



Laboratoire de *Mécanique des Fluides* et d'*Acoustique*
LMFA UMR 5509



Flow stability and Introduction to turbulence

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Centrale Lyon – 2A FLE-1a – 16-12-2024

<http://acoustique.ec-lyon.fr>

● Introduction to turbulence

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● Textbooks

Batchelor, G.K., 1967, An introduction to fluid dynamics, *Cambridge University Press*, Cambridge.

Bailly C. & Comte Bellot G., 2003 *Turbulence*, *CNRS éditions*, Paris (out of print).

———, 2015, *Turbulence* (in english), Springer, Heidelberg.

(360 pages, 147 illustrations, Foreword by Charles Meneveau, 53 € for ECL students)

Bailly C. & Comte Bellot G., 2003, *Turbulence* (in french), *CNRS éditions*, Paris.

———, 2015, *Turbulence* (in english), Springer, Heidelberg.



Springer, ISBN 978-3-319-16159-4,

360 pages, 147 illustrations.

(discount for students, 53 €)

Candel S., 1995, Mécanique des fluides, *Dunod Université*, 2nd édition, Paris.

Cousteix, J., 1989, *Turbulence et couche limite*, Cépaduès, Toulouse.

Davidson P.A., 2004, *Turbulence. An introduction for scientists and engineers*, Oxford University Press, Oxford.

Davidson, P.A., Kaneda, Y., Moffatt, H.K. & Sreenivasan, K.R., Edts, 2011, *A voyage through Turbulence*, Cambridge University Press, Cambridge.

Guyon E., Hulin J.P. & Petit L., 2001, Physical hydrodynamics, *EDP Sciences / Editions du CNRS*, première édition 1991, Paris - Meudon.

● **Textbooks (cont.)**

Hinze J.O., 1975, *Turbulence*, *McGraw-Hill International Book Company*, New York, 1^{ère} édition en 1959.

Landau L. & Lifchitz E., 1971, *Mécanique des fluides*, *Editions MIR, Moscou*.

Also *Pergamon Press*, 2nd edition, 1987.

Lesieur M., 2008, *Turbulence in fluids : stochastic and numerical modelling*, *Kluwer Academic Publishers*, 4th revised and enlarged ed., Springer.

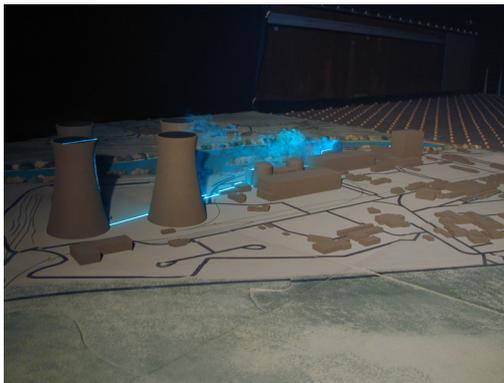
Pope S.B., 2000, *Turbulent flows*, *Cambridge University Press*.

Tennekes H. & Lumley J.L., 1972, *A first course in turbulence*, *MIT Press*, Cambridge, Massachusetts.

Van Dyke M., 1982, *An album of fluid motion*, *The Parabolic Press*, Stanford, California.

White F., 1991, *Viscous flow*, *McGraw-Hill, Inc.*, New-York, first edition 1974.

Introduction



Atmospheric wind tunnel (LMFA)

● **Turbulent flows are part of everyday life!**

- Geophysical flows
astrophysics, climate, wheather, environment, hydraulics
- Transportation industry : space, aeronautics, marine & submarine
and also sport applications
- Transport of fluids (energy industry, chemistry), production of energy
- Biology (physiology, biomechanics, medicine)
- Complex flows (two-phase flows, including solid particles, ...)

External aerodynamics • Noise of turbulent flows (aeroacoustics) • Sound propagation (atmosphere, ocean) • Fluid-solid coupling and vibroacoustics • Combustion (reactive flows)

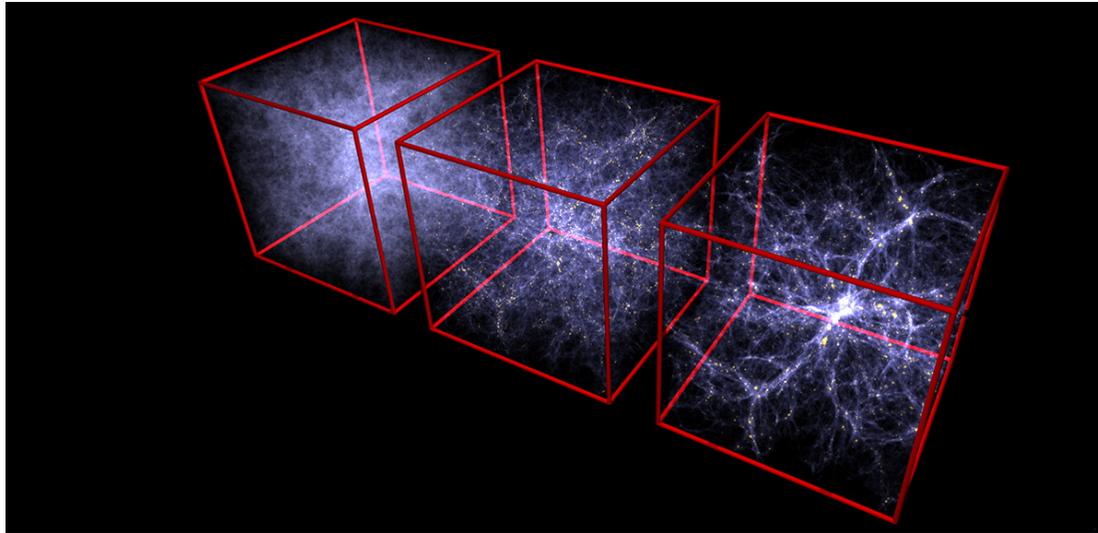
Fluid mechanics is involved in many societal challenges

● Turbulent flows

- unsteady aperiodic motion
- unpredictable behaviour
- presence of a wide range of scales (eddies)

Turbulence appears when the source of the kinetic energy which drives the fluid motion is able to overcome viscosity effects

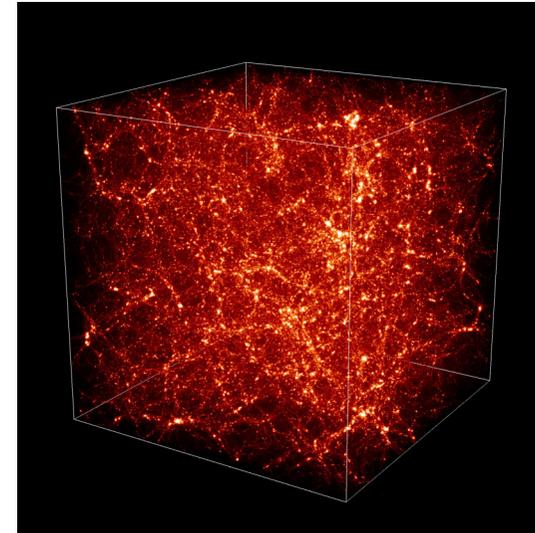
- **Simulation of the growth of cosmic structure (galaxies and voids)**
(cosmological hydrodynamics)



Simulation of the growth of cosmic structure (galaxies and voids) when the Universe was 0.9 billion years old, then 3.2 billion and 13.7 billion years old (today)

Volker Springel / Max Planck Institute for Astrophysics

<https://news.cnrs.fr/articles/euclid-on-a-quest-to-understand-dark-energy>



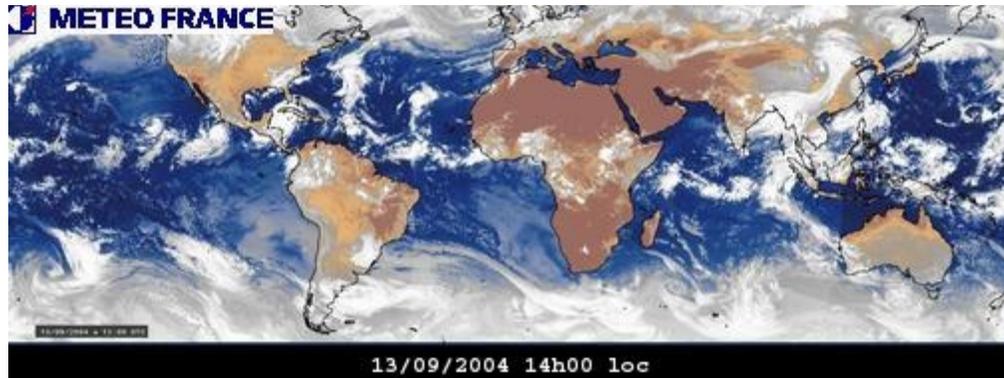
Structure formation in an expanding universe : N -body simulation (red temperature) with 70 billions particles ; 500 million light-years long on each side 

Institut d'Astrophysique de Paris (Pichon) & CEA (Teyssier)

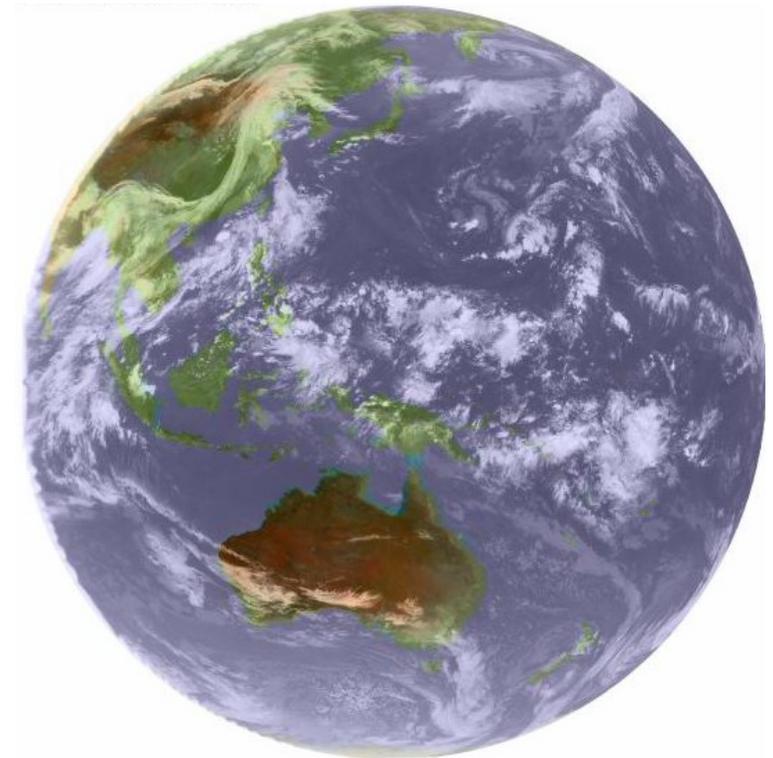
- **Weather satellite images**

www.meteofrance.com • www.meteo-lyon.net

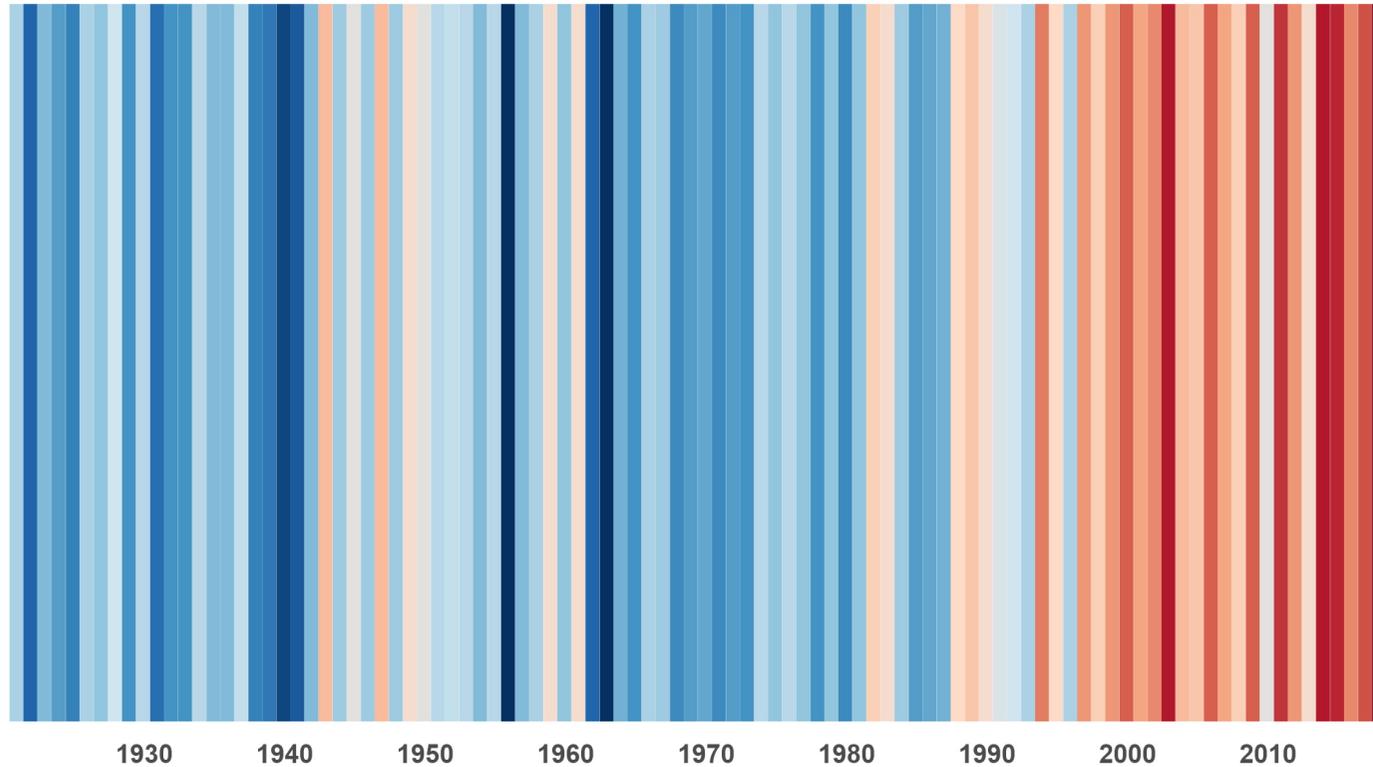
https://www.meteoblue.com/fr/meteo/semaine/lyon_france_2996944



Intertropical convergence zone



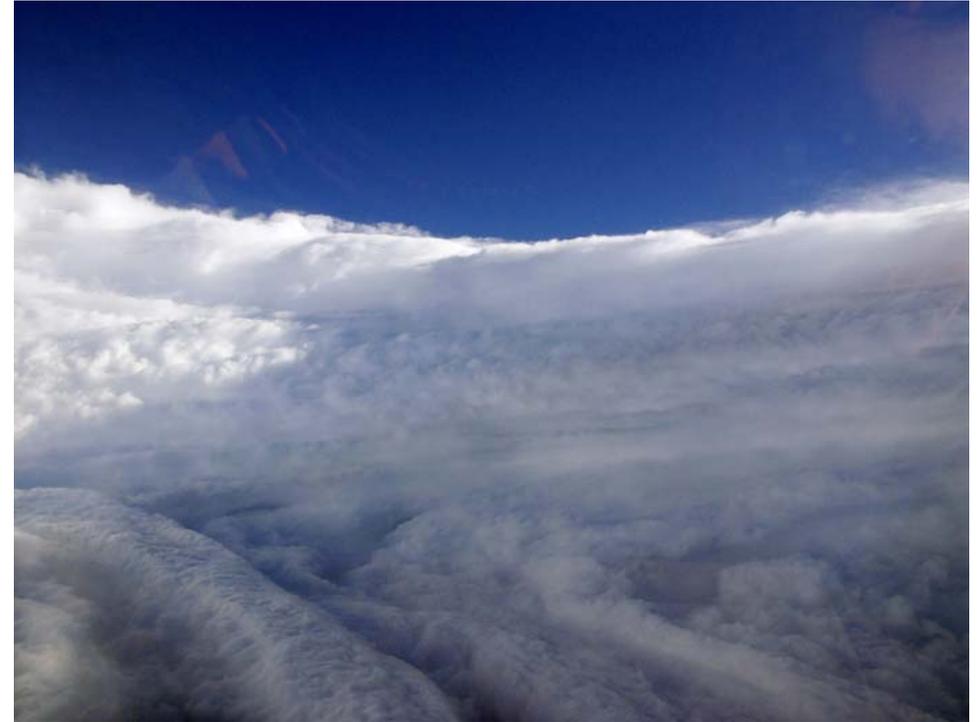
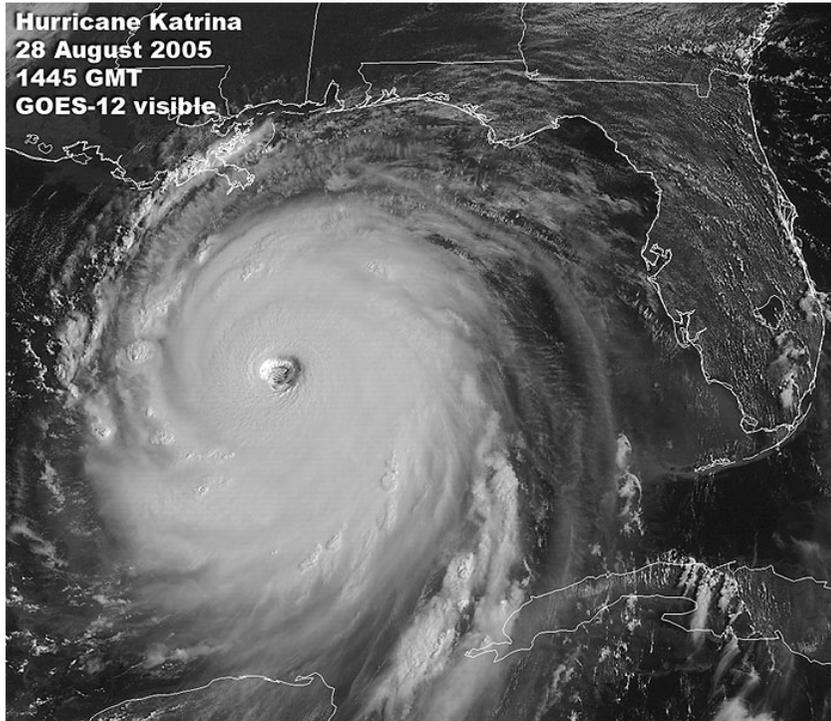
- **Annual mean temperature in Lyon - Bron Airport - from 1921 to 2018**
(from Météo France, Le Monde 08.01.2019)



Evolution de la température par année depuis 1921
Lyon (69) : de 10 °C à 14,5 °C

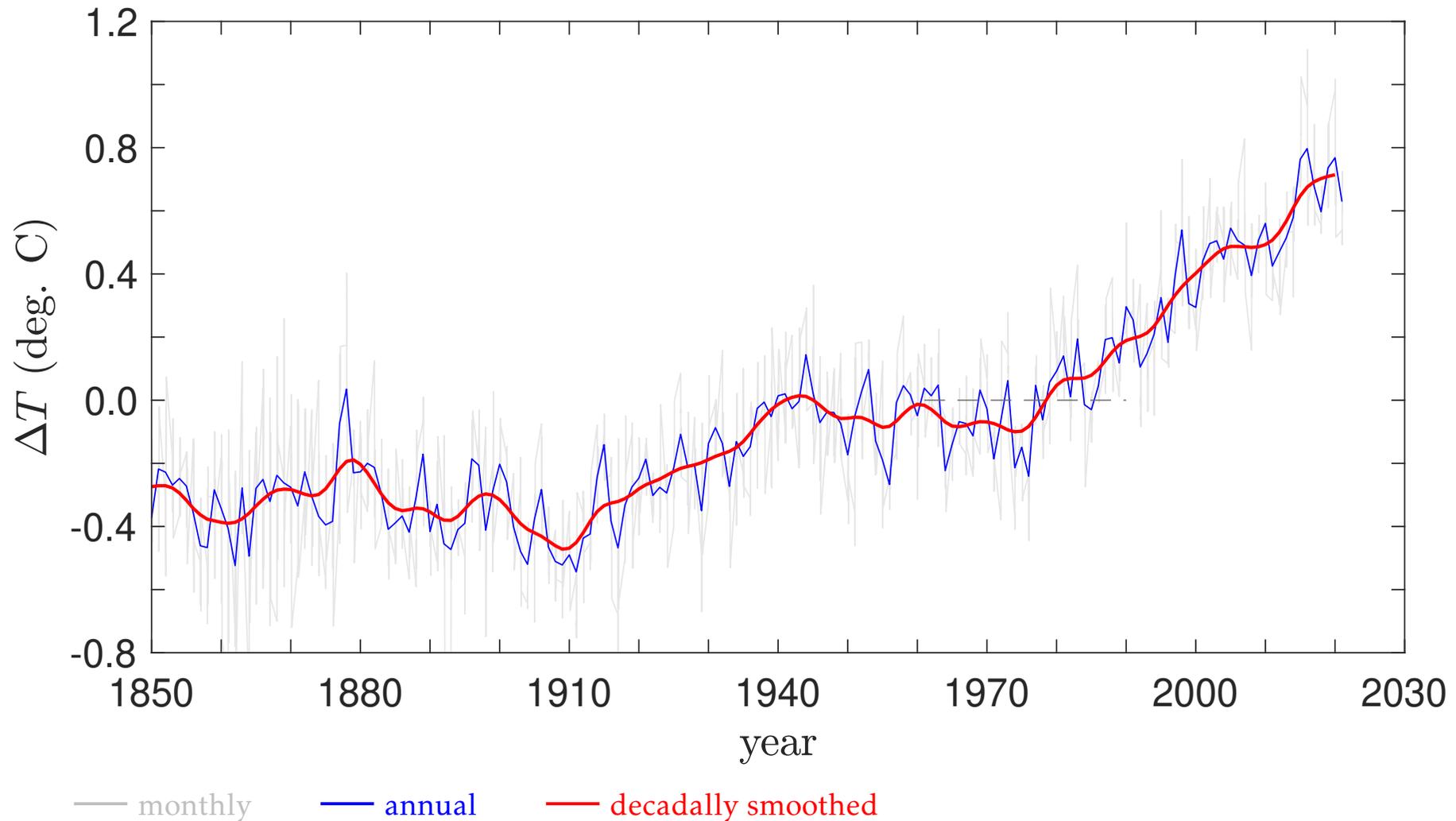


● Cyclone Katrina - Sept. 2005 - Category 5



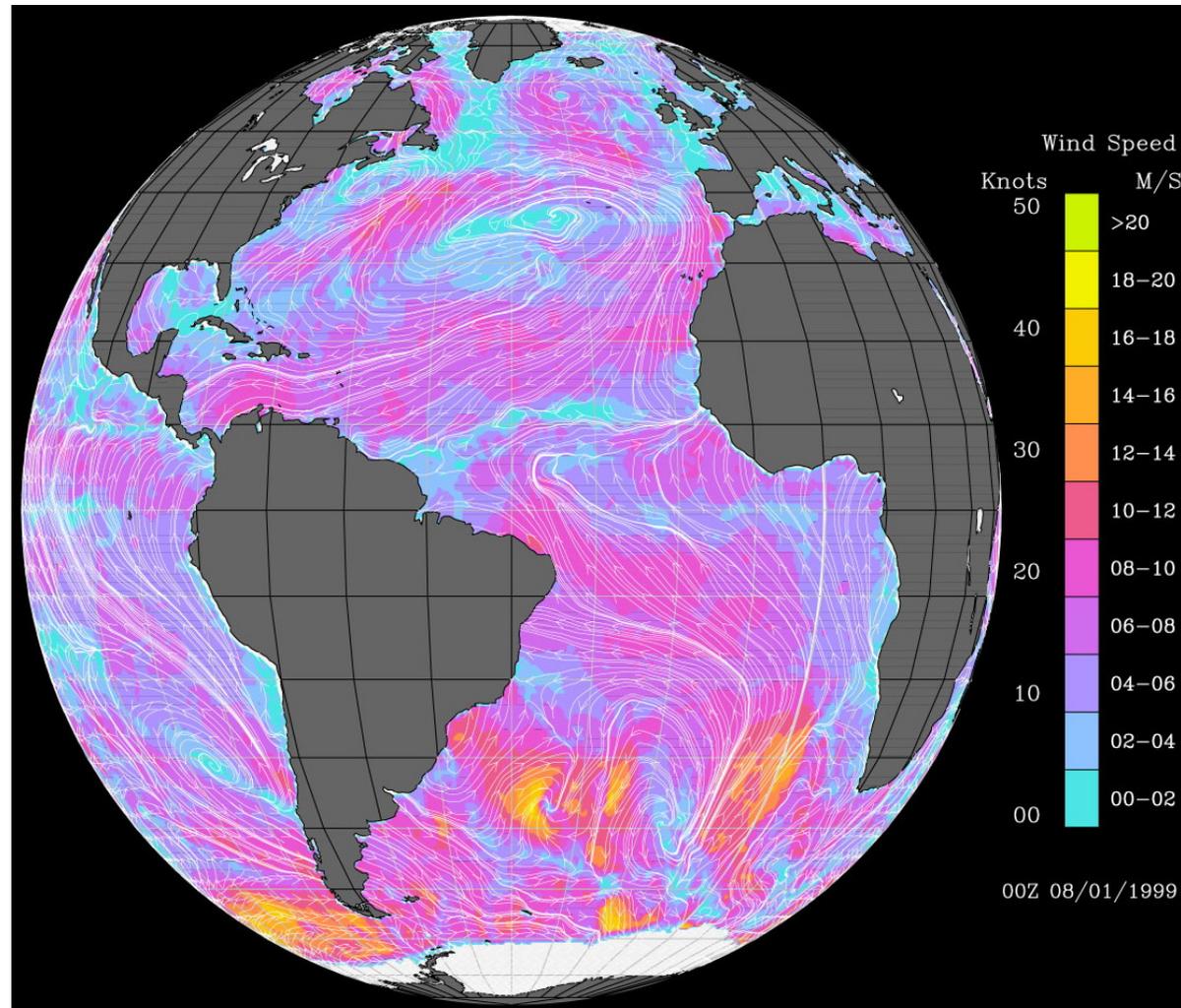
Wind gusts of 280 km/h (average during 1 minute in USA), 80% of New Orleans was flooded, Dixon *et al.*, 2006, *Nature*, 441, 586-587
(1464 people died in the hurricane and subsequent floods according to the Louisiana Department of Health)

- **Earth's (land and marine) surface temperature from 1850 to 2020**
 expressed as 'anomaly' from 1961-90 in dashed line
 data from www.cru.uea.ac.uk



● **Near-surface wind speeds 10 meters above the Atlantic Ocean**

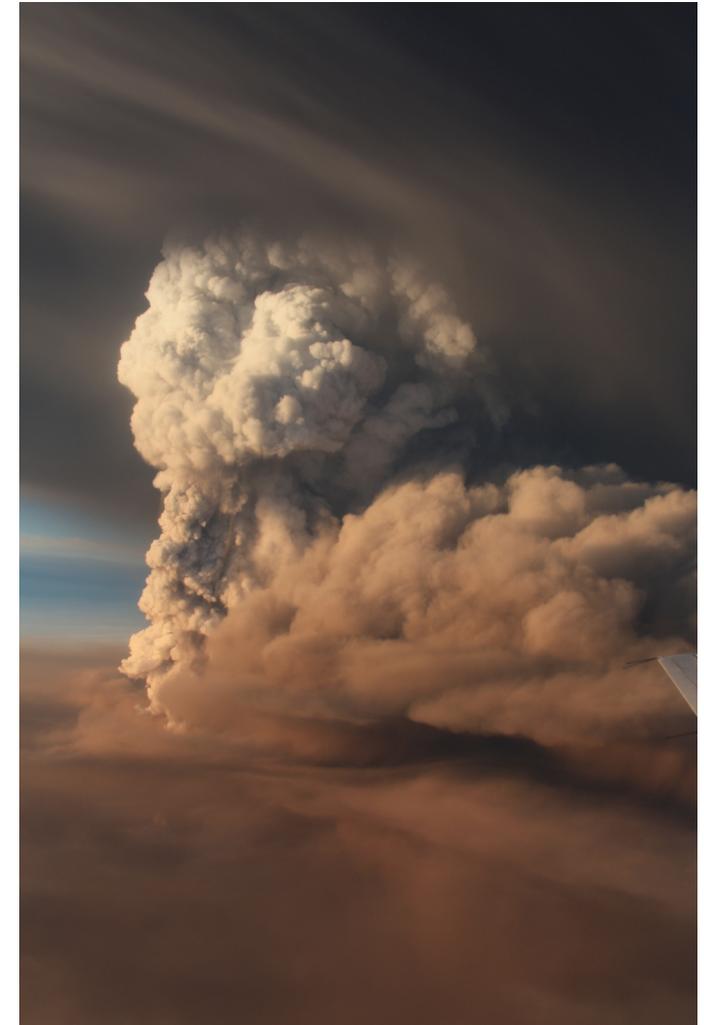
Data collected by the SeaWinds scatterometer on-board NASA's QuikSCAT satellite (NASA's Jet Propulsion Laboratory)



- **Eruption of the subglacial Grimsvötn volcano, Iceland, on 21 May 2011**

An initial large plume of smoke and ash rose up to about 17 km height.

Courtesy of Thördis Högnadóttir, Institute of Earth Sciences, University of Iceland

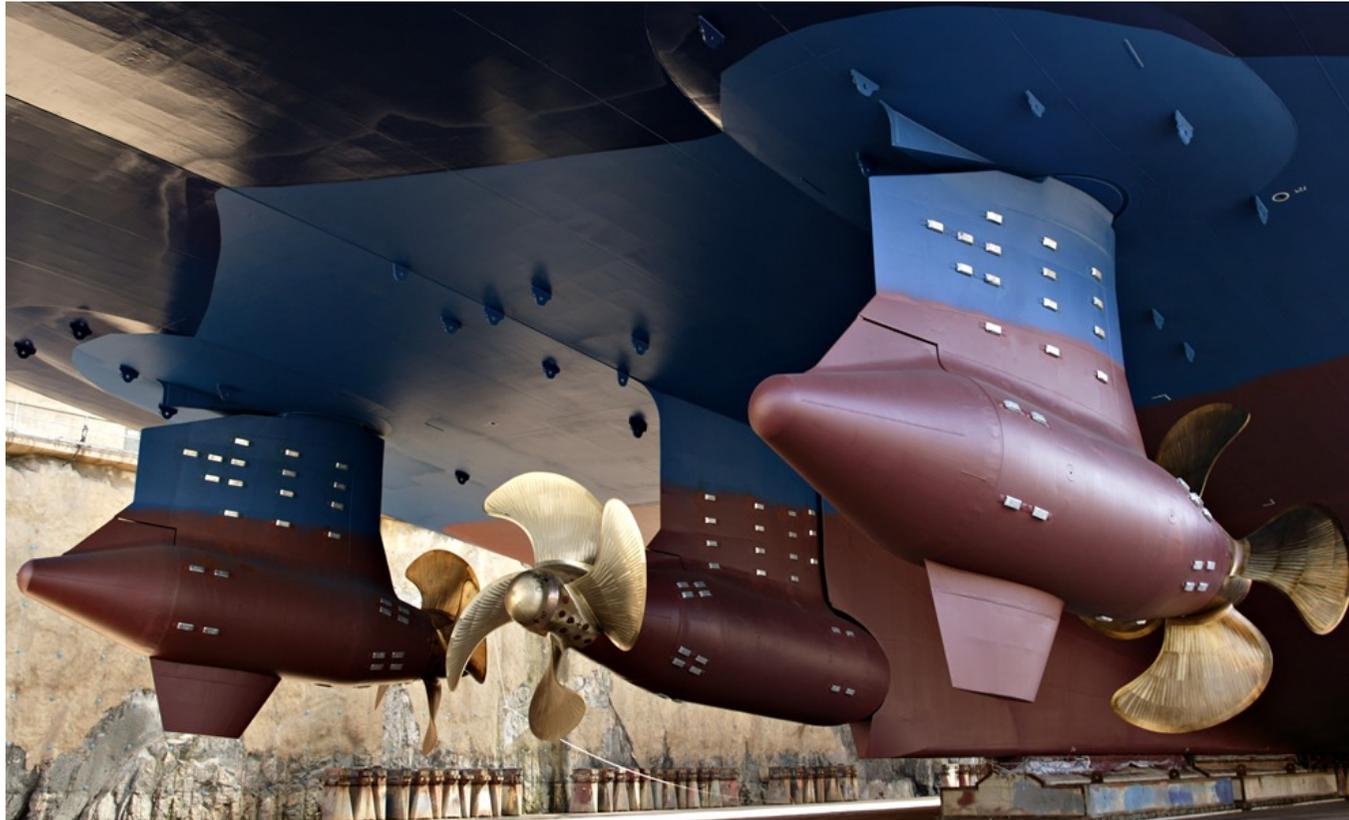


- Propeller hydrodynamics



(propeller cavitation)

● Hydrodynamics : azimuth thruster



Cruise vessel
Harmony of the Seas
(2016)

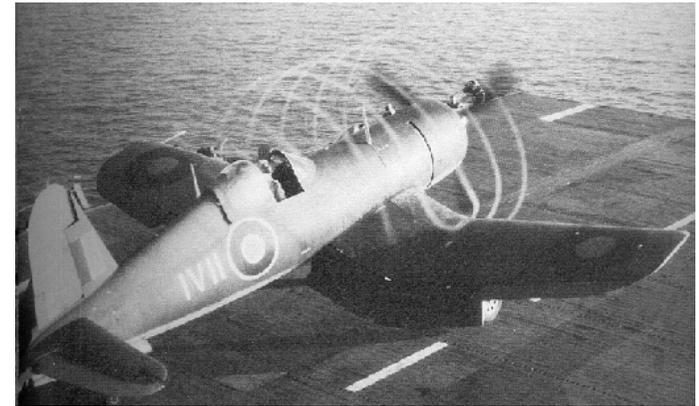
Azimuth thruster : configuration of marine propellers placed in pods that can be rotated to any horizontal angle (azimuth), making a rudder unnecessary. It is equipped with a new-generation exhaust gas cleaning system (multi-stream scrubbers) and also features a hull lubrication system allowing the ship to float on air bubbles (created around the hull) thus reducing drag and increasing fuel efficiency.

● Aeronautics



NASA Wake Vortex Study at Wallops Island
NASA Langley Research Center 5/4/1990 Image # EL-1996-00130

Tip vortex behind an airplane



Fleet Air Arm Corsair III in 1944,
(unintended) visualization of the
propeller wake



Boeing 767-370/ER

● **Emirates A380-800 over Arabian Sea on Jan 7th 2017, wake turbulence sends business jet in uncontrolled descent**

www.avherald.com

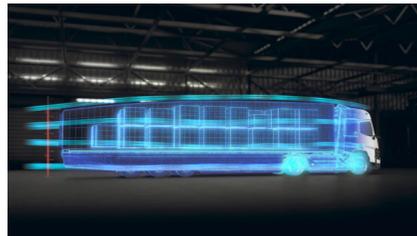
The CL-604 passed 1000 feet below an Airbus A380-800 while enroute over the Arabian Sea, when a short time later (1-2 minutes) the aircraft encountered wake turbulence sending the aircraft in uncontrolled roll turning the aircraft around at least 3 times, both engines flamed out, the Ram Air Turbine could not deploy possibly as result of G-forces and structural stress, the aircraft lost about 10,000 feet until the crew was able to recover the aircraft exercising raw muscle force, restart the engines and divert to Muscat.



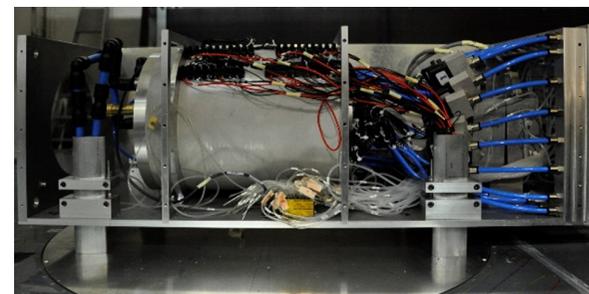
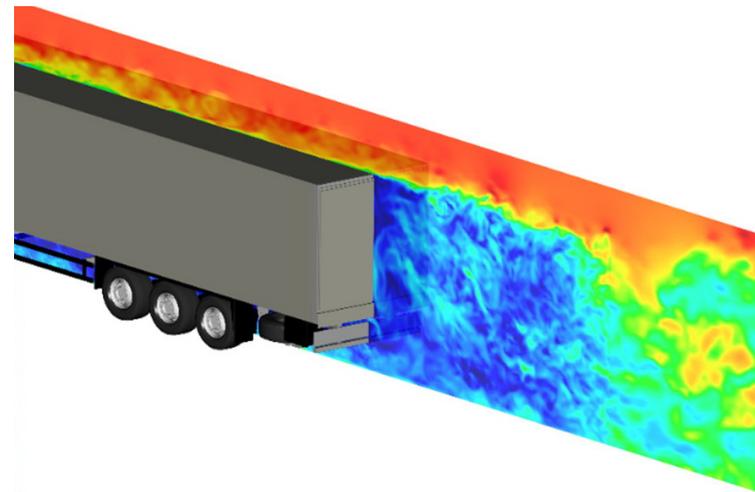
wingspan of 19.6 m (Canadair Challenger 604) versus 79.7 m (A380)

● **Aerodynamics of cars and trucks**

Optifuel Lab 3 - Renault Trucks
laboratory vehicle - aims to reduce
fuel consumption by 13%



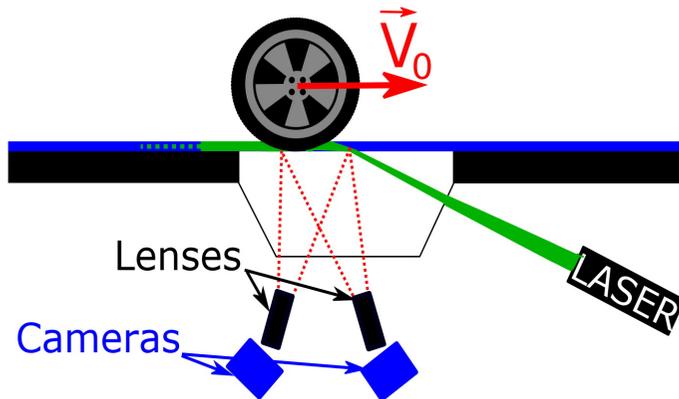
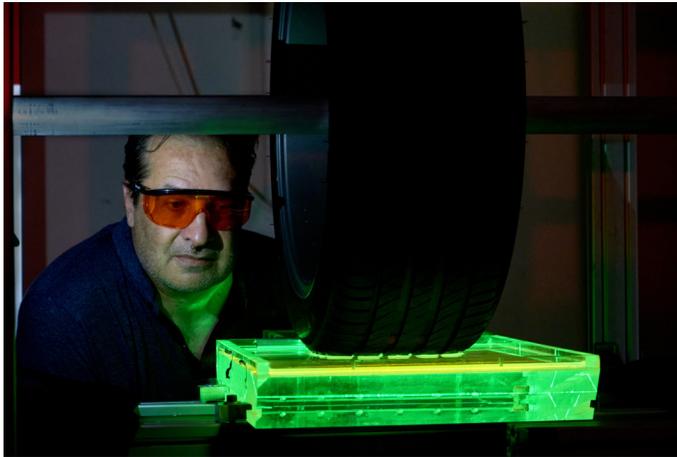
High Reynolds number wake control to improve
acoustic and aerodynamic performance



(LMFA - T. Castelain ; Renault Trucks, Pprime, PSA, Ampère)

● **Aerodynamics of cars and trucks (cont.)**

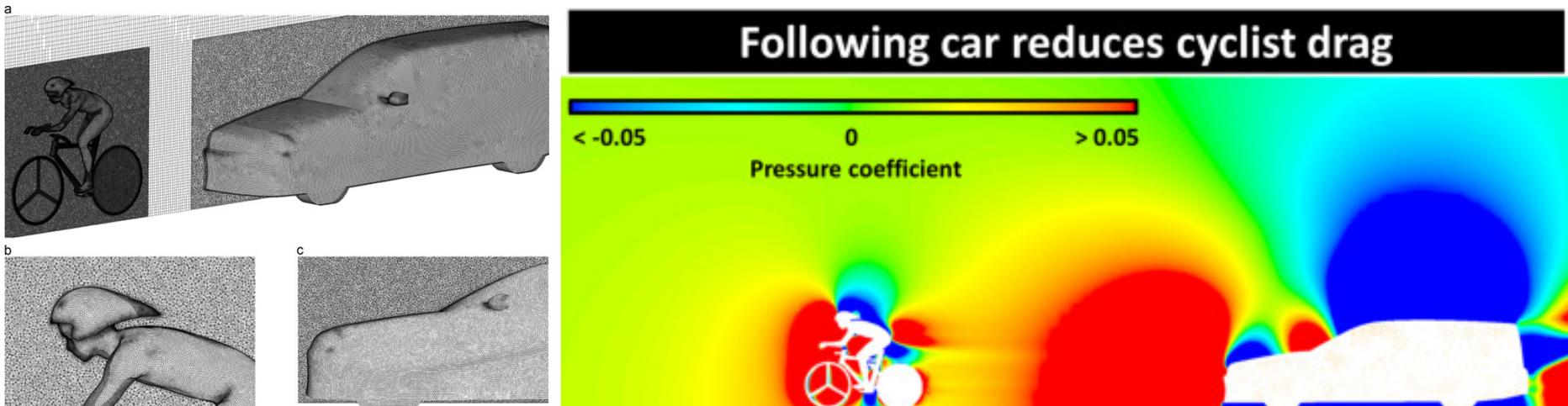
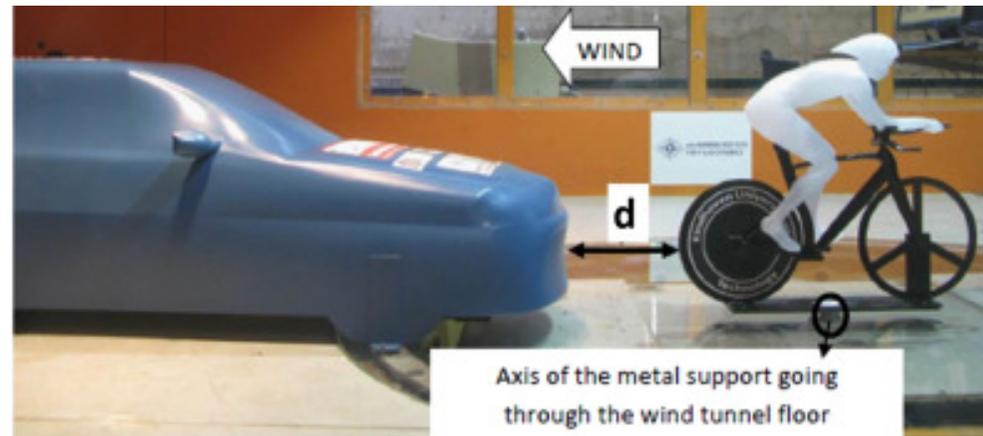
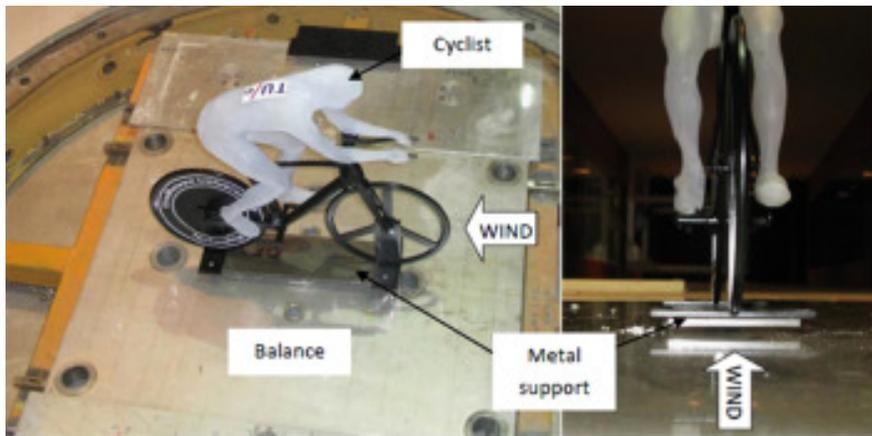
Characterisation of the flow in a water-puddle under a rolling tire with refracted PIV method



(LMFA, LHEEA, Nextflow Software, Michelin ; S. Simoens & M. Michard)

- **Elite cyclist : reduction of drag ...**
 ... when a cyclist rides **in front of** a car

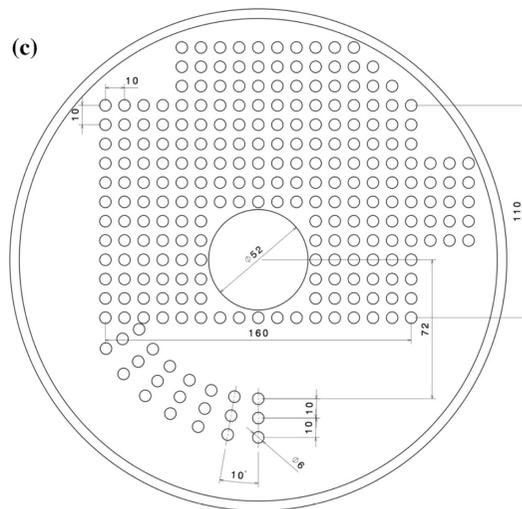
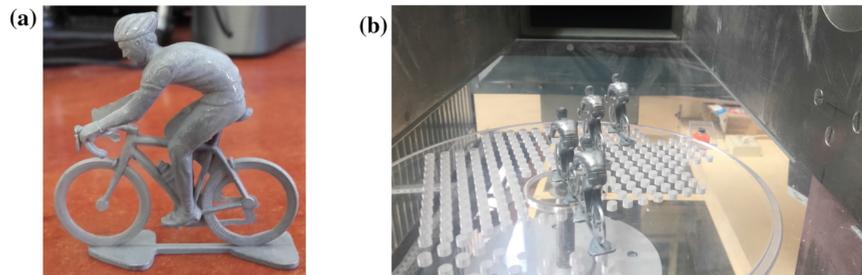
(Blocken & Toparlar, *J. Wing. Eng. Ind. Aerodyn.*, 2015)



For a 50 km individual time trial : $3 \leq d \leq 10 \text{ m} \implies 1 \text{ mm} \rightarrow 4 \text{ s}$ time reduction!
 Recommendation for UCI, $d \geq 30 \text{ m}$

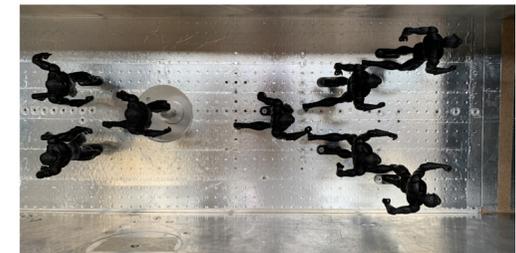
● Sport Aerodynamics

Influence of crosswind on a cyclist formation (Par 2019)



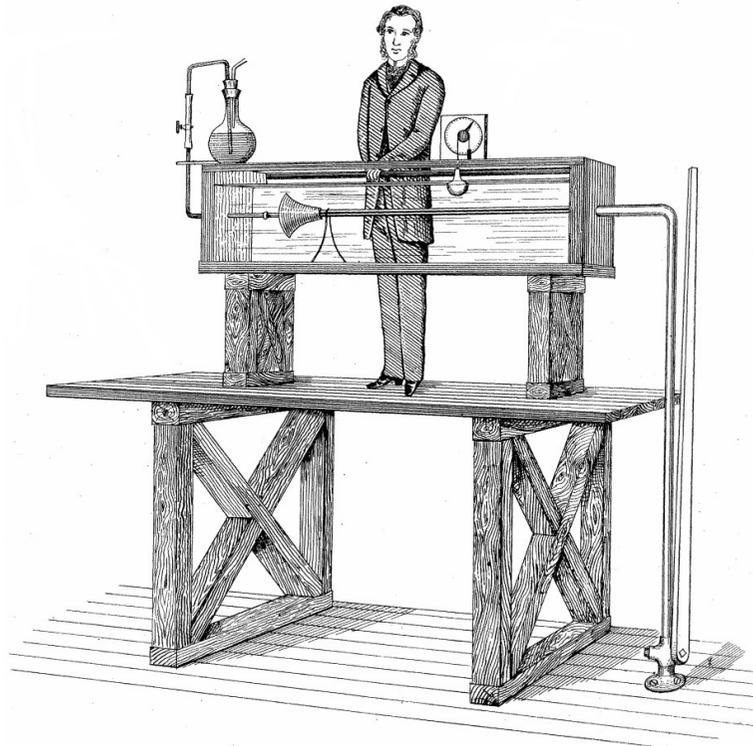
Kraemer *et al.*, 2021, *SN Applied Sciences*

Pacer formations for a top runner (Par 2021)



Massimo Marro, [Jack Leckert[‡]](#), [Ethan Rollier[‡]](#), Pietro Salizzoni and Christophe Bailly
 Wind tunnel evaluation of novel drafting formations for an elite marathon runner
Proc. Roy Soc. A, 479, 2023
[‡] undergraduate students at [Centrale Lyon](#)

● Reynolds' experience (1883) : laminar versus turbulent regime

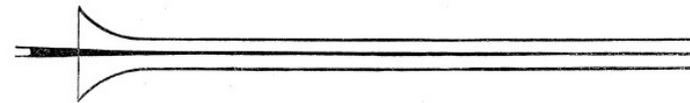


Reynolds, O., 1883, An experimental investigation of the circumstances which determine whether the motion of water shall be direct or sinuous, and of the law of resistance in parallel channels, *Phil. Trans. Roy. Soc.*, 174, 935-982.

The general results were as follows :—

(1.) When the velocities were sufficiently low, the streak of colour extended in a beautiful straight line through the tube, fig. 3.

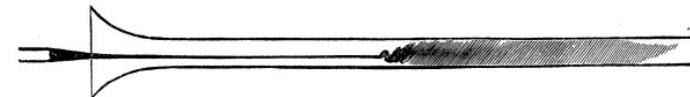
Fig. 3.



(2.) If the water in the tank had not quite settled to rest, at sufficiently low velocities, the streak would shift about the tube, but there was no appearance of sinuosity.

(3.) As the velocity was increased by small stages, at some point in the tube, always at a considerable distance from the trumpet or intake, the colour band would all at once mix up with the surrounding water, and fill the rest of the tube with a mass of coloured water, as in fig. 4.

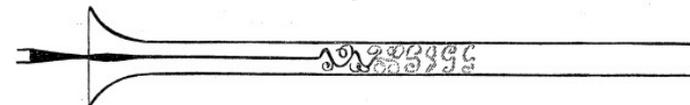
Fig. 4.



Any increase in the velocity caused the point of break down to approach the trumpet, but with no velocities that were tried did it reach this.

On viewing the tube by the light of an electric spark, the mass of colour resolved itself into a mass of more or less distinct curls, showing eddies, as in fig. 5.

Fig. 5.



● **Control parameter : the Reynolds number**

$$\text{Re}_D = \frac{\rho U_d D}{\mu} = \frac{U_d D}{\nu} \sim \frac{\text{diffusion time}}{\text{convection time}} \sim \frac{D^2/\nu}{D/U_d}$$

The transition from a laminar to a turbulent state occurs for

$$\text{Re}_D \sim 2300$$

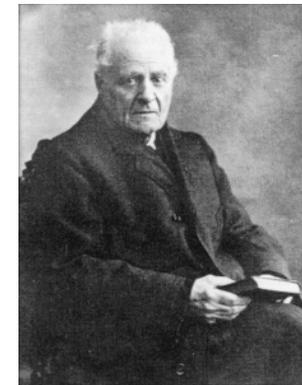
D characteristic length of the mean shear

U_d bulk velocity

The concept of a turbulent regime (wrt a laminar state) was introduced by Boussinesq (1872, 1877) and Reynolds (1883, 1894)



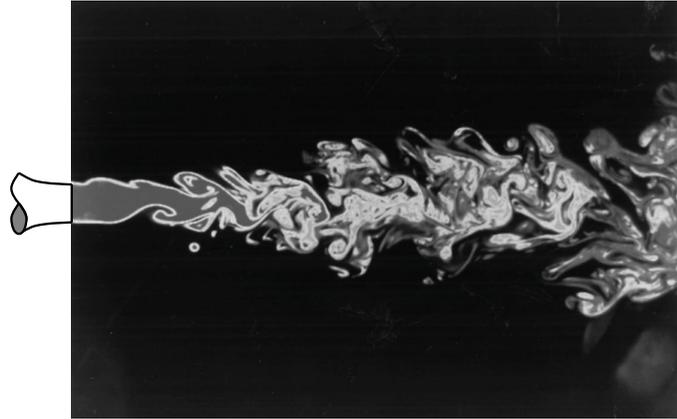
Osborn Reynolds
(1842-1912)



Joseph Boussinesq
(1842-1929)

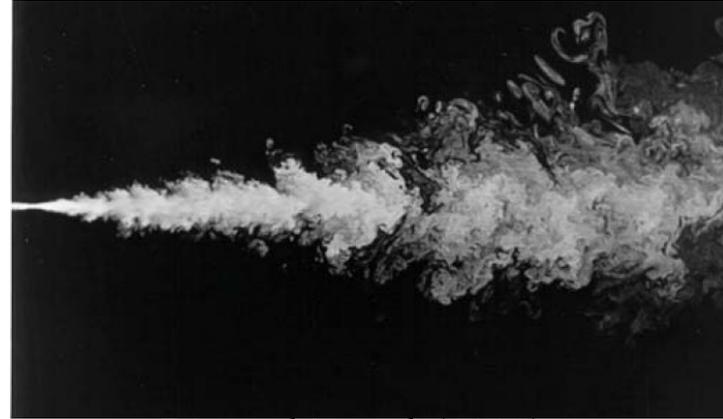
● Turbulent subsonic (round) jet

$$Re_D = u_j D / \nu$$



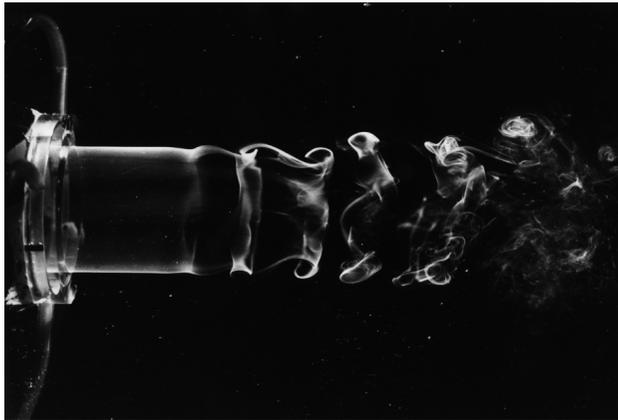
Prasad & Sreenivasan (1989)

$$Re_D \approx 4000$$



Dimotakis *et al.* (1983)

$$Re_D \approx 10^4$$



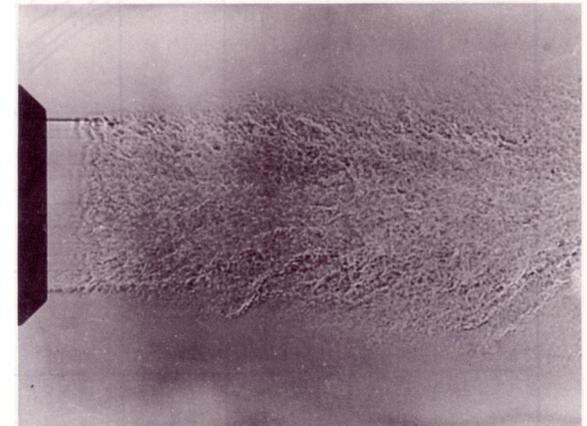
Kurima, Kasagi & Hirata (1983)

$$Re_D \approx 5.6 \times 10^3$$



Ayrault, Balint & Schon (1981)

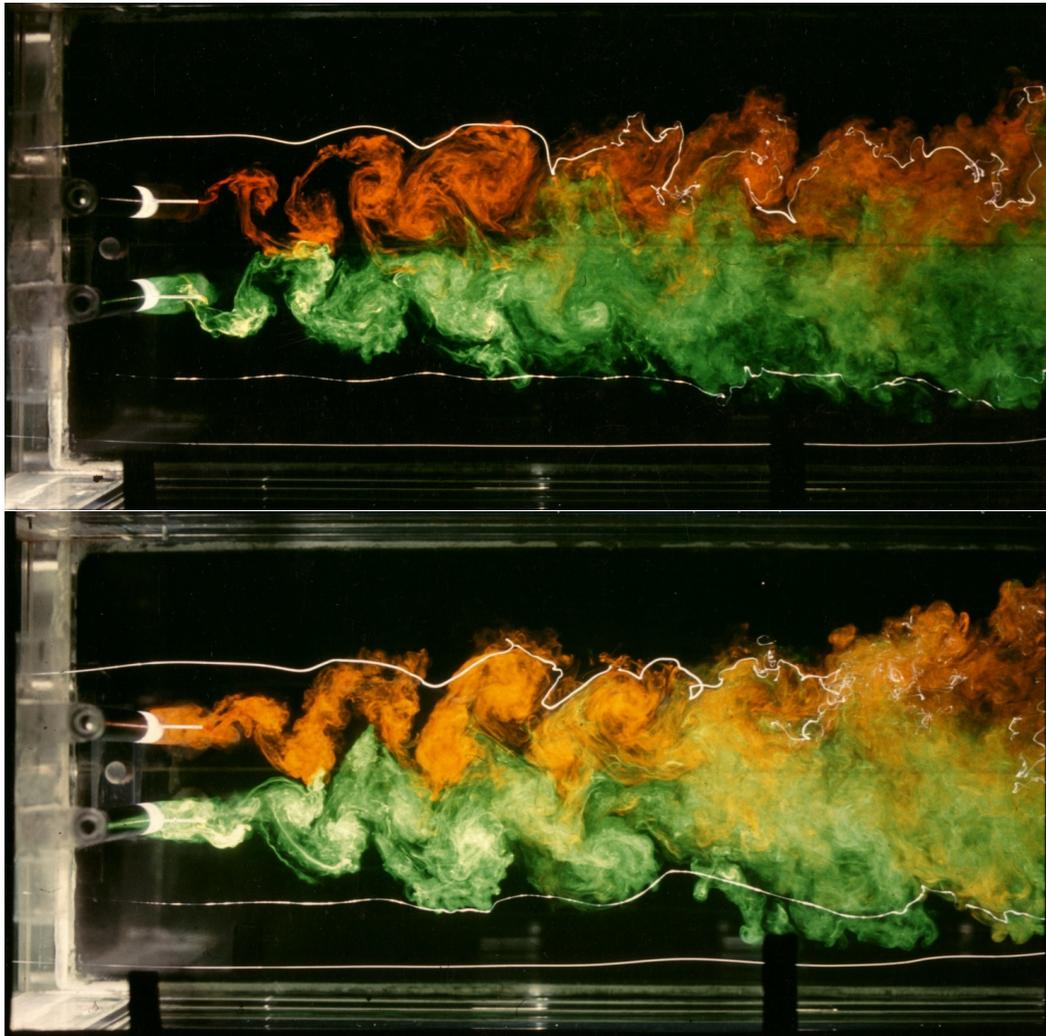
$$Re_D \approx 1.1 \times 10^4$$



Mollo-Christensen (1963)

$$Re_D = 4.6 \times 10^5$$

● Weak interaction between two wakes



Wakes produced by a couple of cylinders with the same diameter : visualization with fluorescein and congo red dye from the trailing edge cylinders, and laser light-sheet for illumination.

Béguier, C. & Fraunié, F., 1991, Double wake flow with heat transfer, *Int. J. of Heat and Mass Transfert*, **34**(4/5), 973-982

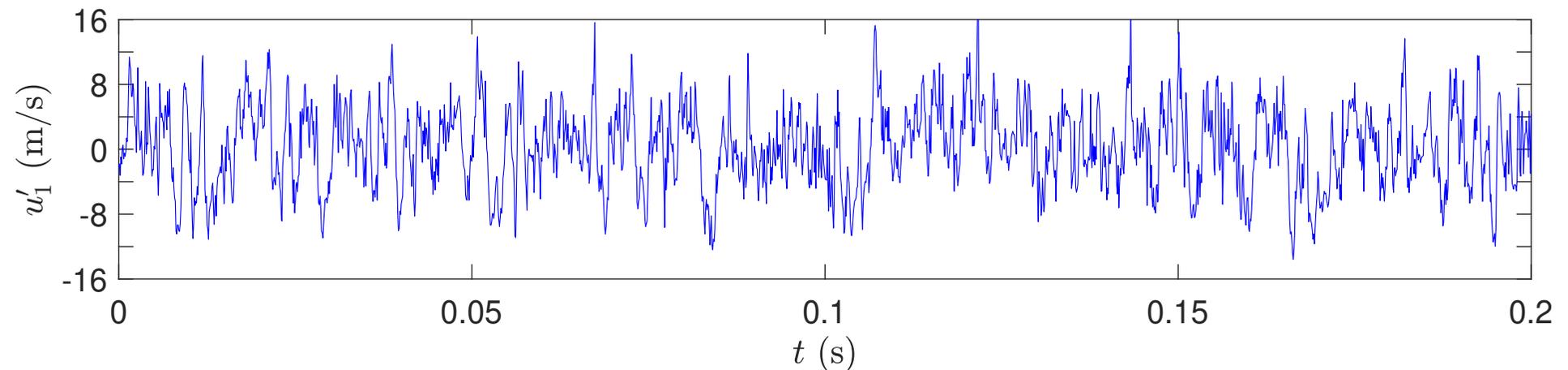
Turbulent signals

- **Fluctuating velocity signal in the shear layer of a subsonic round jet**
(measured by crossed-wire probes at $x_1 = 2D$, $x_2 = D/2$, $x_3 = 0$)

Nozzle diameter $D = 50$ mm, exit velocity $U_j = 30$ m.s⁻¹

↷ Reynolds number $Re_D = 10^5$

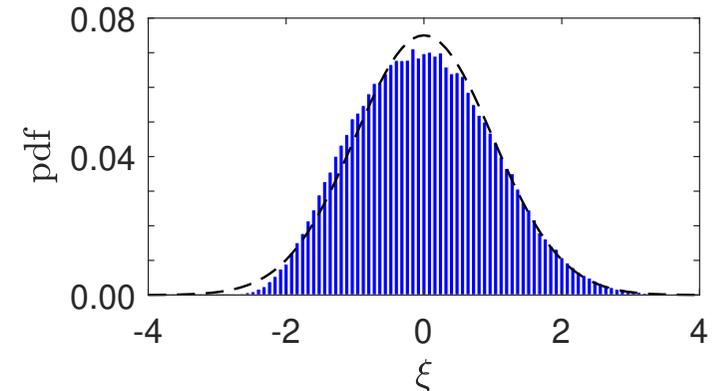
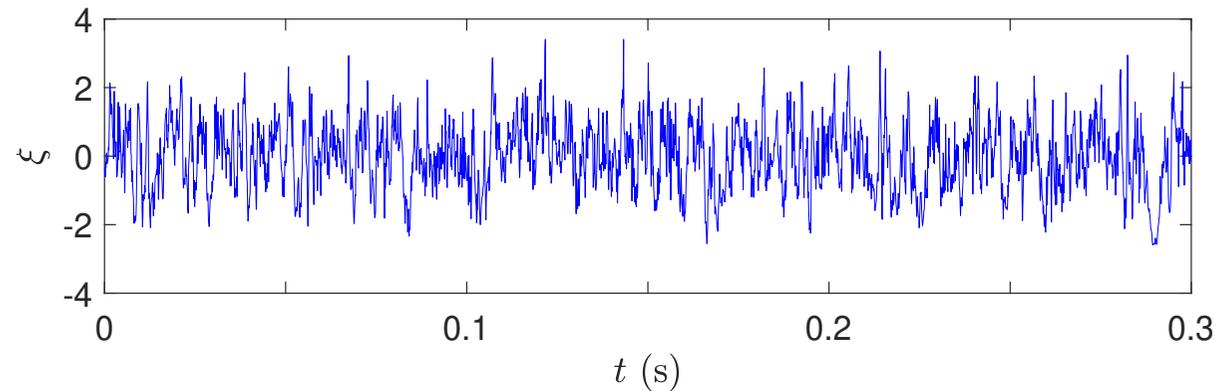
$u'_1(t)$



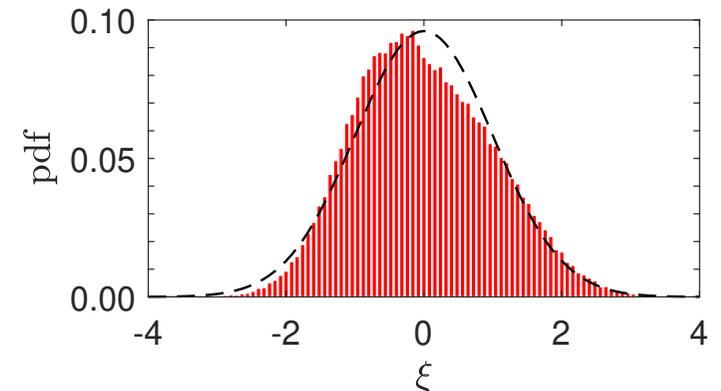
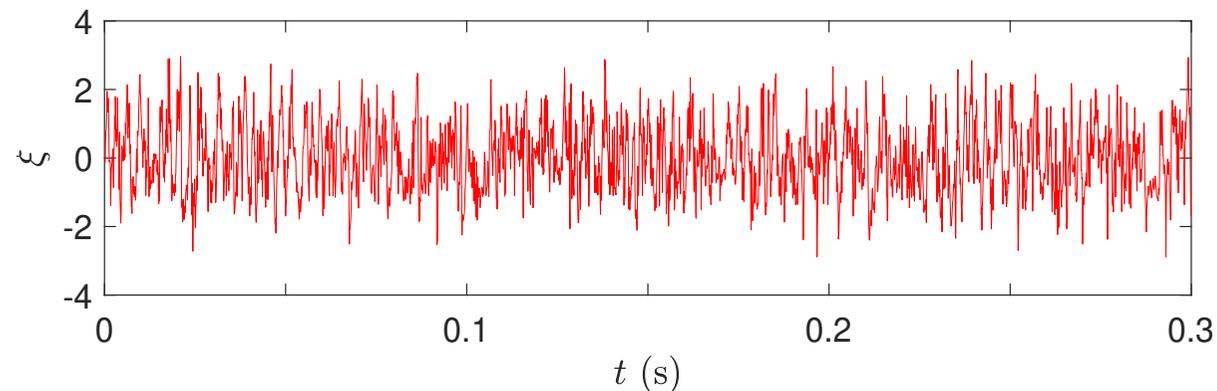
● **Fluctuating velocity signal in the shear layer of a jet (cont.)**

$u'_1(t)$ and $u'_2(t)$ normalised by their rms value, $\xi = u'_\alpha(t)/u_{\alpha rms}$

$S_{u'_1} \simeq 0.21$ $T_{u'_1} \simeq 2.74$



$S_{u'_2} \simeq 0.24$ $T_{u'_2} \simeq 2.76$



● **Probability density function of a random variable**

Probability density function $p(\zeta)$ of a random variable ζ , for instance $\zeta = u'_i$ (centered variables to simplify the writing, $u'_i = u_i - \bar{U}_i$)

$$\int_{-\infty}^{+\infty} p(\zeta) d\zeta = 1 \quad \bar{\zeta}^n = \int_{-\infty}^{+\infty} \zeta^n p(\zeta) d\zeta$$

Standard deviation (root-mean-square), $\zeta_{\text{rms}} \equiv \left(\overline{\zeta^2}\right)^{1/2} = \sigma_\zeta$

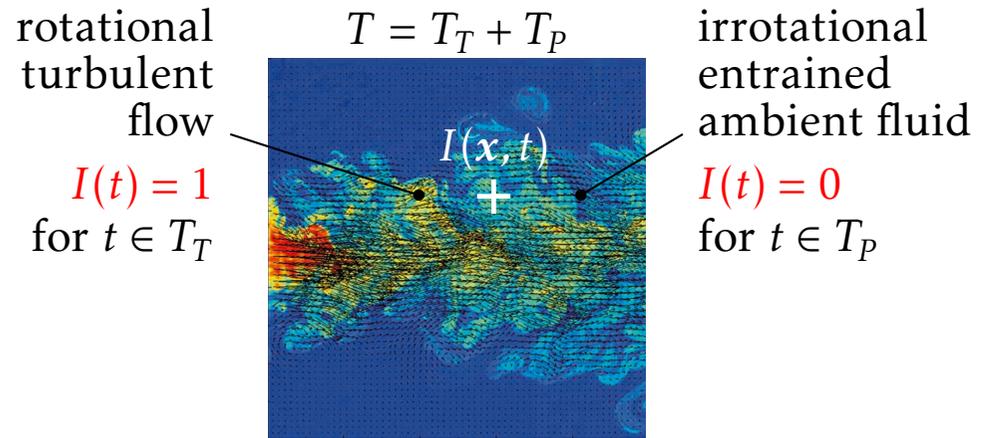
Skewness factor S_ζ , and flatness or kurtosis factor T_ζ ,

$$S_\zeta \equiv \frac{\overline{\zeta^3}}{\zeta_{\text{rms}}^3} \quad T_\zeta \equiv \frac{\overline{\zeta^4}}{\zeta_{\text{rms}}^4}$$

For a **Gaussian (normal) distribution** : $S_\zeta = 0$ and $T_\zeta = 3$

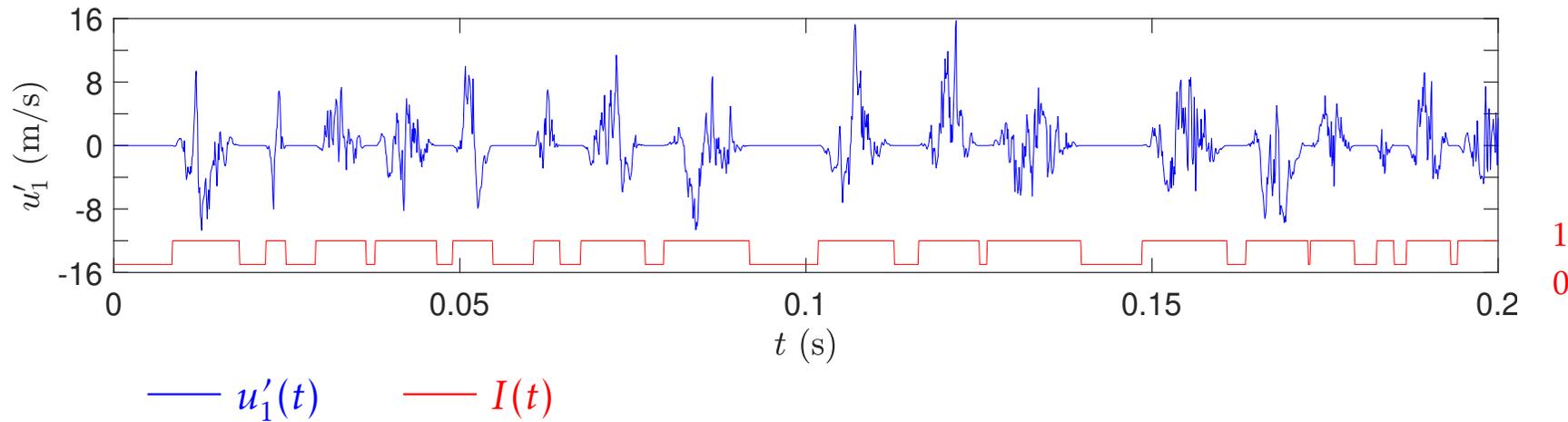
$$p(\zeta) = \frac{1}{\sqrt{2\pi}\sigma_\zeta} \exp\left(-\frac{\zeta^2}{2\sigma_\zeta^2}\right)$$

● Intermittence at the edge of a free shear flow

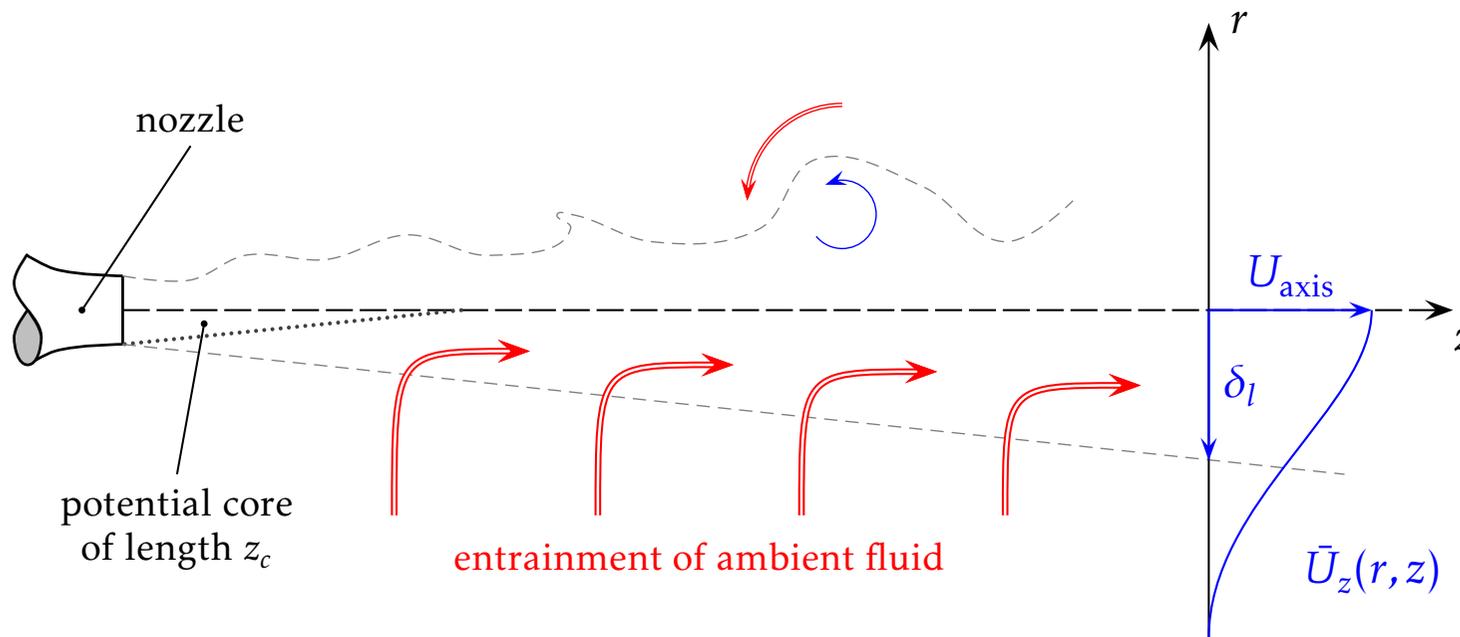


Intermittency factor $\gamma(\mathbf{x})$, the probability that the flow at (\mathbf{x}, t) is turbulent

$$\gamma = \bar{I} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T I(t) dt = \frac{T_T}{T}$$

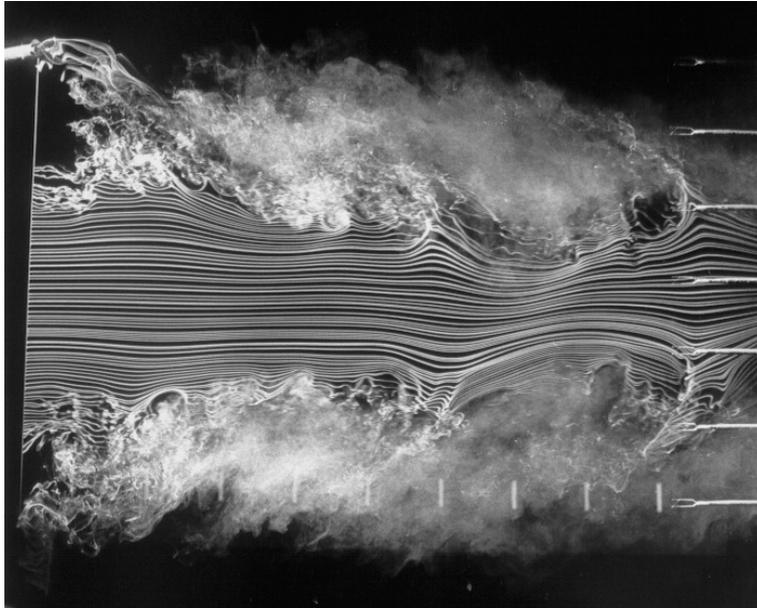


- Intermittence at the edge of a free shear flow (cont.)
Importance of entrainment

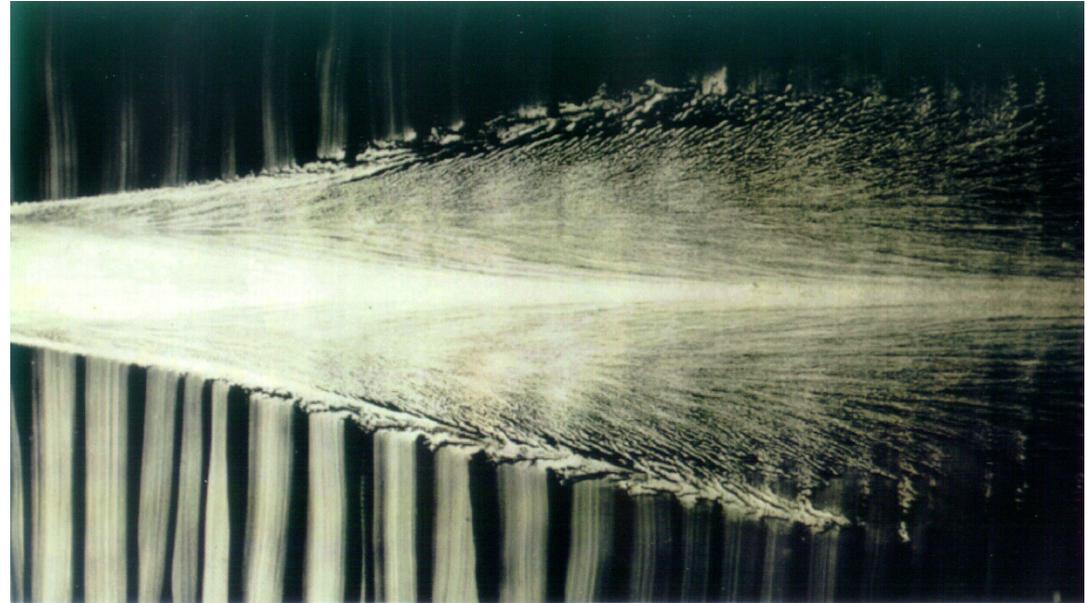


● Intermittence at the edge of a free shear flow (cont.)

Importance of entrainment



Visualization with smoke wires
 $Re_D \simeq 5.4 \times 10^4$
Courtesy of H. Fiedler (1987)



Entrainment by a turbulent round jet from a wall
 $Re_D = 10^6$
Florent, *J. Méc.* (1965)

● **Intermittence**

For a centered fluctuating signal $u'_1 = Iu'_T + (1 - I)u'_p$

where u'_T is the turbulent signal following a **Gaussian** law of variance σ_T^2 , and u'_p is the potential entrained fluid ($u'_p = 0$ to simplify here, laminar flow)

$$\overline{u_1'^2} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T I^2 u_T'^2 dt = \lim_{T \rightarrow \infty} \frac{T_T}{T} \frac{1}{T_T} \int_{T_T} u_T'^2 dt = \gamma \overline{u_T'^2}$$

We thus get the following relationship, $\overline{u_1'^2} = \gamma \overline{u_T'^2} = \gamma \sigma_T^2$, and in the same way, $\overline{u_1'^4} = \gamma \overline{u_T'^4} = \gamma 3\sigma_T^4$ for Gaussian turbulence (see slide 30)

Hence, for the **complete fluctuating signal**,

$$\text{variance : } \sigma^2 = \gamma \sigma_T^2 \quad \text{flatness factor : } T = \frac{3}{\gamma}$$

The flatness (kurtosis) factor is thus larger than for a Gaussian distribution, and the variance is smaller, meaning that very small and very large values of the random variable u'_1 are both more probable (wrt a Gaussian pdf) : this is a feature of an **intermittent signal**

● Intermittence (cont.)

A more general approach requires to include the contribution of the mean flow. We now consider both contributions $u_T = Iu_1$ and $u_P = (1-I)u_1$. The mean velocity when $I \neq 0$ is given by

$$\overline{Iu_1} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T Iu_1 dt = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{T_T} u_1 dt = \frac{T_T}{T} \bar{U}_T = \gamma \bar{U}_T$$

and by a similar way, $\overline{(1-I)u_1} = (1-\gamma)\bar{U}_P$.

Hence, the unconditional mean of $u_1 = u_T + u_P$ reads

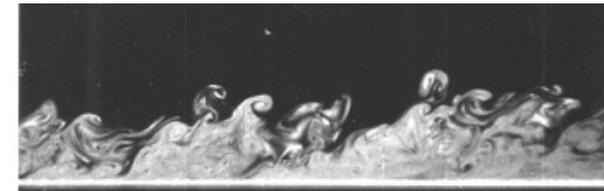
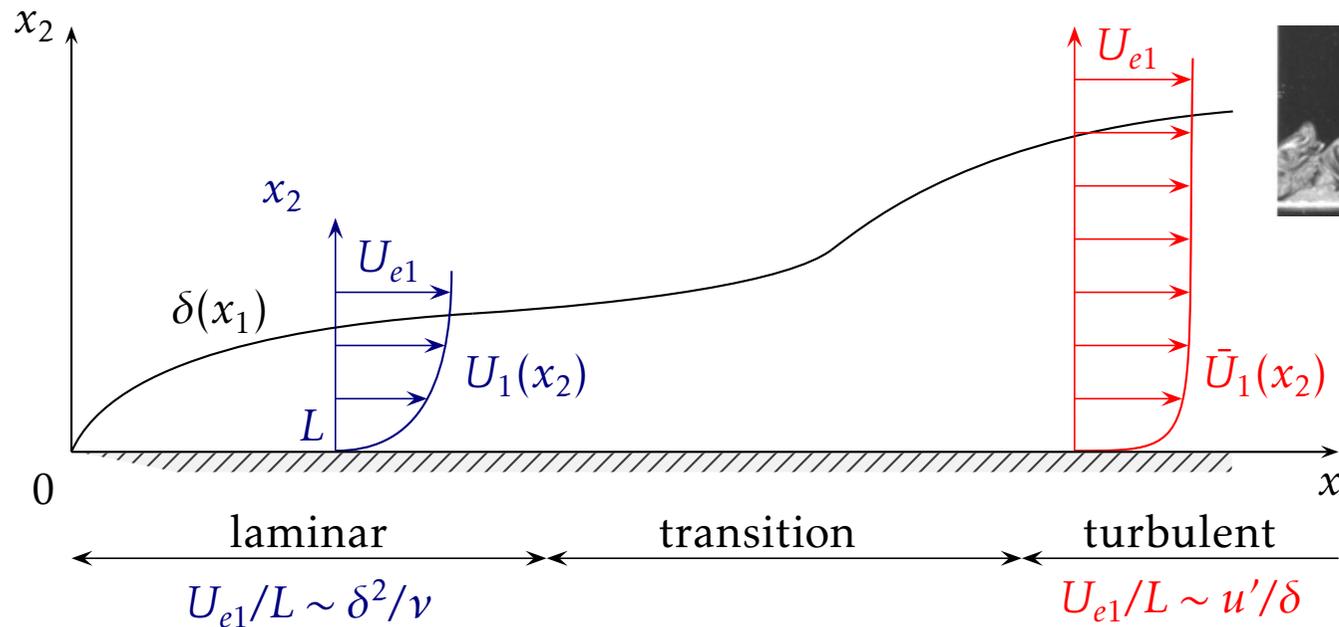
$$\bar{U}_1 = \gamma \bar{U}_T + (1-\gamma)\bar{U}_P$$

By considering $u'_1 = u_1 - \bar{U}_1$ as usual, the variance is given by (see small classes for a demonstration),

$$\overline{u'^2_1} = \gamma \overline{u'^2_T} + (1-\gamma)\overline{u'^2_P} + \gamma(1-\gamma)(\bar{U}_T - \bar{U}_P)^2$$

● Zero-pressure-gradient boundary layer on a flat plate

Transition for $Re_{x_1} \simeq 3.2 \times 10^5$ or equivalently for $Re_\delta = U_{e1} \delta / \nu \simeq 2800$



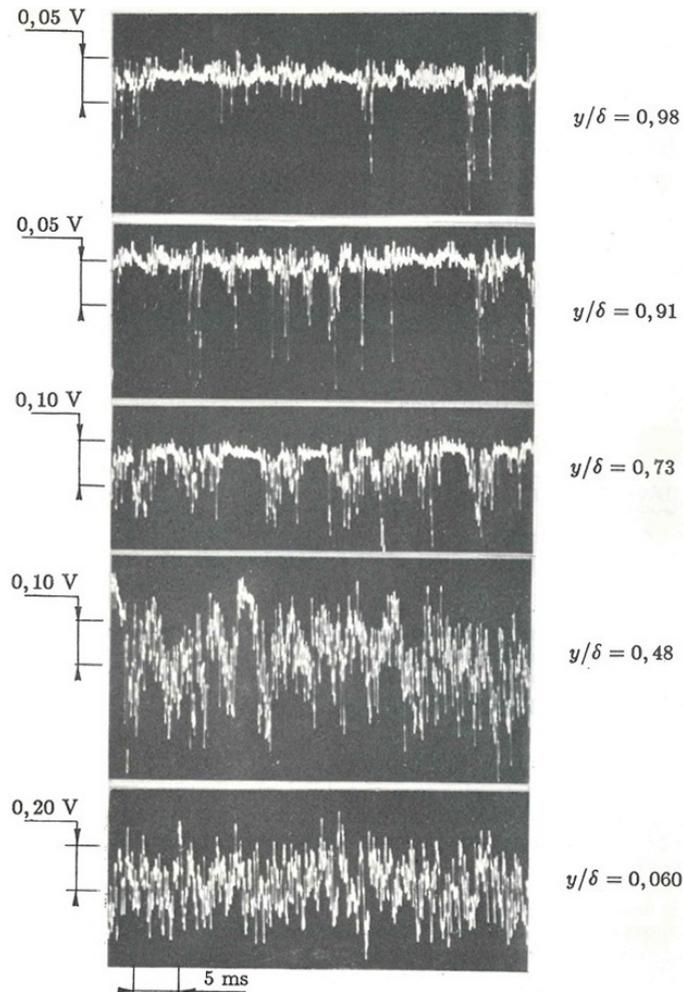
(F. Laadhari, LMFA,
 $Re_{\delta_0} \simeq 1000$
 $U_{e1} = 2.1 \text{ m.s}^{-1}$ $\delta \simeq 7 \text{ cm}$)

In laminar regime, molecular diffusion $\tau \sim \delta^2/\nu$ in the transverse direction, compared with the turbulent regime, turbulent diffusion δ/u' with $u' \simeq 0.1 U_{e1}$

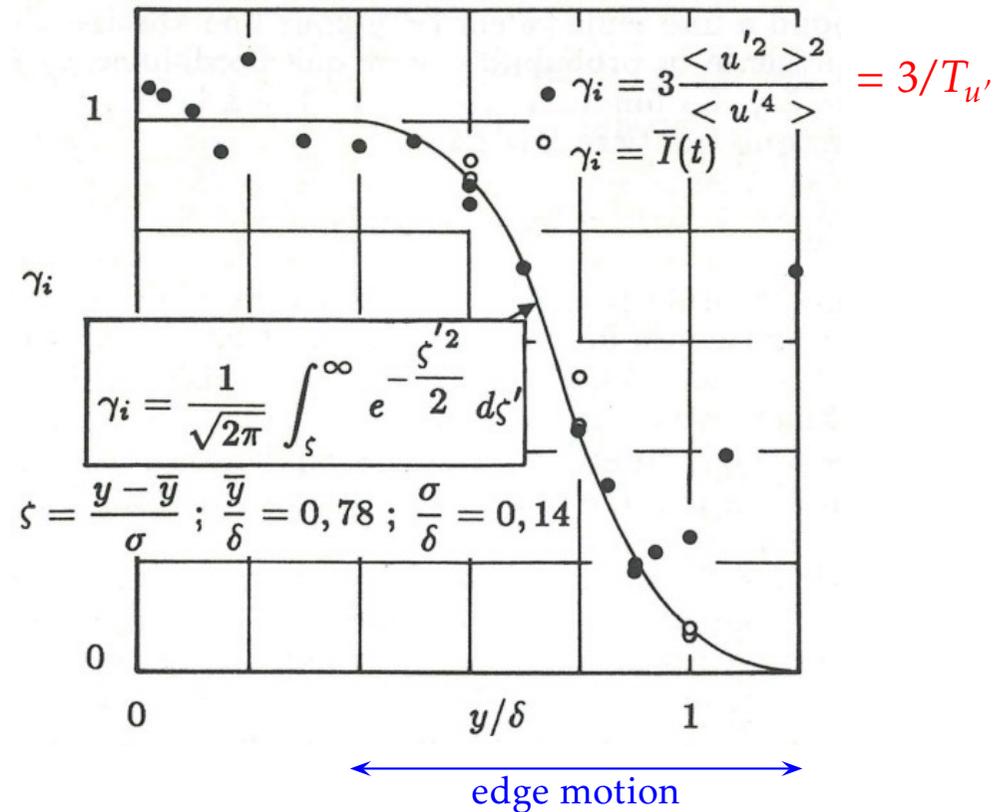
Lee, Kwon, Hutchins & Monty (Melbourne University)



● Intermittence in a boundary layer



$U_\infty = 38 \text{ m.s}^{-1}$ (Cousteix, 1989)

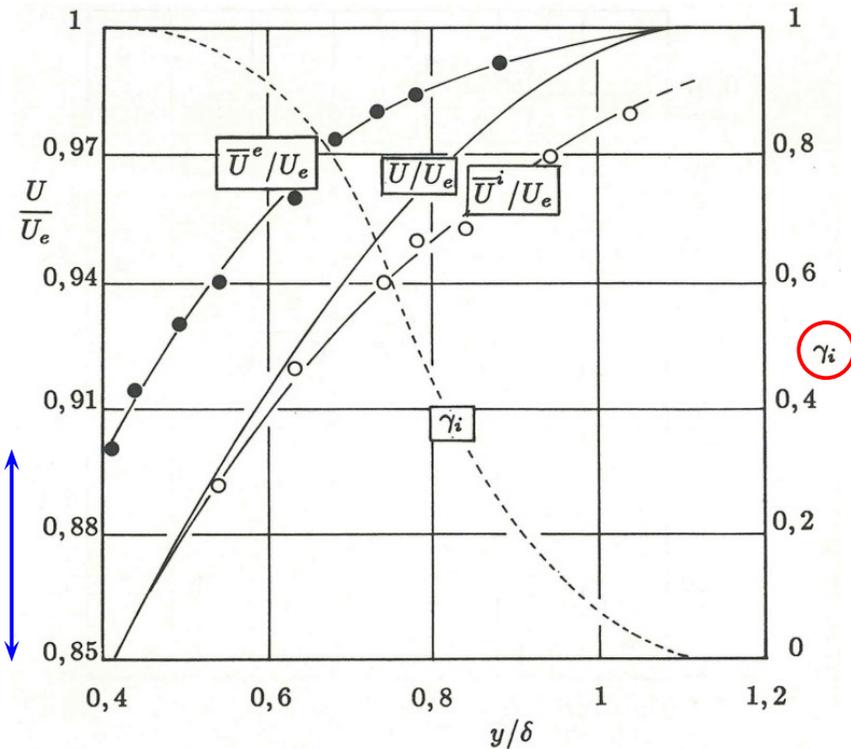


The intermittency factor $\bullet \gamma_i$ (see slide 34) is here defined as the probability to be inside a turbulent burst of the boundary layer (internal flow)

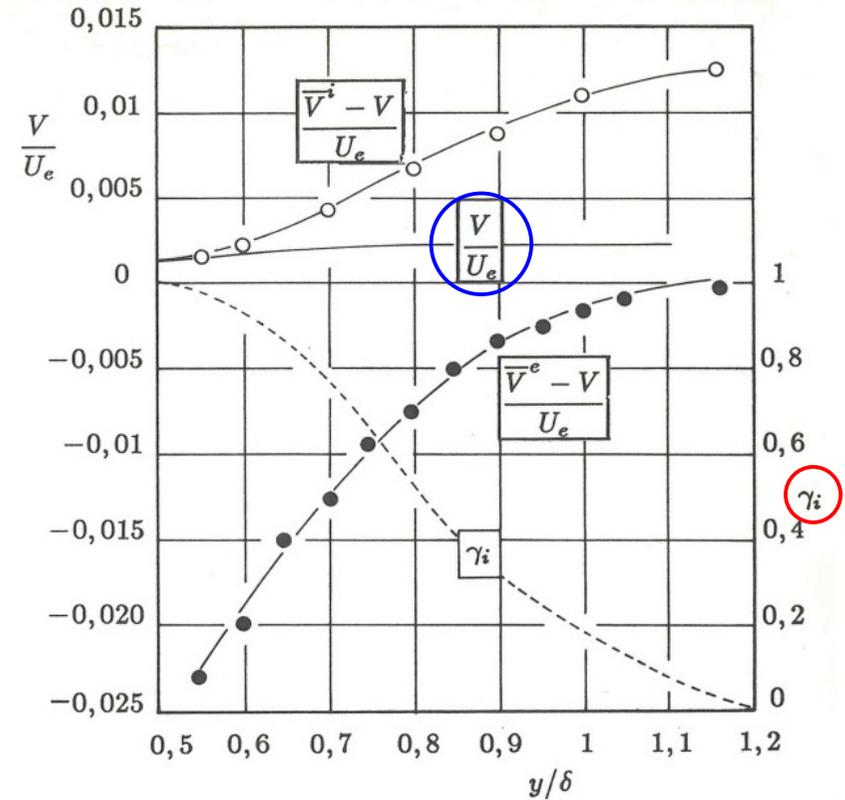
Klebanoff, 1954, NACA TN 3178; See also Cousteix (1989)

● Intermittence in a boundary layer (cont.)

Longitudinal velocity



Transverse velocity



conditional means : $-^i$ inside a turbulent burst, $-^e$ outside a turbulent burst, entrainment of the external flow
 U_e external velocity

Kovasznay, Kibens & Blackwelder, 1970, *J. Fluid Mech.*, 41(2), 283-325; See also Cousteix (1989)