Sonic Boom : an introduction

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Concorde & military fighters



General objective

« ... ensuring that no <u>unacceptable</u> situation for the public is created by sonic boom from supersonic aircraft in <u>commercial</u> service » (ICAO resolution A39–1 - Appendix G, 2016).











What is a sonic boom ?

How does a sonic boom propagate in the atmosphere ?



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Why is a sonic boom annoying ?
What is the future of sonic boom ?



What is a sonic boom ?



What is a sonic boom ?























Aircraft position

at the emission time

at the reception time

What is a sonic boom ?

Sonic boom is the acoustical « trace » at the ground level of the aerodynamic flow past a supersonic body

The Mach cone

Theory : Doppler (1847) Experimental observation (Schlieren) : Mach 1888



In flight observation (NASA - Schlieren - T38 at M 1.1)



The main characteristics of a sonic boom



SR-71, BoomFile database, USAF

Time waveform

Frequency spectrum





impulsive

low frequency



Whitham theory (1952-1956)

Whitham's theory (1952) Axisymetric body (no lift) Slender body \Rightarrow Wave equation (linearized Euler - no shock) Farfield (r >> L)

$$p_{a}(x,y,z,t) = \frac{\rho_{0}c_{0}^{2}M_{av}^{2}}{\sqrt{2r}\sqrt{M_{av}^{2}-1}}F_{W}\left[c_{0}M_{av}(t-x,n/c_{0})\right]$$

$$F_{W}(x) = \frac{1}{2\pi} \int_{-\infty}^{x} \frac{1}{\sqrt{x-y}} \frac{d^{2}S(y)}{dy^{2}} dy$$

Fw : Whitham function, relates the fuselage geometry to the pressure field



x

S (E)

Lav

R

Linearized equations with source term

Fuselage
$$\Rightarrow$$
 Mass flow

 Mass
 $\frac{\partial \rho_a}{\partial t} + \rho_0 \operatorname{div} \vec{v}_a = \rho_0 V_{AV} \frac{dS}{d\xi} (c_0 Mt - x) \delta(y) \delta(z)$

 Momentum
 $\rho_0 \frac{\partial \vec{v}_a}{\partial t} + \operatorname{grad} p_a = 0$

 State
 $p_a = c_0^2 \rho_a$



Solution

Wave equation + source term

$$\frac{1}{c_0^2} \frac{\partial^2 p_a}{\partial t^2} - \Delta p_a = \rho_0 c_0^2 M_{av}^2 \frac{d^2 S}{d\xi^2} (c_0 M_{av} t - x) \delta(y) \delta(z)$$

Exact solution (Green function) + farfield approx.

$$p_{a}(x,y,z,t) = \frac{\rho_{0}c_{0}^{2}M_{av}^{2}}{\sqrt{2r\sqrt{M_{av}^{2}-1}}}F_{W}\left[c_{0}M_{av}(t-\vec{x}.\vec{n}/c_{0})\right]$$



Example : parabolic body



Example : parabolic body

$$p_{a}(x,y,z,t) = \frac{\rho_{0}c_{0}^{2}M_{av}^{2}}{\sqrt{2r\sqrt{M_{av}^{2}-1}}}F_{W}\left[c_{0}M_{av}(t-x,n/c_{0})\right]$$

Pressure dependance

Mach number

Fuselage radius

Fuselage length

Distance

(singular at M = 1, optimum M=1.414) R^2 L^{-3/2}

 $F_{W}(x) = \frac{1}{2\pi} \int_{-\infty}^{x} \frac{1}{\sqrt{x-v}} \frac{d^{2}S(y)}{dy^{2}} dy$

r^{-1/2} (half a line source)

Sharp variations in geometry



Whitham theory, Mach cone & ray theory

 $p_{a}(x,y,z,t) = \frac{\rho_{0}c_{0}^{2}M_{av}^{2}}{\sqrt{2r}\sqrt{M_{av}^{2}-1}}F_{W}\left[c_{0}M_{av}(t-x.n/c_{0})\right]$ Cône normal Cône de Mach Φ a rayon acoustique



Mach cone in-flight visualization





Extension to lifted bodies (Walkden, 1958)

$$p_{a}(x,y,z,t) = \frac{\rho_{0}c_{0}^{2}M_{av}^{2}}{\sqrt{2r\sqrt{M_{av}^{2}-1}}}F_{W}\left[c_{0}M_{av}(t-\vec{x}.\vec{n}/c_{0}),\Phi_{av}\right]$$

$$F_{W}(x,\Phi) = \frac{1}{2\pi}\int_{-\infty}^{x}\frac{1}{\sqrt{x-y}}\frac{d^{2}S_{eq}(y,\Phi)}{dy^{2}}dy \qquad p_{a} \propto \frac{W}{L_{eff}^{2}}$$
If the second second



Elements for low boom design





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The Shaped Sonic Bonic Demonstrator (2005)







1.5





The Shaped Sonic Bonic Demonstrator (2005)




Some key questions on sonic boom

What is a sonic boom ?

How does a sonic boom propagate in the atmosphere ?





Linear speed of sound

$$c_0^2 = \left(\frac{\partial p}{\partial \rho}\right)_s (\rho_0, s_0)$$



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Flow convection

$$c = c_0 + \vec{u}_0 \cdot \vec{n}$$



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Nonlinear speed of sound

$$c = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_{s}}(\rho_{0} + \rho_{a}, s_{0}) + \vec{u}_{a}.\vec{n}$$



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Locally plane wave

$$\rho_a = p_a / c_0^2$$

$$\vec{u}_a = \frac{p_a \vec{n}}{\rho_0 c_0}$$

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Linear speed of sound

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Nonlinear speed of sound

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 ρ_a

Ū,

Locally plane wave

$$= p_a / c_0^2$$
$$= \frac{p_a \vec{n}}{\rho_0 c_0}$$





Nonlinear parameter

 $c = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_{s} (\rho_{0} + \rho_{a}, s_{0}) + \vec{u}_{a}.\vec{n}}$ State equation Convection $B/A = \frac{\rho_0}{c_0^2} \left(\frac{\partial^2 p}{\partial \rho^2} \right) (\rho_0, s_0)$ $\beta = 1 + B/2A$





1D wave equation

 $\left| \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} = 0 = \left(\frac{1}{c_0} \frac{\partial p}{\partial t} + \frac{\partial p}{\partial x} \right) \left(\frac{1}{c_0} \frac{\partial p}{\partial t} - \frac{\partial p}{\partial x} \right) \right|$



1D wave equation

 $\left[\frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2}\right] = 0 = \left(\frac{1}{c_0}\frac{\partial p}{\partial t} + \frac{\partial p}{\partial x}\right)\left(\frac{1}{c_0}\frac{\partial p}{\partial t} - \frac{\partial p}{\partial x}\right)$



1D wave equation

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Nonlinear speed of sound

$$c = c_0 + \frac{\beta p_a}{\rho_0 c_0}$$



1D wave equation

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Nonlinear speed of sound

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Nonlinear "one way" wave eq.

$$\frac{1}{c_0}\frac{\partial p}{\partial t} + \frac{\partial p}{\partial x} = \frac{\beta}{\rho_0 c_0^3} p \frac{\partial p}{\partial t}$$



1D wave equation

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Inviscid Burgers' eq.

$$\tau = t - \frac{x}{c_0} \implies \frac{\partial p}{\partial x} = \frac{\beta}{\rho_0 c_0^3} p \frac{\partial p}{\partial \tau}$$





Inviscid Burgers' eq.

 $\frac{\partial p}{\partial x} = \frac{\beta}{\rho_0 c_0^3} p \frac{\partial p}{\partial \tau}$



Inviscid Burgers' eq.

др др β $\frac{1}{\partial x} = \frac{1}{\rho_0 c_0^3} p$ $\partial \tau$

Dimensionless

$$p = P_0 \overline{P}$$
$$\overline{\tau} = \omega_0 \tau$$
$$\sigma = x/L L?$$



Inviscid Burgers' eq.

др др $\hat{\partial}\tau$ $=\frac{1}{\rho_0 c_0^3} p$ ∂x

Dimensionless

$$p = P_0 \overline{P}$$
$$\overline{\tau} = \omega_0 \tau$$
$$\sigma = x/L L?$$



by choosing

 $\frac{\rho_0 c_0^3}{\beta \omega_0 P_0}$ $\beta k_0 M_0$











 $\frac{\partial P}{\partial \sigma} = P \frac{\partial P}{\partial \tau} + P(\sigma = 0, \tau) = P_0(\tau)$





$$P(\sigma = 0,\tau) = P_0(\tau)$$

Exact implicit Poisson's solution (1802)

$$P(\sigma,\tau) = P_0(\theta) \quad \text{with} \quad \tau = \theta - \sigma P_0(\theta)$$





$$P(\sigma = 0,\tau) = P_0(\tau)$$

Exact implicit Poisson's solution (1802)

$$P(\sigma,\tau) = P_0(\theta)$$
 with $\tau = \theta - \sigma P_0(\theta)$

Single-valued as long as

$$\frac{\partial \tau}{\partial \theta} = 1 - \sigma P_0^{\prime}(\theta) > 0$$

$$\sigma < \frac{1}{\max(P_0^{\prime}(\theta))} \quad (=1 \quad if \quad P_0 = \sin\theta)$$











Nonlinear signal distorsion





Bessel-Fubini solution

Case of an initially sine wave

$$P(\sigma,\tau) = \sin(\theta) \quad avec \quad \tau = \theta - \sigma \sin(\theta)$$

Nonlinearity keeps parity and periodicity ⇒ Fourier series





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Nonlinear distorsion and harmonic generation





Nonlinear distorsion and harmonic generation





Shock position : 3 methods

2) Law of equal areas 1) Rankine-Hugoniot (Landau 1944) (weak shock theory) $W = \frac{1}{2}(c_1 + c_2)$ A+=A-3) Burgers-Hayes potential (1954-1969) $\Phi(\sigma,\tau) = \int_{-\infty}^{\tau} P(\sigma,\tau') d\tau' \text{ monovalued solution}$ For potential

is max of multivalued one

 $\Phi_{mono}(\sigma,\tau) = \max(\Phi_{multi}(\sigma,\tau))$



Burgers-Hayes potential method



16



Burgers-Hayes potential method



16



Saw-tooth wave

$$P = \sin(\theta) \approx \frac{\pi - \tau}{1 + \sigma}$$
$$\tau \in \left[0, \pi\right]$$

 $E \downarrow \sigma^{-2}$

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$$\tau \in \left[0, \pi\right]$$

 $E \downarrow \sigma^{-2}$

N-wave



5 g - 6

18



N-wave



5 g - 6

18


Sonic boom is an N-wave !



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From Whitham function to N-wave

Nonlinear propagation of an N-wave

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From Whitham function to N-wave

Spectre (dB) -20 Pression $\sigma = 1$ 0.5 -40 $\sigma = 1$ -60 Č -80 -0.5 -100 2 -2 0 4 6 10 0 10 20 30 40 50 60 70 90 100 -4 8 80 Temps Fréquence Spectre (dB) -20 $\sigma = 5$ Pression 0.5 -40 -60 $\sigma = 5$ -80 -0.5 -100 -2 0 2 10 10 30 100 4 6 0 20 40 50 60 70 80 90 Temps Fréquence Spectre (dB) $\sigma = 10$ Pression 0.5 -40 -60 $\sigma = 10$ Ċ, -80 -0.5 -10 -100 -2Ó 2 10 10 20 30 40 50 80 90 100 8 60 70

Temps

Nonlinear distortion : time waveform versus frequency spectrum



Fréquence

From 1D to 3D : nonlinear ray theory



3D heterogeneous atmosphere : rays = curves (Fermat's principle : time from source to observer is minimized OBSERVER





Linear ray theory (cf D. Juvé's lecture)





Linear ray theory $p_a^2(l,t - \Psi(l))B(l) = Cte$



Linear ray theory



Linear ray theory

Non linear

$$c \rightarrow c + \frac{\beta p_a}{\rho_0 c_0}$$





Change of variable

 $q(\sigma, \tau = t - \Psi(l)) = \sqrt{B(l)} p_a(l, t)$





Age variable

$$\sigma = \int_0^l \frac{\beta}{\rho_0 c_0 c^2} \frac{dl}{\sqrt{B(l)}}$$

« age variable »

Plane wave	B∝l¹	SO	σ∝l
Cylindrical wave	B∝l¹	SO	$\sigma \propto ^{1/2}$
Spherical wave	B∝l²	SO	σ∝log l

Nonlinear effects much more important for plane waves or line sources (like sonic booms) than for point sources



Sonic boom in a homogeneous atmosphere

$$q(\sigma, \tau = t - \Psi(l)) = \sqrt{B(l)} p_a(l, t)$$

<u>Cylindrical wave</u> $B \propto I^1$ so $\sigma \propto I^{1/2}$

N-wave is farfield solution of inviscid Burgers' equation : $q \propto \sigma^{-1/2}$

$$p = qB^{-1/2} \propto |^{-1/4} \times |^{-1/2} = |^{-3/4}$$

Sonic boom in homogeneous atmosphere decays as power 3/4 of distance



Sonic boom in standard atmosphere





Sonic boom in standard atmosphere





Carpet width



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18

21 G - R



Carpet width vs meteorological variability



« Real » atmosphere (St George Channel, 24/01/1985, ECMWF data)





Standard atmosphere

Carpet width vs meteorological variability





Carpet width vs meteorological variability

Ground impact of sonic boom (Mach 1.6 - Edwards CA - year 1993) Width (km) Peak overpressure (Pa)



East to West

West to East



11 12

Some key questions on sonic boom





Who is annoyed by sonic boom ?

Sonic boom ...



NY/ LA= 3950 km - 2 X 200 km = 3550 km 3550 km ×65 km ×30 hab/km² =**6,922,500 people** *How many highly annoyed ? We don't know yet*



Who is annoyed by sonic boom ?



Sonic boom ...

- A large number of people exposed to an « unfrequent » event (to be quantified)

... opposite to airport noise

- A relatively « small number » of people exposed to a very frequent event (almost continuous)



Amsterdam Schiphol, 2005, 1180 flight movements About 230,000 people highly annoyed by airport noise



Some key questions on sonic boom





What influences a sonic boom ?

The aircraft design / operations



The atmosphere large / medium / small scales

The buildings type & quality

The people



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The sonic boomS (smooth atmosphere)



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Molecular relaxation

Simplified mechanism

$$N_{2}^{*} + H_{2}O \iff N_{2} + H_{2}O + \acute{e}nergie$$
$$O_{2}^{*} + H_{2}O \iff O_{2} + H_{2}O + \acute{e}nergie$$

Relaxation time (chemical kinetics)

 $f_N \sim 300 \text{ Hz}$ $f_O \sim 30 \text{ kHz}$

Dispersion





Atmospheric absorption



9 R

18

8 G X

 $X \in X$











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Focused boom



Focused boom

Low supersonic acceleration \Rightarrow focused, higher amplitude boom

Sensitivity to turbulence?

Off design \Rightarrow robustness to low boom design ?

Relatively local \Rightarrow impacted population ?

Higher amplitude \Rightarrow higher annoyance ?







Focused boom

Low supersonic acceleration \Rightarrow focused, higher amplitude boom

Sensitivity to turbulence?

Off design \Rightarrow robustness to low boom design ?

Relatively local \Rightarrow impacted population ?

Higher amplitude \Rightarrow higher annoyance ?





Blumrich et al. 2005



Some key questions on sonic boom





Field studies

2 field studies

- St Louis (Minnesota) 1961-1962, 76 flights / 45 flights during 9 pm-4 am
- Oklahoma City 1964, 1225 flights, 0 during night



Brown and Sutherland NASA CR-189643, 1992



Western Sonic Boom Survey

1 Community Survey : Western Sonic Boom Survey, Nevada & California, 1992-1995, 20 communities, 1595 booms, 1573 interviews - poor correlation with metrics except # booms / day

34 25,5 17 8,5 0 Startle

Western Sonic Boom Survey :most disturbing aspect of sonic boom

Startle Vibrations/Rattle Possible damages Brown and Sutherland NASA CR-189643, 1992



Why is a conventional sonic boom annoying ?





Expectations for a low sonic boom



Startle Vibrations & rattle Potential damages Direct noise Sleep disturbances ?



Startle Vibrations & rattle Potential damages Direct noise Sleep disturbances ?



Sonic boom vs artillery noise disturbances

Relatively similar response regarding artillery noise



Schomer, Noise Con. 2004



Night noise: WHO Europe 2000 recommendations

During night, World Health Organization (Guidelines for Community Noise 2000, Berglund et al.) recommends « for a good sleep indoor sound pressure level should not exceed approximately 45 dB Lamax more than 10-15 times per night, and most studies show an increase of awakenings at SEL values of 55-60 dBA » (pp.28)

And « ... special attention should be given to the following considerations - environment with a low background noise level

- environments where a combination of noise and vibrations are produced
- sources with low-frequency components. Disturbances may occur even though the sound pressure level during exposure is below 30 dBA » (pp.28)

Note that sonic boom may satisfy **all** these three conditions.

A special attention to sonic boom during night (22:00-07:00) should be given.



European Directive

European Directive 2002/49/EC relative to environmental noise protection

- Selects noise indicator L_{night} (A-weighted long-term average sound level ISO 1996-2; 1987 over night period from 23:00 to 07:00) to assess sleep disturbances
- 2) Special noise indicators may be used in the following cases relevant to sonic booms
 - noise source under a small proportion of the night
 - average number of events is low
 - low frequency content of noise is strong
 - noise has an impulsive character
 - L_{A,max} or SEL for night period protection in case of noise peaks



Some key questions (and answers) on sonic boom

What is a sonic boom ?

Sonic boom is the acoustical « trace » at the ground level of the aerodynamic flow past a supersonic body

How does a sonic boom propagate in the atmosphere ?

Sonic boom propagates nonlinearly along acoustical rays

Who is annoyed by a sonic boom ?

Millions of people will be exposed to the sonic boom of a single aircraft over one transcontinental flight, but the % of annoyed people is unknown

What influences a sonic boom ?

Sonic boom is influenced by the aircraft design, its flight parameters including transonic acceleration, and the local atmosphere below the aircraft including turbulence

Why is sonic boom annoying ?

For classical boom, mostly because of startle, house vibrations, and fear of potential damages. Sleep disturbances and people response to future low sonic booms are unknown



Prospects : key points for future research

Better knowledge is needed about :

- Various booms (lateral boom / focused boom / sensitivity to turbulence)
- Future supersonic trafic to quantify global exposure
- Boom transmission through structures
- Potential damages of low booms to structures (a likely source of unacceptability)
- Effets of low booms on sleep disturbances (a likely source of unacceptability)
- Noise metrics including rattle noise
- Community surveys \Rightarrow need for a demonstrator

Critical issues

- Extrapolation of scientific surveys to global population
- The X % of the population that are the most annoyed are those about whom the most information is needed
- A regulation process (ICAO / FAA...) should be accompanied by an information one



Past, present, future



