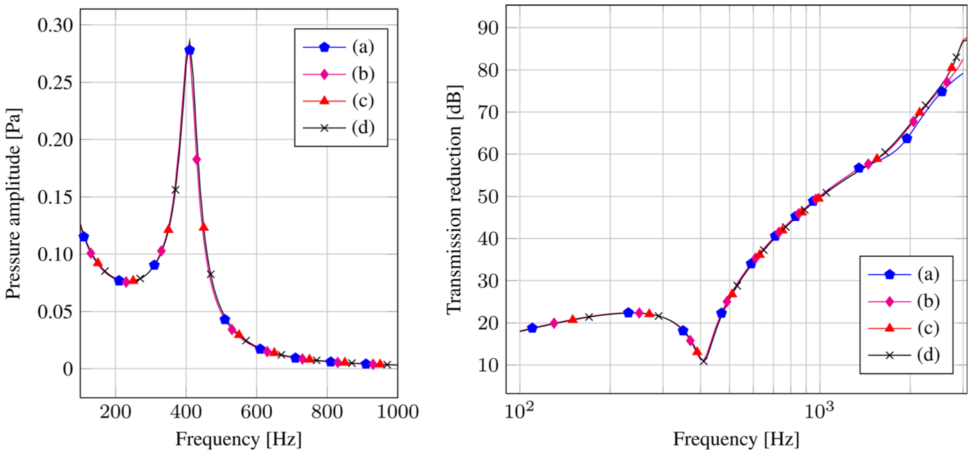

Fig. 6 Transmission results for the passive sandwich panel – the amplitude of acoustic pressure and transmission reduction at point D: (*a*) 3D model with passive piezo-patch, (*b*) 2D coarse mesh with passive piezo-patch, (*c*) 2D coarse mesh with no piezo-patch, (*d*) analytical solution

situated at some distance from the sample. The vibration shapes of panel, shown in Fig. 7 for the passive and active states, are scaled by the same scaling factor. Moreover, in both cases some isolines of the amplitude of (total) displacement are shown. It can be observed that the maximal vibrations in active case are under the PZT actuator and they are more than 8 times larger than the maximal vibrations in passive case. Nevertheless, they are still very small and fully comply with the linear regime. The isolines show the amplitudes of vibrations in micrometers: in the passive case the biggest amplitudes are along both faceplates and the smallest vibrations are in the middle of the core, whereas in the active case the vibrations are reduced nearly to zero at the lower faceplate. The necessary voltage amplitude was estimated as app. 26 V. Since the piezo-patch actuators are fixed to the upper faceplate the phase shift was negligible, and the necessary voltage was found by some parametric analyses involving sweeping of voltage amplitude, after some convergence tests — discussed below — had been carried out. Alternatively, it can be found using the transfer function method described in the next Section, involving two harmonic analysis.

Figure 8 presents some of the p- and h-convergence tests of the active approach performed for various finite element models. The acoustic pressure, sound pressure level, and transmission reduction were computed for the mutual, 400 Hz-harmonic excitations by the acoustic wave with the amplitude of 1 Pa, and the active electrical signal with the amplitude swept from 0 to 50 V or 90 V. The curves (*a*), (*b*), and (*c*) from Fig. 8 present the h-convergence of the active solution — when the mesh density is increased and the approximation remains linear, whereas the curves (*a*) and (*d*), as well as the curves (*c*) and (*e*) present two p-convergence solutions —when the approximation order is increased, from linear to quadratic, and the mesh density remains unchanged, coarse or very dense, respectively in these two cases. It is remarkable that all the curves start from the same point at 0 V, which confirms that for the passive case all the models give the same results. However, when the voltage is increased, the coarse and dense-mesh models with linear approximation in poroelastic domain give solutions — the curves (*a*) and (*b*) — which are significantly different from the results obtained with the very dense-mesh models. Thus, it can be stated that the models with linear shape functions are not appropriate for the active analyses if their meshes are not extremely dense; they give very exaggerated estimations for the proper voltage amplitudes of active reduction signals. On the other hand, the curves (*c*) and (*d*), obtained for the *very dense* mesh with linear approximation in poroelastic domain, and for the coarse mesh with *quadratic* approximation, respectively, are very close to the curve (*e*) obtained when the accurate model with the very dense mesh and quadratic approximation was used. Eventually, a conclusion must be drawn that the model with the coarse mesh [as shown in Fig. 3(*c*)], but with quadratic approximation in poroelastic domain, gives very good results, being at the same time very economical (only 551 DOF).

Finally, the three-dimensional mesh (see Fig. 2) was tested. One should notice that the characteristic lengths of finite elements for this mesh are comparable to the characteristic length for the coarse two-dimensional mesh [see Fig. 3(*c*)]. As a matter of fact, they are twice as big; however, Fig. 9 shows that the results

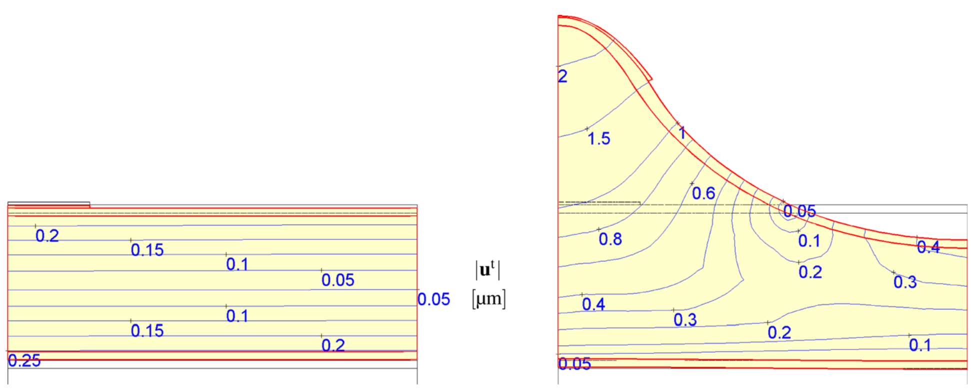

Fig. 7 Passive and active behavior of panel at 400 Hz, with isolines of the amplitude of total displacement (in micrometers)

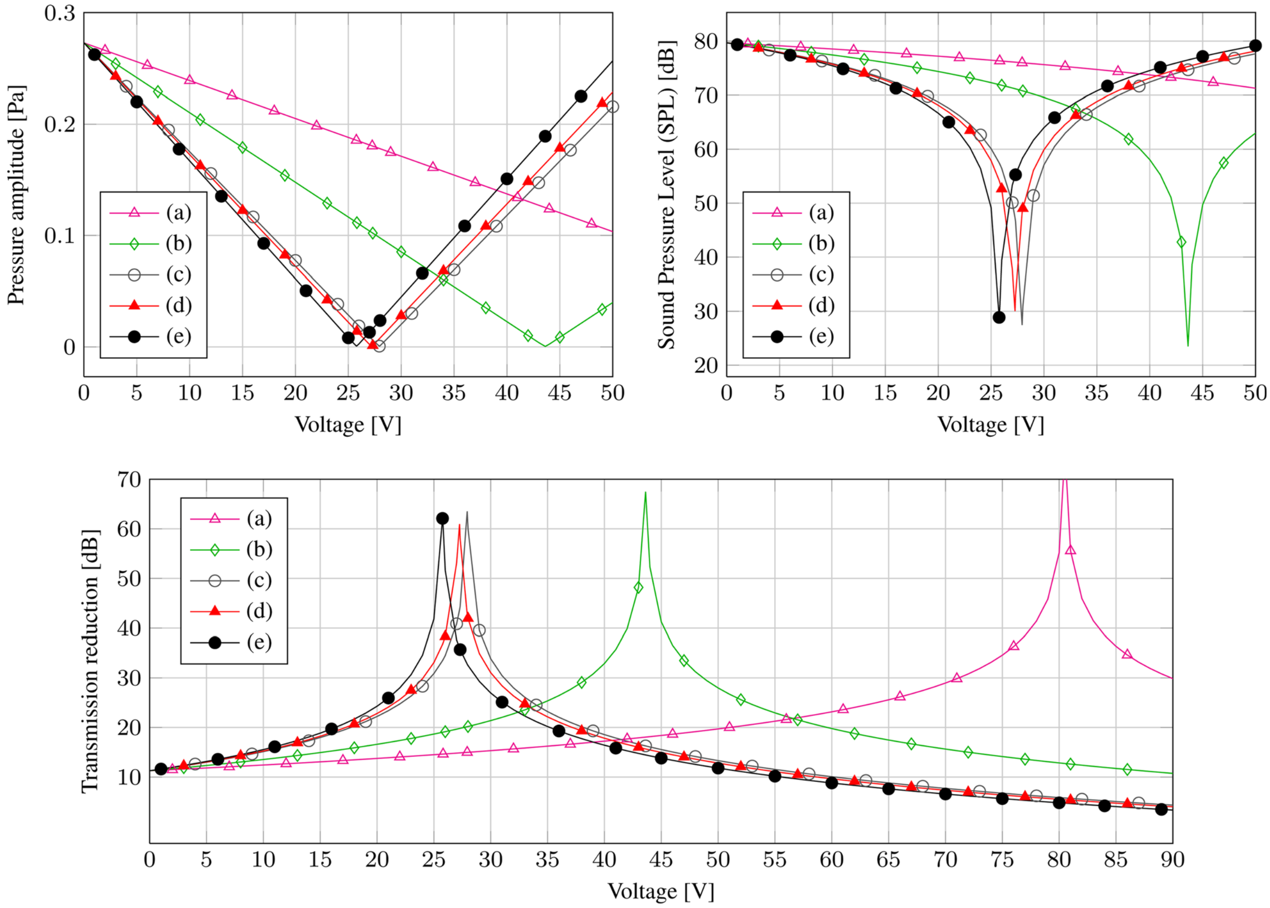

Fig. 8 The *h*- and *p*-convergence solutions for the active reduction at 400 Hz: (*a*) 2D coarse mesh with linear approximation in poroelastic domain (364 DOF), (*b*) 2D dense mesh with linear approximation in poroelastic domain (2738 DOF), (*c*) 2D very dense mesh with linear approximation in poroelastic domain (10040 DOF), (*d*) 2D coarse mesh with quadratic approximation in poroelastic domain (551 DOF), (*e*) 2D very dense mesh with quadratic approximation everywhere (39305 DOF)

obtained from the three-dimensional model with quadratic approximation in poroelastic domain are in practice identical with the results computed using the coarse two-dimensional mesh with such approximation. This is almost obvious when one remembers that the circular panel's vibrations are axially symmetric. Nevertheless, it seems that in a general case when the axial-symmetry is not present, such three-dimensional mesh should be suitable for modeling some characteristic segments of active sandwich panels, provided that the quadratic shape functions are used.

6 Frequency Analyses and a Parametric Survey for the Active Approach

Figure 10 presents parametric sweeps performed in order to estimate optimal voltage amplitudes for the active reduction of vibroacoustic transmission at different frequencies. For each of several computational frequencies from 250 to 800 Hz, the voltage amplitude of active signal was swept to find the minimum of the sound pressure level computed at point D which meant also the

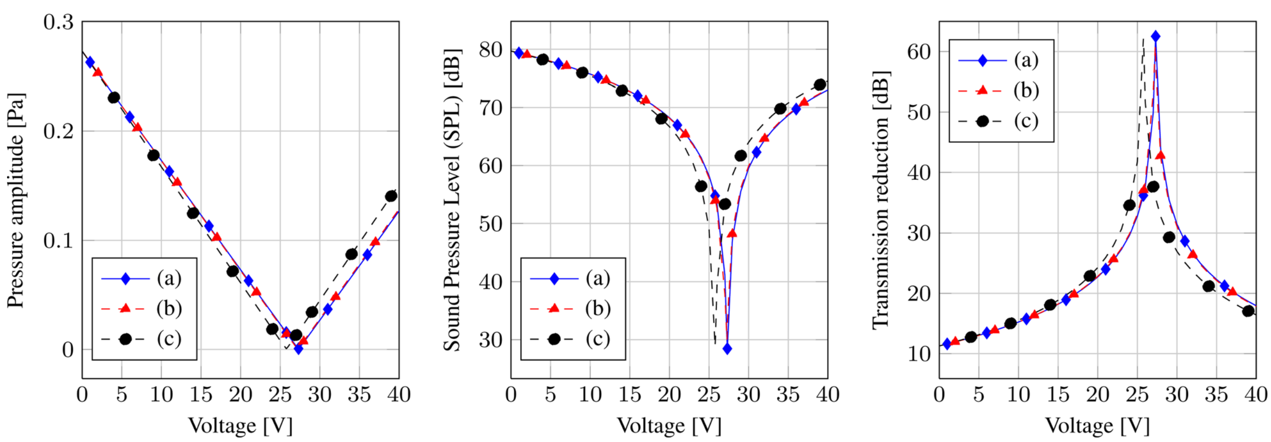

Fig. 9 Validation of 3D modeling with 2D axial-symmetry solution for the active reduction at 400 Hz: (*a*) 3D model with quadratic approximation (12136 DOF), (*b*) 2D coarse mesh with quadratic approximation (551 DOF), (*c*) 2D very dense mesh with quadratic approximation (39305 DOF)

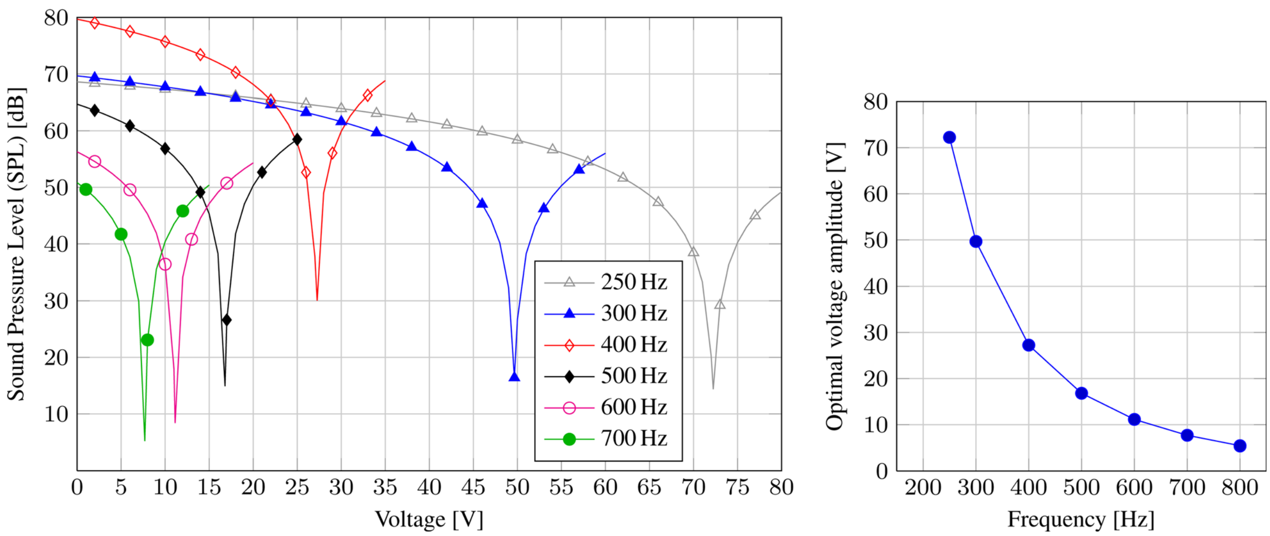

Fig. 10 Optimal voltage amplitudes for the active reduction of vibroacoustic transmission at different frequencies

maximum of the transmission reduction. The optimization of the phase of the signal was neglected since the piezo-patch actuator affects directly the upper faceplate, where the acoustic excitation is also directly applied, and the elastic faceplate is assumed lossless. The results shown in Fig. 10 clearly confirm that at lower frequencies higher voltages for active signals are necessary.

The right-hand side graph in Fig. 10 summarizes the results of optimal voltage amplitudes found for the PZT-patch actuator of radius 10 mm. These results were also obtained by the following calculation procedure. First, a pressure measure, for example, the acoustic pressure at the distance of 60 mm from the lower faceplate is computed for the harmonic excitation by the plane acoustic wave with the amplitude of 1 Pa; let the result be termed as α [Pa]. Then, the pressure measure is calculated for only electrical excitation with the voltage amplitude of 1 V; let the result be termed as β [Pa]. Since the system is linear, the pressure response at 60 mm for simultaneous, acoustic and electric excitations can be computed as follows

$$p_{\text{at 60 mm}} = \frac{\alpha}{1\,\text{Pa}}\hat{p} + \frac{\beta}{1\,\text{V}}\hat{U}$$

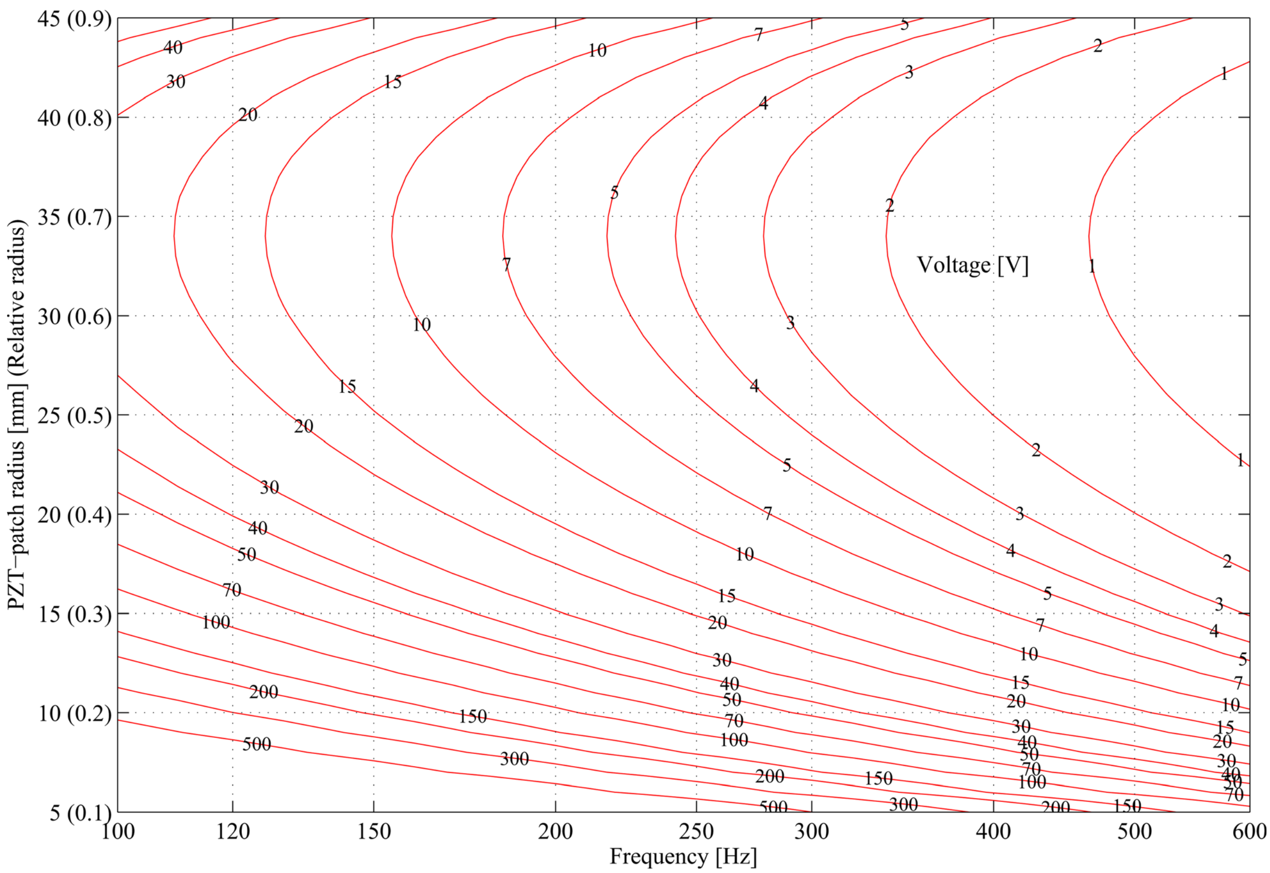

Fig. 11 Optimal voltage amplitudes for the active reduction signals

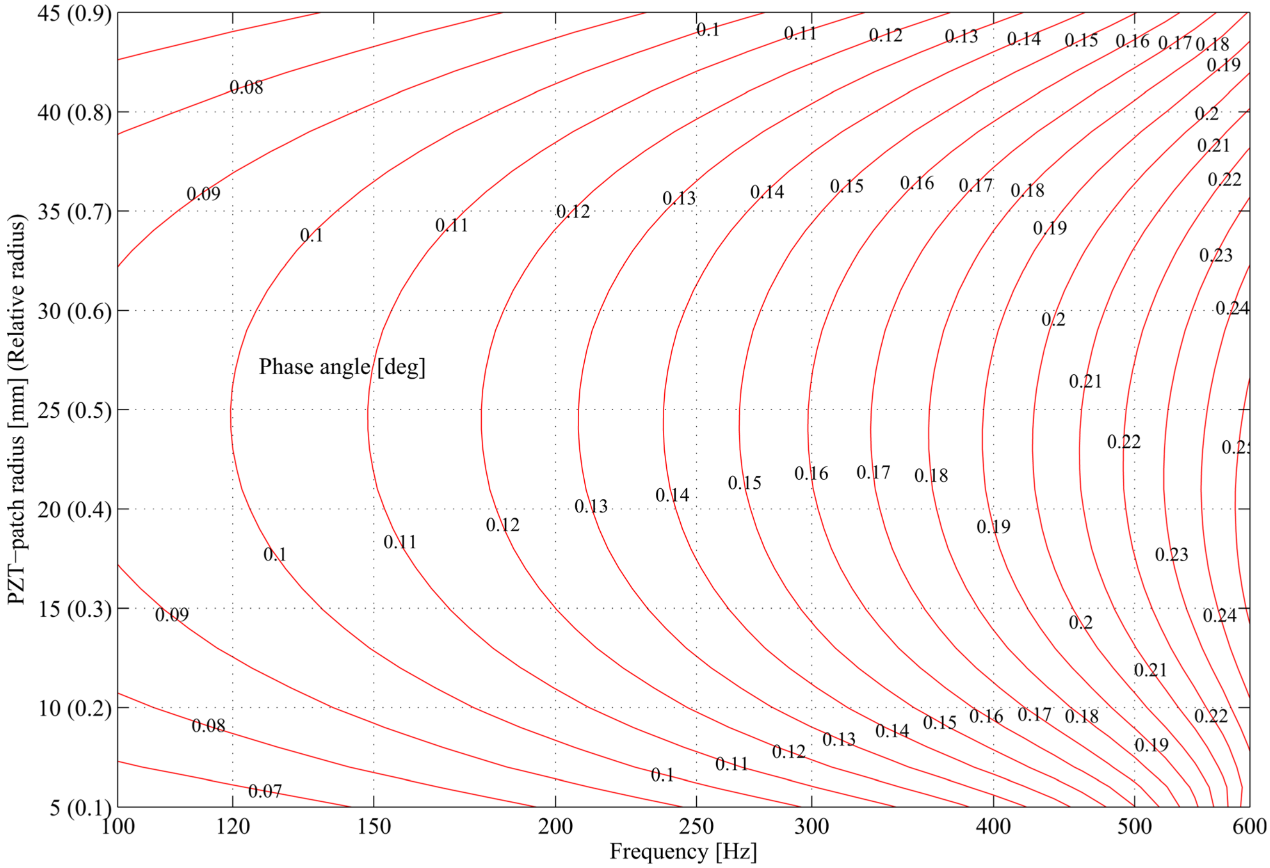

Fig. 12 (Negligible) phase angles for the optimal active reduction signals

where $\hat{p}$ [Pa] and $\hat{U}$ [V] are the amplitudes of the acoustical and electrical excitations, respectively. Now, from the requirement that $p_{\mathrm{at60mm}} \approx 0\mathrm{Pa}$, the necessary amplitude for the active signal may be estimated as:

$$\hat{U} = -\frac{\alpha}{\beta}\frac{\mathrm{V}}{\mathrm{Pa}}\hat{p} = -\frac{\alpha}{\beta}\mathrm{V}.$$

Here, it has been assumed that the amplitude of acoustic excitation is $\hat{p} = 1$ Pa. Notice that α, β, and $\hat{U}$ are in general complex amplitudes. Moreover, notice also that $\beta/1\mathrm{V}$ is the *transfer function between actuator and sensor* (a microphone situated at point D, that is 60 mm from the panel), and α is the *signal at sensor* obtained for the excitation by incident waves of SPL 94 dB (i.e., with the amplitude $\hat{p} = 1$ Pa). This procedure was extensively used for a parametric survey where the optimal voltage amplitudes for active signals were estimated at different frequencies for different sizes of the piezoelectric actuator. The radii of the PZT-patch were taken from 5 to 45 mm, whereas the computational frequencies were from the range of 100 to 600 Hz. The results are given in Figs. 11 and 12. From the first of these two figures it can be observed that the optimal radius of PZT-patch is approximately 34 mm: the smallest voltage amplitudes are required in the whole considered frequency range for such PZT actuator. It means that when the radius is bigger or smaller than 34 mm the optimal voltage amplitude increases. Nevertheless, quite a good performance has a PZT-patch with the radius of only 16 mm, for which — even at the lowest considered frequency of 100 Hz —the required voltage amplitudes for active signal are not bigger than several dozens of volts. Figure 12 clearly illustrates the mentioned above fact that the phase optimization is negligible — for such a disk of active sandwich panel in the considered frequency range —when the PZT actuator directly affects the upper faceplate subjected to the direct wave excitation of the amplitude of 1 Pa.

7 Conclusions

A disk-shaped segment of an active sandwich panel with a poroelastic core was analyzed numerically with respect to some convergence issues of the applied fully coupled finite-element modeling, in order to test the feasibility and performance of the passive and active acoustic attenuation as well as to carry out some frequency and parametric studies. The shape and size of the segment as well as the applied lateral boundary conditions are consistent with the requirements and general conditions met in experiments performed in an impedance or transmission-loss circular tube; however, approximately such a disk of panel may be also considered as a fragment of a larger, regular structure of hybrid sandwich panel.

It was shown that in the case of passive behavior of panel a simple linear approximation in the poroelastic domain of the core can be used with even the coarsest of the considered finite-element meshes. The obtained results are valid, because, as a matter of fact, the passive problem simply involves unidirectional vibrations. However, in the case of active approach the dynamic deformations of panel are much more complex, and the finite-element modeling requires higher-order approximations and/or denser meshes. Namely, some p- and h-convergence tests showed that if the linear shape functions are still used in the poroelastic domain, the mesh must be very dense indeed; however, if the quadratic approximation is applied the coarsest mesh is sufficient. Eventually, the convergence tests led to a selection of a very economical finite-element model, which allowed to carry out some frequency and parametric survey for the axially-symmetric problem of the active reduction of vibroacoustic transmission through the disk-shaped segment of panel. This model was also used to choose a valid three-dimensional model, and thus, general recommendations —gathered in that way — concerning finite-element modeling should be also relevant for three-dimensional models of square-shaped segments of hybrid sandwich panels, suitable for parametric analyses which are not axially-symmetric.

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